ADAPTIVE NOISE CANCELLATION OF DOPPLER SHIFTED NOISE SIGNALS: A LINEAR FRAMEWORK

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ABSTRACT

In this paper we investigate the performance of single channel adaptive noise cancellation techniques for situations where the noise signal received by the two microphones cannot be related by a fixed weight canceller's (linear) digital filter due to Doppler shift on the two signals. A mathematical signal model is produced, which shows that the adaptive filter is in fact required to identify a time-varying system which incorporates Doppler shift, and potential rapid variations in signal power as the Doppler producing source passes the filter microphones. We present theory, simulated performance and real world performance for both the least mean square (LMS) and normalised LMS (NLMS) when operating in a Doppler noise environment.

1 INTRODUCTION

Over the last few years adaptive noise cancellation techniques have been widely adopted for various real world applications including telecommunication noise control, hearing aids, in-flight noise control and so on. In all these environments the simple single channel noise controller in Fig. 1 attempts to identify the correlation between the noise present on the desired microphone, ie. $d[n] = s[n] + m[n]$, where $s[n]$ and $m[n]$ are signal and noise respectively, and on the noise reference microphone, where $x[n] = m'[n]$, and $m[n]$ and $m'[n]$ are (linearly) correlated signals [1]. There are however scenarios where $m[n]$ and $m'[n]$ are not clearly correlated due to a Doppler shift. Such situations can arise where the noise source is not stationary and is moving past both spatially separated microphones in eg. situations where vehicles (cars or trains) are moving past stationary roadside telephones, or where a moving vehicle with an internal telephone is moving past a stationary noise source. In this paper we consider the first case.

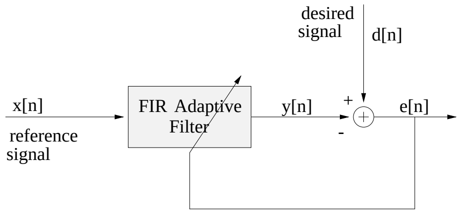


Figure 1: Generic structure of an adaptive filter

Assuming a simple deterministic situation, Sec. 2 overviews the model we have developed for the received noise signals at the roadside which can specify the functional relation between signals received at two different locations. This relation will allow us to determine the optimum coefficients of a two-tap deterministic filter, to which we can compare the trajectories of an LMS adaptive solution as outlined in Sec. 3. Due to the time-varying input power this problem lends itself to applying a normalized LMS, for which superior results are also presented.

2 PROBLEM DESCRIPTION AND ANALYSIS

2.1 Received Signals

For the analysis, we assume a single point noise source, S, moving with constant velocity, $\underline{v} = \hat{v}\underline{e}_x$, emitting a sinusoidal signal $p(t) = \hat{p}\sin(\omega t)$ of constant frequency ω and amplitude $\hat{p}$, producing the vectorised model of the environment shown in Fig. 2. The noise signal received at two stationary microphone positions M_1 and M_2 has the form

$$p_i(t) = \frac{\hat{p}}{r_i(t)} \cdot \sin\left(\omega t - k r_i(t)\right), \quad i \in \{1,2\}, \qquad (1)$$

where r_i are the distances the sound travels from the instance of emission until reception, and $k = \omega/c$ is the wavenumber and c the velocity of sound in air. The time-varying phase causes a Doppler shift in frequency and therefore a difference in instantaneous frequency between both signals, the extent of which depends on the separation of the reception points. Also note that the received signal power depends on $1/r_i^2$ due to the attenuation in air. For the calculation, we introduce the time duration

$$\Theta_i = \frac{r_i}{c}, \quad i \in \{1,2\}, \qquad (2)$$

which the signal received at time t needed to travel since it had been emitted.

Using geometric considerations from Fig. 2, we find

$$r_1 \;\; = \;\; \Theta_1 \cdot c = \sqrt{\left(x - \hat{v}\Theta_1\right)^2 + Y_1^2} \quad , \quad \text{and}$$

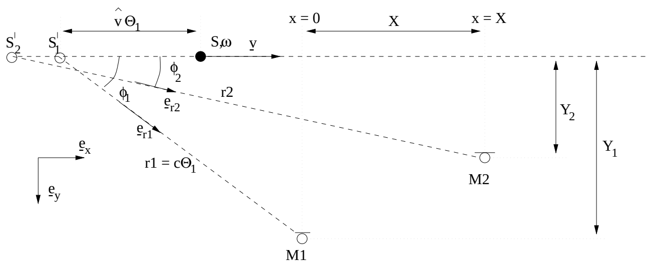


Figure 2: Model of roadside environment

$$r_2 \quad = \quad \Theta_2 \cdot c = \sqrt{(x - X - \hat{v}\Theta_2)^2 + Y_2^2} \quad ,$$

where X is the horizontal separation between the two microphones and Y_1 and Y_2 their distances from the track, and $x(t)$ the current position of the vehicle. Solving the quadratic equations for $\Theta_i, i \in 1,2$ and using the restriction imposed by a causal system of $\Theta_i \geq 0$ yields

$$\Theta_1 \;=\; \frac{x\hat{v} + \sqrt{x^2\hat{v}^2 + (x^2 + Y_1^2)\,(c^2 - \hat{v}^2)}}{c^2 - \hat{v}^2}, \quad \text{and}$$

$$\Theta_2 \;=\; \frac{-(x–X)\hat{v} + \sqrt{(x–X)^2\hat{v}^2 + ((x–X)^2 + Y_2^2)\,(c^2 - \hat{v}^2)}}{c^2 - \hat{v}^2}$$

with $x = \hat{v}t$ being time dependent. Exploiting (2) now yields all the parameters for the received signals $p_i(t)$ of (1) in terms of the geometric arrangement $\{X, Y_1, Y_2\}$ and the vehicle parameters $\{\hat{v}, \omega\}$.

2.2 Consistency with Doppler

Differentiating the arguments of the sine terms in (1) with respect to time t

$$\begin{aligned} \frac{d}{dt}\left(\omega t - \frac{\omega}{c}r_i\right) \;&=\; w\left(1 - \frac{1}{c}\cdot\frac{dr_i}{dt}\right) \\ &=\; \omega\cdot\left(1 \;-\; \frac{v_i}{c}\right) \;=\; \tilde{\omega}_i, \quad i \in \{1,2\} \end{aligned} \quad (3)$$

we yield a term $\tilde{\omega}_i$ commonly known as Doppler frequency, where v_1 and v_2 are the velocity components of the source velocity $\underline{v}$ at the virtual locations S_1' and S_2', where the signal received at time t had been emitted, in direction of the microphones:

$$v_1 \;=\; \hat{v}\cos\phi_1 = \hat{v}\cos\left(\arctan\left(\frac{Y_1}{x - \hat{v}\Theta_1}\right)\right)\operatorname{sgn}(x - \hat{v}\Theta_1),$$

$$v_2 \;=\; \hat{v}\cos\left(\arctan\left(\frac{Y_2}{x - X - \hat{v}\Theta_2}\right)\right)\operatorname{sgn}(x - X - \hat{v}\Theta_2).$$

Here, the signum function has been introduced to correct for the side branches of the arctangent. As the v_i are depending on time t, the terms

$$\tilde{\omega}_i = \omega\left(1 - \frac{v_i}{c}\right), \quad i \in \{1,2\} \qquad (4)$$

have to be interpreted as instantaneous frequencies. Due to the different location of the microphones M_1 and M_2, these instantaneous frequencies are different for both recorded signals. For applying an adaptive filter, reference and desired or error signal would thus show a shift in instantaneous frequency due to the Doppler phenomenon of the received signals.

2.3 Derivation and Classification of the Underlying System

If adaptive filtering is employed to suppress p_2, a filter with input p_1 will ideally have to identify the function f: $p_2 = f(p_1)$. For the identification of f, a reformulation of p_2 in terms of p_1 can be performed:

$$\begin{aligned} p_2(t) \;&=\; \frac{\hat{p}}{r_2}\sin(\omega t - kr_2) = \frac{\hat{p}}{r_2}\sin(\omega t - kr_1 - k(r_2 - r_1)) \\ &=\; \frac{r_1}{r_2}\cos(k(r_1 - r_2))\cdot p_1(t) \;+ \\ &\quad +\; \frac{\hat{p}}{r_2}\cos(\omega t - kr_1)\cdot\sin(k(r_1 - r_2)) \quad . \end{aligned} \qquad (5)$$

To relate the second summand directly to $p_1(t)$, we differentiate p_1 with respect to time t

$$\dot{p}_1(t) \;=\; -\frac{v_1}{r_1}\cdot p_1(t) \;+\; \hat{p}\frac{\omega(1 - v_1/c)}{r_1}\cos(\omega t - kr_1)$$

where the identity $\dot{r}_1 = v_1$ has been used, and hence

$$\cos(\omega t – kr_1) = \left(\dot{p}_1(t) + \frac{v_1}{r_1}\cdot p_1(t)\right)\frac{1}{\hat{p}}\cdot\frac{r_1}{\omega(1 - v_1/c)}. \qquad (6)$$

Inserting (6) into (5) gives

$$\begin{aligned} p_2(t) \;&=\; \frac{r_1}{r_2}\cos(k(r_1 - r_2))\cdot p_1(t) \;+\; \frac{r_1}{r_2}\frac{1}{\omega(1 - v_1/c)}\;\cdot \\ &\quad\cdot\;\sin(k(r_1 - r_2))\left(\dot{p}_1(t) + \frac{v_1}{r_1}\cdot p_1(t)\right), \end{aligned} \qquad (7)$$

so that f can be expressed as a *linear* first order differential equation

$$p_2(t) \;= f(p_1) \;=\; a_0(t)\cdot\dot{p}_1(t) \;+\; a_1(t)\cdot p_1(t) \qquad (8)$$

with *time varying* parameters

$$a_0(t) \;=\; \frac{r_1}{r_2}\frac{1}{\omega(1 - v_1/c)}\sin(k(r_1 - r_2)) \qquad (9)$$

$$a_1(t) \;=\; \frac{r_1}{r_2}\cos(k(r_1 - r_2)) \;+\; \frac{v_1}{r_1}\cdot a_0(t) \quad . \qquad (10)$$

As (9) and (10) no longer includes ωt, there cannot be any further p_1 terms extracted from a_0 or a_1. Thus the functional context of the system (1) is completely governed by (8)-(10), revealing the *linear, time varying* nature of the function $f\!: p2 = f(p_1)$.

3 TRACKING AND CANCELLATION RESULTS OF ADAPTIVE FILTER

Based on the previous analysis, it is now possible to perform the filtering task with *linear* adaptive filters. The main question remaining is whether the adaptive algorithm can track the time varying parameters of the system.

3.1 Discrete Time Model and Filtering

If the sound pressure signals are acquired in discrete time, i.e. $t \rightarrow n \cdot T_s$, the resulting discrete time sequences are determined by a set of parameters $\mathbb{P} = \{X, Y_1, Y_2, \hat{v}, \omega, f_s\}$ consisting of

- local microphone arrangement (X,Y_1, and Y_2),
- moving source speed $\hat{v}$,
- source angular frequency $\omega = 2\pi f$, and
- sampling frequency $f_s = 1/T_s$,

which yield discrete sequences $p_1[n]$ and $p_2[n]$. An adaptive filter as shown in Fig. 1 for noise cancellation is supplied with these sampled pressure signals as reference and desired signal, such that $x[n] = p_1[n]$ and $d[n] = p_2[n]$.

3.2 Optimal Filtering and Trajectories

For a first order filter, there is a unique set of optimum coefficients $w_{i,\mathrm{opt}}[n], i \in \{0, 1\}$, such that

$$y[n] \overset{!}{=} p_2[n] = w_{0,\mathrm{opt}} \cdot p_1[n] + w_{1,\mathrm{opt}} \cdot p_1[n-1] \quad (11)$$

is satisfied, where the optimum coefficients can be evaluated in an approach analogous to Sec. 2.3 as

$$w_{0,\mathrm{opt}}[n] = \frac{r_1[n] \cdot \sin(k(r_2[n] - r_1[n-1]) - \omega/f_s)}{r_2[n] \cdot \sin(k(r_1[n] - r_1[n-1]) - \omega/f_s)}\,, \quad (12)$$

$$w_{1,\mathrm{opt}}[n] = \frac{-r_1[n-1] \cdot \sin(k(r_1[n] - r_2[n]))}{r_2[n] \cdot \sin(k(r_1[n] - r_1[n-1]) - \omega/f_s)}\,. \quad (13)$$

Thus, the optimum filter has a dynamic, non-stationary solution. The shape of the optimum trajectories, $w_{0,\mathrm{opt}}$ and $w_{1,\mathrm{opt}}$, depends on the parameter set $\mathbb{P}$. An example for the curve of these trajectories is illustrated in Fig. 3 for an either vertical (in y- direction), horizontal (in x-direction) or a combined diagonal spatial separation by 20cm between the microphones recording reference and desired signal. There, the strongest variations in the coefficients occur during the transition of the noise source at $t = 0$.

The influences of different parameters of the set $\mathbb{P}$ are described in Fig. 3(c) and (d). Generally for variations within realistic limits, the vehicle velocity $\hat{v}$ and the distance from the vehicle trajectory, Y_1, determine the width and sharpness of the highly non-stationary transition period around $t = 0$, while the parameters f_s, $Y_2 - Y_1$, and X, are responsible for the magnitude of the trajectory values. The emitted frequency ω has little influence on the curvature of the trajectories, which cannot be generalized. Thus, the further apart the reception points M_1 and M_2 are, the stronger the non-stationarities become.

3.3 Adaptive Solution

As (11)-(13) represents a fully determined system, an adaptive first order filter has to adapt to and keep track of the previously described optimum trajectories. Using an LMS algorithm, the error in the adaptive solution either can be seen in terms of the deviation of the coefficients from the optimum values, or the excess mean squared error (MSE) caused by the weights' misadjustment. It has been shown that the misadjustment of LMS adaptive filters operating in non-stationary environments can be decoupled into a lag error term, ie. the difference with which the ensemble average of the coefficients lags behind the optimum solution, being proportional to μ, and into gradient noise, ie. the difference between the actual weight vector and the ensemlbe average, being proportional to $1/\mu$, thus creating a trade-off for the step size μ [2, 3, 4, 1].

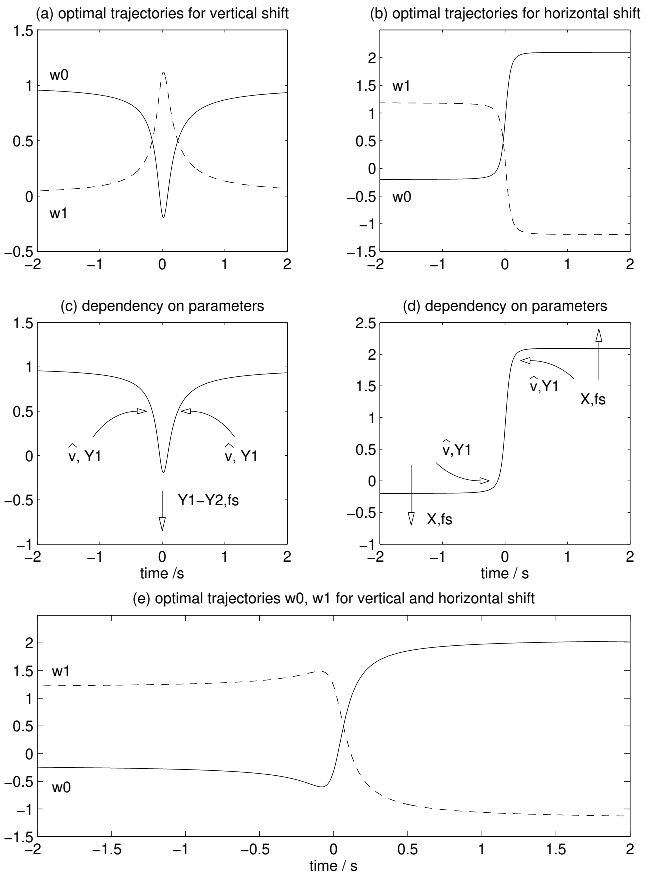
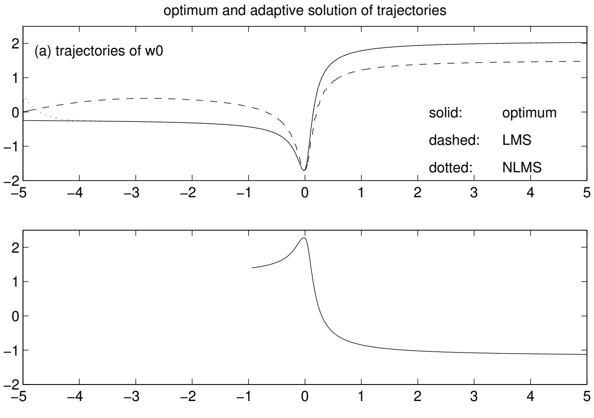


Figure 3: typical optimum trajectories $w_{0,\mathrm{opt}}$ and $w_{1,\mathrm{opt}}$ for (a) vertical and (b) horizontal shift of the microphones, along with the dependency on set-up parameters (c,d). A combination of shifts is shown in (e).

Figs. 4(a)-(d) give an example of the deviation of the trajectories of an adaptive first order filter compared from the optimum curves and the resulting excess MSE and reductions for a set-up of $\mathbb{P} = \{3m, 3.5m, 2cm, 33ms^{-1}, 100Hz, 2kHz\}$. Using the LMS algorithm, the results are reasonably good during transition, but very poor in the approach and departure stage of the noise source, as due to the input power depending on $1/r^2$, the step size has to be limited such that no instability occurs during transition when power levels are maximum, thus adapting very slowly in the other phases. The uncancelled curve in Fig. 4(c) under-


optimum and adaptive solution of trajectories
(a) trajectories of w0
solid: optimum
dashed: LMS
dotted: NLMS
2
1
0
−1
−2
−5
−4
−3
−2
−1
0
1
2
3
4
5
2
1
0
−1
−2
−5
−4
−3
−2
−1
0
1
2
3
4
5