Chapter 6. Measured compressional wave properties of inter-tidal sediments

This section presents the measured group velocities, effective attenuation coefficients and quality factors of the inter-tidal sediments described in *Section 4.4.3* and examines their dependence on frequency and geotechnical parameters. Velocity, attenuation coefficient and quality factor will be plotted relative to the mean central frequency of the received signals f_r (see *Equation 5.4, Section 5.1*). This frequency has been selected as the best estimate of the central frequency of the acoustic wave propagated through the sediment. These frequencies vary from 15.98 to 96.62 kHz, with errors from $\pm$ 0.48 to $\pm$ 2.89 kHz and are lower than the central frequencies of the *voltage output pulse* f_o. The downshift in frequency is a result of the preferential attenuation of higher frequency components within the sediment. The frequency range for which compressional wave properties are presented varies from location to location, owing to the rejection of spurious data, *Section 5.2.2.*

6.1. Frequency-dependence of compressional wave properties

The inter-tidal sediments examined can be classified into two groups, medium to fine sands and medium to fine silts, with the range of geotechnical properties of each group presented in *Section 4.4.3.* The frequency-dependence of velocity, attenuation coefficient and quality factor will therefore be examined for the sandy and silty sediments separately. These results are presented in *Figures 6.1* to *6.7*, with errors in frequency omitted to allow the frequency-dependent trend to be more clearly observed.

6.1.1. Sandy sediments

Velocities, attenuation coefficients and quality factors are displayed for the three inter-tidal sites which possessed sandy sediments, *i.e.* Studland *(Figure 6.1)*, Branksome (*Figure 6.2*) and Lilliput (*Figure 6.*3). Data obtained at certain locations and frequencies were considered to be anomalous and hence omitted from the following analysis, see *Appendix D* for details.

Velocities from the remaining frequencies and locations ranged from 1590 to 1880 $m \cdot s^{-1}$, with typical errors ranging from $\pm$ 20 to $\pm$ 70 $m \cdot s^{-1}$. This range extends higher than that previously measured in sands, *i.e.* 1580 to 1797 $m \cdot s^{-1}$ for frequencies of 125 Hz to 100 kHz (see *Table 2.1*, *Section 2.3.1)*. Though velocities measured at the three sites display a considerable degree of overlap, values are generally greater for the coarser, less porous Branksome and Studland sites than the Lilliput site.

Values of attenuation coefficient from the selected frequencies and locations ranged from 2 to 52 dB·m^{-1}, with errors from $\pm$ 2 to $\pm$ 7 dB·m^{-1}. These values agree with published values that have been measured in sands using *in situ* techniques which span a similar frequency range to this project, *i.e.* 0.28 to 60 dB·m^{-1} for a frequency range of 3.5 to 100.0 kHz and 8 to 60 dB·m^{-1} for a frequency range of 25 to 100 kHz (see *Table 2.2).*

Quality factors were greater at the Branksome site (54 to 120) than at the Lilliput site (14 to 62) or Studland site (25 to 85). Errors were less than $\pm$ 35 for all frequencies and locations, except for Branksome 2 at frequencies less than 38 kHz and Lilliput 4 at frequencies less than 30 kHz. This reflects the manner in which quality factor was calculated from attenuation coefficient, velocity and frequency *(*see *Section 5.4).* Though quality factors of greater than 10 ensure that this approximation equation is valid (*Section 2.*1) the low values of attenuation coefficient which give rise to larger quality factors result in a large fractional error being transmitted through *(*see *Section 5.4)* and a large error in quality factor. The calculated quality factors for Studland and Lilliput broadly agree with previously reported values (*i.e.* less than 45 for medium to fine sands measured at frequencies from 1.5 to 100.0 kHz, *Figure 2.12*), with quality factors at Branksome exceeding this published range.

The variability of compressional wave properties across each site was also examined, with the maximum variation in velocity, attenuation and quality factor observed at a single frequency recorded, *Table 6.1.* The variability of compressional wave properties agreed with spread of the geotechnical properties over each site, with the Studland site possessing the largest spread in both compressional and geotechnical properties and the Branksome site possessing the least spread in both. Despite the minimal changes in the geotechnical properties at the Branksome site, *i.e.* a variation of 0.1 % in porosity and 0.27 ϕ in mean grain diameter, significant variations in compressional wave properties are still observed between the two locations examined, with velocity varying by 60 m·s^{-1}, attenuation coefficient by 9 dB·m^{-1} and quality factor by 25. This displays the non-uniqueness of marine sediment, with a single value of porosity or mean grain diameter being able to correspond to a range of frame structures (see *Section 2.4.1*).

Site	Variability of property				
	Porosity (%)	M (ϕ)	Velocity ($m\cdot s^{-1}$)	Attenuation coefficient ($dB\cdot m^{-1}$)	Quality factor
Studland	16.2	0.71	140	25	60
Branksome	0.1	0.27	60	9	25
Lilliput	7.6	0.03	110	20	46

Table 6.1. Variability of geotechnical and compressional wave properties at sand sites, including range of porosity and mean grain diameter across uncorrupted locations and maximum range of compressional wave properties at a single frequency.

The frequency-dependencies of velocity, attenuation coefficient and quality factor were tested using two basic hypotheses, namely that the compressional wave property is either independent of frequency, *i.e.* "adequately" described by a mean value, or proportional to frequency. These hypothesis were tested using a weighted mean, *Equation 5.26,* and a weighted linear least-square fit respectively (Barlow, 1989). The weighted linear least-square fits for velocity, attenuation coefficient and quality factor are displayed in *Equations 6.1* to *6.3* respectively;

$$v = v_A \cdot f + c_1 \qquad 6.1,$$

$$\alpha = k_A \cdot f + c_2 \qquad 6.2,$$

$$Q = Q_A \cdot f + c_3 \qquad 6.3,$$

where v_A, k_A and Q_A are the constants of proportionality and c_1, c_2 and c_3 are respective values of velocity, attenuation coefficient and quality factor corresponding to a frequency of zero. Note that *Equation 6.2* is a modified version of *Equation 2.11,* with q=1 and c_2 included to account for the possibility of a non-linear relationship between attenuation coefficient and frequency outside the frequency range examined within this project. The standard deviations in the mean values and constants of proportionality were also obtained and are quoted as the relevant errors in the remainder of *Section 6.1*.

For each hypothesis the deviation of the measured compressional wave property from weighted fit was converted into a confidence limit using the χ^2 distribution, see *Appendix E* for details. Confidence limits greater than or equal to 95 % are considered to indicate an acceptable fit between the weighted fit and measured values, with this threshold value used by the majority of statistical literature. The use of both χ^2 statistics and weighted fits assumes that the measured values are independent of one another. The pulses used in the *in situ* experiments possessed finite bandwidths, varying from 3 to 18 kHz for the *voltage output pulse* (see *Section 4.2.2)*, while the increments of the central frequencies used are approximately 2 kHz. Hence the measured velocity at a certain frequency is not independent of the velocities at neighbouring frequencies. Though this does not invalidate the approach adopted, it indicates that the "true" number of degrees of freedom is less that used, and hence the confidence limits obtained will be over-estimated, while resulting errors in mean velocities will be underestimated. This over-estimation in confidence limits and under-estimation in errors will also apply to the frequency-dependent examination of attenuation coefficient and quality factor

The resulting confidence limits for velocity are displayed in *Table 6.2*. The application of the more complex linear fit only improves confidence for one of the nine locations examined, with confidence in the weighted mean greater than a threshold value of 95 % for seven of the nine locations. These high confidence limits support negligible velocity dispersion within the errors observed for sandy sediments. Though the weighted linear fit possessed χ^2 values less than those for the weighted mean, these were not sufficiently different to improve the range of confidence values which the χ^2 value corresponded to (see *Appendix E)*. The weighted mean is observed to agree with the measured velocities within errors for all locations and frequencies except at frequencies from 20 to 26 kHz at Lilliput 1, less than 26 kHz at Branksome 2 and greater than 70 kHz at Studland 2. Concern is raised with respect to frequencies greater than 70 kHz at Studland 2, due to a low SNR. This may explain the larger and more uncertain values of velocity and quality factor and lower values of attenuation coefficient observed at these higher frequencies with respect to central frequencies below 70 kHz.

Location	Mean velocity ($m.\cdot s^{-1}$)	Confidence in weighted mean (%)	Confidence in weighted linear fit (%)
Studland 1	1668 $\pm$ 5	> 97.5	> 99.5
Studland 2	1712 $\pm$ 6	> 99.5	> 99.5
Studland 3	1783 $\pm$ 5	> 99.5	> 99.5
Studland 4	1760 $\pm$ 4	< 80.0	< 80.0
Branksome 1	1718 $\pm$ 6	> 99.5	> 99.5
Branksome 2	1782 $\pm$ 4	> 97.5	> 97.5
Lilliput 1	1631 $\pm$ 4	> 95.0	> 99.5
Lilliput 3	1703 $\pm$ 6	< 80.0	> 80.0
Lilliput 4	1647 $\pm$ 4	> 99.5	> 99.5

Table 6.2. Weighted mean velocity for nine uncorrupted sand locations examined. Confidence in weighted mean and weighted linear fit are included for comparison purposes.

As attenuation coefficient is clearly dependent on frequency, only the hypothesis that attenuation coefficient was proportional to frequency was tested, with the resulting fits plotted in *Figure 6.1E, 6.2E* and *6.3E.* For seven of the nine locations examined the linear fit possessed a confidence greater than 95 % (*Table 6.3*), which indicate an attenuation coefficient which is proportional to frequency in sands. As both global viscous losses and squirt flow are able to generate attenuation coefficients which are proportional to frequency (*Section 2.5.2*), no conclusions can be drawn concerning the dominant attenuation mechanism. The issue of energy loss mechanisms is discussed in more detail in *Section 7.2.*

The sharp nature of the deviations from the measured attenuation coefficients at Studland 4 from the weighted fit imply that these deviations are due to scatter in the measured values. Constants of proportionality k_A ranged from 0.48 to 0.52 $dB\cdot m^{-1}\cdot kHz^{-1}$ at Lilliput, 0.19 to 0.29 $dB\cdot m^{-1}\cdot kHz^{-1}$ at Branksome and 0.14 to 0.51 $dB\cdot m^{-1}\cdot kHz^{-1}$ at Studland, with errors less than 0.05 $dB\cdot m^{-1}\cdot kHz^{-1}$, *Table 6.3.* The weighted linear fit is observed to agree with the measured attenuation coefficients within errors for all frequencies and locations except for Lilliput 4 at frequencies greater than 85 kHz, Branksome 2 from 42 to 50 kHz, Studland 1 from 20 to 30 and 38 to 55 kHz and Studland 4 from 28 to 55 kHz.

The results of the examination of the frequency-dependence of quality factor are displayed in *Table 6.3.* The weighted linear fit, *Figures 6.1F, 6.2F* and *6.3F,* resulted in a higher confidence than the weighted mean, with confidence greater than 95 % for eight out of nine locations for the weighted linear fit and only four out of nine for the weighted mean. However the observation of constants of proportionality which possess errors greater than or equal to the absolute value at three locations, and negative constants of proportionality at four locations, introduces a significant degree of doubt to the reliability of the linear fit. Hence no conclusions can be drawn concerning the frequency-dependence of the quality factor in sands.

Location	Weighted linear fit in α		Weighted mean in Q		Weighted linear fit in Q	
	k_A ($dB \cdot m^{-1} \cdot kHz^{-1}$)	Confidence (%)	Mean Q	Confidence (%)	$Q_A \cdot 10^{-2}$ (kHz^{-1})	Confidence (%)
Studland 1	0.51 ± 0.02	< 80.0	30.3 ± 0.5	< 80.0	8.9 ± 2.5	> 95.0
Studland 2	0.14 ± 0.03	> 99.5	50.6 ± 1.5	< 80.0	46.0 ± 1.2	> 99.5
Studland 3	0.27 ± 0.03	> 99.5	56.5 ± 1.1	> 99.5	-4.2 ± 7.2	> 99.5
Studland 4	0.41 ± 0.01	< 80.0	39.9 ± 0.5	< 80.0	4.3 ± 2.7	< 80.0
Branksome 1	0.29 ± 0.04	> 99.5	60.7 ± 1.4	> 99.5	-12.3 ± 12.6	> 99.5
Branksome 2	0.19 ± 0.01	> 97.5	68.6 ± 1.6	> 80.0	36.4 ± 8.7	> 99.5
Lilliput 1	0.52 ± 0.02	> 99.5	33.6 ± 0.5	> 99.5	-7.4 ± 2.6	> 99.5
Lilliput 3	0.50 ± 0.05	> 99.5	18.5 ± 0.4	< 80.0	22.2 ± 3.8	> 99.5
Lilliput 4	0.48 ± 0.03	> 95.0	38.3 ± 0.7	> 97.5	-10.2 ± 6.8	> 99.0

Table 6.3. Investigation of frequency-dependence of attenuation coefficient and quality factor for nine uncorrupted sand locations examined including; constants of proportionality k_A and Q_A, weighted mean of quality factor and corresponding confidences.

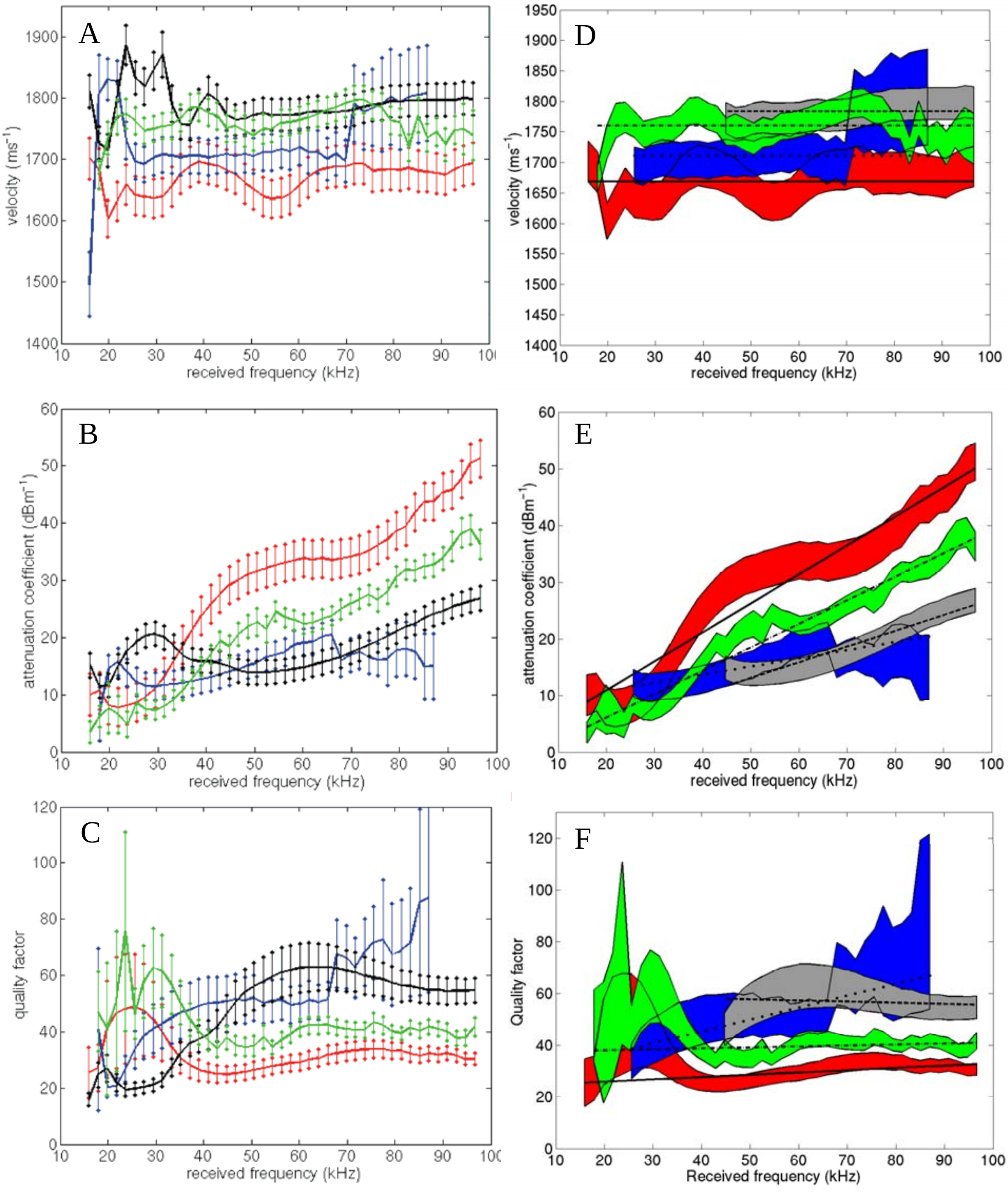

Figure 6.1. Compressional wave properties of sediment examined at the Studland site including; velocity (A and D), attenuation coefficient (B and E) and quality factor (C and F) from Locations 1 (red), 2 (blue), 3 (black/grey) and 4 (green). Weighted means are plotted for Location 1 (solid line), Location 2 (dotted line), Location 3 (dash-dot line) and Location 4 (dashed line). Errors are denoted by vertical bars (A, C and E) and shaded regions (B, D and F).

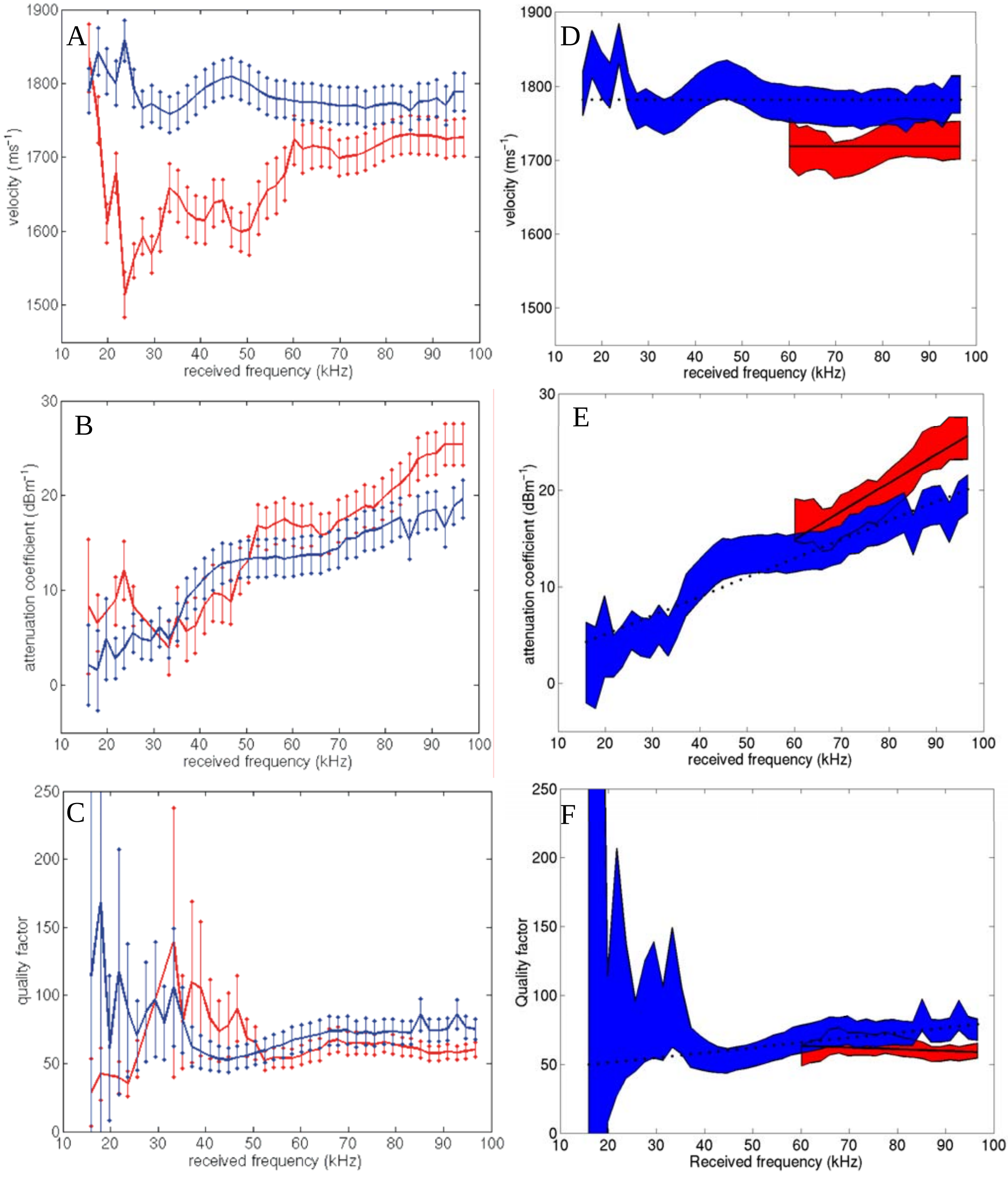

Figure 6.2. Compressional wave properties of sediment examined at the Branksome site including; velocity (A and D), attenuation coefficient (B and E) and quality factor (C and F) from Locations 1 (red) and 2 (blue). Least-square weighted linear fits are plotted for Location 1 (solid line) and Location 2 (dotted line). Errors are denoted by vertical bars (A, C and E) and shaded regions (B, D and F).

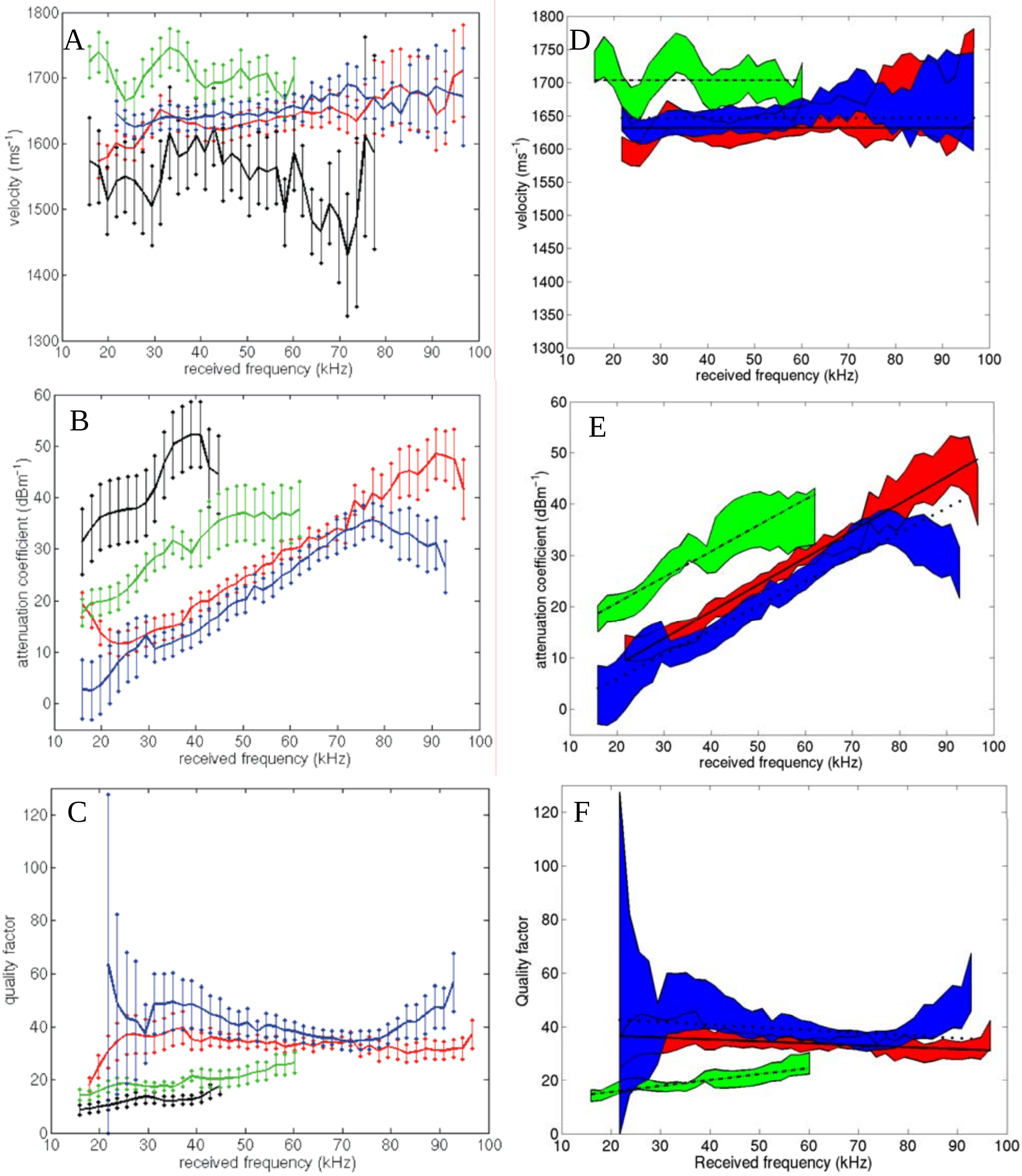

Figure 6.3. Compressional wave properties of sediment examined at the Lilliput site including; velocity (A and D), attenuation coefficient (B and E) and quality factor (C and F) from Locations 1 (red), 2 (black), 3 (green) and 4 (blue). Least-squares weighted linear fits are plotted for Location 1 (solid line), Location 3 (dash-dot line) and Location 4 (dotted line). Errors are denoted by vertical bars (A, C and E) and shaded regions (B, D and F).