

## INTRODUCTION TO WG2

### TOOLS AND TECHNOLOGIES IN MATHEMATICAL DIDACTICS

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This set of papers and posters is the work of the Thematic Group Tools and Technologies in Mathematical Didactics. The papers build on the work of the group at CERME1, where three embedded levels were distinguished when analysing the use of tools and technologies:

- the level of interactions between tool and knowledge;
- the level of interactions between knowledge, tool and the learner;
- the level of integration of a tool in a mathematics curriculum and in the classroom.

Bearing in mind these three embedded levels of analysis, the Thematic Group, as represented by this collection of papers, worked on the following questions:

- How can ideas of representation, metaphor and tool help us to understand how learners interact with technologies?
- How do tools and technologies mediate learning?
- What might be the parallels and contrasts between computer algebra system (CAS) use in algebra and calculator use in arithmetic?
- How does CAS use compare with use of such other tools as dynamic-geometry software or statistics packages?

#### Theoretical Ideas

Theoretical ideas, such as representations, metaphors, mediation and tools, proved useful when looking at how students link representatives (artifacts-in-use in an activity) to the mathematical function they are meant to represent, through actions of *interweaving* representatives and rejecting those that are incompatible with their concept image. The paper by Landa illustrates the mediation of a spreadsheet, showing how students misunderstanding can be clarified not only for the teacher but also for the students. In Cerulli's work can be seen, in the software that he designed, the equivalence of a button (a software functionality) and a statement in algebra. Algebraic knowledge can then be build and used in

the same way that a theory is built from axioms by way of proof. Across these papers, technology can be seen as means to allow the use of a wider variety of metaphors and representatives (see Jones, and Schwarz & Hershkowitz).

### **Algebraic Knowledge when using Spreadsheet and CAS**

The papers by Chiocca and Hošpesová show two contrasting cases of the educational use of spreadsheet. The former case showed conceptual difficulties shifting from the mathematical content at stake (statistics) to a “spreadsheet” concept (the distinction between absolute and relative references). In the latter, the spreadsheet was reported as facilitating the access to mathematical concepts and fostering positive attitudes. In this apparent contradiction, technologies can be seen as facilitators or obstacles, depending on the point of view. Spreadsheets are, in a sense, quite intuitive. Nevertheless, their use implies understanding more or less their operation, especially when the problem is not the direct equivalent of a paper and pencil task. Obstacles reported in a task using such software do not necessarily imply that this task is to be banished. Educators have to reflect on the students’ actual difficulties and on the knowledge involved in their resolution.

Routitsky and Lagrange both present the results from large-scale surveys. Routitsky’s is about teachers’ attitudes towards the use of calculators. This, it seems, does not have a straightforward relationship with the period of time over which teachers have used calculators. When the period is short, the attitude is generally good. Then, over a longer period, it declines. Then it grows slowly again. This implies that teachers are generally inclined to use calculators at the beginning, then they come up against difficulties and they need time to overcome these. Lagrange’s paper surveys the literature on the use of technology, especially Computer Algebra Systems, to teach and learn mathematics. It shows a variety of works and trends, some very optimistic, others more aware of difficulties that students and teachers might meet.

This raises the question of the “instrumental” and “institutional” approach to the use of technology. The instrumental approach refers to a technological tool as a mental construction by the user. The institutional dimension considers the tasks, techniques and theories in a given institution (classroom, educational system, ...) and the impact of the introduction of technology on these. These approaches are complementary to the “epistemological approach” that relies on the study of the knowledge in relation to the introduction of technology.

## **Dynamic Geometry Systems (DGS)**

The paper by Gallopin and Zuccheri focuses on how to improve the teaching of deductive reasoning. They give an example from their work of how to use as didactical instruments, strictly linked, technological tools and mathematical theories and concepts. The work of Mogetta relates to the forming of conjectures from visual images in DGS, what she called ‘dynamic’ definitions. Olivero and Robutti give examples from their research on the role of measuring in the proving process.

## **Open Questions**

The Thematic Group finished with the following open questions:

1. The influence of the use of technology on proof.

It appears that we have to look at the process of proving as a specific theoretical activity. General software like DGS and CAS can tend to encourage empirical activity. Specific settings are necessary to reach a more theoretical level. Proving is often difficult, as students may be easily convinced by empirical evidence. Working on proof probably implies the use of more specific software like Cerulli’s “L’Algebrista”.

2. The notions of “tool”

A technology can be used by the teacher as a “didactical tool” or by the learner as a means to do tasks and learn mathematics. The notion of “tool” is also used for a host of different “immaterial” entities like concepts or theories. We have to distinguish these notions when analysing the use of technology. Looking at a technological tool for the learner as an “instrument” is beneficial because it accounts for the links between the appropriation of the tool and the learning of mathematics.

## **Concluding Comments**

Overall, the work of the Thematic Group covered a variety of software technologies (DGS, spreadsheet, CAS, multimedia, distance education, calculators), school levels (from primary to university students) and methodologies (small scale case studies, software design, big surveys...). The outcomes for the participants were that a range of common notions were developed and a variety of concerns shared. We hope that this variety is captured in this set of papers.