PRE-PRINT: Arsene, C., M. A. Strickland, et al. (2008). Comparison of two methods for probabilistic finite element analysis of total knee replacement. 8th International Symposium on Computer Methods in Biomechanics and Biomedical Engineering Porto, Portugal.

COMPARISON OF TWO PROBABILISTIC METHODS FOR FINITE ELEMENT ANALYSIS OF TOTAL KNEE REPLACEMENT

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1. ABSTRACT

Probabilistic Finite Element (FE) models have recently been developed to assess the impact of experimental variability present in knee wear simulator on predicted Total Knee Replacement (TKR) mechanics by determining the performance envelope of joint kinematics and contact mechanics. The gold standard for this type of analysis is currently the Monte Carlo method, however, this requires a larger number of trials and is therefore computationally expensive. Alternatively, probabilistic methods exist, such as response surface methods that can offer considerable savings in computational cost. The aim of the current study was to compare the performance envelopes obtained for three metrics (Anterior-Posterior (AP) translation, Internal-External (IE) rotation and peak Contact Pressure (CP)) for a FE model of TKR mechanics using two different probabilistic methods: the Monte Carlo technique and the Response Surface Method (RSM), implemented with PamCrash FE solver and PamOpt optimization/probabilistic software. The influence of implant alignment was considered, based on a study from the literature. The results of a 1000 trial Monte Carlo analysis were compared to predictions from 25, 50 and 100 trial response surface calculations. Overall, the Response Surface Method (RSM) was capable of predicting similar results to the Monte Carlo method, but with a substantially reduced computational cost (RSM-50 4 hours as compared to 4 days with the Monte Carlo method).

2. INTRODUCTION

Computer methods in bioengineering have been used since the late 1970s and one of the methods of choice has been the finite element (FE) technique (Huiskes and Chao, 1983; Prendergast, 1997; Mackerle, 1992). Many orthopedic studies using implicit or explicit FE modeling techniques have been implemented (Lee, 1987; Godest et al., 2002). Modeling of TKR with the explicit FE method has allowed the simultaneous calculation of joint kinematics and contact mechanics during a gait cycle and under force-controlled loading conditions (Godest et al 2002, Halloran et al., 2005a; Halloran et al., 2005b).

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However, individual deterministic FE studies cannot include the influence of input variability on the outputs of the FE model.
The application of probabilistic techniques in combination with FE modeling has the potential to assess the performance envelope of joint replacements under the influence of input variability such as implant positioning, loading and soft tissue (muscle, tendon and ligament) properties. Probabilistic FE techniques were initially applied in the assessment of structural reliability (Melis et al., 1999; Zhang and Liu, 2002) and more recently in the reliability of orthopedic components (Nicolella et al., 2001; Browne et al., 1999; Dar et al., 2002; Laz et al., 2006).
The aim of this paper is to compare the performance envelopes obtained for three mechanical 'output' metrics for total knee replacement (Anterior-Posterior (AP) translation, Internal-External (IE) rotation and peak Contact Pressure (CP)) using explicit FE analysis in combination with two different probabilistic methods: the Monte Carlo technique and the Response Surface Method (RSM). The influence of implant alignment will be the primary source of input variability. The results of a 1000 trial Monte Carlo analysis will be compared to predictions from 25, 50 and 100 trial RSM calculations. The FE model of TKR will be implemented with the PamCrash FE solver (ESI, France) and the Monte Carlo method will be implemented with the PamOpt optimization/probabilistic software. The RSM will use the *regress* function from Matlab, which implements a linear regression.

2. DETERMINISTIC FE MODEL OF TKR

The three dimensional explicit FE model of a PFC Sigma DePuy International, Leeds, UK) was analyzed. The loading and boundary conditions represent the force-controlled knee wear simulator (Walker et al., 1997; ISO Standard 14243-1, 2000).

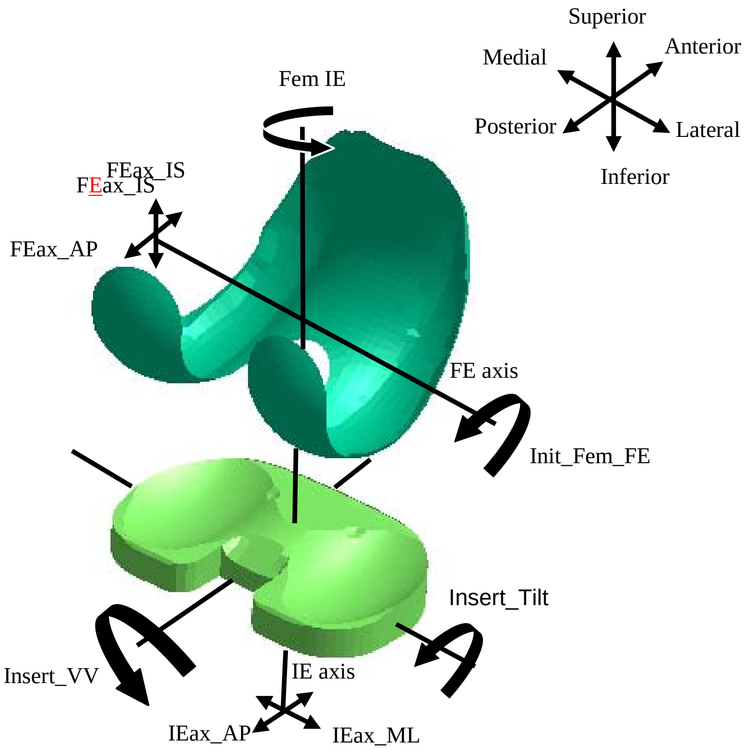


Fig 1. Finite element model of TKR illustrating the study parameters.

Both the femoral component and the tibial polyethylene insert were modeled as rigid bodies using four-noded shell elements (Fig. 1). An advanced penalty method based contact algorithm was used to model the contact between the two components (Godest et al., 2002).
Eight input experimental parameters dealing with component alignment were evaluated. The input parameters included four translations (standard deviation 0.5 mm) and four rotations (standard deviation 1^o) of the femoral component and tibial insert with the notations and the values adopted from the original study by Laz et al. (2006).The values represent estimates of the uncertainties in the knee simulator experimental setup, but a similar approach could be used to assess surgical positioning variability. They define the position of the femoral component and tibial insert relative to the fixed rotational axes. The mean values were the deterministic values representing the neutral position of the implant in the Stanmore knee simulator.

3. PROBABILISTIC METHODS

Using the Monte Carlo method (Metropolis and Ulam, 1949; Fishman, 1995) each input parameter assumes a Gaussian probability distribution from which a number of random samples are generated (e.g. 1000 points). For every point from the probability distributions, a FE simulation is carried out and the output variables are obtained. There are 1000 data sets for each of the output performance metrics (e.g. AP translation), in comparison to a single data set obtained with a single deterministic FE analysis. Going through the 1000 data sets at each time step of the gait cycle, appropriate statistical measures (e.g. mean, range, standard deviation, or specific percentile levels such as 1-99%) can be used to express the degree of output variability observed.
The second probabilistic method used in this study was the Response Surface Method (RSM) (Isukapalli et al., 2004). The RSM fits an analytic function of the input variables to approximate the output parameter, across the full range of the sample space. Typically, this will be a low-order polynomial (called the Response Surface Equation, RSE), and regression techniques are used to select the term coefficients. The method comprises three steps: first, a response vector *y* (i.e. AP translation, IE rotation, Peak Contact Pressure) is obtained from a probabilistic FE simulation which uses the Monte Carlo simulation with few input points X (e.g. 20, 25, 100, etc). Trials could be random, but a better result is achieved by distributing the trials regularly across the sample space. Second step, with the least square method we have:

$$b = \left(X^T X\right)^{-1} X^T y \qquad (1)$$

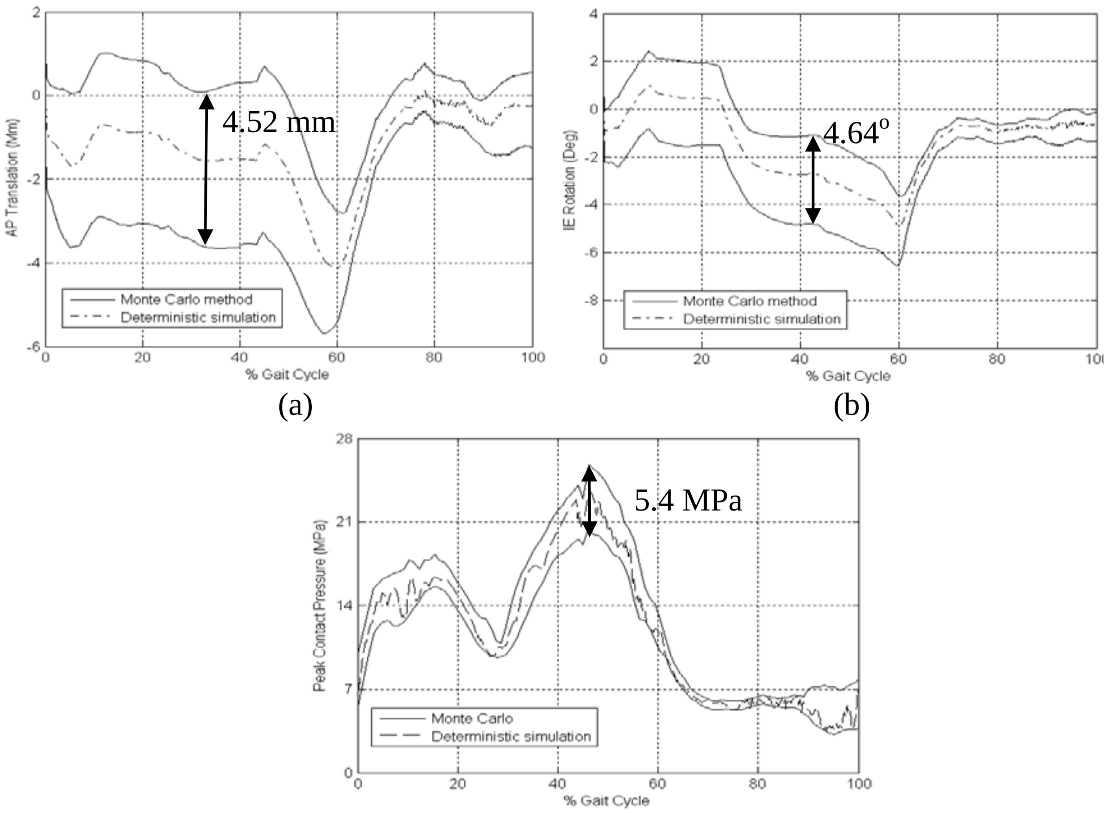
where b denotes the coefficients of the RSE.
Third, the RSE (*b*) together with 1000 Gaussian distributed samples for each input variable (4 translations, 4 rotations) forming matrix X_1 will generate the response vector of interest y_1:

$$y_1 = b\,X_1 \qquad (2)$$

Similar statistical measures can be applied to the response vector y_1 (e.g. mean, range, standard deviation, etc). This method works best when the true output is well represented by the analytic function, for example relatively linear, smooth and monotonic models can easily be fitted and the TKR model is expected to be relatively linear with the small perturbation range being studied here; highly non-linear functions are not well-represented. The higher the order of the RSE used, the more terms that are needed; hence the more samples needed to achieve a good fit with the regression. Clearly, higher order response surfaces require more runs according to the power of the highest polynomial term. Beyond quadratic terms this is impractical for many models. In this first proof of concept study we will fit the TKR behavior with a linear RSE, so that fewer trials (25-100) can be tested.

4. RESULTS

The predicted response envelopes utilizing the Monte Carlo probabilistic method is shown at the 1 and 99 percentile levels for each metric over the entire gait cycle (Fig. 2).

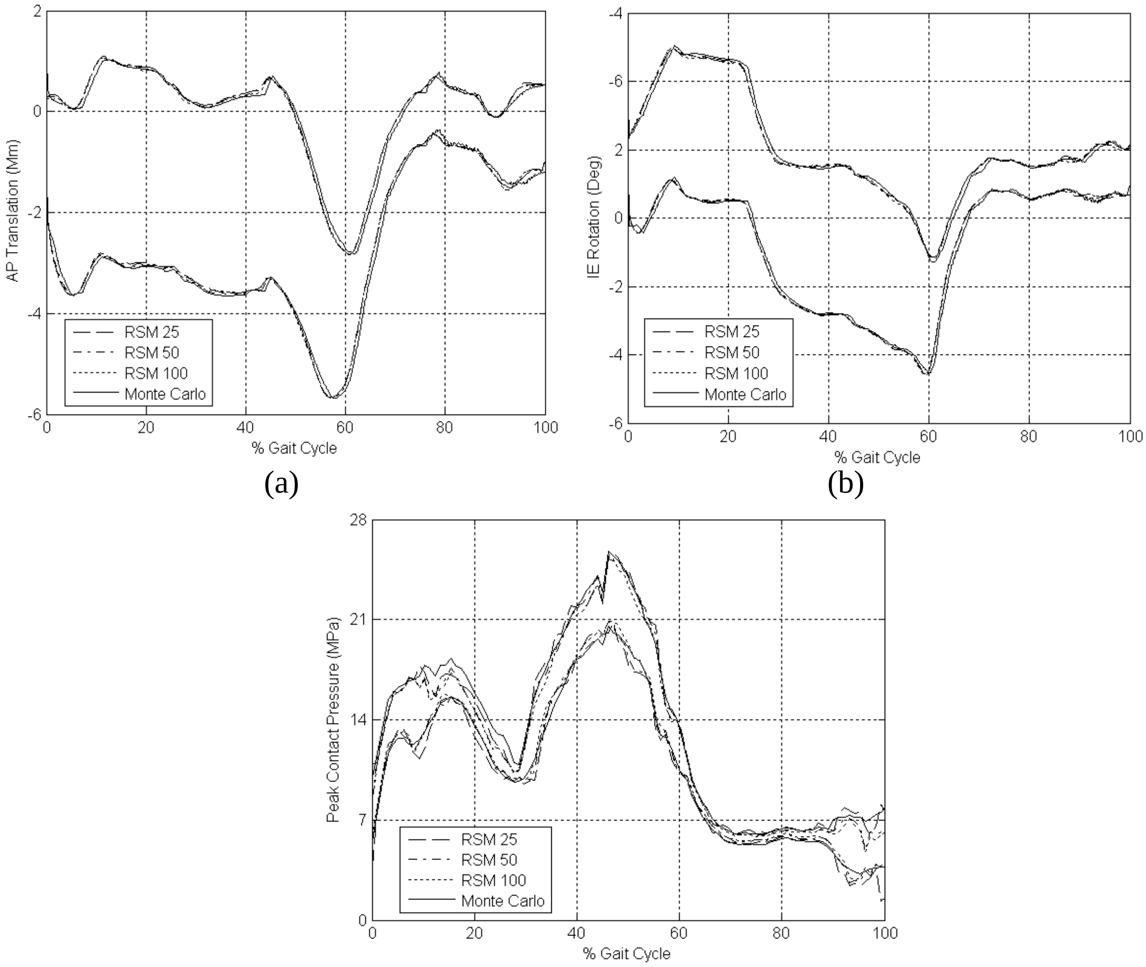


(c)

Fig. 2 Monte Carlo model-predicted envelope (solid line) (1-99%) and deterministic FE simulation (dashed line):
a) AP translation (mm); b) IE rotation (°); c) Peak Contact Pressure (MPa).

The deviations as a result of mal-positioning resulted in a maximum predicted range of 4.52 mm for AP translation, 4.64° for IE rotation and 5.4 MPa for peak contact pressure, which are closed to similar predictions reported in the literature (Laz et al., 2006).
The computation time for a single trial was approximately 5 min on a ~3 GHz Intel computer; the Monte Carlo results for 1000 trials required approximately 4 days.

The results obtained using the RSM (using either 25, 50 or 100 trials to build the response surface) showed similar predictions to the Monte Carlo method. For example, using the Monte Carlo results as the benchmark against which the RSM results were compared, the maximum difference between RSM-50 and Monte Carlo method was 0.17 mm for AP translation, 0.26º for IE rotation and 2 MPa for peak CP. Overall, the RSM was capable of predicting similar results to the Monte Carlo method, but with a greatly reduced computational cost: 4 hours with RSM-50 as compared to 4 days with the Monte Carlo method. In cases where similar analyses need to be repeated, the RSM can substantially reduce the computation time, which may have potential applications where rapid solution times are required.



(a) (b)

(c)

Fig. 4 Monte Carlo simulation (solid line) and RSM model-predicted envelope (1-99%) for 25, 50 and 100 trial response surface calculations:
a) AP translation (mm); b) IE rotation (º); c) Peak Contact Pressure (MPa).

5. CONCLUSIONS

The scope of the paper was to study two probabilistic FE methods (Monte Carlo with FE, RSM with FE) by inspecting the performance envelope for kinematics and contact mechanics of TKR because of mal-alignment. The results obtained with RSM were close to the ones predicted with the Monte Carlo method (0.17 mm for AP translation, 0.26º for IE rotation and 2 MPa for peak CP). It was confirmed that the performance envelopes of the TKR model can be predicted with a linear RSM model and a reduced number of points (25). In conclusion, the paper reinforces the value of probabilistic methods and demonstrated alternative statistical approaches for low computational-cost studies.

6. REFERENCES

1. Laz, P.J., Pal S., Fields, A., Petrella, A.J. and Rullkoetter, P. J., Effects of knee simulator loading and alignment variability on predicted implant mechanics: a probabilistic study, J. Ortho. Res., 2006, Vol. 24, 2212-2221.
2. Huiskes, R. and Chao, E. Y. S., A survey of finite element analysis in orthopaedics biomechanics: the first decade, J. Biomech., 1983, Vol. 16, 385-409.
3. Prendergast, P.J., Finite element models in tissue mechanics and orthopedic implant design, Clin. Biomech., 1997, Vol. 12, No. 6, 343-366.
4. Mackerle, J., Finite and boundary element methods in biomechanics: a bibliography (1976-1991), Eng. Comput., 1992, Vol. 9, 403-435.
5. Lee, A.J.C., Finite element analysis and its significance in total hip replacement, The Art of Total Hip Arthroplasty, ed. W.T. Stillwell, Grune and Stratton, Orlando, 1987, 33-39.
6. Godest, A.C., Beaugonin, M., Haug, E., Taylor, M., Gregson, P.J., Simulation of a knee joint replacement during gait cycle using explicit finite element analysis." J. Biomech, 2002, Vol. 35, 267-275.
7. Halloran, J.P., Petrella, A.J., Rullkoetter, P.J., Explicit finite element modelling of total knee replacement mechanics, J. Biomech Eng., 2005a, Vol. 38, 323-331.
8. Halloran, J.P., Easley, S.K., Petrella, A.J., Rullkoetter, P.J., Comparison of deformable and elastic foundation finite element simulations for predicting knee replacement mechanics, J. Biomech Eng., 2005b.,Vol. 127, 813-818.
9. Melis, M.E., Zaretsky, E.V., August, R., Probabilistic analysis of aircraft gas turbine disk life and reliability, 1999, Journal of Propulsion and Power , Vol. 15(5), 658-666.
10. Zhang, Y., Liu, Q., Reliability-based design of automobile components. Proceedings of the Institute of Mechanical Engineers, Part D., Journal of Automobile Engineering, 2002, Vol. 216(6), 455-471.
11. Nicolella, D.P., Thacker, B.H., Katoozian, H., Davy, D.T., Probabilistic risk analysis of a cemented hip implant, 2001, ASME Bioengineering Division, Vol. 50, 427-428.
12. Browne, M., Langley, R.S., Gregson, P.J., Reliability theory for load bearing biomedical implants, 1999, Biomaterials, Vol. 20, 1285-1292.
13. Dar, F.H., Meakin, J.R., Aspden, R.M., Statistical methods in finite element analysis Journal of Biomechanics, 2002, Vol. 35(9), 1155-1161.
14. Metropolis, N. and Ulam, S., The Monte Carlo Method, J. Amer. Stat. Assoc., 1949, Vol. 44, No. 247, 335-341.
15. Fishman, G.S., Monte Carlo: Concepts, Algorithms, and Applications, 1995, Springer Verlag, New York.
16. Isukapalli, S.S., Balakrishnan, S. and Georgopoulos, P.G., Computationally efficient uncertainty propagation and reduction using the stochastic response surface method, 43rd IEEE Conference on Decision and Control, 2004, Bahamas, US.