Refinement in Practice

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Abstraction

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- The simplification should:
  - focus on the intended purpose of the system
  - ignore details of how that purpose is achieved.
- The modeller needs to make judgements about what they believe to be the key features of the system.
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- If the purpose is to provide some service, then
  - model what a system does from the perspective of the service users
  - ‘users’ might be computing agents as well as humans.
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- If the purpose is to provide some service, then
  - model what a system does from the perspective of the service users
  - ‘users’ might be computing agents as well as humans.
- If the purpose is to control, monitor or protect some phenomenon, then
  - the abstraction should focus on those phenomenon
  - in what way should they be controlled or protected?
Access control system

- Users are authorised to engage in activities
- User authorisation may be added or revoked
- Activities take place in rooms
- Users gain access to a room using a one-time token provided they have authority to engage in the room activities
- Tokens are issued by a central authority
- Tokens are time stamped
- A room gateway allows access with a token provided the token is valid
This model is unnecessarily complex to specify the main access control policy.
Extracting the essence

• **Access Control Policy**: *Users may be in a room only if they are authorised to engage in all activities that may take place in that room*

• To express this we only require **Users, Rooms, Activities** and relationships between them

• **Abstraction**: focus on key entities in the problem domain
Diagrammatic representation of an abstract model

USER --- authorised --- ACTIVITY

ROOM --- location --- takeplace
Variables and invariants of Event-B model

Variables of Event-B model

@inv1  authorised ∈ User ↔ Activity  // relation
@inv2  takeplace ∈ Room ↔ Activity  // relation
@inv3  location ∈ User ↦ Room  // partial function

Access control invariant:
if user u is in room r,
then u must be authorised to engaged in all activities that can take place in r

@inv4  ∀ u,r . u ∈ dom(location) ∧ location(u) = r  ⇒
takeplace[r] ⊆ authorised[u]
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  - augment the functionality being modelled, or
  - explain how some purpose is achieved
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- Event-B provides a notion of *consistency* of a refinement:
  - We use proof to *verify* the consistency of a refinement step
  - Failing proof can help us *identify inconsistencies* in a refinement step
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- Abstraction and refinement together should allow us to **manage system complexity** in the design process
Modelling Components and Refinement

machine $m$

variables $v$

invariants $l$

events $e_1, e_2, ...$

context $ctx$

sets $s$

constants $c$

axioms $x$

sees
Modelling Components and Refinement

- **machine** \( m \)
- **variables** \( v \)
- **invariants** \( I \)
- **events** \( e_1, e_2, \ldots \)

- **context** \( ctX \)
- **sets** \( s \)
- **constants** \( c \)
- **axioms** \( x \)

- **sees**
- **refines**
- **extends**
Extension Refinement in Event-B

A refined machine has the following form:

```plaintext
machine M2
refines M1
variables ...
invariants ...
events...
```

Extension refinement can be used to extend or add new features to a model.

- Add variables and invariants
- Extend existing events to act on additional variables
- Add new events to act on additional variables

All events must maintain the new invariants.
Event Extension

Event $E_1$ of $M_1$ may be extended by $E_2$ in $M_2$:

$$E_2 \text{ extends } E_1 \equiv \begin{array}{l}
\text{any } \langle \text{additional parameters} \rangle \quad \text{where} \\
\quad \langle \text{additional guards} \rangle \\
\text{then} \\
\quad \langle \text{additional actions} \rangle \\
\text{end}
\end{array}$$
Event Extension

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any $\langle$ additional parameters $\rangle$ where $\langle$ additional guards $\rangle$

then $\langle$ additional actions $\rangle$

end

Extending an event means

- adding parameters
- adding guards
- adding actions
Extension example: add ownership to secure database
Class diagram for secure database

- **object** \( \subseteq \) **OBJECT**
- **data** \( \rightarrow \) **DATA**
- **LEVEL**
- **clear** \( \rightarrow \) user \( \subseteq \) **USER**
context $c_1$
sets $OBJECT$ $DATA$ $USER$
constants $LEVEL$
axioms $LEVEL = 1..10$

machine $SecureDB_1$
sees $c_1$
variables $object$, $user$, $data$, $class$, $clear$

invariants

$$object \subseteq OBJECT$$
$$user \subseteq USER$$
$$data \in object \rightarrow DATA$$
$$class \in object \rightarrow LEVEL$$
$$clear \in user \rightarrow LEVEL$$
Adding object ownership

Extend the database specification so that each object has an owner.

The clearance associated with that owner must be at least as high as the classification of the object.

Only the owner of an object is allowed to delete it.

What additional variables are required?

What events are affected?
Class diagram with ownership

object \subseteq \text{OBJECT} \rightarrow \text{class} \rightarrow \text{LEVEL}

data \rightarrow \text{owner} \rightarrow \text{user} \subseteq \text{USER}

\text{clear} \rightarrow \text{LEVEL}
Refinement

\hspace{1cm} \textbf{machine} \hspace{0.5cm} SecureDB2
\hspace{1cm} \textbf{refines} \hspace{0.5cm} SecureDB1
\hspace{1cm} \textbf{variables} \hspace{0.5cm} object, user, data, class, clear, owner
\hspace{1cm} \textbf{invariants}

\hspace{4cm} \text{owner} \in \text{object} \rightarrow \text{user}

Note we must list \textbf{all} the variables: those from M1 that we wish to retain as well as new ones

Here \textit{owner} is a new variable.

We do not repeat invariants of M1 in M2.
Adding users

\[
AddUser \overset{\triangleq}{=} \\
\text{any } u, c \text{ where } \\
u \in \text{USER} \\
u \notin \text{user} \\
c \in \text{LEVEL} \\
\text{then} \\
\text{user} := \text{user} \cup \{u\} \\
clear(u) := c \\
\text{end}
\]

Do we need to modify this?
Adding objects

\[ \text{AddObject} \triangleq \]
\[ \text{any } o, d, c \text{ where } \]
\[ o \in \text{OBJECT} \]
\[ o \notin \text{object} \]
\[ d \in \text{DATA} \]
\[ c \in \text{LEVEL} \]
\[ \text{then} \]
\[ \text{object} := \text{object} \cup \{o\} \]
\[ \text{data}(o) := d \]
\[ \text{class}(o) := c \]
\[ \text{end} \]

Do we need to modify this?
Event Extension

AddObject extends AddObject ≜
  any u where
    u ∈ user
    clear(u) ≥ class(o)
  then
    owner(o) := u
end
This is equivalent to

\[
\text{AddObject \, refines \, AddObject} \quad \triangleq \\
\text{any } o, d, c, u \text{ where} \\
o \in \text{OBJECT} \\
o \notin \text{object} \\
d \in \text{DATA} \\
c \in \text{LEVEL} \\
u \in \text{user} \\
clear(u) \geq \text{class}(o) \\
\text{then} \\
\text{object} := \text{object} \cup \{o\} \\
data(o) := d \\
\text{class}(o) := c \\
\text{owner}(o) := u \\
\text{end}
\]
Other events to consider

- Read
- Write
- ChangeClass
- ChangeClear
- RemoveUser, RemoveObject

Do we need new events?
Forms of Event-B Refinement

1. Extension:
   - Add variables and invariants
   - Extend existing events to act on additional variables
   - Add new events to act on additional variables
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2. Extension with Guard Modification:
   - Similar to model extension, except that we modify guards of existing events

Verification of 2, 3 and 4 requires gluing invariants that link abstract and concrete variables.
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   - Replace some variables with other variables, i.e., replace abstract variables with concrete variables
   - Modify existing events, add new events

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Verification of 2, 3 and 4 requires gluing invariants that link abstract and concrete variables.
Extension example: add ownership to secure database
Extension with Guard Modification example: add tokens to buildings access system
Variable replace example: simple data sampling system
Abstract model of building access

- register \( \subseteq \text{USER} \)
- permission
- location
- BUILDING
Refine this by introducing a token mechanism
Refinement of access control with tokens

\[ EnterBuilding \triangleq \text{any } u, b \text{ where } \]
\[ u \notin \text{dom}(\text{location}) \]
\[ u \leftrightarrow b \in \text{permission} \]
\[ \text{then} \]
\[ \text{location}(u) := b \]
\[ \text{end} \]

\[ \text{RefinedEnterBuilding} \triangleq \text{any } u, b, t \text{ where } \]
\[ u \notin \text{dom}(\text{location}) \]
\[ t \in \text{valid} \]
\[ \text{tusr}(t) = u \]
\[ \text{tloc}(t) = b \]
\[ \text{then} \]
\[ \text{location}(u) := b \]
\[ \text{end} \]
We need to prove that the guard of a refined event is not weaker than the guard of the abstract event. E.g., the refined enter event should not weaken the conditions under which a user may enter a room.
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GRD Proof obligation:
Assume: guard(RefinedEnter) + invariants
Prove: guard(AbstractEnter)
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GRD Proof obligation:
Assume: \( \text{guard(RefinedEnter)} \) + invariants
Prove: \( \text{guard(AbstractEnter)} \)

For the access control refinement, we need this invariant:

\[
\forall t \cdot t \in \text{valid} \implies \text{tusr}(t) \leftrightarrow \text{tloc}(t) \in \text{permission}
\]
Simple data sampling system

machine MaxSet1
variables samples
invariants samples ⊆ N
initialisation samples := {0}
events

\[ Add \overset{\triangle}{=} \text{any } x \text{ where } x \in \mathbb{N} \]
then
\[ samples := samples \cup \{x\} \]
end

GetMax \overset{\triangle}{=} \text{any } result \text{ where } result = \max(samples) 
end
Refine to a more optimal design

machine MaxSet2
refines MaxSet1
variables $m$ we only need to store the maximum so far
invariants $m \in \mathbb{N}$
initialisation $m := 0$
events

$$ Add \triangleq \text{any } x \text{ where }\begin{align*}
x & \in \mathbb{N} \\
\text{then} & \\
& m := \max(\{m, x\}) \\
\text{end} \end{align*} $$

$$ GetMax \triangleq \text{any result where }\begin{align*}
\text{result} & = m \\
\text{end} \end{align*} $$
Gluing invariant

What is the relationship between \( m \) and \( \text{samples} \)?
Gluing invariant

What is the relationship between \( m \) and \( \text{samples} \)?

```
machine MaxSet2
refines MaxSet1
variables \( m \)
invariants \( m = \max(\text{samples}) \)
events...
```

This is called a gluing invariant: it specifies the relationship between the abstract and refined variables.
Proving that the gluing invariant is maintained

Abstract Add: \( samples := samples \cup \{x\} \)

Refined Add: \( m := \max(\{m, x\}) \)
Proving that the gluing invariant is maintained

Abstract Add: \( \text{samples} := \text{samples} \cup \{x\} \)

Refined Add: \( m := \max(\{m, x\}) \)

Assume: \( m = \max(\text{samples}) \)

Prove: \( \max(\{m, x\}) = \max(\text{samples} \cup \{x\}) \)
Proving that the gluing invariant is maintained

Abstract Add: \( \text{samples := samples } \cup \{x\} \)

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Assume: \( m = \max(\text{samples}) \)

Prove: \( \max(\{m, x\}) = \max(\text{samples } \cup \{x\}) \)

This is valid since:

\( \max(s \cup \{x\}) = \)
Proving that the gluing invariant is maintained

Abstract Add: \( \text{samples} := \text{samples} \cup \{x\} \)

Refined Add: \( m := \max(\{m, x\}) \)

Assume: \( m = \max(\text{samples}) \)

Prove: \( \max(\{m, x\}) = \max(\text{samples} \cup \{x\}) \)

This is valid since:

\[
\max(s \cup \{x\}) = \max(\{\max(s), x\})
\]
Closing Messages

- Role of **formal modelling:**
  - increase understanding
  - decrease errors
- Role of **refinement:**
  - manage complexity through multiple levels of abstraction
- Role of **verification:**
  - improve quality of models (consistency, invariants)
- Role of **tools:**
  - make verification as automatic as possible, pin-pointing errors