LEARNING MATHEMATICS COLLABORATIVELY - LEARNING THE SKILLS

Julie-Ann Edwards

Centre for Research in Mathematics Education, University of Southampton

The research in this study explores the learning of mathematics through collaborative activity in two pedagogically identical environments but with distinct and different skill-learning agendas. Evidence is provided through analysis of small group discussion in the naturalistic emancipatory and socioconstructivist setting of both classes. Audio tapes are used as evidence for both qualitative and quantitative analysis. Findings indicate that groups which explored mathematical problems collaboratively sought solutions which reflected the input of the whole group. A greater degree of 'helping' activity was exhibited amongst groups of students who were overtly taught collaborative skills. Results suggest that a coconstructed group environment is more conducive to learning mathematics than an imposed structure.

TALK IN THE CLASSROOM

The study of talk in the classroom involves three disciplines: linguistics, psychology, and sociology. Whilst overlaps in interests are represented by, for example, psycholinguistics and sociolinguistics, each discipline contributes a component to the overall picture. Educationists have, more recently, tended to a sociological interpretation of talk and its relation to learning in classrooms, though the psycholinguistic analysis of classroom talk has done much to move thinking forward and validate the use of peer talk in the classroom. In a review of small group talk, Good, Mulryan and McCaslin (1992) describe "clear and compelling evidence that small group work can facilitate student achievement as well as more favourable attitudes towards peers and subject matter" (p167). This evidence validates further research into small group talk in the classroom.

Research in the area of psychology has offered a relation between talk amongst peers and metacognitive activity. Peterson and Swing (1985) investigated students' cognitions as mediators of the effectiveness of small group work. They use the quality of explanations as a measure of metacognitive activity and relate achievement by outcome to the level of these explanations, both given and received. Their study found that small groups which engaged in high order explanations achieved higher scores in individual work.

Similarly, Larson *et al* (1985) use metacognition as a focus for their study. Their stance is that knowledge about one's own cognition leads to effective self-monitoring activity and thus to more purposive cooperative learning with effective transfer of this learning to the individual. They conclude that metacognitive activity facilitates cooperative learning and that elaborative activity facilitates transfer to individual learning. They summarise a need to "tailor cooperative learning strategies to instructional goals". This involves the use of "cooperative learning with a focus on metacognitive activity" and "elaborative activity within cooperative learning emphasised".

There are difficulties inherent in attempting to assimilate findings such as those above into a study which is essentially naturalistic, though comparative. Firstly, very few studies have undertaken similar psychological research using small group work in mathematics classrooms. The control of variables is such that there are serious implications for the validity of these research findings in naturalistic settings. Furthermore, these studies were especially designed research situations and undertaken in a matter of hours. One of the factors implicit in effective small groups is the time involved in working together (Laborde 1994). This is not accounted for in psychological studies because it is a social factor, though Peterson and Swing acknowledge the possible impact of the nature of the group on cognition.

COLLABORATIVE AND COOPERATIVE SMALL GROUPS

Cooperative and collaborative learning is perhaps the greatest change to be seen in mathematics education in recent years. Classroom discussion, often involving cooperative or collaborative group work, has become an increasing focus for research. Debate still rages about whether discussion *per se* is effective in the mathematics classroom (see, for example, Sfard, Nesher, Streefland, Cobb and Mason, 1998). The consensus is that there are positive effects but questions remain about the nature of discussion which actually generates a positive outcome on mathematical learning.

A good deal is known about cooperative small group learning (for reviews, see Good *et al ibid*, or Cohen 1994). Discussion of cooperative learning in these reviews begins from the assumption that research has already established it as a legitimate means of teaching and learning. For example, peer collaboration is effective for mathematical tasks which require reasoning but not for tasks which require rote learning. Both the above reviews advocate a future focus for research on the socially situated learning which occurs in small groups. They argue that research on small groups has gone beyond a need to justify its benefits through improved learning outcomes. They emphasise the need for work on the factors which affect *discourse processes* rather than factors which affect *achievement outcomes*. There is suggestion of a paucity of process data because of a lack of classroom observations and student interviews which would help to describe what happens during small group interactions and provide information about these processes. There is also a perceived need for cooperative group participants to develop interpersonal skills relevant to group work. Team-building activities conducive to prosocial behaviours are recommended *before* cooperative group work takes place.

Much less is known about collaborative small group work than cooperative small group work (Lyle 1996). This may be due to the divide between US and UK approaches to both cooperative and collaborative learning. The US work tends to test hypotheses in experimental situations whereas the UK work tends to be more ethnographic. As a result, little has been reported about a range of issues such as how the composition and dynamics of collaborative groups affect their ability to function effectively (for a recent report, see Barnes 1998), or whether the students themselves find it an effective way of working. One study which does address this issue is reported by Edwards and Jones (1999). The research discussed above suggests a need for more research to be undertaken on small group work in mathematics classrooms relating to the influence of the task, processes of group interaction, the status of group participants and the need to overtly teach skills for cooperation, the focus of the study described here.

COLLABORATIVE AND COOPERATIVE LEARNING

In an overview of cooperative learning research by Davidson and Kroll (1991) one definition of cooperative learning is “learning that takes place in an environment where students in small groups share ideas and work collaboratively to complete academic tasks” (p362). However, they recognise that this definition encompasses a broad range of practices and meanings applied to the same terms. Most definitions of cooperative learning do not encompass the essence of collaborative learning. This demonstrates the need for a careful definition of terms.

Damon and Phelps (1989) consider it imperative that distinctions are made between types of peer interaction. They describe cooperative learning as utilising distinct principles and practices such as specific role assignments in a group, division of tasks and goal-related accountability of both individuals and the group. It is defined by Damon and Phelps as high on equality and variable on mutuality. Peer collaboration, in contrast, involves groups of novices working together to solve challenging problems which none of the participants could do prior to the collaboration. Relationships in peer collaboration are described as high on equality and high on mutuality.

Much of the research into cooperative learning has not made the necessary distinction between cooperative and collaborative; indeed many studies interchange the terms. Most such studies have centred on outcome objectives as a measure of success. Few have studied the internal dynamics of small group work and the relation of this aspect to cognition. More recent studies that do so include those reported by Webb (1991) and Cobb and Bauersfeld (1995).

For the purposes of this study, I define collaborative learning as that which is constructed amongst student peers working together in self-selected groups (defined in this study as coconstruction). The process involved in mathematical endeavour is as important a focus to the group as the end outcome. Though the aim is to provide a solution to the activity, the lack of an outcome is not seen as failure, as the process is an end in itself because it is viewed as valid mathematical activity.

Some studies of small group work explore ways of supporting such mathematical reasoning by students through problem-solving (for example, Gravemeijer, McClain and Stephan 1998). In a response to Gravemeijer *et al*, Groves (1998 p210) outlines three critical aspects in developing students' powerful problem-solving: "the role of the teacher, the 'design and enactment' of sequences of instructional tasks, and the development of a classroom culture which supports students in explaining and justifying their thinking".

Jaworski (1994 p56), furthermore, describes four aspects of mathematics learning which characterise an investigative approach to learning. These descriptions by Groves and by Jaworski point to the need to develop an environment (or culture or ethos) conducive to the processes of mathematical activity.

THE PEDAGOGIC APPROACH

The pedagogic approach in the study reported here can be described as an emancipatory socioconstructivist model in which a variety of open-ended situations are used as learning contexts. It is emancipatory, firstly, because the teacher is constantly involved in reflective action research (Jaworski, 1992). Secondly, the direction of classroom mathematical activity is directed by students within small groups and assessment outcomes are negotiated between the class and the teacher. The role of the teacher is that of a facilitator of ideas, challenger of decisions, and scaffolder for content learning. The influence of direct teaching is minimal, carefully timed, and designed so as not to affect the power relations established in the classroom.

The classroom approach is socioconstructivist, based on the teacher's underlying belief that mathematics learning cannot occur in isolation of the learner's culture, gender, politics, environment and social history and that these influences will affect the way in which learners construct learning and how they interpret learning situations (see, for example, Ernest 1991). The use of collaborative classroom activity amongst students supports the belief in such an approach.

Research has suggested that the direct teaching of cooperative or collaborative skills may be beneficial to learning (see, for example, Sharan 1990). Some studies in cooperative mathematics learning in small groups call for the direct teaching of collaborative skills or team-building skills to enable groups to work effectively (see Good *et al ibid*, Cohen *ibid*). The study described in this report addresses this call within collaborative groups

DATA COLLECTION AND ANALYSIS

Students in this study attend an inner-city comprehensive secondary (11-16) school. A Year 8 group (12-13 year olds) whom the teacher had taught for the previous two terms received the 'normal' pedagogy. Any references to collaboration were made within the context of the mathematics or mathematics learning. A Year 7 group (11-12 year olds) provided the comparative group. This class received direct instruction (after Mercer *et al* 1999) on collaborative learning skills as a means of learning mathematics. Reference to collaborative organisation was made during lessons where necessary to reinforce this initial introduction to collaboration.

The whole class session for Year 7 was audio-recorded during the 'teaching' of collaborative skills which took the form of a class discussion to establish shared meanings for collaboration. No recording was made for Year 8 at this time as a parallel session did not take place. Throughout the ensuing 10 teaching weeks, pilot sessions of single groups were audio-recorded to accustom students to being recorded. These tapes were not considered as data for this study because of their possible contamination of outcomes in an experimental situation. After 10 weeks all small groups were audio-recorded in each class for three parallel lessons during one week.

To fit with the sociolinguistic approach to the analysis, two methods of analysis are used: constant comparative analysis and categorisation/typology. Data from audio-tapes and field notes was categorised in sections of various lengths, to reflect the nature of the talk. Such sections were defined as 'episodes'. Results were recorded as numerical data to indicate frequency of occurrence with a proviso that each occurrence may not be equivalent to another in terms of representative time. This is a task for further analysis of the data. One aspect of group work that is intimately related to talk which is not exposed by the use of audio tapes is the skill of listening to others. While there may be some evidence in the form of responses to each other that group members are listening to each other, it is a feature which is not directly measurable.

FINDINGS

There were 30 categories of talk identified, some of which were common to both classes, some of which were particular to Year 7 and some to Year 8. These categories are summarised in Appendix 1 with a brief description of each.

There are distinct trends in the results which suggest that the Year 8 class tends to focus on the mathematical skills involved in group work, rather than the organisational skills. Categories 1 and 2 (the organisational categories) are negligible in Year 8. For year 8, category 5 (talking aloud) is a feature which may function also as category 6 (checking with each other). It may serve as a means of keeping members jointly informed of the direction of thinking within the group. This is borne out in a decrease in necessity for category 6 as the amount of talking aloud increases. It is also reflected in a decrease in the number of periods of silence indicating individual work.

Helping behaviours, prevalent in cooperative groups, (categories 7, 8 and 9) were only evident in the Year 7 class. The degree to which assertions were made (category 18) was much lower in Year 8. This is juxtaposed against the tendency for this class to use more questioning, explaining and challenging modes of communication (categories 10, 11 and 12). Categories involving more sophisticated group talk, such as justifying decisions, accepting another's reasoning, confirming behaviours, argumentation and making collective decisions (categories 13, 14, 15, 19 and 22) were also only represented amongst the Year 8 class. Predicting or hypothesising about the work (category 21) and demonstrating some pleasure in the work (categories 23 and 24) were almost equally represented between the classes. However, of 50 instances of 'negative' behaviours (categories 27, 28, 29 and 30) only 4 were exhibited by Year 8.

SUMMARY

The Year 8 class whose learning is implicit through mathematics exhibited many more features of positive mathematical activity. Results therefore indicate that the coconstructed group environment is more conducive to learning mathematics together than an imposed structure. Previous discussion has suggested that the direct teaching of group skills is a lengthy process. A mathematics teacher needs to consider the most efficient *mathematical* use of time in the classroom. The findings from this research suggest that a more efficient use of mathematics lesson time is a focus on the nature of mathematical processes, mathematical actions and the building of mathematical knowledge as a means of promoting effective group skills rather than on the group skills themselves. This is evident in the extent to which the Year 7 class, taught directly, exhibited negative and 'off task' behaviours compared with the Year 8 class.

There is evidence that when group skills are taught directly, it is these skills that remain overt in group practice. The social activity is emphasised more than the mathematical activity in this situation. The directly taught groups referred to group processes during the mathematical activity. Similarly, this class were able to give specific examples of instructions to encourage group activity. In contrast, the class who learned group skills through their mathematical practices, could not think of explicit examples, though indicated that they were aware of these skills being part of their mathematical activity. This suggests a greater focus on the mathematical activity.

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Appendix 1

Categories of talk

1: *Organising (non-work)*: of equipment, paper and other requirements
2: *Organising (work)*: administrative activity related to the functioning of the group;
3: *Comparing data*: occurred when students were generating initial data for analysis;
4: *Assigning tasks*: allocation of specific areas of the activity to each of the members;
5: *Talking aloud*: a 'stream of consciousness' activity by one or more members;
6: *Checking with each other*: ensuring the group were working in the same direction;
7 : *Requesting help*: indicated a request for a peer tutoring arrangement;
8 : *Receiving help*: a member of the group responded positively to a request for help;
9 : *Receiving no help*: a member of the group asking for help was ignored;
10: *Questioning to compare*: assertive roles sometimes requiring explanations;
11: *Explaining*: involved the information directly available to group members;
12: *Challenging*: involved a request to explain or justify findings or decisions;
13: *Justifying*: use of information beyond that worked with at the time;
14: *Accepting reasons*; verbal indication of understanding and/or acceptance;
15: *Confirming*: an offer of a finding with an explanation and a request for agreement;
16: *Agreeing*: a verbal indication that a result or decision was agreed upon;
17: *Seeking agreement without explanation*: an offering made without justification;
18: *Asserting*: a statement made which was not responded to by any other member;
19: *Argumentation*: an argument with a positive direction and a positive outcome in the form of agreement, though not structured as a challenging argument;
20: *Collective act*: two or more members of the group are in verbal unison about evidence or decisions;
21: *Predicting/Hypothesising*: the group is working towards a solution for the activity
22: *Making decisions*: related to outcomes of the mathematical activity and involved agreement amongst members of the group;
23: *Excitement*: reflected by an indication of joy in a finding or decision;
24: *Enjoyment*: reflected by an overt statement to this effect;
25: *Silence*: a period of time when members of the group were working individually;
26: *Comment on group work*: awareness of whether the group was collaborating;
27: *Arguing*: negative interactions, often about assertions about results;
28: *Boasting*: about achieving a result that others in the group had not;
29: *Correcting*: another group member's suggestion without offering an explanation;
30: *Off task work*: chatter about anything other than the task in hand;