Critical train speeds and associated track movements –
a case study

Vitesse critique et mouvements de la voie associés – Etude de cas

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ABSTRACT As train speeds are increased, ballasted railway tracks that have previously performed acceptably may experience large movements as a result of what are commonly termed critical velocity effects. These occur when the train speed approaches the speed of surface (Rayleigh) waves in the underlying ground, and can lead to increased rates of track geometry degradation, poor ride quality and increased maintenance costs. Critical velocity effects are also a potential concern for new high-speed lines. An improved understanding of the causes of the ground and track movements, through field instrumentation and modelling, will help to identify potentially problematic locations and to develop more cost-effective remediation methods. This paper presents the results of a study of the ability of a semi-analytical model (TGV) to calculate realistic ground movements at train speeds approaching the critical velocity. Several ground geometries and loading cases are considered, and a site on the classic railway network is used as a case study for validation purposes. The track at the study site experienced large displacements following an increase in line speed from 160 km/h to 200 km/h. Geotechnical investigations showed that the site is underlain by a horizon of peat of low stiffness. By refining the parameters used in the model, assuming that the peat horizon was the primary cause of the large track movements at high speed, it was possible to obtain reasonably close agreement between site measurements and the model in both quasi-static and dynamic analyses.

RÉSUMÉ Lorsque la vitesse des trains est augmentée, des voies ferrées ballastées n’ayant pas posé de problèmes lors de leur utilisation habituelle se déplaceront de façon importante. C’est le résultat de ce que l’on appelle les effets de la vitesse critique. Ils surviennent quand la vitesse du train approche celle des ondes de surface (Rayleigh) du sol sous le ballast. Ces effets engendrent une dégradation accélérée de la voie, un inconfort pour le passager et des coûts de maintenance élevés. Une compréhension approfondie des causes des mouvements du sol et de la voie soumis aux effets de la vitesse critique, par l’intermédiaire de modélisation et d’instrumentation, permettra de repérer plus facilement les zones à risques et d’améliorer sensiblement le rapport coût-efficacité des méthodes de maintenances actuelles. Cet article présente les résultats d’une étude sur les capacités d’un modèle semi-analytique à prédire les mouvements de terrains lorsqu’un train s’approche de la vitesse critique du sol. Différents chargements et natures des sols ont été considérés et un site du réseau ferré a été utilisé pour évaluer les résultats du modèle. Sur le site, l’amplitude des déplacements de la voie pour des trains roulants à 200 km/h furent nettement plus importants que pour des trains à 160 km/h. Une étude géotechnique a montré que le site était situé sur une tourbière peu rigide. Les paramètres utilisés dans le programme furent modifiés, en supposant que la faible rigidité de la tourbière était la cause principale des déplacements importants mesurés à haute vitesse. Des résultats relativement proches avec les mesures sur le site furent obtenus, en analyse quasi-statique comme en analyse dynamique.

1 INTRODUCTION

In many countries high-speed rail is playing a growing role in improving the capacity, availability and carbon footprint of national infrastructure. There is pressure to increase speeds on existing lines as well as constructing new routes. Higher speeds require straighter railway lines, which often mean crossing areas of soft ground that have traditionally been avoided. The soils involved are typically peats, organic clays and soft marine clays with shear wave velocities as low as 30 m s−1. Trains running at speeds of up to 300 km/h (83 m s−1) are therefore increasingly likely to approach or exceed the speed of the ground-borne surface waves, known as Rayleigh waves. When the train speed approaches the Rayleigh wave...
speed of the underlying ground, large vertical track movements can occur, increasing track degradation rates and maintenance costs as well as affecting ride quality. These so-called critical velocity effects have been observed in many countries worldwide. An improved understanding of the cause, and the ability to model potentially problematic locations would allow more cost-effective mitigation measures to be implemented.

Critical velocity effects were first reputed by De Nie (1948) after making experimental observations of large deflections of railway lines on soft soil. Theoretical work in this area was then developed by several authors (e.g. Kenney, 1954), with Cole and Huth (1958) presenting a solution for a moving load exciting a homogeneous elastic half-space, for speeds below, at and above the ground’s Rayleigh wave speed. Later authors such as Krylov (1998) and Sheng et al. (1999) incorporated layered ground geometry. This theoretical work has since been substantiated by various case studies, for example by Karlström and Boström (2006), Kaynia et al. (2000), Krylov (1995). Recently 2.5D models, based on a wavenumber transform in the longitudinal direction, have been used in an attempt to reach a balance between model-efficiency and flexibility (e.g. Sheng et al., 2006). There are still, however, very few sites for which data and observations are available in the literature to allow model validation. This paper presents an investigation into the ability of an existing model (TGV-Sheng et al., 2004) to calculate velocity-dependent displacements, up to and beyond the critical velocity, using relatively simple ground geometry models and estimates of soil parameters. This was carried out in conjunction with measurements at a site in the UK. The model set-up and the site measurements are described, and example results are given and discussed.

2 CASE STUDY BACKGROUND

2.1 Site Information

The study site was found to experience large track displacements following an increase in line speed from 160 km/h to 200 km/h. Geotechnical investigations showed the site to be underlain by a horizon of peat, over layers of stiffer sand, clay and gravel (Table 1). The low stiffness of the peat is believed to be the main cause of the large track movements when the train speed was increased. Borehole results were only available along the line of the track; no information was available in the direction perpendicular to the track. A range of trains run on this line, with the fastest (Class 390) being selected for this case study.

Table 1. Site borehole results summary. 10 m spacing between boreholes.

<table>
<thead>
<tr>
<th>Ground Type</th>
<th>BH1</th>
<th>BH2</th>
<th>BH3</th>
<th>BH4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Made Ground-Gravel</td>
<td>0 to 1.0</td>
<td>0 to 1.0</td>
<td>0 to 0.8</td>
<td>0 to 0.9</td>
</tr>
<tr>
<td>Made Ground-Sand</td>
<td>1.0 to 1.3</td>
<td>1.0 to 1.2</td>
<td>0.8 to 1.0</td>
<td>0.9 to 1.3</td>
</tr>
<tr>
<td>Peat</td>
<td>1.3 to 3.2</td>
<td>1.2 to 3.0</td>
<td>1.0 to 5.0</td>
<td>1.3 to 3.0</td>
</tr>
<tr>
<td>Sand</td>
<td>3.2 to 4.4</td>
<td>3.0 to 5.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Clay</td>
<td>4.4 to 6.7</td>
<td>-</td>
<td>5.0 to 6.0</td>
<td>-</td>
</tr>
<tr>
<td>Gravel</td>
<td>-</td>
<td>-</td>
<td>6.0 to 7.0</td>
<td>3.0 to 5.0</td>
</tr>
</tbody>
</table>

2.2 Geophone Monitoring

Track movements at the site were monitored using geophones, small seismic devices which output a voltage proportional to velocity. The signal is then filtered and integrated to obtain the displacement. The monitoring and analysis methods used are described in Bowness et al. (2006). Geophones were attached to nine alternate sleepers allowing movements over a 10 m length of the track to be recorded. Figure 1 shows the site with geophones installed.

Figure 1. Case study site, with geophones installed on sleepers.

Vertical deflection measurements were taken during the passage of 11 trains, of varying type (class) and consisting of between 3 and 11 cars. Figure 2 shows typical processed geophone data in the form of vertical displacement against time. Owing to the high-pass filtering applied before integration, the trace apparently contains both upward and downward displacements whereas in reality the motion of the sleepers is predominantly downwards. The parameter
of interest, which is obtained correctly from the analysis, is the peak-to-trough displacement amplitude under each axle passage, which in this example is more than 6 mm.

![Geophone trace showing vertical displacement of a sleeper when passed by a 9-car Class 390 at 195 km/h (54 m/s).](image)

**Figure 2.** Geophone trace showing vertical displacement of a sleeper when passed by a 9-car Class 390 at 195 km/h (54 m/s).

3 BASIS OF THE MODEL

Analyses were carried out using TGV, a semi-analytical model for ground vibration in the wave-number domain developed at the University of Southampton (Sheng et al., 2004).

The ground is modelled as a layered half-space using flexibility matrices in terms of wavenumbers \( k_x \) and \( k_y \) in the \( x \) and \( y \) directions. TGV requires inputs for the moving axle loads and vertical rail irregularities.

![Schematic of TGV model for track-ground system](image)

**Figure 3.** Schematic of TGV model for track-ground system (Sheng et al., 2003)

Vehicles are represented as multiple rigid body systems and only the vertical dynamics is considered. The track is modelled as multiple beams supported by vertical springs with identical mass (Figure 3), where \( P_1(t), \ldots, P_n(t) \) represent the vertical wheel-rail forces for \( n \) wheelsets. It is assumed that each wheelset is always in contact with the rails.

4 MODEL SET-UP

4.1 Ground Geometry and Loading Types

Three types of ground geometry can be adopted within TGV:

1. A homogeneous half-space,
2. One or more layers of ground of specified thickness above a rigid foundation,
3. One or more layers of ground of specified thickness above a stiffer half-space.

As the soft peat layer is the ground material of interest on the site, three alternative ground geometries were chosen to represent the site (Figure 4). The rigid foundation and the stiffer half-space are both methods of representing the comparatively stiffer layers of sand, gravel and clay beneath the peat.

![Model ground geometry types.](image)

**Figure 4.** Model ground geometry types.

For this study the following loading configurations were used in TGV:

1. A single moving load (a moving point load producing quasi-static deflection),
2. A single vehicle (allowing interaction of the displacements from the two wheelsets in a bogie),
3. Four vehicles (representing a train, including dynamic excitation due to track unevenness).

For case 3, as well as the moving load, excitation frequencies from 0.25 to 120 Hz with 80 logarithmically spaced steps were used. By limiting the overall model size and mesh density it was possible to run all models on a desktop computer. A length and width of 268 m was sufficient to accommodate a 4 car train length (116 m) and the decaying waves.

4.2 Initial Ground Parameters

Limited measurements of the ground wavespeeds made beside the track at the study site provided an initial estimate of the dilational (P) and shear (S)
wave speeds for the lower layers of sand and gravel, with a P wave speed of 768 ms\(^{-1}\) and an S wave range of 95-200 ms\(^{-1}\). The S wave speed of the sand layer, 95 ms\(^{-1}\), was selected as representative of the ground beneath the peat.

It was not possible to estimate the wave speed of the peat from the site measurements, so the S wave speed was set to 53 ms\(^{-1}\) (the train speed causing the largest measured movements on the site). The Poisson’s ratio was set to 0.11 (Rowe et al., 1984) and the density to 1050 kg/m\(^3\), allowing the P wave speed and Young’s modulus to be derived. The derived Young’s modulus of 6.3 MPa is reasonable for slightly clayey peat, with all other ground parameters assigned typical values for that ground type (Table 2).

With ground geometry types 2 and 3, the thickness of the peat layer was set at 2 m, representative of the range in the boreholes of 1.4 to 4 m.

Table 2. Initial Ground Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Peat</th>
<th>Stiffer Half-space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, kg/m(^3)</td>
<td>1050</td>
<td>2000</td>
</tr>
<tr>
<td>Damping loss factor – constant, (Twice the damping ratio)</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>S wave speed, ms(^{-1})</td>
<td>53</td>
<td>95</td>
</tr>
<tr>
<td>P wave speed, ms(^{-1})</td>
<td>79</td>
<td>768</td>
</tr>
</tbody>
</table>

4.3 Track and Train Parameters

Parameter values typical of UK track were used (Table 3) for UIC60 rail and mono-block concrete sleepers. Where possible train parameters specific to Class 390’s were adopted, otherwise values typical of higher speed passenger trains were used (Table 4).

Table 3. Track Parameters (for a single track, i.e. two rails).

<table>
<thead>
<tr>
<th>Track Parameter</th>
<th>Value</th>
<th>Track Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail mass, kg/m</td>
<td>120</td>
<td>Rail pad damping loss factor</td>
<td>0.2</td>
</tr>
<tr>
<td>Sleeper mass, kg/m</td>
<td>461.5</td>
<td>Rail damping loss factor</td>
<td>0.01</td>
</tr>
<tr>
<td>Ballast mass, kg/m</td>
<td>1740</td>
<td>Ballast damping loss factor</td>
<td>0.04</td>
</tr>
<tr>
<td>Rail bending stiffness, Nm(^2)</td>
<td>1.29x10(^7)</td>
<td>Track width, m</td>
<td>2.5</td>
</tr>
<tr>
<td>Ballast stiffness, N/m(^2)</td>
<td>4.64x10(^8)</td>
<td>Ballast width, m</td>
<td>3.1</td>
</tr>
<tr>
<td>Rail pad stiffness, N/m(^2)</td>
<td>3.69x10(^8)</td>
<td>Track roughness, FRA class</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4. Class 390 Vehicle Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axle spacing, m</td>
<td>2.7</td>
<td>Wheelset mass, kg</td>
<td>1750</td>
</tr>
</tbody>
</table>

5 RESULTS AND DISCUSSION

5.1 Initial Parameters

Using the initial parameters given in Section 4, all three ground geometry types were run with all three load cases. Loading case 1 (a single load) is unable to account for interaction between train wheelsets, thus the calculated displacements were significantly less than those calculated using loading case 2 (a single vehicle). Figure 5 shows the maximum peak-to-trough displacements calculated using loading case 2 for all three ground types, plotted as a function of train speed. Measured results are also included for comparison.

It is clear that introducing a stiffened or rigid ground element beneath the peat has a considerable effect, with the full peat half-space (type 1) substantially over-estimating the displacements. Although this model is useful for carrying out parametric studies, it is too simple to represent the site accurately.

The peat layer above a rigid foundation (type 2) and the peat layer over a stiffer half-space (type 3) both match the shape of the curve of the site displacements well, but over-predict the displacements. Although ground model types 2 and 3 give similar results for this site, it is unlikely that type 2 would be suitable for other sites unless there were a similarly extreme stiffness disparity between the different ground layers.

With ground model type 3, the displacements are consistently around 30% greater than those measured on site, thus the basic parameter estimates used need to be adjusted to match the site. The excessive displacements may be partly attributable to the lack of confinement of the peat layer in the TGV model as each layer is considered to be of infinite lateral ex-
tent. The inability to represent the varying thickness of the peat horizon along the track evident in the borehole records may also have had an impact on the results.

Figure 5. Displacement vs. load speed for a single vehicle running across ground types 1, 2 and 3 using initial parameters. Geophone measurement results for the site also presented.

5.2 Refining the Parameters

To test further the potential of TGV to model critical velocity effects, model type 3 (a peat layer above a stiffer half-space) was re-run with a large number of parameter combinations in an attempt to match more closely the site measurements. The objective was to reduce both the low speed deflections and the critical velocity. Several parameters may be adjusted to reduce the calculated displacements; first the Young’s modulus, density and Poisson’s ratio for the peat were varied. When adjusted individually the parameters have different effects on the displacement-speed curve.

Increasing the Young’s modulus reduces the displacements but also increases the critical speed, whereas increasing the density maintains the general shape of the displacement curve while reducing the critical velocity and slightly decreasing displacements, owing to its impact on wave speeds. Increasing the peat damping reduces the slope of the curve and hence the peak displacements, but it also increases the critical velocity. An increase in the Poisson’s ratio reduces displacements, especially at lower speeds.

Secondly the S-wave speed of the stiffer half-space was increased to 130 ms$^{-1}$. This is still within the range measured on site and is equivalent to a Young’s modulus of 100 MPa, a reasonable value to represent a combination of sands and gravels. Table 5 shows the parameter combination giving the best fit.

Table 5. Refined Ground Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Peat Value</th>
<th>Stiffer Half-space Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1600 kg/m$^3$</td>
<td>2000 kg/m$^3$</td>
</tr>
<tr>
<td>Damping loss factor - constant</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>S wave speed</td>
<td>35 ms$^{-1}$</td>
<td>130 ms$^{-1}$</td>
</tr>
<tr>
<td>P wave speed</td>
<td>1400 ms$^{-1}$</td>
<td>768 ms$^{-1}$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>5.88 MPa</td>
<td>100.01 MPa</td>
</tr>
</tbody>
</table>

Figure 6 shows the resulting displacement curve for the refined parameters. A close match to the site measurements is achieved. The calculated critical velocity of 67 ms$^{-1}$ is greater than the speed of the fastest trains on the site and so cannot be validated. The variation in the fastest trains measured deflections is thought to have been due to variations in load, not that critical velocity is being reached. The parameters required to produce this result are generally reasonable except for the density and Young’s modulus of the peat, which are unusually high. However, this could be interpreted as aggregating and averaging out the properties needed to reproduce the measured behaviour for a lens of peat confined by much stiffer, denser materials.

Figure 7 shows the displacement of the ground surface, including the wave behind the train. This illustrates the importance of allowing for the superposition of displacements between axles. Figure 8 shows the results of running the full vibration analysis including the dynamic excitation. The frequency spec-
trum of the vibration velocity is shown, and a close match to the site measurements is achieved. The response to quasi-static excitation is dominant at low frequencies and the dynamic component at high frequencies. At this train speed (53 ms\(^{-1}\)) the transition frequency between these two regimes is 30Hz. As train speed decreases so does this transition; e.g. it is found to be at 20 Hz at 25 ms\(^{-1}\).

**Figure 7.** 3D plot of ground surface displacement, for a single vehicle (type 2 model) travelling at 67 ms\(^{-1}\) across a 2 m peat layer (refined peat parameters) above a stiffer half-space.

**Figure 8.** Full train pass-by vibration response for 4 vehicles running at 53 ms\(^{-1}\) across ground type 3, using refined peat parameters. Site measurements also presented.

### 6 CONCLUSIONS

A study has been presented of critical speed effects using a site in the UK. Initial calculations made using best estimates of ground properties from the site and the 2.5D TGV model led to much larger deflections than were measured. However, by refining the soil properties it was possible to achieve a much closer agreement between the modelled and measured track displacements and frequency spectra. It is proposed these refined properties represent an aggregation of a lens of peat confined by much stiffer denser material. This aggregation of properties was necessary due to the limitations of the 2.5D model in being unable to reproduce the limited lateral extent of the peat layer.

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