

# Modeling Frequency-Dependent Losses in Ferrite Cores

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**Abstract**—We suggest a practical approach for modeling frequency-dependent losses in ferrite cores for circuit simulation. Previous work has concentrated on the effect of eddy-current losses on the shape of the  $B$ – $H$  loop, but in this paper we look at the problem from the perspective of energy loss and propose a different network for accurately modeling power loss in ferrite cores. In power applications, the energy loss across the frequency range can have a profound effect on the efficiency of the system, and a simple ladder network in the magnetic domain is not always adequate for this task. Simulations and measurements demonstrate the difference in this approach from the  $RL$  ladder network models both in the small-signal and large-signal contexts.

**Index Terms**—Circuit simulation, energy loss, Jiles–Atherton, magnetic component modeling.

## I. INTRODUCTION

### A. Background

THE accurate prediction of the losses in magnetic cores is crucial for a number of applications, especially power electronics and in the use of ferrite materials to absorb unwanted harmonics. In the first case, the frequency dependence of the ferrite material has a significant bearing on the design of the magnetic component and the resulting performance of the system as a whole. For ferrite beads and other filter devices, it is the behavior of the lossy material that determines how effective the material will be at removing unwanted signals. In both cases, it is important to be able to predict losses across the desired frequency range if possible using computer simulations that are fast and accurate. It is important for the model to be practically useful that it is able to be included within standard electronic circuit simulation.

### B. Modeling Core Loss in Magnetic Components

The approaches used for modeling core loss partly depend on whether the model is linear or nonlinear and whether the frequency is high enough to cause eddy-current or other frequency-dependent effects. The total core loss consists of two parts, the basic low frequency core hysteresis loss and the higher frequency eddy-current or other frequency-dependent losses. In the low frequency case, the core behavior may be implemented using a linear or a nonlinear model. Cherry [1], Laithwaite [2], and Carpenter [3] show how electromagnetic components may be implemented using equivalent circuit elements in either, or

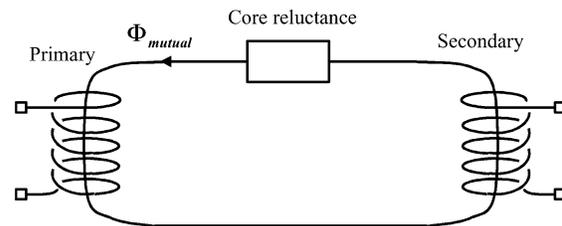


Fig. 1. Basic mixed-domain transformer model.

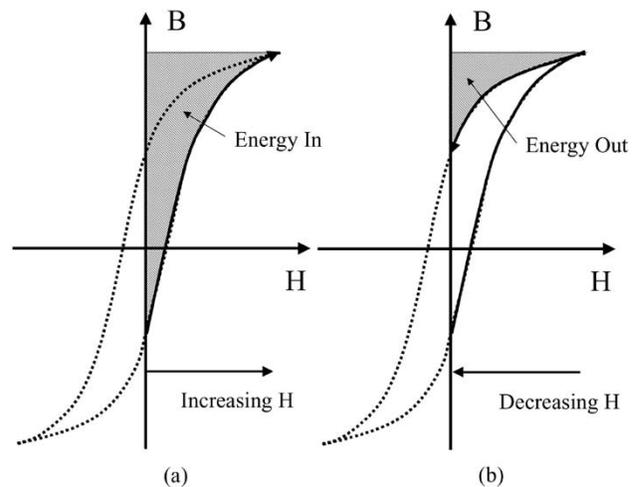


Fig. 2. Energy loss in  $B$ – $H$  loop showing the transfer of energy into and out of the core with changing  $H$ .

both, the electrical and magnetic domains. For example, a magnetic reluctance can be modeled as a resistive element in the magnetic domain.

Taking a simple transformer as an example (Fig. 1), a model may easily be developed that has models for each winding of the transformer connected to a magnetic model of the core. This is an example of a “mixed-domain” model for circuit simulation.

The core reluctance may be modeled as a simple linear element—this corresponds to a perfect lossless core. In practice, of course, there usually needs to be an accurate model of the  $B$ – $H$  loop of the material which may also vary with frequency. The area inside the  $B$ – $H$  loop corresponds to the energy lost in the core, as shown in Fig. 2.

If a linearized model is required (useful for frequency domain analysis), the average core loss can be implemented as a resistor in the electrical domain, or as an inductor in the magnetic domain as shown in Fig. 3. In the magnetic domain, the energy loss per cycle may be found by integrating,  $H$  over  $B$  for a complete cycle and multiplying by the core volume.

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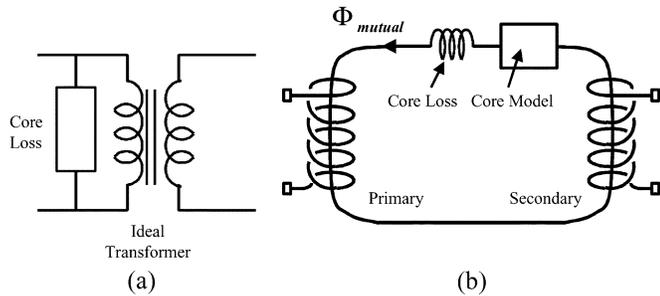


Fig. 3. Core loss implemented in the electrical and magnetic domains.

When a nonlinear model of hysteresis is required, the ubiquitous Jiles–Atherton (JA) [10]–[12], Preisach [13] or Chan and Vladimirescu [14] models are all in common use.

In each case, the nonlinear  $B$ – $H$  curve is characterized at a specific frequency and temperature and used under those conditions. Frequency-dependent (otherwise known as rate-dependent) models which model the change in  $B$ – $H$  loop with frequency also exist, such as the Hodgdon model [16]–[18] or Carpenter’s differential equation approach [15].

### C. Modeling High Frequency (Eddy-Current) Losses

In conducting core materials, currents are induced which flow in loops. These eddy currents act against the externally applied magnetic field, causing decreased flux and increased losses as the frequency is increased. Konrad [4] discusses eddy currents generally in some detail and Zhu, Hui, and Ramsden [5]–[8] propose methods of implementing the effects of eddy currents in a simulation model of a magnetic core. The basic modeling concept is to treat the magnetic material as a series of zones. This approach is also described in detail by Brown *et al.* [9] to model magnetic components for sensor applications. The currents in these zones approximate the eddy-current behavior as the frequency increases. Each zone is modeled as a resistance–inductance ( $RL$ ) element, in a network in the magnetic domain, as shown in Fig. 4.

For each zone, the eddy-current loop can be considered to be a single turn winding around the cross section. In each cross section, the resistor represents the reluctance of the core material in the magnetic domain and is calculated using

$$R = \frac{l}{A\mu_0\mu_r} \quad (1)$$

where  $l$  is the magnetic path length,  $A$  is the cross-sectional area of the zone, and  $\mu_r$  is the relative permeability of the core material. The magnetic-domain inductance representing the core loss for the lamination is calculated using

$$L = \frac{A_{cs}\sigma}{l} \quad (2)$$

where  $A_{cs}$  is the height of the zone multiplied by the magnetic path length,  $l$  is the length around the eddy-current loop, and  $\sigma$  is the conductivity of the magnetic core material.

Using this approach, the number of zones can be controlled for the required accuracy, and the  $RL$  components derived easily. This technique can also be used with nonlinear core

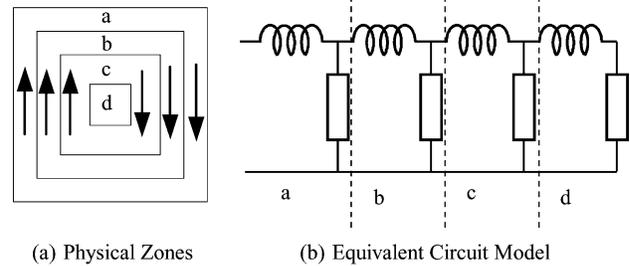


Fig. 4. Zones and equivalent core loss circuit model—where each  $RL$  circuit model corresponds to a physical zone in the magnetic material. (a) Physical zones. (b) Equivalent circuit model.

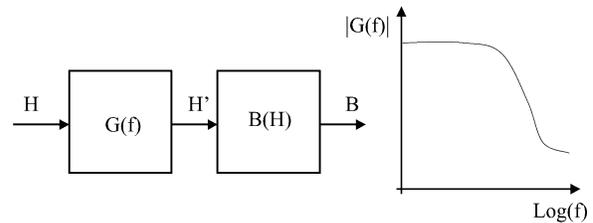


Fig. 5. Behavioral model to implement frequency-dependent hysteresis.

reluctances, simply by replacing the linear models with a suitable nonlinear model with the correct physical dimensions of the lamination.

### D. Behavioral Modeling of High Frequency Losses

Another approach to implementing the eddy-current behavior described in the previous section is to modify the  $B$ – $H$  loop behavior by modifying the applied magnetic field strength ( $H$ ) prior to calculating  $B$ . As the frequency of the applied magnetic field strength  $H$  increases, a low-pass filter function  $G(f)$  causes the apparent magnetic field strength  $H'$  to decrease as illustrated in Fig. 5. The effect of this is to widen the  $B$ – $H$  loop.

The advantage of this type of approach (as implemented in the Saber simulator) is the simplicity of implementation, as opposed to the complex network of individual nonlinear core models required in the network method. This gives a resulting increase in simulation speed and reliability due to the reduced number of equations and nonlinearities to be solved.

## II. MODEL FOR FREQUENCY-DEPENDENT LOSSES

The drawback for a model such as the  $RL$  ladder approach for predicting high frequency losses is that, in electrical terms, it only has real poles. In order to model many soft ferrite materials, a more complex network is required. Even using a ladder network of the form described previously, the effect is only to add more poles at a higher frequency. It does not help model the more complex characteristics of many commercial soft ferrites such as Philips 3E5 or 3F3 material, where the complex permeability of the material (and the permeance of the core) varies with frequency as illustrated in Fig. 6.

The concept behind the proposed model is to design a network that represents the small-signal variation in permeability but that can also be used with a nonlinear reluctance model to predict the variation in loss with frequency. In order to accomplish this, the basic  $RL$  network is modified to include a

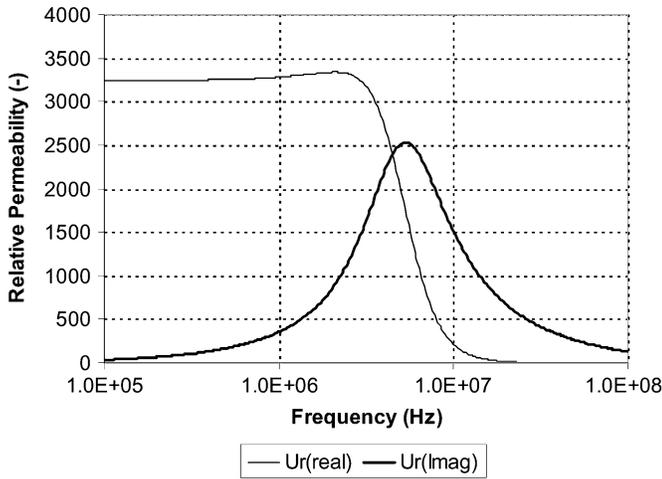


Fig. 6. Frequency variation of permeability.

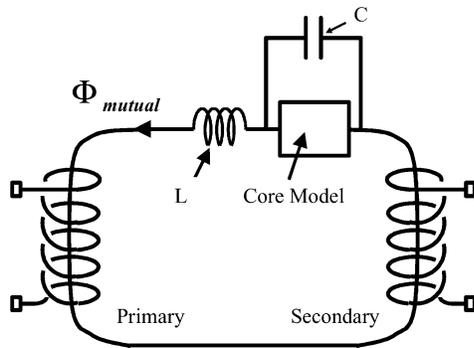


Fig. 7. Modified model structure.

“magnetic” capacitor. The modified model structure is shown in Fig. 7.

The concept of a “magnetic capacitor” is not physically realistic (a capacitor in the magnetic domain would be an energy source!), but in combination with the standard loss component it gives a second-order response with frequency for the reluctance. The resulting small-signal behavior in the frequency domain may now represent the behavior shown in Fig. 6. Note that the relative permeability characteristic is a second-order response rather than the first-order response achievable using a basic *RL* ladder network.

The complex permeance of the network may be written as in

$$G(\omega) = \frac{(j\omega + \beta)\alpha}{R(\omega^2 - \alpha\beta - j\omega\beta)} \quad (3)$$

where

$$\alpha = R/L$$

$$\beta = 1/(CR)$$

*R* is the reluctance (the “magnetic resistance”), *L* is the “magnetic inductance,” and *C* is the “magnetic capacitance.” The requirement that the network is a net energy loss imposes the condition that the imaginary part of the complex permeance is negative. This requires that  $\beta$  is greater than  $\alpha$ . The reluctance of the core (*R*) is found directly from the low frequency permeability. In order to choose the values of *L* and *C*, appropriate

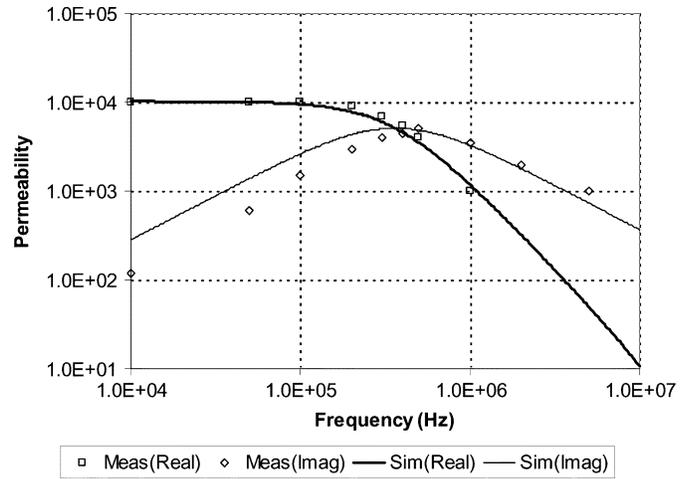


Fig. 8. The 3E5 complex permeability curves.

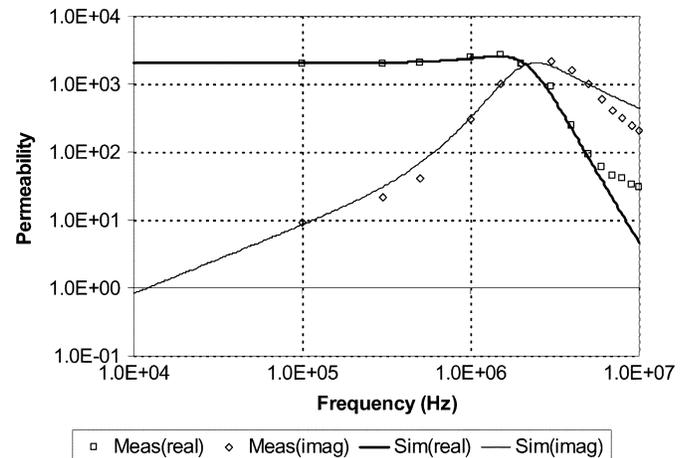


Fig. 9. The 3F3 complex permeability curves.

values of  $\alpha$  and  $\beta$  must be chosen. This may be done by reference to published data for the complex permeability. The rolloff frequency is primarily determined by  $\alpha$ , with the slope of the curves being determined by  $\beta$ .

To illustrate the approach two materials, 3E5 and 3F3 were chosen that have different permeability characteristics. In each case, the magnetic *L* and *C* parameters were derived from the Ferroxcube data sheet curves for complex permeability and simulations carried out in the frequency domain to calculate the model’s response across a similar frequency range. Fig. 8 shows the response for 3E5 and Fig. 9 shows the same analysis for 3F3.

It is interesting to note that even for this basic lumped model with the simplest network, that the complex permeability is reasonably accurate across the majority of the frequency range. If greater accuracy is required at the higher end of the frequency range, then extra network components could be added as required, but it then becomes more difficult to characterize the model.

In order to provide an accurate nonlinear model for time-domain circuit simulation, the linear magnetic reluctance model can be replaced with a nonlinear model of hysteresis such as that of Jiles–Atherton. A strength of this approach is that the linear model can be used to empirically define the loss terms (*L*

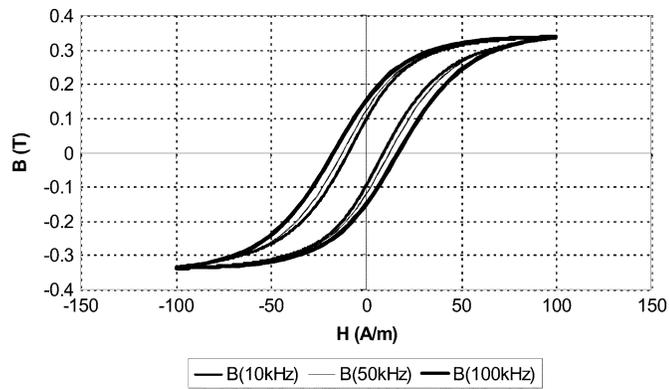


Fig. 10. Frequency-dependent core model simulation results.

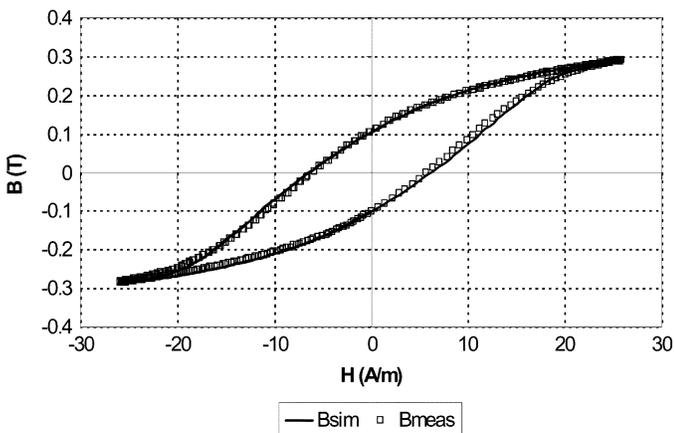


Fig. 11.  $B$ - $H$  curves (measured and simulated) at 10 kHz.

and  $C$ ) and then the core model can be changed into the nonlinear core model required for large-signal analysis.

Using the Jiles–Atherton model of hysteresis in the place of the linear core model used previously, large-signal time-domain simulations were carried out over the frequency range 10–100 kHz. The resulting  $B$ - $H$  curves for the modified complete core model are shown in Fig. 10 (Philips 3F3 material).

Taking the 3E5 example, measurements were made of the  $B$ - $H$  curve using a simple toroid core (cross section  $4.44 \text{ mm}^2$ , effective length 22.9 mm) with four turns on the primary and secondary and at frequencies of 10, 50, and 100 kHz. These measurements were compared with a modified Jiles–Atherton model characterized at 10 kHz using the methods described by Wilson, Ross and Brown [19]–[22], with the resulting parameters  $a = 6.4$ ,  $k = 8.5$ ,  $c = 0.42$ ,  $M_s = 300 \text{ k}$ ,  $\alpha = 6 \text{ u}$ , and  $ECrate = 9.1$ . The parameters of the model are related to physical properties of the core magnetic material and are briefly summarized as follows:

- $k$  controls the irreversible loss;
- $a$  defines the anhysteretic behavior;
- $c$  controls the reversible/irreversible proportions;
- $\alpha$  influences the internal effective field strength;
- $M_s$  defines the saturation magnetization;
- $ECrate$  controls the rate of loop closure.

The value of  $L$  was estimated from a small signal analysis to be 4 mH and  $C$  estimated to be 10 pF (in the magnetic domain).

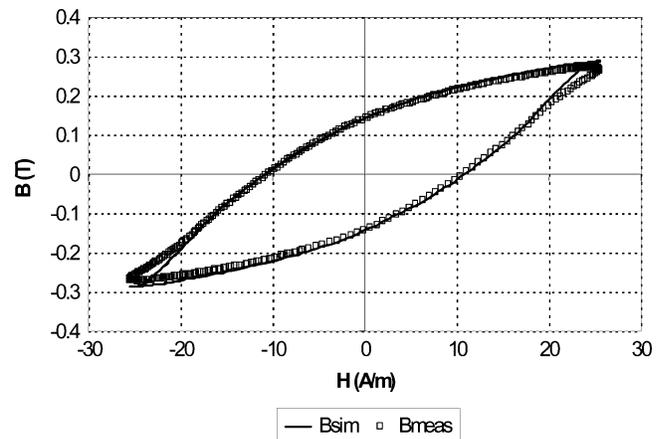


Fig. 12.  $B$ - $H$  curves (measured and simulated) at 50 kHz.

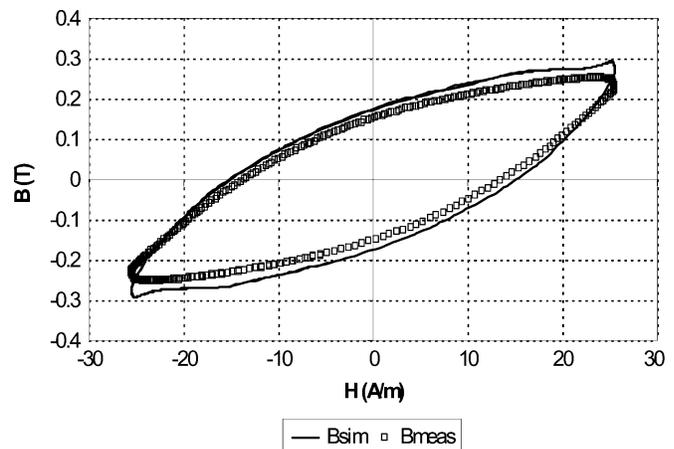


Fig. 13.  $B$ - $H$  curves (measured and simulated) at 100 kHz.

The resulting measured and simulated  $B$ - $H$  curves are shown in Figs. 11–13.

It can be seen that the model accurately predicts the change in  $B$ - $H$  loop shape as the frequency increases (the slight dc offset in Fig. 13 is probably due to measurement error).

There is good agreement between measured and simulated results for major and symmetric minor loops. However, for asymmetric minor loops, the Jiles–Atherton model behavior is less satisfactory. In these cases, the minor loop will relax rapidly to a point such that the loop becomes symmetric about the anhysteretic and this behavior is not observed experimentally. Modifications to the original Jiles–Atherton model have been proposed by Carpenter [25] and Jiles [26] that go some way to improving the modeling of minor loops, but no totally satisfactory solution exists.

### III. CONCLUSION

The results of the simulations demonstrate that this simple network model can accurately replicate the complex permeability characteristics observed in typical soft ferrites for power, signal, or EMI applications. The model taken in conjunction with a suitable hysteresis model can be used for linear (frequency-domain) or nonlinear (time-domain) analyses with no modification of the basic loss terms. This provides a systematic

approach to the characterization of the ferrite material model based on data-sheet information or empirical measurement.

The technique could easily be extended to encompass more complex network models for increased accuracy, but given the relatively wide variability of material tolerances in general for soft ferrites, this approach is generally adequate for most applications. This approach offers a mechanism for the inclusion of model parameters that may depend on environmental factors such as temperature or stress.

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