

# DIFFERENTIAL SPACE-TIME SPREADING USING FOUR TRANSMIT ANTENNAS AND ITERATIVELY DETECTED SPHERE PACKING MODULATION

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**Abstract** - This paper presents a novel Differentially encoded Space-Time Spreading (DSTS) scheme using four transmit antennas that can be readily combined with PSK, QAM, as well as Sphere Packing (SP) modulation schemes. The advocated SP-aided system has simple encoding and decoding algorithms that requires no channel knowledge and outperforms the DSTS dispensing with SP. Further improvement to the system performance can be obtained by serially concatenated convolutional coding and then performing SP-symbol-to-bit demapping as well as channel decoding iteratively. Explicitly, the proposed turbo-detected DSTS-SP scheme exhibits an  $E_b/N_0$  gain of 18dB at a Bit Error Rate (BER) of  $10^{-5}$  over an uncoded identical-throughput system and an  $E_b/N_0$  gain of 2dB over an equivalent 1 bits/symbol effective throughput QPSK-modulated turbo-detected DSTS scheme dispensing with SP.

## 1. INTRODUCTION

The detrimental effects of channel fading may be significantly reduced by employing space-time block code (STBC) aided transmit diversity invoking multiple antennas [1]. STBC is an effective multi-input-multi-output (MIMO) scheme that provides a good performance in conjunction with a simple decoding scheme over slow fading channels [2, 3]. In the ensuing era, the design of meritorious space-time modulation schemes has attracted considerable research attention [4, 5]. Inspired by the philosophy of Space-Time Block Codes (STBC), Hochwald *et al.* [6] proposed the transmit diversity concept known as Space-Time Spreading (STS) for the downlink of Wideband Code Division Multiple Access (WCDMA) [7] that is capable of achieving the highest possible transmit diversity gain. As a further advance, the concept of combining orthogonal transmit diversity designs with the principle of sphere packing modulation was introduced by Su *et al.* in [8], where it was demonstrated that the proposed Sphere Packing (SP) aided STBC system was capable of outperforming the conventional orthogonal design based STBC schemes of [2, 3].

A common feature of all the above-mentioned schemes is that they use coherent detection, which assumes channel knowledge at the receiver. In practice, the channel state information (CSI) of each link between each transmit and each receive antenna pair has to be estimated at the receiver either blindly or using training symbols. However, channel estimation invoked for all the antennas substantially increases both the cost and complexity of the receiver. Furthermore, when the CSI fluctuates dramatically from burst to burst, an increased number of training symbols has to be transmitted, potentially resulting in an undesirably high transmission overhead and wastage of transmission power. Therefore, it is beneficial to develop low-complexity techniques that do not

require any channel information. Differential space-time block codes (DSTBC) were designed by Tarokh *et al.* for two transmit antennas and then they were extended to a higher number of transmit antennas [5, 9]. Afterwards, the authors of [10, 11] developed a DSTBC scheme that supports non-constant modulus constellations combined with both two and four transmit antennas.

Iterative decoding of spectrally efficient modulation schemes was considered by several authors [1]. In [12], the employment of the turbo principle was considered for iterative soft demapping in the context of multilevel modulation schemes combined with channel decoding, where a soft symbol-to-bit demapper was used between the multilevel demodulator and the binary channel decoder. The iterative soft demapping principle of [12] was extended to SP-aided STBC schemes in [13], where the sphere packing demapper of [8] was modified in [13] for the sake of accepting the *a priori* information passed to it from the channel decoder as extrinsic information.

*Motivated by the performance improvements reported in [13] and [6], we develop a novel Differential Space-Time Spreading (DSTS) scheme using four transmit antennas, which exploits the combined advantages of the differential encoding, the multi-user support capability of the STS, as well as the benefits of SP modulation and those of iterative symbol-to-bit demapping and decoding. The proposed scheme can be readily combined with PSK, QAM, as well as SP modulations schemes. Moreover, with the advent of multiple receive antennas, further receive diversity gains can be achieved in addition to the attainable transmit diversity gain.*

This paper is organised as follows. In Section 2, a brief system overview is presented. In Section 3.1, the encoding and decoding algorithms designed for DSTS schemes employing four transmit antennas with real-valued constellations are described. In Section 3.2, we demonstrate how the proposed DSTS scheme is designed using complex-valued constellations. The DSTS design using SP signal constellations is described in Section 4. Section 5 demonstrates how the DSTS-SP demapper is modified for the sake of exploiting the *a priori* knowledge provided by the channel decoder, while our simulation results and discussions are provided in Section 6. Finally, we conclude in Section 7.

## 2. SYSTEM OVERVIEW

The schematic of the entire system is shown in Figure 1, where the transmitted source bits are convolutionally encoded and then interleaved by a random bit interleaver. A rate  $R = \frac{1}{2}$  recursive systematic convolutional code was employed. After channel interleaving, the SP mapper first maps  $B_{sp}$  channel-coded bits  $\mathbf{b} = b_{0, \dots, B_{sp}-1} \in \{0, 1\}$  to a sphere packing symbol  $s^l \in S$ ,  $l = 0, 1, \dots, L - 1$ , such that we have  $s^l = \text{map}_{sp}(\mathbf{b})$ , where  $B_{sp} = \log_2 L$  and  $L$  represents the number of modulated symbols in the sphere-packed signalling alphabet, as described in [13]. Subsequently, each of the four components of a SP symbol is then transmitted using DSTS via four transmit antennas during a single

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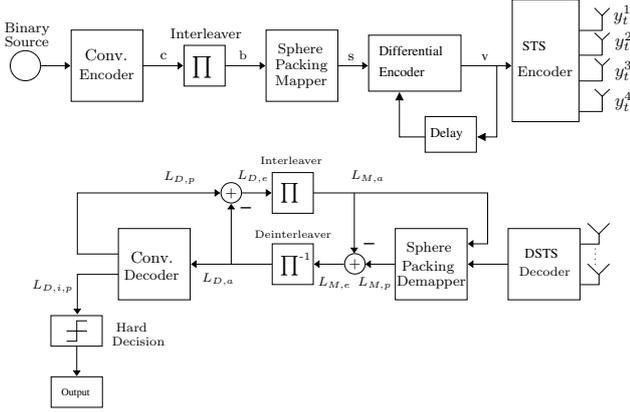


Figure 1: The Turbo Detection Aided DSTS-SP System. time slot, as it will be detailed in Section 4.

In this treatise, we considered transmission over a correlated narrowband Rayleigh fading channel, associated with a normalised Doppler frequency of  $f_D = f_d T_s = 0.01$ , where  $f_d$  is the Doppler frequency and  $T_s$  is the symbol duration. The complex Additive White Gaussian Noise (AWGN) of  $n = n_I + jn_Q$  contaminates the received signal, where  $n_I$  and  $n_Q$  are two independent zero-mean Gaussian random variables having a variance of  $\sigma_n^2 = \sigma_{n_I}^2 = \sigma_{n_Q}^2 = N_0/2$  per dimension, with  $N_0/2$  representing the double-sided noise power spectral density expressed in  $W/Hz$ .

As shown in Figure 1, the received complex-valued symbols are demapped to their Log-Likelihood Ratio (LLR) representation for each of the  $B$  channel-coded bits per sphere packing symbol. The *a priori* LLR values of the demodulator are subtracted from the *a posteriori* LLR values for the sake of generating the extrinsic LLR values  $L_{M,e}$ , and then the LLRs  $L_{M,e}$  are deinterleaved by a soft-bit deinterleaver, as seen in Figure 1. Next, the soft bits  $L_{D,a}$  are passed to the convolutional decoder in order to compute the *a posteriori* LLR values  $L_{D,p}$  provided by the Max-Log MAP algorithm [14] for all the channel-coded bits. During the last iteration, only the LLR values  $L_{D,i,p}$  of the original uncoded systematic information bits are required, which are passed to the hard decision decoder of Figure 1 in order to determine the estimated transmitted source bits. As seen in Figure 1, the extrinsic information  $L_{D,e}$ , is generated by subtracting the *a priori* information from the *a posteriori* information according to  $(L_{D,p} - L_{D,a})$ , which is then fed back to the DSTS-SP demapper as the *a priori* information  $L_{M,a}$  after appropriately reordering them using the interleaver of Figure 1. The DSTS-SP demapper of Figure 1 exploits the *a priori* information for the sake of providing improved *a posteriori* LLR values, which are then passed to the channel decoder and then back to the demodulator for further iterations.

### 3. DIFFERENTIAL STS USING FOUR TRANSMIT ANTENNAS

As widely recognised, coherent detection schemes require CSI, which is acquired by transmitting training symbols. However, high-accuracy multi-antenna channel estimation imposes a high complexity on the receiver. This renders differential encoding and decoding an attractive design alternative, despite the associated  $E_b/N_0$  loss.

In this paper, we assume that no CSI is available at the receiver. The transmitted and received DSTS symbols are encoded and decoded based on the differential relationship among subsequent symbols, as illustrated for classic QAM in Chapter 11 of [15]. For the sake of simplicity, we consider having a single receive antenna, although the extension to systems having more than one receive antenna is straightforward.

### 3.1. Real-Valued Constellations

According to Figure 1, it becomes clear that the DSTS encoder can be divided into two main stages. The differential encoding takes place before the space-time spreading and the differentially encoded symbols are then spread. The differential encoding scheme follows Chapter 11 of [15]. Moreover, the basic principle of STS was exemplified in simple graphical terms on page 303 of [7], where a 4-chip orthogonal spreading code was used for spreading each bit of duration  $T_b$  to an interval of  $4T_c$ .

The DSTS encoding/decoding algorithms operate as follows. At time instant  $t = 0$ , the arbitrary dummy reference symbols  $v_0^1, v_0^2, v_0^3$  and  $v_0^4$  are transmitted from antennas one, two, three and four, respectively. These symbols usually carry no information and are known to the receiver. At time instants  $t \geq 1$ , a block of  $4B$  bits arrives at the SP mapper of Figure 1, where each set of  $B$  bits is mapped to a real-valued modulated symbol  $a_t^k$ ,  $k = 1, 2, 3, 4$ , selected from a  $2^B$ -ary constellation. Assume that  $v_t^k$ ,  $k = 1, 2, 3, 4$ , are the symbols transmitted from the four antennas. Then, differential encoding is carried out as follows:

$$\mathbf{V}_t = \begin{pmatrix} v_t^1 \\ v_t^2 \\ v_t^3 \\ v_t^4 \end{pmatrix} = a_t^1 \times \begin{pmatrix} v_{t-1}^1 \\ v_{t-1}^2 \\ v_{t-1}^3 \\ v_{t-1}^4 \end{pmatrix} + a_t^2 \times \begin{pmatrix} v_{t-1}^2 \\ -v_{t-1}^1 \\ v_{t-1}^4 \\ -v_{t-1}^3 \end{pmatrix} + a_t^3 \times \begin{pmatrix} v_{t-1}^3 \\ -v_{t-1}^4 \\ -v_{t-1}^1 \\ v_{t-1}^2 \end{pmatrix} + a_t^4 \times \begin{pmatrix} v_{t-1}^4 \\ v_{t-1}^3 \\ -v_{t-1}^2 \\ -v_{t-1}^1 \end{pmatrix}. \quad (1)$$

The vector  $\mathbf{V}_t$  of Equation 1 is normalised by the magnitude of the previously computed vector  $\mathbf{V}_{t-1}$  before transmission in order to limit the peak power and hence the out-of-band power emissions.

The differentially encoded symbols are then spread with the aid of the spreading codes  $\underline{c}_1, \underline{c}_2, \underline{c}_3$ , and  $\underline{c}_4$ , which are generated from the same user-specific spreading code  $\underline{c}$  by ensuring that they are orthogonal using the simple code-concatenation rule of Walsh-Hadamard codes, yielding longer codes and hence a proportionately reduced per antenna throughput according to:

$$\underline{c}_1^T = [\underline{c} \quad \underline{c} \quad \underline{c} \quad \underline{c}], \quad \underline{c}_2^T = [\underline{c} \quad -\underline{c} \quad \underline{c} \quad -\underline{c}] \quad (2)$$

$$\underline{c}_3^T = [\underline{c} \quad \underline{c} \quad -\underline{c} \quad -\underline{c}], \quad \underline{c}_4^T = [\underline{c} \quad -\underline{c} \quad -\underline{c} \quad \underline{c}], \quad (3)$$

where the subscript  $T$  denotes the transpose of the vector.

The differentially encoded data is then divided into four quarter-rate substreams and the four consecutive symbols are then spread to the four transmit antennas using the mapping of:

$$y_t^1 = \frac{1}{\sqrt{4}} (\underline{c}_1 \times v_t^1 - \underline{c}_2 \times v_t^2 - \underline{c}_3 \times v_t^3 - \underline{c}_4 \times v_t^4) \quad (4)$$

$$y_t^2 = \frac{1}{\sqrt{4}} (\underline{c}_1 \times v_t^2 + \underline{c}_2 \times v_t^1 + \underline{c}_3 \times v_t^4 - \underline{c}_4 \times v_t^3) \quad (5)$$

$$y_t^3 = \frac{1}{\sqrt{4}} (\underline{c}_1 \times v_t^3 - \underline{c}_2 \times v_t^4 + \underline{c}_3 \times v_t^1 + \underline{c}_4 \times v_t^2) \quad (6)$$

$$y_t^4 = \frac{1}{\sqrt{4}} (\underline{c}_1 \times v_t^4 + \underline{c}_2 \times v_t^3 - \underline{c}_3 \times v_t^2 + \underline{c}_4 \times v_t^1), \quad (7)$$

which follows the concept of page 303 of [7].

Assuming the channel to be non-dispersive, the received signal at the output of the single receiver antenna can be represented as:

$$r_t = h_1 \times y_t^1 + h_2 \times y_t^2 + h_3 \times y_t^3 + h_4 \times y_t^4 + n_t, \quad (8)$$

where  $h_1, h_2, h_3$ , and  $h_4$  denote the non-dispersive complex-valued CIRs corresponding to the four transmit antennas, while  $n_t$  represents the AWGN having a variance of  $\sigma_n^2$ .

The received signal  $r_t$  is then correlated with  $\underline{c}_1, \underline{c}_2, \underline{c}_3,$  and  $\underline{c}_4$  according to the following operation:  $d_t^k = \underline{c}_k^H \times r_t$ , with  $k = 1, 2, 3, 4$  and  $H$  representing the Hermitian of the vector. After the correlation operation we arrive at four data symbols represented by:

$$d_t^1 = h_1 \times v_t^1 + h_2 \times v_t^2 + h_3 \times v_t^3 + h_4 \times v_t^4 + \underline{c}_1^H \times n_t \quad (9)$$

$$d_t^2 = -h_1 \times v_t^2 + h_2 \times v_t^1 - h_3 \times v_t^4 + h_4 \times v_t^3 + \underline{c}_2^H \times n_t \quad (10)$$

$$d_t^3 = -h_1 \times v_t^3 + h_2 \times v_t^4 + h_3 \times v_t^1 - h_4 \times v_t^2 + \underline{c}_3^H \times n_t \quad (11)$$

$$d_t^4 = -h_1 \times v_t^4 - h_2 \times v_t^3 + h_3 \times v_t^2 + h_4 \times v_t^1 + \underline{c}_4^H \times n_t. \quad (12)$$

To derive the decoder equations of the DSTS receiver, the received signals in Equations (9)-(12) are rearranged in vectorial form as follows:

$$R_t^1 = (d_t^1, d_t^2, d_t^3, d_t^4) \quad (13)$$

$$R_t^2 = (-d_t^2, d_t^1, d_t^4, -d_t^3) \quad (14)$$

$$R_t^3 = (-d_t^3, -d_t^4, d_t^1, d_t^2) \quad (15)$$

$$R_t^4 = (-d_t^4, d_t^3, -d_t^2, d_t^1) \quad (16)$$

$$R_{t+1} = (d_{t+1}^1, d_{t+1}^2, d_{t+1}^3, d_{t+1}^4). \quad (17)$$

To decode the transmitted symbols  $a_t^k$ ,  $k = 1, 2, 3, 4$ , the decoder uses Equations (13)-(17) and computes:

$$Re\{R_{t+1} \times R_t^k\} = 2 \times \sum_{i=1}^4 |h_i|^2 \times \sqrt{\sum_{j=1}^4 |v_{t-1}^j|^2} \times a_t^k + N_k, \quad (18)$$

where  $Re\{\cdot\}$  denotes the *real* part of a complex number and  $N$  denotes the noise term. The receiver estimates  $a_t^k$  based on Equation (18) by employing a Maximum Likelihood (ML) decoder.

According to Equation (18), the receiver only has to estimate  $\sum_{i=1}^4 |h_i|^2$  in order to decode non-constant modulus real-valued constellations, such as Pulse Amplitude Modulation (PAM). In other words, the receiver does not have to estimate the individual CIR tap values of  $h_i$ ,  $i = 1, 2, 3, 4$ , only  $\sum_{i=1}^4 |h_i|^2$  and  $\sqrt{\sum_{j=1}^4 |v_{t-1}^j|^2}$  has to be estimated in order to recover PAM modulated information from the received signal of Equation (18). A simple channel power estimator may be derived by computing the autocorrelation of the received signal as follows [10]:

$$E\{d_t^{i*} \times d_t^i\} = \sum_{i=1}^4 |h_i|^2 + \sigma_n^2. \quad (19)$$

The power of the previously transmitted symbols  $\sqrt{\sum_{j=1}^4 |v_{t-1}^j|^2}$  can be estimated from the previous output of the decoder [10].

Again, the above analysis has been carried out for the case of a single receive antenna, but it can be readily extended for an arbitrary number of receive antennas, where the resultant signals are appropriately combined, before passing them to the decoder.

### 3.2. Complex-Valued Constellations

Signals represented by complex-valued constellations can also be transmitted using the proposed DSTS algorithm. Accordingly, we assume that at time  $t$ , a block of  $4B$  bits arrives at the encoder, where each  $2B$  bits are modulated using an  $M$ -ary complex-valued constellation, so that we have  $2B = \log_2 M$ . The modulator outputs the two complex symbols  $x_t^{m_1}$  and  $x_t^{m_2}$ , conveying the original  $4B$  bits, where we have  $m_1 = 0, 1, \dots, M-1$  and  $m_2 = 0, 1, \dots, M-1$ . Now,  $x_t^{m_1}$  and  $x_t^{m_2}$  are mapped to  $a_t^k$ ,  $k = 1, 2, 3, 4$ , as follows

$$(a_t^1, a_t^2, a_t^3, a_t^4) = (Re\{x_t^{m_1}\}, Im\{x_t^{m_1}\}, Re\{x_t^{m_2}\}, Im\{x_t^{m_2}\}). \quad (20)$$

Similarly to real-valued constellations,  $a_t^k$ ,  $k = 1, 2, 3, 4$ , can be estimated using Equation (18), which is then used to recover the original complex symbols,  $x_t^{m_1}$  and  $x_t^{m_2}$ , according to Equation (20).

## 4. DSTS DESIGN USING SPHERE PACKING

It was shown in [8] that the so-called diversity product quantifying the achievable coding advantage<sup>1</sup> of an orthogonal transmit diversity scheme is determined by the minimum Euclidean distance of the transmitted signal vectors. Hence, in order to maximise the achievable coding advantage, it was proposed in [8] to use sphere packing schemes that maximise the minimum Euclidean distance of the transmitted signal vectors.

According to Equation (18), the decoded signals represent scaled versions of  $a_t^1, a_t^2, a_t^3,$  and  $a_t^4$  corrupted by the complex-valued AWGN. This observation implies that the diversity product of DSTS systems is determined by the minimum Euclidean distance of all legitimate vectors  $(a_1, a_2, a_3, a_4)$ , where the time index is removed for notational simplicity. The idea is to jointly design the legitimate vectors  $(a_1, a_2, a_3, a_4)$  so that they are represented by a single phasor point selected from a sphere packing constellation corresponding to a 4-dimensional real-valued lattice having the best known minimum Euclidean distance in the 4-dimensional real-valued space  $R^4$ . For the sake of generalising our treatment, let us assume that there are  $L$  legitimate vectors  $(a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4})$ ,  $l = 0, 1, \dots, L-1$ , where  $L$  represents the number of sphere-packed modulated symbols. The transmitter, then, has to choose the modulated signal from these  $L$  legitimate symbols, where the four elements  $a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}$  are differentially space-time spread and transmitted from the four transmit antennas. The throughput of the system is  $(\log_2 L)/2$  bits per channel use.

In the four-dimensional real-valued Euclidean space  $R^4$ , the lattice  $D_4$  is defined as a sphere packing having the best minimum Euclidean distance from all other  $(L-1)$  legitimate constellation points in  $R^4$  [16]. More specifically,  $D_4$  may be defined as a lattice that consists of all legitimate sphere-packed constellation points having integer coordinates  $[a_1 \ a_2 \ a_3 \ a_4]$  subjected to the sphere packing constraint of  $a_1 + a_2 + a_3 + a_4 = k$ , where  $k$  is an even integer. Assuming that  $S = \{s^l = [a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}] \in R^4 : 0 \leq l \leq L-1\}$  constitutes a set of  $L$  legitimate constellation points from the lattice  $D_4$  having a total energy of  $E \triangleq \sum_{l=0}^{L-1} (|a_{l,1}|^2 + |a_{l,2}|^2 + |a_{l,3}|^2 + |a_{l,4}|^2)$ , and upon introducing the notation

$$C_l = \sqrt{\frac{2L}{E}} (a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}), \quad l = 0, 1, \dots, L-1, \quad (21)$$

we have a set of constellation symbols,  $\{C_l : 0 \leq l \leq L-1\}$ , leading to the design of DSTS signals, whose diversity product is determined by the minimum Euclidean distance of the set of  $L$  legitimate constellation points in  $S$ .

## 5. ITERATIVE DEMAPPING FOR SPHERE PACKING CONSTELLATIONS

Again, for the sake of simplicity, a system having a single receive antenna is considered, although its extension to several receive antennas is feasible. As already discussed in Section 3, the detected DSTS signals can be represented by Equation (18), where a received sphere-packed symbol  $\tilde{s}$  is then constructed from the estimates  $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3$  and  $\tilde{a}_4$ . The received sphere-packed symbol  $\tilde{s}$  can be written as

$$\tilde{s} = h \cdot \sqrt{\frac{2L}{E}} \cdot s^l + N, \quad (22)$$

<sup>1</sup>The diversity product or coding advantage was defined as the estimated gain over an uncoded system having the same diversity order as the coded system [8].

Sphere Packing Modulation	$L = 16$
No. of Transmitter Antennas	4
No. of Receiver Antennas	1
Channel	Correlated Rayleigh Fading
Normalised Doppler frequency	0.01
Outer channel Code	RSC (2, 1, 5)
Generator	$(G_r, G) = (35, 23)_8$
Spreading Code	Walsh Code
Spreading Factor	8
Number of users	4
Effective throughput	1 bit/symbol

Table 1: System parameters

where we have  $h = 2 \times \sum_{i=1}^4 |h_i|^2 \times \sqrt{\sum_{j=1}^4 |v_{t-1}^j|^2}$ ,  $s^l \in S$ ,  $0 \leq l \leq L - 1$ , and  $N$  is a four-dimensional Gaussian random variable having a variance of  $\sigma_N^2$ , since the SP symbol constellation  $S$  is four-dimensional.

The SP symbol  $\tilde{s}$  carries  $B_{sp}$  channel-coded bits  $\mathbf{b} = b_0 \dots b_{B_{sp}-1} \in \{0, 1\}$ . As discussed in [13], the max-log approximation of the LLR value of bit  $k$  for  $k = 0, \dots, B_{sp} - 1$  can be written as [12]

$$\begin{aligned}
& L(b_k / \tilde{s}) \\
&= L_a(b_k) \\
&+ \max_{s^l \in S_1^k} \left[ -\frac{1}{2\sigma_N^2} (\tilde{s} - \alpha \cdot s^l)^2 + \sum_{j=0, j \neq k}^{B-1} b_j L_a(b_j) \right] \\
&- \max_{s^l \in S_0^k} \left[ -\frac{1}{2\sigma_N^2} (\tilde{s} - \alpha \cdot s^l)^2 + \sum_{j=0, j \neq k}^{B-1} b_j L_a(b_j) \right], \tag{23}
\end{aligned}$$

where  $\alpha = h \cdot \sqrt{\frac{2L}{E}}$ ,  $S_1^k$  and  $S_0^k$  are subsets of the symbol constellation  $S$ , where we have  $S_1^k \triangleq \{s^l \in S : b_k = 1\}$  and likewise,  $S_0^k \triangleq \{s^l \in S : b_k = 0\}$ . In other words,  $S_i^k$  represents all symbols of the set  $S$ , where we have  $b_k = i \in \{0, 1\}$ ,  $k = 0, \dots, B_{sp} - 1$ .

## 6. RESULTS AND DISCUSSION

Without loss of generality, we considered a SP modulation scheme associated with  $L = 16$  and using four transmit antennas as well as a single receiver antenna in order to demonstrate the performance improvements achieved by the proposed system. All simulation parameters are listed in Table 1, where RSC stands for recursive systematic convolutional code.

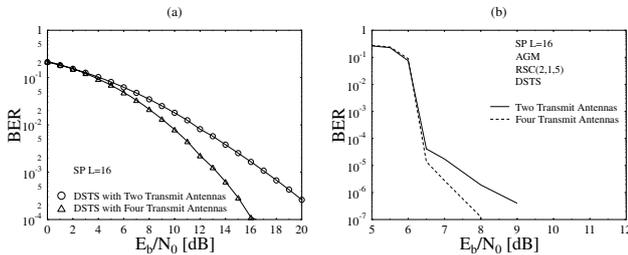


Figure 2: Performance comparison of DSTS-SP in conjunction with  $L=16$  when employing two and four transmit antennas and using the system parameters outlined in Table 1. Figure (a) compares the uncoded systems' performance while Figure (b) shows the performance comparison of Anti-Gray Mapping (AGM) based convolutional-coded scheme when employing two and four transmit antennas, while using an interleaver length of  $D = 1,000,000$  bits and 10 iterations.

It was shown in [16] that there is a total of 24 legitimate symbols<sup>2</sup> hosted by  $D_4$  having an identical minimum energy of  $E = 2$ .

<sup>2</sup>In simple terms, the sphere centred at  $(0, 0, 0, 0)$  has 24 spheres

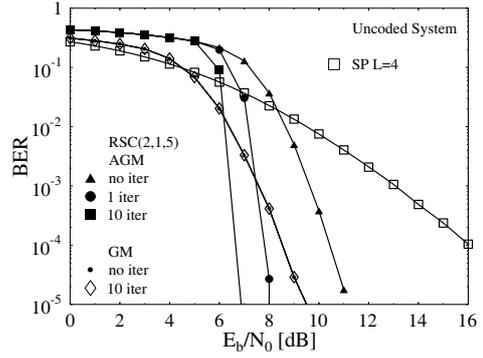


Figure 3: Performance comparison of the Anti-Gray Mapping (AGM) based  $\frac{1}{2}$ -rate RSC-coded DSTS-SP scheme in conjunction with  $L = 16$  against the identical-throughput 1BPS uncoded DSTS-SP scheme, while using an interleaver length  $D=1,000,000$ , a variable number of iterations and the system parameters outlined in Table 1.

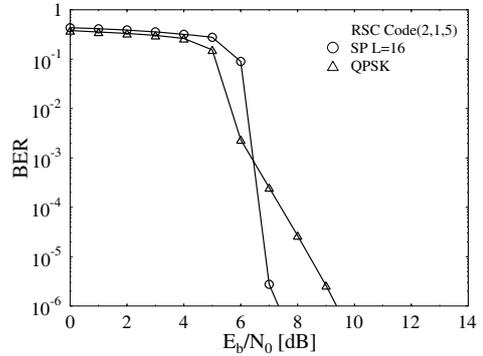


Figure 4: Performance comparison of the Anti-Gray Mapping (AGM) based convolutional-coded DSTS scheme in conjunction with  $SP L = 16$  against an identical-throughput 1BPS convolutional-coded DSTS-QPSK scheme, while using an interleaver length of  $D = 1,000,000$  bits,  $I = 10$  iterations and the system parameters outlined in Table 1.

It is desirable to choose a specific subset of  $L = 16$  points from the entire set of legitimate constellation points hosted by  $D_4$ , which results in the minimum total energy. Moreover, it is also desirable to choose a specific constellation set which possesses the highest minimum Euclidean distance, hence minimising the error probability. Therefore, we used a computer search for determining the optimum choice of the  $L = 16$  points out of the possible 24 points.

Both Figures 2-a and 2-b provide a performance comparison between two 1 BPS-throughput DSTS schemes, namely those employing two<sup>3</sup> and four transmit antennas, without and with RSC-coding respectively. The results are presented for the SP constellation of  $L = 16$  and using the parameters of Table 1. The results of Figure 2-a clearly illustrate the achievable BER performance improvement due to the increased diversity order (*i.e.* the increased number of transmit antennas). By contrast, Figure 2-b plots our performance comparison of Anti-Gray Mapping (AGM)<sup>4</sup> based  $\frac{1}{2}$ -rate RSC-coded DSTS-SP schemes in conjunction with  $L = 16$ , when employing two and four transmit antennas, while using an interleaver length of  $D = 1,000,000$  bits, 10 iterations and the system parameters outlined in Table 1. The four transmit antenna aided system outperforms its two-antenna counterpart by approx-

around it, centred at the points  $(+/- 1, +/- 1, 0, 0)$ , where any choice of signs and any ordering of the coordinates is legitimate [[16], p.9].

<sup>3</sup>The design of DSTS with two transmit antennas is described in [17].

<sup>4</sup>Any mapping different from the classic Gray mapping may be referred to as AGM. We tested the performance of all legitimate AGM schemes in order to find the best performer.

imately  $1.2dB$  at a BER of  $10^{-6}$ . It has been reported in [18] that the capacity of a MIMO system is determined by the minimum of the transmit and receive antennas. Therefore, the systems using two transmit and one receive as well as four transmit and one receive antennas have the same capacity. Thus, we can see that the performance of the coded systems using two and four transmit antennas becomes similar since the coded system having two transmit antennas has already approached the system capacity quite closely and the extra  $1.2dB$  of the four transmit antennas is a benefit of the higher diversity gain of the four transmit antennas.

Figure 3 compares the attainable performance of the proposed  $\frac{1}{2}$ -rate RSC-coded DSTS-SP scheme employing both AGM and Gray Mapping (GM) of the bits to the SP symbol, which are also contrasted to that of an identical-throughput 1 BPS uncoded DSTS-SP scheme using  $L = 4$ , when communicating over a correlated Rayleigh fading channel. In Figure 3, an interleaver depth of  $D = 10^6$  bits was employed and a normalised Doppler frequency of  $f_D = 0.01$  was used. Observe in the figure that the two GM-based DSTS-SP BER curves are exactly the same, regardless, whether no iterations or  $I = 10$  turbo-detection iterations were employed. By contrast, AGM achieved a substantial performance improvement in conjunction with iterative demapping and decoding. Explicitly, the figure demonstrates that a coding advantage of about  $18dB$  was achieved at a BER of  $10^{-5}$  after  $I = 10$  iterations by the  $\frac{1}{2}$ -rate RSC-coded AGM DSTS-SP system over the uncoded DSTS-SP scheme. Additionally, a coding advantage of approximately  $2.5dB$  was attained over the 1BPS-throughput  $\frac{1}{2}$ -rate RSC-coded GM DSTS-SP scheme.

Finally, the proposed turbo-detected DSTS-SP scheme provides an improved performance over an equivalent DSTS scheme dispending with SP modulation, as evidenced in Figure 4, demonstrating that the proposed DSTS-SP scheme using  $L = 16$  exhibits an  $E_b/N_o$  gain of around  $2dB$  at a BER of  $10^{-6}$  over the identical-throughput 1BPS DSTS-QPSK scheme.

## 7. CONCLUSION

In this paper we proposed a novel system that exploits the advantages of both iterative demapping and turbo detection [12], as well as those of the DSTS-SP scheme developed. The proposed DSTS scheme benefits from a substantial diversity gain, while using four transmit antennas without the need for any CSI. The DSTS scheme using four transmit antennas outperforms that employing two transmit antennas due to the increased diversity order. Moreover, our investigations demonstrated that significant performance improvements may be achieved, when the AGM DSTS-SP scheme is combined with outer channel decoding and iterative SP-symbol-to-bit demapping, as compared to the Gray-Mapping based systems. When using an appropriate bit-to-SP-symbol mapping scheme and 10 turbo detection iterations, an  $E_b/N_0$  gain of about  $18dB$  was obtained by the convolutional-coded DSTS-SP scheme over the identical-throughput 1 bit/symbol uncoded DSTS-SP benchmarker scheme. Our future research will consider the design of a multiple transmit antennas DSTS schemes that can be generalised to work with arbitrary modulation schemes while using a variable number of transmit antennas. Additionally, optimised bits-to-SP-symbol mapping and precoding schemes will be designed with the aid of Extrinsic Information Transfer (EXIT) Charts [19].

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