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**JAMES RAYMER, ANDREI ROGERS**

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# Using Age and Spatial Flow Structures in the Indirect Estimation of Migration Streams

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## ABSTRACT

This paper presents a modeling strategy for describing and estimating interregional migration flows. The categorical log-linear model is used to demonstrate various approaches to estimation, including direct and indirect methods. And estimates of known data on interdivisional migration patterns in the United States during the 1995-2000 period are used to illustrate the effectiveness of the various log-linear models. The important aspects of the modeling strategy presented in this paper include parameter interpretation, incorporation of auxiliary or a priori information, and assessment of the various model predictions. The results show that capturing the interactions between origins and destinations are very important for accurate predictions.

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## 1. INTRODUCTION

In the absence of adequate migration data for a particular study region, demographers generally have turned to estimates obtained by combining related information (drawn from several sources, time periods, and geographical areas) with models that impose regularities exhibited by past migration patterns in the particular study region or elsewhere. These efforts often seek to produce estimates of a migration variable on the basis of information that may be only indirectly related to its value, or that may be out of date.

This paper sets out to provide an effective strategy for describing, analyzing, and estimating interregional migration flows by focusing on the problem of developing formal methods for inferring migration flows from birthplace-specific population stocks. The latter have in the past been used to infer patterns of mortality and fertility and, indeed, of net migration. But no one has developed a workable formal method for using population stock data to infer directional (i.e., origin-destination-specific) migration flows. The categorical log-linear model is used for estimating such patterns, and the key to the modeling strategy is the incorporation of auxiliary, or a priori, information for origin-destination interactions in the patterns. This strategy comes from several recent publications on this topic (Lin 1999; Rogers and Jordan 2004; Rogers and Raymer 1998; Rogers, Raymer and Newbold 2003; Rogers et al. 2002; Rogers, Willekens and Raymer 2001, 2002, 2003; Rogers and Wilson 1996; Sweeney 1999; Willekens 1999).

Interdivisional migration data for the United States during the 1995-2000 period are used to illustrate the formal method.

Migration estimation techniques are needed to provide the necessary inputs for better understanding population redistribution patterns and to improve current or projected estimates of population totals. At present, only residual methods to obtain net migration totals and, to a lesser extent, model migration schedules to estimate age patterns of migration have been incorporated into books on demographic methods (e.g., Morrison, Bryan and Swanson 2004; Preston, Heuveline and Guillot 2000; Rowland 2003). On the other hand, there exists a vast literature on methods for estimating fertility and mortality. For most countries in the world, migration is a major component of population change. It is therefore somewhat surprising that more attention is not given to methods for analyzing and estimating migration flows. To find models of migration, one generally has to go to texts on quantitative geography and look for gravity or spatial interaction models (e.g., Fotheringham, Brunsdon and Charlton 2000; Plane and Rogerson 1994; Robinson 1998). Much of the background for this paper comes from developments in spatial interaction modeling made by geographers in the late 1970s and early 1980s (Plane 1981, 1982; Snickars and Weibull 1977; Willekens 1980, 1982, 1983).

This paper begins with an exposition of a multiplicative component model for describing interregional migration patterns. The link is then made with the log-linear model. This is followed by some strategies and examples for estimating migration flow patterns. The paper concludes with a summary and a discussion.

## 2. ORIGIN, DESTINATION, AND AGE STRUCTURES OF INTERDIVISIONAL MIGRATION IN THE UNITED STATES

The data used for this study include the observed age-specific interdivisional migration patterns in the United States during the 1985-1990 and 1995-2000 periods obtained from the 1990 and 2000 censuses, respectively. The emphasis, however, is on analyzing and estimating the migration patterns of the latter period. The 1985-1990 migration patterns are applied as a priori information in the direct estimation model set out in Section 3 of this paper. The U.S. migration data obtained from the 1990 and 2000 censuses represent transition or status migration data, that is, place of residence at the time of the census and five-years prior to the census. These can be distinguished from event migration data obtained from population registers (Morrison, Bryan and Swanson 2004; Willekens 1999).

### 2.1 Describing Age and Spatial Structures with Multiplicative Components

Interdivisional migration flows (without age) can be disaggregated into four separate components (Rogers et al. 2002): an *overall* component representing the level of migration, an *origin* component representing the relative “pushes” from each region, a *destination* component representing the relative “pulls” to each region, and a two-way *origin-destination interaction* component representing the physical or social distance between places (not explained by the overall and main effects). This breakdown is multiplicative, such that

$$n_{ij} = T * O_i * D_j * OD_{ij} \quad (1)$$

where  $n_{ij}$  is an observed flow of migration from region  $i$  to region  $j$ ,  $T$  is the total number of migrants (i.e.,  $n_{++}$ ),  $O_i$  is the proportion of all migrants leaving from region  $i$  (i.e.,  $n_{i+} / n_{++}$ ), and  $D_j$  is the proportion of all migrants moving to region  $j$  (i.e.,  $n_{+j} / n_{++}$ ). The interaction component  $OD_{ij}$  is defined as  $n_{ij} / (T * O_i * D_j)$  or the ratio of observed migration to expected migration (for the case of no interaction). This general type of model is termed a *multiplicative component model*.

To illustrate the advantages of analyzing migration in terms of multiplicative components, consider the U.S.-born migration flows between the nine Census Bureau-defined divisions during the 1995-2000 time period set out in Panel A of Table 1. Note that non-migrants (i.e.,  $n_{ii}$ ) are not included in the table. During this period, 14.6 million U.S.-born persons over the age of 5 years made an interdivisional migration. Nearly half of all migrants came from the East North Central, South Atlantic, and Pacific divisions and about a quarter of all migrants went to the South Atlantic division. The largest origin-destination-specific flow was from the Middle Atlantic division to the South Atlantic division.

Table 1. The spatial structure of U.S.-born interdivisional migration (in thousands), 1995-2000

Origin	Destination									Total
	NE	MA	ENC	WNC	SA	ESC	WSC	MTN	PAC	
<b>A. Observed flows</b>										
NE	0	167	61	22	298	23	41	59	100	771
MA	245	0	199	54	1,084	74	105	145	191	2,097
ENC	68	161	0	297	674	280	223	273	241	2,217
WNC	25	48	270	0	185	63	205	215	145	1,157
SA	168	437	413	139	0	393	314	215	301	2,380
ESC	18	40	185	47	379	0	159	54	67	947
WSC	37	76	184	188	358	179	0	235	226	1,482
MTN	43	72	154	166	197	53	222	0	472	1,379
PAC	92	151	230	180	397	101	310	766	0	2,227
Total	696	1,150	1,696	1,093	3,573	1,165	1,581	1,962	1,741	14,657
<b>B. Multiplicative components</b>										
NE	0.000	2.755	0.686	0.388	1.583	0.374	0.494	0.573	1.092	0.053
MA	2.464	0.000	0.820	0.344	2.120	0.445	0.466	0.516	0.765	0.143
ENC	0.647	0.926	0.000	1.794	1.247	1.589	0.934	0.920	0.913	0.151
WNC	0.460	0.523	2.015	0.000	0.658	0.687	1.647	1.390	1.054	0.079
SA	1.486	2.341	1.500	0.786	0.000	2.075	1.225	0.674	1.063	0.162
ESC	0.393	0.532	1.688	0.664	1.641	0.000	1.554	0.425	0.592	0.065
WSC	0.519	0.652	1.071	1.704	0.992	1.516	0.000	1.185	1.281	0.101
MTN	0.653	0.662	0.967	1.612	0.587	0.481	1.495	0.000	2.883	0.094
PAC	0.873	0.862	0.892	1.082	0.732	0.570	1.291	2.570	0.000	0.152
Total	0.047	0.078	0.116	0.075	0.244	0.080	0.108	0.134	0.119	14,657

Note: NE = New England, MA = Middle Atlantic, ENC = East North Central, WNC = West North Central, SA = South Atlantic, ESC = East South Central, WSC = West South Central, MTN = Mountain, and PAC = Pacific.

The multiplicative components corresponding to the migration flows discussed above are set out in Panel B of Table 1. Note that the overall component (T) is set out in the total sum (i.e.,  $n_{++}$ ) location of the table, the origin components ( $O_i$ ) are set out in the row-sum locations (i.e.,  $n_{i+}$ ), the destination components ( $D_j$ ) are set out in the column-sum locations (i.e.,  $n_{+j}$ ), and the origin-destination interaction components ( $OD_{ij}$ ) are set out in the cells inside the marginal totals (i.e.,  $n_{ij}$ ). For example, consider the Middle



Atlantic to South Atlantic flow of 1,084 thousand persons disaggregated into the four multiplicative components:

$$\begin{aligned}
 n_{25} &= T * O_2 * D_5 * OD_{25} \\
 &= n_{++} * \frac{n_{2+}}{n_{++}} * \frac{n_{+5}}{n_{++}} * \frac{n_{25}}{\left(n_{++}\right) \left(\frac{n_{2+}}{n_{++}}\right) \left(\frac{n_{+5}}{n_{++}}\right)} \\
 &= 14,657 * \frac{2,097}{14,657} * \frac{3,573}{14,657} * \frac{1,084}{511} \\
 &= 14,657 * 0.143 * 0.244 * 2.120 \\
 &= 1,084
 \end{aligned}$$

where the subscripts 2 and 5 denote the Middle Atlantic and South Atlantic divisions. The interpretations of these components are relatively simple. The overall component is the reported total number of U.S.-born interdivisional migrants aged 5 years and over; 14.6 million persons made an interdivisional move between 1995 and 2000. The origin component represents the shares of all migrants from each division; 14 percent of all migrants originated in Middle Atlantic division. The destination component represents the shares of all migrants to each division; 24 percent of all migrants moved to the South Atlantic division. And, finally the interaction component represents the ratio of observed migration to expected migration; there were roughly two observed migrants for every one expected migrant. The expected flow is based on the marginal total information, i.e.,  $T * O_2 * D_5$ .

The ratio of observed to expected flows captures the relative association or “interaction” between divisions, so the interaction component value of 2.12 indicates a strong association between the Middle Atlantic and South Atlantic regions. Other flows that exhibited high levels of association (over 2.0) were New England-Middle Atlantic, Middle Atlantic-New England, West North Central-East North Central, South Atlantic-Middle Atlantic, South Atlantic-East South Central, Mountain-Pacific, and Pacific-Mountain. In all of these cases, the divisions shared borders with each other.

Next, consider age-specific migration between divisions. The multiplicative component model for this table is specified as:

$$n_{ijx} = T * O_i * D_j * A_x * OD_{ij} * OA_{ix} * DA_{jx} * ODA_{ijx} \quad (2)$$

where the superscript A denotes age and x denotes a five-year age group. The age groups for the U.S. migration data start with 5-9 years and end with 85+ years and are measured at the time the census was taken. In total, there are seventeen age groups. This model is more complicated because there are now three two-way interaction components and one three-way interaction component between the variables origin, destination, and age. However, the interpretations of the parameters remain relatively simple and follow the same format as presented for the two-way table. That is, interaction components represent observed flows or marginal totals to expected ones. For example, the origin-age interaction component is calculated  $n_{i+x} / (T * O_i * A_x)$  and represents ratios of observed age profiles of out-migration from each division divided by the overall age profile of migration (as demonstrated below).

The age and spatial structures of U.S.-born interdivisional migration during the 1995-2000 period are described next using the multiplicative components set out above. The analysis follows a hierarchical format starting with the overall level component and ending with the two-way interaction components. The three-way interactions between origin, destination, and age are not analyzed for two reasons. The first is that most of the structure found in the migration patterns is captured by the overall, main, and two-way interaction effects. The second reason is, while there are often patterns found in the three-way interactions, it is tedious to incorporate these into the modeling process and their interpretation is more difficult. Therefore, we just focus on the more simple and powerful aspects of the model represented by the other seven components.

The overall level, origin main effect, destination main effect, and origin-destination interaction components of U.S.-born interdivisional migration have already been presented in Table 1. The values and interpretations remain the same regardless of the number of dimensions in the data, as long as the components are calculated in reference to the total sum. The age-specific shares of all migrants (i.e., the age main effect component) are set out in Figure 4, which shows that the shares of migration were higher for persons in age groups 0-4 to 40-44 years, with a peak occurring in the 25-29 year old age group and the low point occurring for persons in the last age group.

The origin-age interaction components, which represent the observed age-specific out-migration flows from each division relative to the corresponding expected flows (i.e.,  $T*O_i*A_x$ ), are presented in Figure 1. Some interesting patterns are found. First, children were a lot more likely to leave the Pacific division and a lot less likely to leave the New England division. Second, young adults were more likely to leave the New England,

Middle Atlantic, East North Central, and West North Central divisions and less likely to leave the South Atlantic and Pacific divisions. Finally, retirement migrants were more likely to come from the New England, Middle Atlantic, and East North Central divisions.

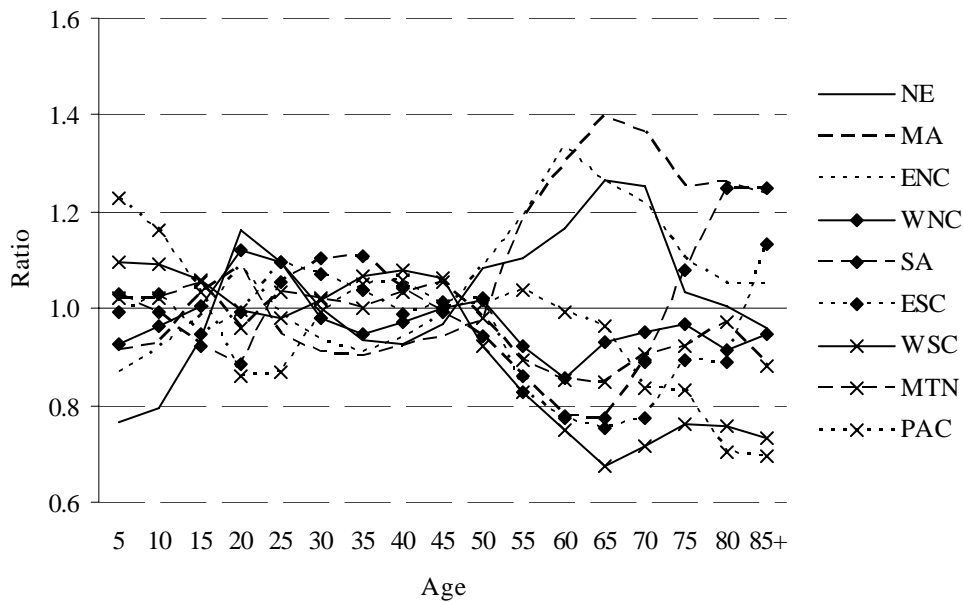


Figure 1. The origin-age interaction components of U.S.-born interdivisional migration, 1995-2000

The destination-age interaction components are presented in Figure 2. Here we find large variations particularly in the retirement age groups. Retirement-aged migrants clearly preferred the South Atlantic and Mountain divisions to other regions. However, this was not true of migrants in other age groups. Young adults were particularly attracted to the New England, Middle Atlantic, and Pacific divisions.

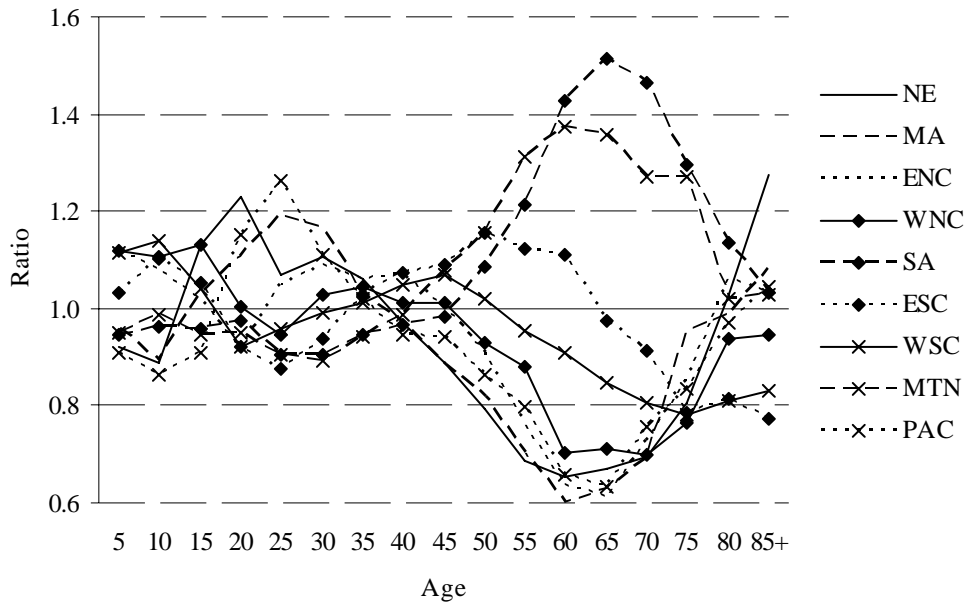


Figure 2. The destination-age interaction components of U.S.-born interdivisional migration, 1995-2000

## 2.2 The Log-Linear Model for Analyzing Structures in Migration Flow Tables

The multiplicative component model set out in Equation 2 used for describing migration age and spatial structures can be expressed as a saturated log-linear model:

$$\ln(n_{ijx}) = \lambda + \lambda_i^O + \lambda_j^D + \lambda_x^A + \lambda_{ij}^{OD} + \lambda_{ix}^{OA} + \lambda_{jx}^{DA} + \lambda_{ijx}^{ODA} \quad (3)$$

or in multiplicative form:

$$n_{ijx} = \tau \tau_i^O \tau_j^D \tau_x^A \tau_{ij}^{OD} \tau_{ix}^{OA} \tau_{jx}^{DA} \tau_{ijx}^{ODA} \quad (4)$$

where the  $\lambda$ 's or  $\tau$ 's denote the parameters or “effects” of the model. When expressed in this form, migration structures can be modeled using standard statistical techniques for categorical data (see, e.g., Agresti 1996). Also, specific effects can be taken out to identify contributions of the various structures in the data identified by goodness-of-fit measures.

Reduced forms of the models set out in Equations 3 and 4 are considered unsaturated models. For example, the model that only includes the main effects of origin, destination, and age is specified as

$$\hat{n}_{ijx} = \tau \tau_i^O \tau_j^D \tau_x^A \quad (5)$$

This model assumes independence between each of categories of origin, destination, and age and is designated [O][D][A], using the notation set out in Knoke and Burke (1980). A model that includes the interaction between origin and destination plus all of the main effects is designated as [OD] rather than its longer form, [O][D][A][OD]. Similarly the saturated model is expressed as [ODA], which in its longer form would be expressed as [O][D][A][OD][OA][DA][ODA]. The simpler notations are used because these models are hierarchical, that is, for two-way interaction terms, the main effect parameters must be included and for three-way interaction terms all the main effects and two-way interactions must be included.

Since migration flow tables often do not include the diagonal elements (i.e., non-migrants), structural zeros can be inserted to represent non-migrants (Willekens 1983).

This model is considered quasi-independent. To do this, an offset is required. This model is specified as:

$$\hat{n}_{ijx} = n_{ijx}^* v_i^O v_j^D v_x^A \quad (6)$$

where the offset,  $n_{ij}^*$ , includes 0's in the diagonal elements and 1's in the off-diagonal elements. The  $v$ 's denote the parameters of the log-linear-with-offset model (refer to Rogers, Willekens and Raymer 2003:60-61).

The eight unsaturated models set out in Table 2 all include structural zeros to remove non-migrants from the predictions. We use the likelihood ratio statistic ( $G^2$ ) to compare model fits:

$$G^2 = 2 \sum n_{ijx} \ln(n_{ijx} / \hat{n}_{ijx}), \quad (7)$$

where  $\hat{n}_{ijx}$  denotes the predicted age-specific migration flows. The most obvious finding in Table 2 is that the origin-destination interaction term is very important for accurately predicting the age-specific migration flows. Most of the flows do not contain a large retirement peak or major deviations from the overall age profile of migration. However, the fits are slightly improved when the origin-age or destination-age interactions (with the latter doing a better job) are included. This is further demonstrated by the several age- and origin-destination-specific flows illustrated in Figure 3. Of course, to capture retirement peaks found in some of the flows, origin-age or destination-age interactions have to be included.

Table 2. Unsaturated log-linear model fits: Age-specific U.S.-born interdivisional migration, 1995-2000

Model	Likelihood Ratio	Df	Likelihood Ratio / df
[O][D][A]	4,068,146	1,191	3,416
[OD]	545,855	1,136	481
[OA]	3,909,131	1,063	3,677
[DA]	3,817,146	1,063	3,591
[OD][OA]	386,839	1,008	384
[OD][DA]	294,855	1,008	293
[OA][DA]	3,678,153	935	3,934
[OD][OA][DA]	163,392	880	186

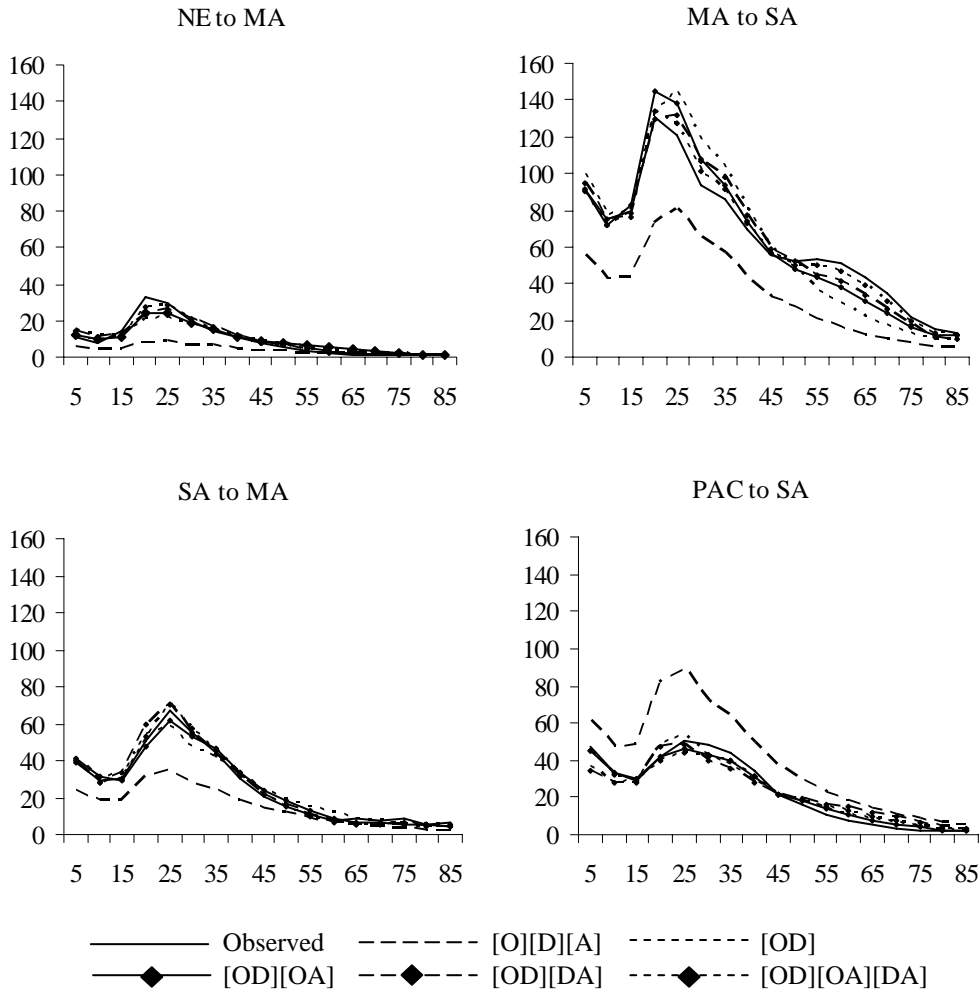


Figure 3. A comparison of various unsaturated log-linear model predictions: Selected age-specific U.S.-born interdivisional migration flows (in thousands), 1995-2000



### 3. DIRECT ESTIMATION

The 1995-2000 age-specific interdivisional migration patterns are directly estimated in this section using some of the structures found in the previous census. Much of this work follows recent developments (e.g., see Rogers, Willekens and Raymer 2002, 2003), but the idea of updating tables of migration using iterative proportional fitting algorithms goes back at least to the 1970s (Rogers 1973; Willekens 1977). Efforts to estimate migration flows using *historical tables* have been shown to be very effective (Isserman et al. 1985; Plane 1982; Rogerson and Plane 1984). Indeed a study by Snickars and Weibull (1977) found that historical data outperformed spatial interaction approaches in such efforts.

To demonstrate the continuity of spatial structures over time, ratios of the 1995-2000 to the 1985-1990 spatial structures (see Equation 1) have been calculated and are set out in Table 3. The ordering of the ratios is the same that was used in Table 1B. Ratios with values close to one indicate continuity in spatial structures over time. For example, consider a comparison over time of the Middle Atlantic to South Atlantic flow multiplicative components:

$$\begin{aligned} \frac{n_{25}^{95-00}}{n_{25}^{85-90}} &= \frac{T^{95-00} * O_2^{95-00} * D_5^{95-00} * OD_{25}^{95-00}}{T^{85-90} * O_2^{85-90} * D_5^{85-90} * OD_{25}^{85-90}} \\ &= \frac{14,657 * 0.143 * 0.244 * 2.120}{14,341 * 0.145 * 0.263 * 1.977} \\ &= 1.022 * 0.988 * 0.927 * 1.073 \\ &= 1.004 \end{aligned}$$

As can be seen with this calculation, there was not much change observed between the 1985-1990 and 1995-2000 periods for the Middle Atlantic to South Atlantic flow. Both the level and the multiplicative components remained relatively close over time. This was true for most, but not all of the multiplicative components. For example, the share of migration from the South Atlantic and Pacific divisions increased by 17 percent and 24 percent, respectively, and decreased by nearly 25 percent in the West South Central division. The proportions of migrants going to the West South Central and Mountain divisions increased substantially, whereas those going to the Pacific division declined. For the origin-destination interaction components, the extremes were those corresponding with the New England to Pacific flow, which increased by 26 percent, and with the West North Central to Mountain flow, which decreased by 18 percent.

*Table 3. A comparison of U.S.-born interdivisional migration: Ratio of 1995-2000 spatial structure to 1985-1990 spatial structure*

Origin	Destination									Total
	NE	MA	ENC	WNC	SA	ESC	WSC	MTN	PAC	
NE		0.960	0.968	0.983	1.019	1.088	1.105	1.070	1.259	0.971
MA	0.980		1.033	1.014	1.073	1.097	0.922	1.089	1.063	0.988
ENC	0.941	0.959		1.071	0.972	1.067	0.923	1.020	0.994	0.946
WNC	0.896	0.956	1.099		1.079	1.109	0.922	0.817	0.930	0.944
SA	0.996	0.979	0.923	1.064		0.924	1.000	1.054	1.080	1.172
ESC	0.995	0.994	1.054	0.953	0.988		0.972	1.128	1.117	0.995
WSC	0.893	0.925	0.975	1.022	1.047	0.985		1.026	1.036	0.752
MTN	1.159	1.054	1.033	1.031	1.241	1.160	1.025		1.034	0.972
PAC	0.909	0.860	0.870	0.959	0.973	0.938	0.897	0.909		1.237
Total	0.912	0.981	0.985	1.056	0.927	1.089	1.196	1.272	0.800	1.022

The age structures also showed consistencies over time. The age main effect components for the 1985-1990 and 1995-2000 periods are set out in Figure 4. The main difference between the two periods is that the labor force peak became slightly wider in the later period. A comparison of New England and South Atlantic's origin-age and destination-age interaction components for the two migration periods are set out in Figure 5. Here, the most noticeable differences are found in the retirement years where the patterns of the 1995-2000 period are less extreme than in the 1985-1990 period. Overall, the comparisons of the age and spatial structures of migration between the two periods show continuity over time and suggest that a model relying on the 1990 census data used to estimate the 1995-2000 patterns should perform well.

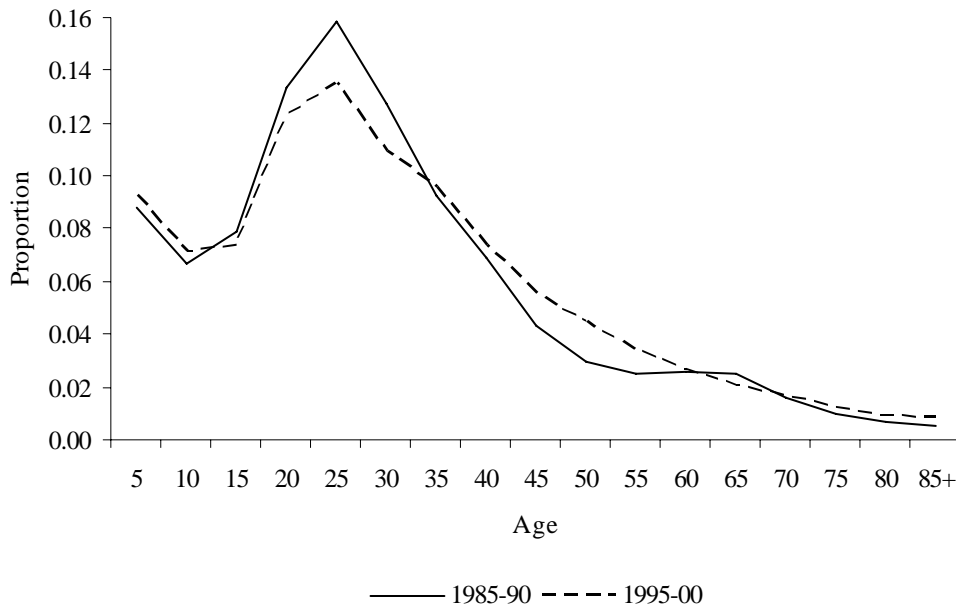
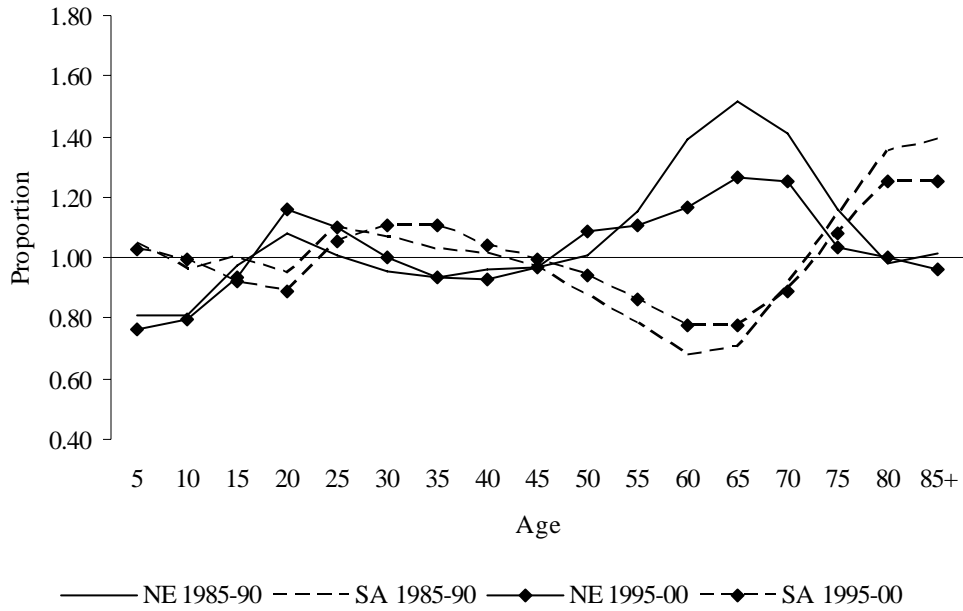


Figure 4. The age main effect components U.S.-born interdivisional migration, 1985-1990 and 1995-2000

A. Origin-Age



B. Destination-Age

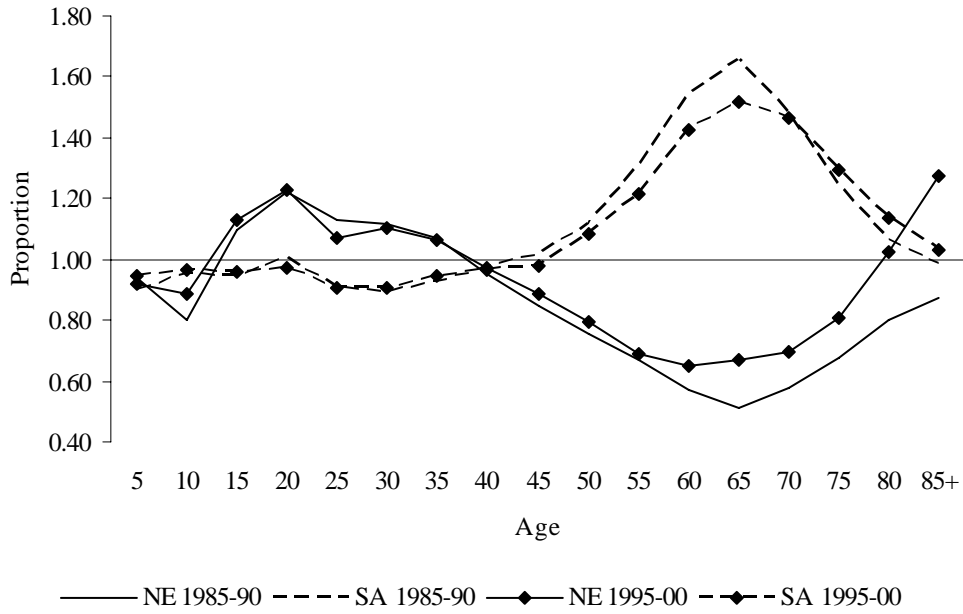


Figure 5. The origin-age and destination-age interaction components of U.S.-born interdivisional migration, 1985-1990 and 1995-2000: New England and South Atlantic

We apply the log-linear-with-offset model (i.e., Equation 6) to directly estimate the 1995-2000 age-specific interdivisional migration flows. The offset in this case is the observed 1985-1990 age-specific interdivisional migration flows. Depending on the available data, the estimation can focus on (1) migrants or (2) both migrants and non-migrants. The first implies that the aggregate numbers of persons in-migrating and out-migrating for each division are known, whereas the second implies that only the beginning and ending divisional population stocks are known (a more common situation). For the second case,  $T$  denotes the overall population size of persons aged 5+ years,  $O_i$  denotes the proportion of the population residing in division at the beginning of the interval,  $D_j$  denotes the proportion of the population residing in division at the end of the interval, and  $A_x$  denotes the proportions of the total population in each age group.

The main concern with modeling migrants and non-migrants is the tendency of non-migrants to dominate the results. During the 1985-1990 and 1995-2000 periods, about 93 percent of the populations were considered non-migrants. For direct estimation modeling, this means that any substantial changes in the non-migrant origin-destination interaction components will have a sizeable impact on the predicted flows of migration. To check this, two offsets were used to estimate the 1995-2000 age-specific interdivisional migration flows: one that included only migrants and another that included both migrants and non-migrants. Both models performed well, as illustrated in Figure 6. In terms of likelihood ratios, the migrant-only-model fitted the observed data better ( $G^2 = 236,326$  vs.  $-425,830$ ).

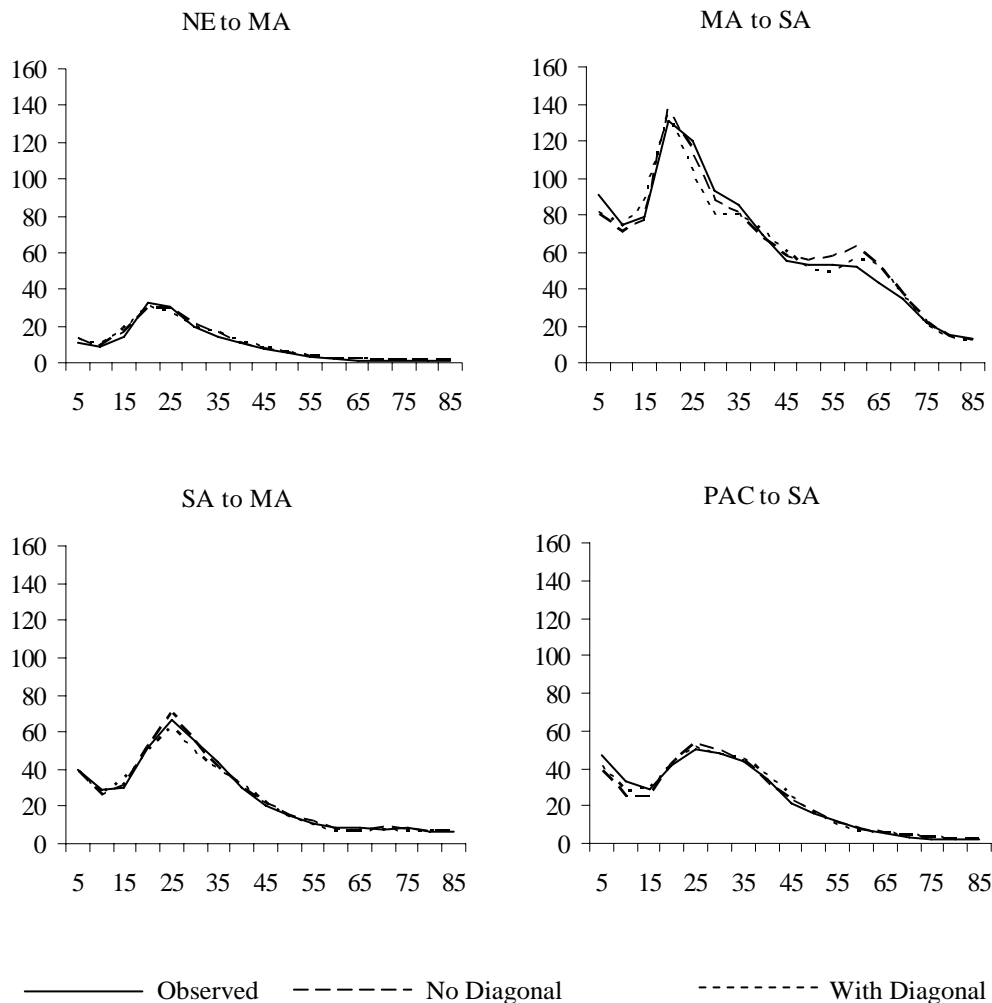


Figure 6. A comparison of direct log-linear model predictions: Selected age-specific U.S.-born interdivisional migration flows (in thousands), 1995-2000

#### 4. INDIRECT ESTIMATION: ESTIMATING INTERDIVISIONAL FLOWS BASED ON INFANT BIRTHPLACE-SPECIFIC POPULATION STOCKS

Estimations of period migration patterns based on lifetime migration patterns have been attempted for many years with limited success (Eldridge and Kim 1968; George 1971; Nair 1985; Rogers and von Rabenau 1971). More recently, a new effort has been presented by Rogers and Jordon (2004), in which birthplace-specific population stock

data of 0-4 year olds was used to predict age-specific interregional patterns of migration in the United States. The first age group was used because it roughly corresponds with the five-year interval migration question. That is, if a child is living in a different place than his or her place of birth, that child must have migrated at least once during the past five years. The same cannot be said for other age groups. And the reason why a single age group can predict other age groups comes from knowledge of the age regularities found in observed migration patterns.

#### 4.1 Model Migration Schedules

Migration propensities differ greatly according to age. Typically, an age-specific profile of migration shows a downward slope from the early childhood age groups to about age sixteen followed by a rise to a peak in the young adult age groups (usually around age twenty-two), then a gradual tapering off to the oldest age groups. This “standard” age profile of migration can be fully described using a multiexponential model migration schedule (Rogers and Castro 1981; Rogers and Little 1994).

The most often used model migration schedule is the seven parameter version:

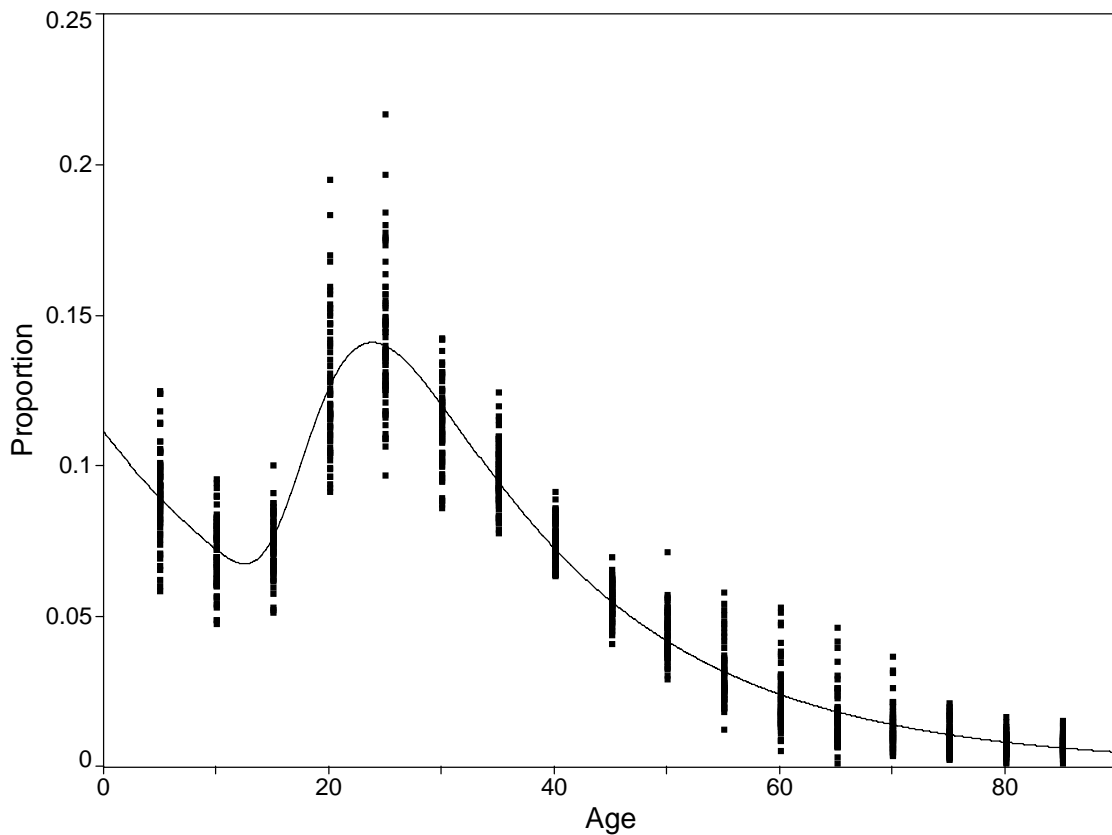
$$N_{ijx} = a_0 + a_1^{(-\alpha_1 x)} + a_2^{\{-\alpha_2(x-\mu_2) - \exp[-\lambda_2(x-\mu_2)]\}}, \quad i \neq j \quad (8)$$

where  $N_{ijx}$  denotes standardized age profiles of migration from  $i$  to  $j$ . The  $a_0$ ,  $a_1$ , and  $a_2$  are level parameters, whereas the  $\alpha_1$ ,  $\alpha_2$ ,  $\mu_2$ , and  $\lambda_2$  parameters are shape parameters.

The age profiles of interdivisional migration during the 1995-2000, standardized to unit area, were fitted with a model migration schedule. The estimated parameters are:

$$N_{ijx} = 0.0000 + 0.1113^{(-0.0429x)} + 0.1885^{(-0.0612(x-19.24) - \exp[-0.2326(x-19.24)])}$$

And the predicted curve is set out in Figure 7. Associated with this curve is an  $R^2$  of 0.934, which means that nearly all of the 72 age-specific profiles of migration can be explained by a single age profile.



*Figure 7. Model migration schedule fit to standardized age profiles of age-specific U.S.-born interdivisional migration, 1995-2000*

This predicted age profile of migration closely approximated the overall age profile of migration (see Figure 4). A model schedule fitted to this profile resulted in the following parameters:



$$N_{++x} = 0.0000 + 0.1147^{(-0.0463x)} + 0.1603^{(-0.0501(x-18.37) - \exp[-0.2660(x-18.37)])}$$

and curve set out in Figure 8. All of the parameters were statistically highly significant for both model schedule fits, with the exception of the constant  $a_0$  parameter.

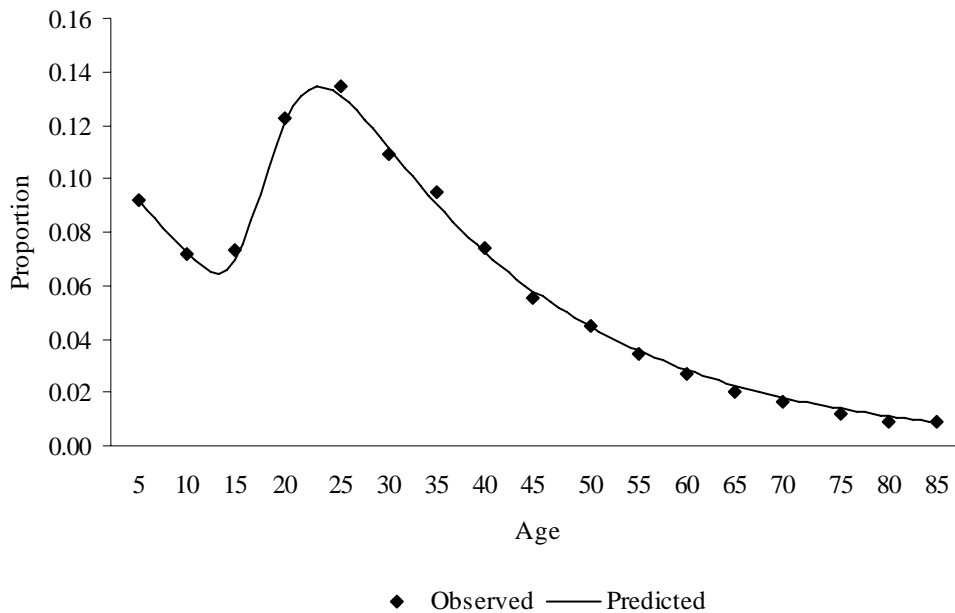


Figure 8. Model migration schedule of the age main effect component, 1995-2000

The multiexponential model migration schedule is useful for describing or inferring age-specific migration patterns when data are incomplete or missing. Yet, because of its relative complexity (i.e., large number of parameters, which do not have simple interpretations), this model has yet to be widely accepted for estimating age-specific migration flows in situations where data are incomplete or inaccurate. It has also been demonstrated that model migration schedules can potentially lead to substantial misrepresentation of age-specific migration patterns if the wrong schedule is applied

(McDevitt 1996). Congdon (1993) posits that a relational approach for modeling age patterns of migration, similar to Zaba's (1987), might be more practical. Next, the knowledge that a single age profile of migration can capture most of the age profiles of migration suggests a relational-type model to predict the interdivisional migration patterns.

#### 4.2 A Relational Approach for Modeling Migration Patterns

The log-linear-with-offset model can be thought of as a relational model (Rogers, Willekens and Raymer 2003). In this situation, the offset is the 0-4 year old migration patterns. As can be seen in Table 4, the spatial structure of these lifetime migrants closely resembles that of the period migrants set out in Table 1. A log-linear-with-offset model can be specified which uses the infant migration patterns to predict the aggregate patterns (assuming the marginal totals are known):

$$\hat{n}_{ij} = n_{ij}^* v v_i^O v_j^D, \quad (8)$$

where the offset  $n_{ij}^*$  contains the “migration” patterns of those aged 0-4 years at the time of the census. The predicted aggregate flows from New England and South Atlantic are set out in Figure 9. Two offsets were used: (1) migrants and (2) migrants and non-migrants. While both models appear to predict the observed data well, the migrants-only model did considerably better. The likelihood ratio statistics for the two models were 132,799 and -1,632,755, respectively. The corresponding  $R^2$  values were 0.985 and 0.955, respectively. Note, these goodness-of-fit measures only compared observed and predicted migration flows (i.e., non-migrants were excluded from the calculations).

Table 4. The spatial structure of U.S.-born interdivisional lifetime migration of 0-4 year olds (in thousands), 2000

Origin	Destination									Total
	NE	MA	ENC	WNC	SA	ESC	WSC	MTN	PAC	
<b>A. Observed flows (in thousands)</b>										
NE	0	14	5	2	17	2	3	3	6	52
MA	17	0	19	5	62	6	10	8	14	140
ENC	5	15	0	25	39	24	17	15	18	158
WNC	2	5	36	0	15	6	19	14	11	106
SA	12	42	44	13	0	35	30	15	28	219
ESC	2	5	27	6	34	0	18	4	7	103
WSC	4	8	20	19	33	15	0	23	26	147
MTN	4	8	13	14	17	5	20	0	43	123
PAC	8	15	25	18	42	10	32	65	0	214
Total	54	111	189	102	258	102	148	147	152	1,264
<b>B. Multiplicative components</b>										
NE	0.000	3.126	0.678	0.500	1.572	0.464	0.505	0.469	0.915	0.041
MA	2.824	0.000	0.886	0.472	2.152	0.536	0.594	0.467	0.843	0.111
ENC	0.708	1.063	0.000	1.918	1.204	1.903	0.929	0.835	0.958	0.125
WNC	0.471	0.501	2.242	0.000	0.681	0.674	1.502	1.096	0.867	0.084
SA	1.317	2.166	1.344	0.743	0.000	1.974	1.160	0.594	1.063	0.173
ESC	0.506	0.506	1.756	0.735	1.626	0.000	1.450	0.373	0.550	0.082
WSC	0.587	0.653	0.920	1.591	1.099	1.231	0.000	1.328	1.438	0.117
MTN	0.729	0.733	0.726	1.438	0.657	0.475	1.380	0.000	2.862	0.098
PAC	0.882	0.779	0.771	1.017	0.967	0.559	1.277	2.617	0.000	0.170
Total	0.043	0.088	0.149	0.081	0.204	0.080	0.117	0.116	0.121	1,264

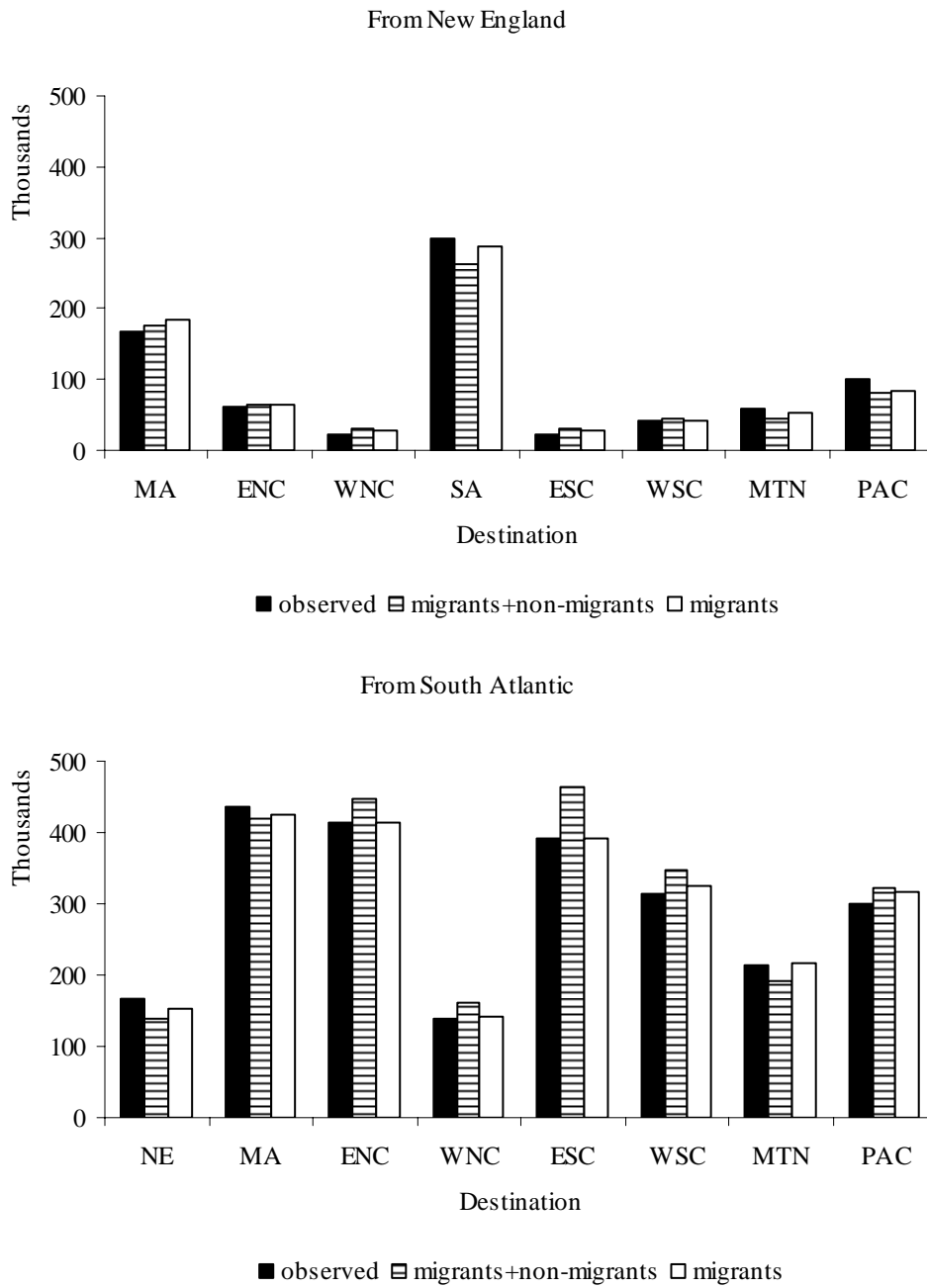


Figure 9. A comparison of indirect log-linear model predictions: U.S.-born interdivisional migration (in thousands) from New England and South Atlantic, 1995-2000

Two models were used to produce the age-specific predictions set out in Figure 10. The first one is analogous to Equation 6, but with an offset that contains structural zeros ( $i = j$ ) and the infant migration patterns set out in Table 4. To obtain reasonable projections of the flows that included non-migrants an additional interaction term was required. This model assumes that the aggregate age-specific proportions of migrants and non-migrants are known and is specified as:

$$\hat{n}_{ijxz} = n_{ijxz}^* v v_i^O v_j^D v_x^A v_z^M v_{xz}^{AM} \quad (9)$$

where M denotes migrant status (i.e., migrant or non-migrant status). This specification is required to distinguish between the age profiles of migrants and non-migrants. Examples of such observed and predicted non-migrants are set out in Figure 11. As can be seen in Figures 10 and 11, both models perform well, capturing the levels and most of the age profiles. The problems occur mainly when the observed age profile differs substantially from the overall age profile. For example, the retirement peak is not estimated at all for the Middle Atlantic to South Atlantic flow.

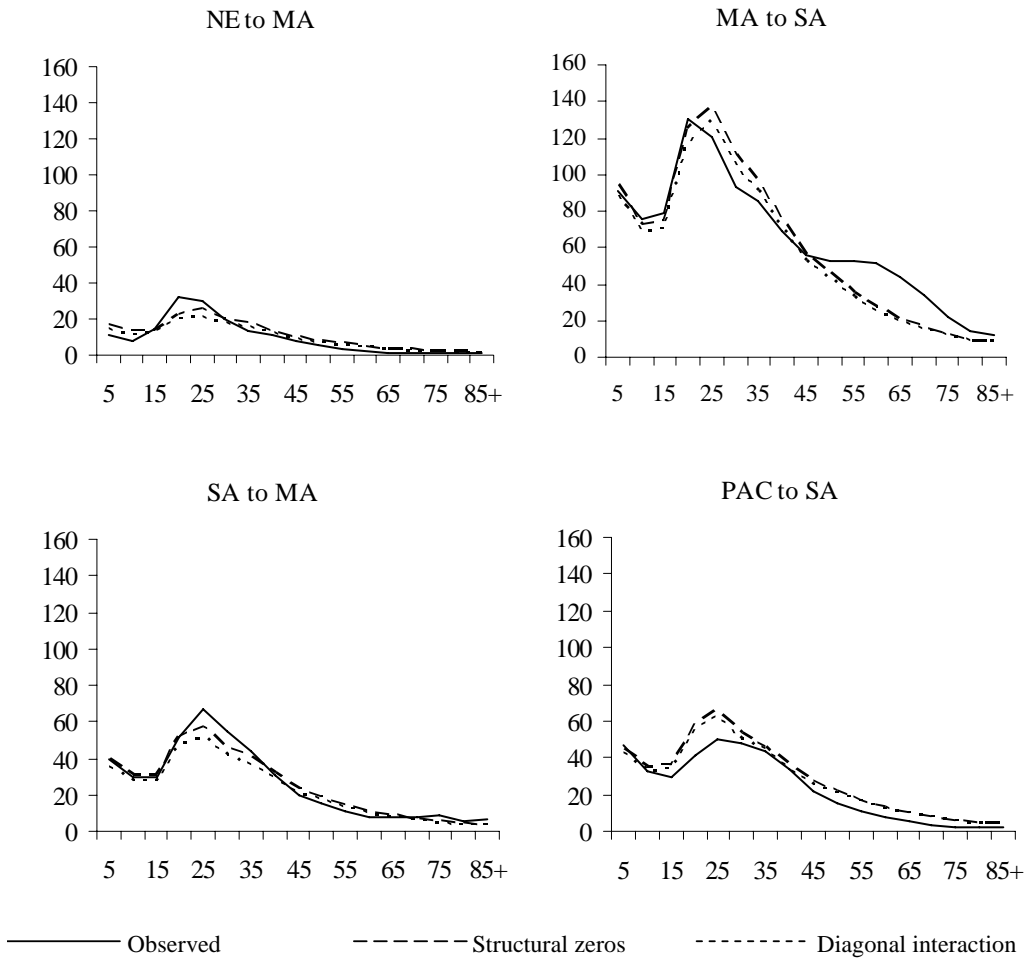


Figure 10. A comparison of indirect log-linear model predictions: Selected age-specific U.S.-born interdivisional migration flows (in thousands), 1995-2000

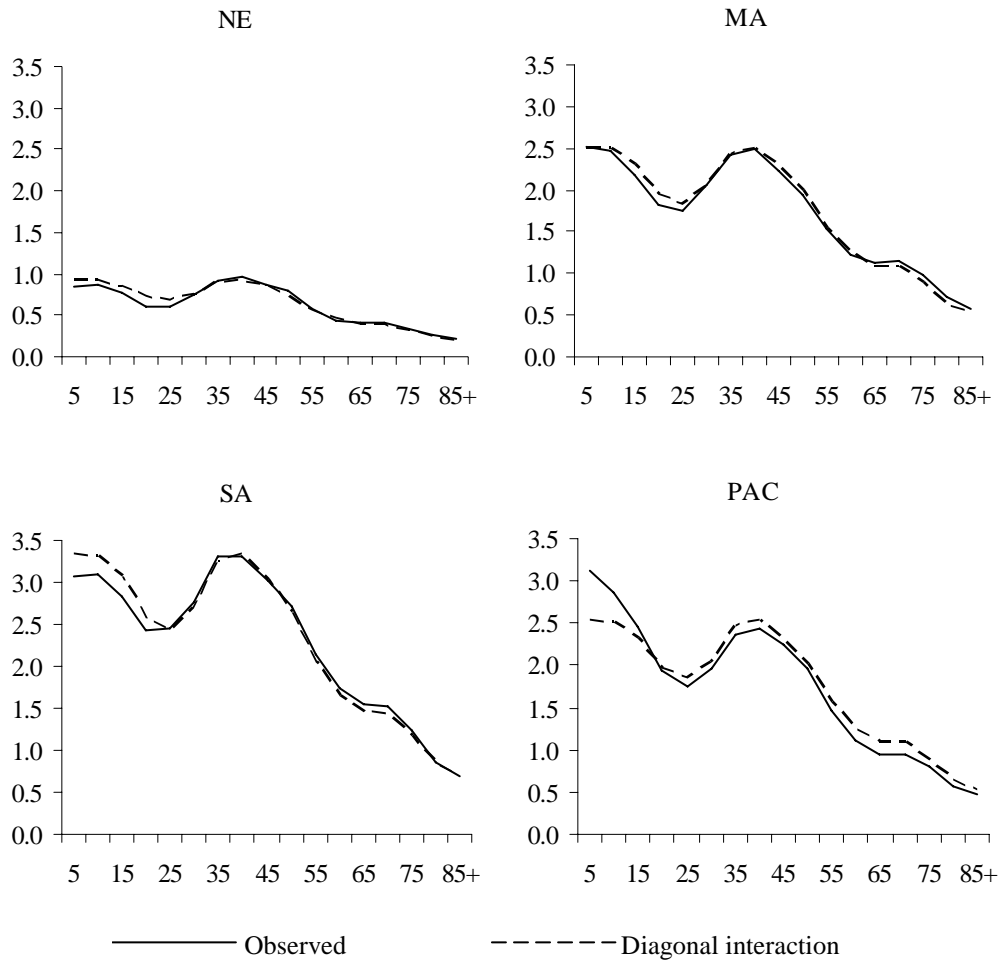


Figure 11. A comparison of indirect log-linear model predictions: Selected age-specific U.S.-born divisional non-migrants (in millions), 1995-2000

## 5. SUMMARY AND DISCUSSION

The topic of indirect estimation has received wide attention from demographers studying the fertility and mortality patterns of countries with incomplete or inaccurate vital registration data. The Population Division of the United Nations has been an especially significant contributor to the collection, description, and dissemination of the assortment of techniques developed by demographers such as William Brass, Ansley Coale, James Trussell, Donald McNeil, Paul Demeny, and others. In 1983 it published a manual that is still used today. Unfortunately, the indirect estimation of migration was ignored:

A further limitation of the Manual is that it deals mainly with the estimation of fertility and mortality in developing countries. There are other demographic processes affecting the populations of these countries (migration for example) which are not treated here (United Nations 1983:1).

An effort has been made in this paper to contribute to that small body of literature that has addressed the subject of the indirect estimation of migration (Hill 1989; Zaba 1987). Our effort builds on previous studies that have outlined log-linear models for describing the spatial structures of directional migration flows (Rogers et al. 2002), adopting model schedules for describing the age structures of these source flows (Rogers and Castro 1981). It then has used these models to adjust and predict the likely spatial and age patterns than underlie observed multiregional distributions of population stocks. Finally, just as in mortality studies infant mortality has been used in efforts to indirectly estimate mortality at all ages, so too we have used a proxy for infant migration to estimate migration at all ages.



Our efforts are preliminary in several respects, but the results that we have obtained so far and presented in this paper are encouraging. They suggest a number of questions that need to be addressed in future work. First, we have dealt with numbers, yet the arithmetic of demography generally is carried out with rates or probabilities. Second, we have assumed the availability of historical data from which we might identify and borrow regularities in patterns, applying them to current data. In the absence of such historical data, the methods presented here would need to posit structures, possibly on the basis of “explanatory” models that link the evolution of patterns to a handful of covariates. For example, we are working on methods that use population age compositions to predict the age compositions of migrants (Little and Rogers 2004; Schmertmann 1992).

Finally, we have used the spatial structure of infant migration that is implied by the birthplace-specific distributions of regional stocks of infants as the basis for indirectly estimating the migration streams of migrants of all ages. In the future we will experiment with using alternative means of obtaining such estimates—means such as, for example, the decomposition of net migration flows (Rogers and Liu forthcoming) or the adoption of regularities found in ratios of secondary to primary migration flows (Rogers and Raymer 2005).

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