A Unified Exact BER Performance Analysis of Asynchronous DS-CDMA Systems Using BPSK Modulation over Fading Channels

Xiang Liu and Lajos Hanzo

***Abstract*— An asynchronous binary DS-CDMA system using random spreading sequences is considered when communicating over various fading channels. New closed-form expressions are derived for the conditional Characteristic Function (CF) of the multiple access interference. A unified analysis is provided for calculating the exact average Bit Error Rate (BER) expressed in the form of a single numerical integration based on the CF approach. The numerical results obtained from our exact BER analysis are verified by our simulation results and are also compared to those obtained by the Standard Gaussian Approximation (SGA), confirming the accuracy of the SGA for most practical conditions, except for high Signal-to-Noise Ratios (SNR) and for a low number of interferers.**

***Index Terms*— BER analysis, CDMA, fading, Rayleigh, Ricean, Hoyt, Nakagami-m, random spreading sequence.**

I. INTRODUCTION

CODE Division Multiple Access (CDMA) has been one of the most successful radio access techniques since the 1990s and Direct Sequence (DS) CDMA has been integrated into the third generation mobile systems. The Bit Error Rate (BER) is one of the most important performance metrics in communication systems and hence it has been extensively studied in various DS-CDMA systems.

For the sake of computational simplicity, the most widely used approach invoked for calculating the average BER of DS-CDMA systems is that of assuming that the Multiple Access Interference (MAI) is Gaussian distributed or conditional Gaussian distributed based on the Central Limit Theorem (CLT). Various Gaussian approximation techniques have been proposed, such as the Standard Gaussian Approximation (SGA) [1]–[7], the Improved Gaussian Approximation (IGA) [3]–[5], [7], the Simplified IGA (SIGA) [4], [7], as well as the Improved Holtzman Gaussian Approximation (IHGA) [8]. However, Gaussian approximation techniques tend to become less accurate, when a low number of users is supported or when there is a dominant interferer [5].

Therefore the exact BER analysis dispensing with the previous assumptions on the MAI distribution is desirable. Several exact BER evaluation techniques have been developed without assuming a Gaussian MAI distribution, such as the series expansion of [5], [9], or the employment of Moment Generating Functions (MGF) [10] and Characteristic Functions (CF) [6], [7], [9]. These techniques typically achieve more accurate BER evaluation at the cost of a high computational complexity.

In the existing literature, most results are reported for the BERs of DS-CDMA systems communicating over Additive White Gaussian Noise (AWGN) channels [1]–[5], [9], and a few studies also considered both Rayleigh channels [5], [7] as well as Nakagami-m channels [6], [10]. Cheng and Beaulieu extended the CF based AWGN results of [9] to both Rayleigh [7] and Nakagami-m [6] channels. However, the results of [6] only apply to Nakagami-m fading associated with an integer fading parameter m.

The novel contribution of this paper is that we provide a unified exact BER performance analysis of asynchronous DS-CDMA systems using BPSK modulation for transmissions over Rayleigh, Ricean, Hoyt and Nakagami-m fading channels. This paper is organized as follows. In Section II the features of Rayleigh, Ricean, Hoyt and Nakagami-m fading channels are summarized. In Section III a general asynchronous DS-CDMA system using BPSK modulation and communicating over fading channels is presented. Then, in Section IV its exact BER performance using random spreading sequences is investigated based on the CF approach. Our numerical results are presented in Section V and finally our conclusions are provided in Section VI.

II. FADING CHANNELS

We consider a range of different fading channels in our analysis, namely the Rayleigh, Ricean, Hoyt and Nakagami-m channels. Note that a table of the corresponding fading PDFs and CFs has been given in [11]. We provide two new closed-form expressions in Table I for the CFs of the Ricean and Hoyt fading, respectively, rather than in the form of the sum of infinite series, as given by Table II of [11]. In Table I, $\Gamma(x)$ is the Gamma function [12], $\mathbb{I}_0(x)$ is the zero-order modified Bessel function of the first kind [12], ${}_1\mathbb{F}_1(\alpha;\beta;x)$ is the confluent hypergeometric function [12], $\Psi_2(\alpha;\gamma,\gamma';x,y)$ and $\mathbb{H}_7(\alpha,\gamma,\delta,x,y)$ are two of Horn's confluent hypergeometric functions of two variables [13], [14].

The fading is characterized by the complex-valued random variable $\widetilde{h} = he^{j\varphi} = h_r + jh_i$, where h and φ are the fading amplitude and phase, respectively, while h_r and h_i are the real and imaginary part, respectively.

The Rayleigh fading phase φ is uniformly distributed over $[0, 2\pi)$ and independent of the fading amplitude h [15]. One of the important properties of the complex-valued Rayleigh variable is that its real and imaginary parts, i.e. h_r and h_i, are mutually independent, zero-mean and real-valued Gaussian variables, both having a variance of σ^2 [15].

The Ricean distribution is also known as the Nakagami-n distribution [16], [17]. The Ricean fading process physically

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TABLE I
PDF AND CF OF THE FADING AMPLITUDE h FOR A RANGE OF DIFFERENT FADING CHANNELS [11].

Rayleigh	PDF: $f_h(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right), x \geq 0,$ CF: $\Phi_h(\omega) = {}_1\mathbb{F}_1\left(1; \frac{1}{2}; -\frac{1}{2}\sigma^2\omega^2\right) + j\sigma\omega\sqrt{\frac{\pi}{2}}\exp\left(-\frac{1}{2}\sigma^2\omega^2\right),$ where $\sigma > 0$ and $\Omega = 2\sigma^2$ is the average fading power.
Ricean (Nakagami-n)	PDF: $f_h(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2+\mu^2}{2\sigma^2}\right)\mathbb{I}_0\left(\frac{x\mu}{\sigma^2}\right), x \geq 0,$ CF: $\Phi_h(\omega) = \exp\left(-\frac{\mu^2}{2\sigma^2}\right)\Psi_2\left(1; 1, \frac{1}{2}; \frac{\mu^2}{2\sigma^2}, -\frac{1}{2}\sigma^2\omega^2\right)$ $+ j\sqrt{2}\sigma\omega\exp\left(-\frac{\mu^2}{2\sigma^2}\right)\Psi_2\left(\frac{3}{2}; 1, \frac{3}{2}; \frac{\mu^2}{2\sigma^2}, -\frac{1}{2}\sigma^2\omega^2\right),$ where $\mu \geq 0$ and $\sigma > 0$, $\Omega = \mu^2 + 2\sigma^2$ is the average fading power.
Hoyt (Nakagami-q)	PDF: $f_h(x) = \frac{(1+q^2)x}{q\Omega}\exp\left[-\frac{(1+q^2)x^2}{4q^2\Omega}\right]\mathbb{I}_0\left[\frac{(1-q^4)x^2}{4q^2\Omega}\right], x \geq 0,$ CF: $\Phi_h(\omega) = \frac{2q}{1+q^2}\mathbb{H}_7\left[1, 1, \frac{1}{2}, \frac{(1-q^2)^2}{4(1+q^2)^2}, -\frac{q^2\Omega}{(1+q^2)^2}\omega^2\right]$ $+ j\omega\frac{2q^2\sqrt{\pi\Omega}}{(1+q^2)^2}\mathbb{H}_7\left[\frac{3}{2}, 1, \frac{3}{2}, \frac{(1-q^2)^2}{4(1+q^2)^2}, -\frac{q^2\Omega}{(1+q^2)^2}\omega^2\right],$ where $\Omega > 0$ is the average power and $0 \leq q \leq 1$ is the Hoyt fading parameter.
Nakagami-m	PDF: $f_h(x) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1}\exp\left(-\frac{m}{\Omega}x^2\right), x \geq 0,$ CF: $\Phi_h(\omega) = {}_1\mathbb{F}_1\left(m; \frac{1}{2}; -\frac{\Omega}{4m}\omega^2\right) + j\omega\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\sqrt{\frac{\Omega}{m}}\,{}_1\mathbb{F}_1\left(m+\frac{1}{2}; \frac{3}{2}; -\frac{\Omega}{m}\omega^2\right),$ where $\Omega > 0$ is the average power and $m \geq \frac{1}{2}$ is the Nakagami-m fading parameter.

consists of a direct LOS component having a power of μ^2 and many weaker components having a total power of $2\sigma^2$. The Ricean K-factor is defined as $\kappa = \frac{\mu^2}{2\sigma^2}$. The Ricean phase φ is neither uniformly distributed nor independent of the fading amplitude h [17]. One of the important properties of the complex-valued Ricean distribution is that its real and imaginary parts, i.e. h_r and h_i, are mutually independent Gaussian variables [17]. The mean and variance of h_r and h_i are $\{\mu_r, \sigma^2\}$ and $\{\mu_i, \sigma^2\}$, respectively, where we have $\mu_r^2 + \mu_i^2 = \mu^2$.

The Hoyt distribution is also known as the Nakagami-q distribution [16], [18]. Similar to the Ricean fading, the Hoyt fading phase φ is neither uniformly distributed nor independent of the fading amplitude h [18]. One of the important properties of the complex-valued Hoyt distribution is that its real and imaginary parts, i.e. h_r and h_i, are mutually independent zero-mean Gaussian variables having a variance of σ_r^2 and σ_i^2, respectively [18], which satisfy $\Omega = \sigma_r^2 + \sigma_i^2$ and $q = \frac{\sigma_i}{\sigma_r}$.

The distribution of the Nakagami-m fading phase φ is still unknown at the time of writing, but typically it is assumed to be uniform over $[0, 2\pi)$ [16].

III. SYSTEM MODEL

We consider a general asynchronous BPSK modulated DS-CDMA system communicating over fading channels mentioned in Section II. There are K simultaneously transmitting users in the system. Binary random spreading sequences having L chips and a rectangular chip waveform are employed. Both the spreading sequence $\{a_{k,m}\}_{m=0}^{L-1}$ and the data sequence $\{b_{k,m}\}_{m=-\infty}^{\infty}$ are mutually independent and symmetrically Bernoulli distributed [15].

Without loss of generality, we assume that the 0th user's signal is the desired one. The decision statistic Z at the output of the coherent receiver is given by [7]:

$$Z = h_0 L b_{0,0} + \sum_{k=1}^{K-1}\Re\left\{X_k\widetilde{h}_k e^{j\Delta_k}\right\} + \eta, \tag{1}$$

where $\Re\{\widetilde{x}\}$ denotes the real part of the complex number $\widetilde{x}$. The received complex equivalent signals $\{\widetilde{h}_k\}_{k=0}^{K-1}$ are mutually independent and conform to one of the fading distributions mentioned in Section II and have an average fading power of $\{\Omega_k\}_{k=0}^{K-1}$. The noise component η is a zero-mean Gaussian random variable having a variance of $\sigma_\eta^2 = \frac{N_0 L}{T_c}$, where N_0 is the double-sided power spectral density of the zero-mean Additive White Gaussian Noise (AWGN) and T_c is the chip duration. The phase shift difference $\Delta_k = -\omega_c(\tau_k - \tau_0) + (\theta_k - \theta_0)$ between the kth and 0th user is uniformly distributed in $[0, 2\pi)$, where τ_k and θ_k are the time delay and the carrier phase of the kth user, respectively. The random variable X_k may be further expressed as [2], [7]:

$$X_k = \sum_{m=0}^{L-2} Y_{k,m}\left[(1-\nu_k) + a_{0,m}a_{0,m+1}\nu_k\right] + Y_{k,L-1}\nu_k + Y_{k,L}(1-\nu_k), \tag{2}$$

where the $(L+1)$ random variables $\{Y_{k,m}\}_{m=0}^{L}$ are mutually independent and symmetric Bernoulli distributed, conditioned on the 0th user's spreading sequence $\{a_{0,m}\}_{m=0}^{L-1}$. Furthermore, the relative chip shifts $\{\nu_k\}_{k=1}^{K-1}$ between the kth and 0th user normalized by the chip duration are mutually independent and uniformly distributed in $[0, 1)$ [2], [7].

IV. BER ANALYSIS

Let B and A denote the number of chip boundaries both with and without chip-value transitions within the 0th user's spreading sequence, respectively, and define two sets $\mathcal{A}$ and $\mathcal{B}$ as follows [2], [7]:

$$\begin{aligned} \mathcal{A} &= \{-A, -(A-2), ..., A-2, A\}, \\ \mathcal{B} &= \{-B, -(B-2), ..., B-2, B\}. \end{aligned} \tag{3}$$

Then we have $A + B = L - 1$.

Upon defining the Co-Channel Interference (CCI), $I_k = \Re\left\{X_k \widetilde{h}_k e^{j\Delta_k}\right\}$, incurred by the kth user, the CCIs imposed by different interferers are mutually independent conditioned on B [2], [7]. It may be readily shown that both the PDF, $f_{I_k|B}(x)$, and the CF, $\Phi_{I_k|B}(\omega)$, of the CCI incurred by the kth user are even functions for all fading channels presented in Section II.

Applying Parseval's theorem [11], the 0th user's BER $P_{e|B}$ conditioned on B may be shown to be:

$$P_{e|B} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega}\Phi_\eta(\omega)\Im\left\{\Phi_{h_0}(\omega L)\right\}\prod_{k=1}^{K-1}\Phi_{I_k|B}(\omega)\mathrm{d}\omega, \tag{4}$$

where $\Im\{\Phi_{h_0}(\omega)\}$ is the imaginary part of the CF of the 0th user's fading amplitude h_0. When the Rayleigh or Nakagami-m fading is considered, Equation 4 reduces to Equation 39 of [7] and to Equation 21 of [6], respectively.

Finally, the overall average BEP is obtained by averaging $P_{e|B}$ over all spreading sequences, yielding:

$$P_e = 2^{-(L-1)}\sum_{B=0}^{L-1}\binom{L-1}{B}P_{e|B}. \tag{5}$$

For the sake of simplicity, we will only consider the CF range spanning $\omega \geq 0$ in our later discussions in the context of Equation 4. Nevertheless, the CF defined for the range $\omega < 0$ can be readily derived from that given for the range $\omega \geq 0$ by exploiting the following property of the CF [15]:

$$\Phi(-\omega) = \Phi^*(\omega), \tag{6}$$

where $\Phi^*(\omega)$ denotes the complex conjugate of $\Phi(\omega)$.

The only task that remained unsolved so far is the determination of the conditional CF, $\Phi_{I_k|B}(\omega)$, of the CCI incurred by the kth user. The CF of I_k conditioned on X_k, $\Phi_{I_k|X_k}$, may be derived by the specific properties outlined in Section II. Upon averaging $\Phi_{I_k|X_k}(\omega)$ over $\{Y_{k,m}\}_{m=0}^{L}$ and ν_k, we arrive at the CF of I_k conditioned on B in the following form:

$$\begin{aligned} \Phi_{I_k|B}(\omega) = {} & 2^{-(L+1)}\sum_{d_1\in\mathcal{A}}\sum_{d_2\in\mathcal{B}}\binom{A}{\frac{d_1+A}{2}}\binom{B}{\frac{d_2+B}{2}} \\ & \times \sum_{Y_{k,L-1},Y_{k,L}\in\{\pm1\}}\Phi_{I_k|\lambda_0,\lambda_1}(\omega), \end{aligned} \tag{7}$$

where the coefficients λ_0 and λ_1 are defined as:

$$\lambda_0 = d_1 + d_2 + Y_{k,L} \tag{8}$$

$$\lambda_1 = -2d_2 + Y_{k,L-1} - Y_{k,L}. \tag{9}$$

The conditional CFs, $\Phi_{I_k|X_k}(\omega)$ and $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$, will be provided for various fading scenarios in the following subsections.

A. Rayleigh

Upon exploiting the specific properties of the Rayleigh distribution which were outlined in Section II, it is readily shown that I_k conditioned on X_k and Δ_k is a zero-mean Gaussian random variable having a variance of $X_k^2\sigma_k^2$. Hence we have the CF of I_k conditioned on X_k in the following form:

$$\Phi_{I_k|X_k}(\omega) = \Phi_{I_k|X_k,\Delta_k}(\omega) = \exp\left(-\frac{1}{2}X_k^2\sigma_k^2\omega^2\right). \tag{10}$$

Finally, the conditional CF $\Phi_{I_k|B}(\omega)$ is expressed as Equation 7 and the conditional CF $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$ in Equation 7 may be shown to be:

$$\Phi_{I_k|\lambda_0,\lambda_1}(\omega) = \begin{cases} \dfrac{1}{\lambda_1\sigma_k\omega}\sqrt{\dfrac{\pi}{2}}\mathrm{erf}\left(\dfrac{x\sigma_k\omega}{\sqrt{2}}\right)\Bigg|_{\lambda_0}^{\lambda_0+\lambda_1}, & \lambda_1 \neq 0 \\ \exp\left(-\dfrac{1}{2}\lambda_0^2\sigma_k^2\omega^2\right), & \lambda_1 = 0, \end{cases} \tag{11}$$

where $\mathrm{erf}(x)$ is the error function [12] and $f(x)|_{x_1}^{x_2} = f(x_2) - f(x_1)$.

This result has been derived in [7] for the Rayleigh fading. However, we represent it here in a unified approach so that Equations 7 - 9 may be used for other fading channels in our later discussions.

B. Ricean (Nakagami-n)

Upon exploiting the properties of the Ricean distribution characterized in Section II or exploiting the results of [19], we have the CF of I_k conditioned on X_k in the following form:

$$\Phi_{I_k|X_k}(\omega) = \exp\left(-\frac{1}{2}X_k^2\sigma_k^2\omega^2\right)\mathbb{J}_0\left(X_k\mu_k\omega\right), \tag{12}$$

where $\mathbb{J}_0(x)$ is the zeroth-order Bessel function of the first kind [12]. Finally, the conditional CF $\Phi_{I_k|B}(\omega)$ is also expressed as Equation 7 and the conditional CF $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$ in Equation 7 may be shown to be:

$$\Phi_{I_k|\lambda_0,\lambda_1}(\omega) = \begin{cases} \dfrac{x}{\lambda_1}\mathbb{F}_{1:0;0}^{1:0;1}\left(\begin{matrix}\frac{1}{2}:-;-; \\ \frac{3}{2}:-;1;\end{matrix} -\dfrac{1}{2}\sigma_k^2\omega^2x^2, -\dfrac{1}{4}\mu_k^2\omega^2x^2\right)\Bigg|_{\lambda_0}^{\lambda_0+\lambda_1}, & \lambda_1 \neq 0, \\ \exp\left(-\frac{1}{2}\lambda_0^2\sigma_k^2\omega^2\right)\mathbb{J}_0\left(\lambda_0\mu_k\omega\right), & \lambda_1 = 0, \end{cases} \tag{13}$$

where $\mathbb{F}_{C:D;D'}^{A:B;B'}\left(\begin{matrix}(a):(b);(b'); \\ (c):(d);(d');\end{matrix} x, y\right)$ is the Kampé de Fériet function [14], [20].

C. Hoyt (Nakagami-q)

Upon exploiting the property of the Hoyt distribution in Section II, it can be readily shown that I_k conditioned on X_k and Δ_k is a zero-mean Gaussian random variable having a variance of $\left(\sigma_{kr}^2\cos^2\Delta_k + \sigma_{ki}^2\sin^2\Delta_k\right)X_k^2$. Hence we have the CF of I_k conditioned on X_k and Δ_k in the following form:

$$\Phi_{I_k|X_k,\Delta_k}(\omega) = \exp\left[-\frac{1}{2}\left(\sigma_{kr}^2\cos^2\Delta_k + \sigma_{ki}^2\sin^2\Delta_k\right)X_k^2\omega^2\right]. \tag{14}$$

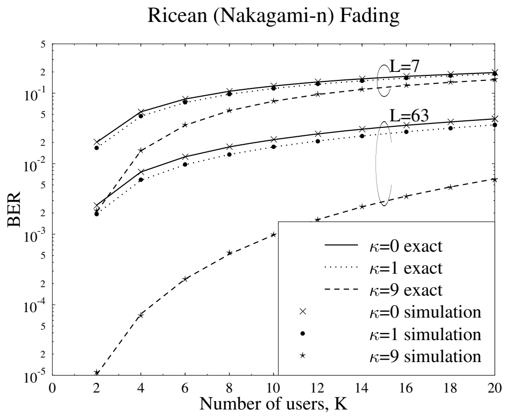

Fig. 1. BER versus the number of users K in an asynchronous DS-CDMA system exposed to Ricean fading using random spreading sequences and BPSK modulation. The length of the random spreading sequences is $L = 7$ and 63. The Ricean K-factor is $\kappa = 0$ (Rayleigh), 1 and 9, which is common to all users. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\mathrm{SNR}} = \infty$.

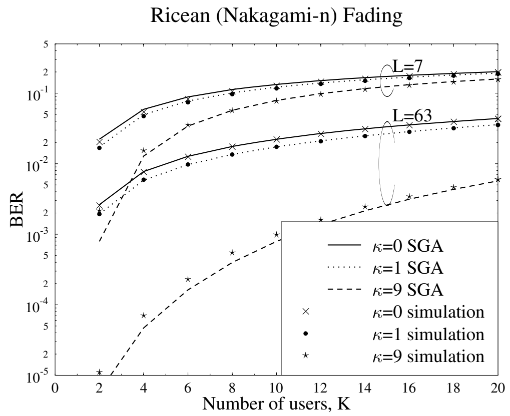

Fig. 2. BER versus the number of users K in an asynchronous DS-CDMA system exposed to Ricean fading using random spreading sequences and BPSK modulation. The length of the random spreading sequences is $L = 7$ and 63. The Ricean K-factor is $\kappa = 0$ (Rayleigh), 1 and 9, which is common to all users. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\mathrm{SNR}} = \infty$.

Applying the integral identity of Equation 3.339 in [12], we arrive at the CF of I_k conditioned on X_k by averaging $\Phi_{I_k|X_k,\Delta_k}(\omega)$ over $\Delta_k \in [0, 2\pi)$ in the form of:

$$\Phi_{I_k|X_k}(\omega) = \exp\left[-\frac{1}{4}\left(\sigma_{kr}^2 + \sigma_{ki}^2\right) X_k^2 \omega^2\right] \times \mathbb{I}_0\left[\frac{1}{4}\left(\sigma_{kr}^2 - \sigma_{ki}^2\right) X_k^2 \omega^2\right]. \quad (15)$$

Finally, the conditional CF $\Phi_{I_k|B}(\omega)$ is also expressed in the form of Equation 7 and the conditional CF $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$ in Equation 7 may be shown to be in the form of Equation 16 seen at the top of the next page.

D. Nakagami-m

Upon exploiting the results of [10], [21], we have the CF of I_k conditioned on X_k in the following form[1]:

$$\Phi_{I_k|X_k}(\omega) = {}_1\mathbb{F}_1\left(m; 1; -\frac{\Omega}{4m} X_k^2 \omega^2\right). \quad (17)$$

Finally, the conditional CF $\Phi_{I_k|B}(\omega)$ is still expressed as in Equation 7, while the conditional CF $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$ in Equation 7 may be shown to be:

$$\Phi_{I_k|\lambda_0,\lambda_1}(\omega) = \begin{cases} \frac{x}{\lambda_1} {}_2\mathbb{F}_2\left(m, \frac{1}{2}; 1, \frac{3}{2}; -\frac{\Omega}{4m} x^2 \omega^2\right)\Big|_{\lambda_0}^{\lambda_0+\lambda_1}, & \lambda_1 \neq 0, \\ {}_1\mathbb{F}_1\left(m; 1; -\frac{\Omega}{4m} \lambda_0^2 \omega^2\right), & \lambda_1 = 0, \end{cases} \quad (18)$$

where ${}_p\mathbb{F}_q(\alpha_1, \alpha_2, ..., \alpha_p; \beta_1, \beta_2, ..., \beta_q; x)$ is the generalized hypergeometric function [12]. The MAI analysis of DS-CDMA systems communicating over Nakagami-m channels has been studied in [6], but the results of [6] only apply to the scenarios where the Nakagami-m fading parameter m is an integer. By contrast, Equation 18 applies to arbitrary values of m.

[1]The authors would like to thank the anonymous reviewer for pointing out that a similar result may also be found in Nakagami's original paper [16], although the paper was not addressing the same problem.

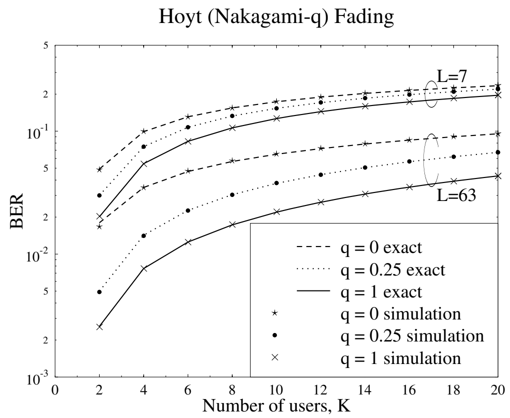

Fig. 3. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L = 7$ and 63. The Hoyt fading parameter is $q = 0$ (one-sided Gauss), 0.25 and 1 (Rayleigh), which is common to all users. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\mathrm{SNR}} = \infty$.

V. Numerical Results

Figures 1 and 2 illustrate the average BER performance versus the number of users in the context of Ricean fading channels, when the effects of background noise are ignored. Figure 1 compares the results obtained from our exact BER analysis to our simulation results and shows that they match very well both for various spreading sequence lengths and for various Ricean K-factors. On the other hand, Figure 2 compares the results obtained using the SGA to our simulation results and shows an interesting phenomenon. It is widely recognized that the SGA slightly over-estimates the average BER, when the Ricean K-factor is $\kappa = 0$, i.e. for Rayleigh fading. This has also been reported in [7]. By contrast, when κ increases to 9, the SGA under-estimates the average BER. Although not shown explicitly here, if we have $\kappa \to \infty$, which corresponds to having no fading and no noise, only CCI, the SGA will more severely under-estimate the average BER, which has been reported in the context of AWGN channels [5].

$$\Phi_{I_k|\lambda_0,\lambda_1}(\omega) = \begin{cases} \frac{x}{\lambda_1}\,\mathbb{F}^{1:0;0}_{1:0;1}\left(\begin{matrix}\left[(\frac{1}{2}):1,2\right]:-\,; & - & ; \\ \left[(\frac{3}{2}):1,2\right]:-\,; & [(1):1]\,; \end{matrix}\; -\frac{1}{4}(\sigma_{kx}^2+\sigma_{ky}^2)\omega^2x^2, \frac{1}{64}(\sigma_{kx}^2-\sigma_{ky}^2)^2\omega^4x^4\right)\Bigg|_{\lambda_0}^{\lambda_0+\lambda_1}, & \lambda_1 \neq 0, \\ \exp\left[-\frac{1}{4}\left(\sigma_{kx}^2+\sigma_{ky}^2\right)\lambda_0^2\omega^2\right]\mathbb{I}_0\left[\frac{1}{4}\left(\sigma_{kx}^2-\sigma_{ky}^2\right)\lambda_0^2\omega^2\right], & \lambda_1 = 0, \end{cases} \tag{16}$$

where $\mathbb{F}^{A:B^{(1)};...;B^{(n)}}_{C:D^{(1)};...;D^{(n)}}\left(\begin{matrix}\left[(a):\theta^{(1)},...,\theta^{(n)}\right]:\left[(b^{(1)}):\phi^{(1)}\right];...;\left[(b^{(n)}):\phi^{(n)}\right]; \\ \left[(c):\psi^{(1)},...,\psi^{(n)}\right]:\left[(d^{(1)}):\delta^{(1)}\right];...;\left[(d^{(n)}):\delta^{(n)}\right];\end{matrix} x_1,...,x_n\right)$ is the generalized Lauricella function of n variables defined as Equations 21 - 23 of [14].

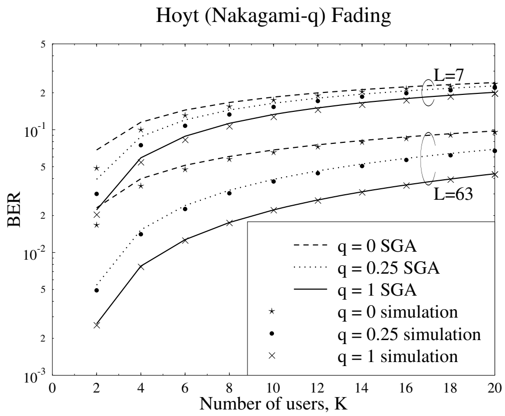

Fig. 4. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L = 7$ and 63. The Hoyt fading parameter is $q = 0$ (one-sided Gauss), 0.25 and 1 (Rayleigh). The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\text{SNR}} = \infty$.

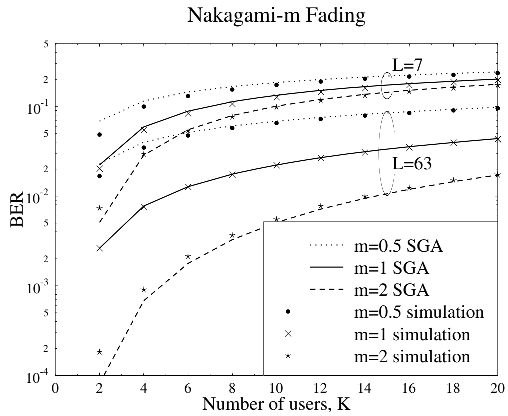

Fig. 6. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Nakagami-m channels. The length of the random spreading sequences is $L = 7$ and 63. The Nakagami-m fading parameter is $m = 0.5$ (one-sided Gauss), 1 (Rayleigh) and 2. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\text{SNR}} = \infty$.

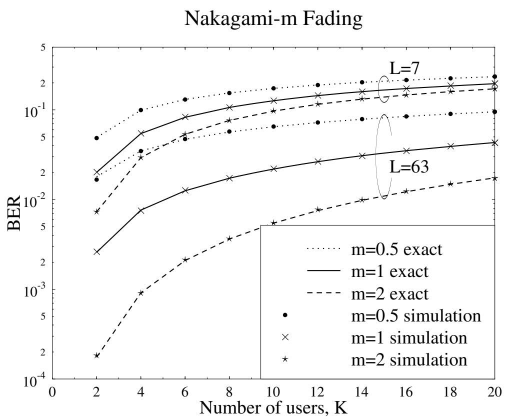

Fig. 5. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Nakgami-m channels. The length of the random spreading sequences is $L = 7$ and 63. The Nakgami-m fading parameter is $m = 0.5$ (one-sided Gauss), 1 (Rayleigh) and 2, which is common to all users. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\text{SNR}} = \infty$.

Figures 3 and 4 illustrate the average BER performance versus the number of users in the context of Hoyt fading channels, when the effects of background noise are ignored. Figure 3 compares the results obtained from our exact BER analysis to our simulation results and shows that they match very well both for different spreading sequence lengths and for various Hoyt fading parameters. On the other hand, Figure 4 compares the results obtained using the SGA to our simulation results and demonstrates that the SGA over-estimates the average BER, especially in the scenarios where either there is a limited number of interferers, or when the Hoyt fading parameter q is small or when short spreading sequences are used.

Figures 5 and 6 illustrate the achievable average BER performance versus the number of users in the context of Nakagami-m fading channels, when the effects of background noise are ignored. Figure 5 compares the results obtained from our exact BER analysis to our simulation results and shows that they match well both for various spreading sequence lengths and for various Nakagami-m fading parameters. On the other hand, Figure 6 compares the results obtained using the SGA to our simulation results and shows a similar phenomenon to that seen in Figure 2. The SGA slightly over-estimates the average BER when the Nakagami-m fading parameter is low, while it marginally under-estimates the average BER, when the Nakagami-m fading parameter is high.

VI. Conclusion

A unified approach has been proposed for the exact average BER analysis of an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation for communicating over various fading channels. Several closed-form expressions were derived for the conditional CFs of the interfering signals in various fading channels. A unified exact BER expression was provided, which requires only a single numerical integration. Our simulation results verified the accuracy of our exact BER analysis for various combinations of the spreading sequence length and the fading parameters. Furthermore, the SGA remains fairly accurate for most practical scenarios, although slightly over-estimates the average BER when the fading is severe, while it under-estimates the average BER, when the fading is benign, especially when either there is a low number of interferers, or the SNR is high and short spreading sequences are used.

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