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Elements of the Visualisation Process within a Dynamic Geometry Environment

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The advent of powerful computer graphic packages has coincided with renewed interest in all forms of visual representation in mathematics. As a result, we need to be clear about what we mean by the visual processing necessary to solve mathematical problems involving visual phenomena. Visual processing involves the ability to mentally manipulate and transform visual representations and visual imagery. This paper describes three different elements of the visualisation process: crude visualisation, visualisation as the reading of visual information, and visual processing. Illustrations are given of how the use of a dynamic geometry package such as Cabri-Géomètre *both needs and contributes to developing visualisation in all these three senses.*

La llegada de poderosos paquetes gráficos informáticos ha coincidido con un renovado interés en todo tipo de representaciones visuales en matemáticas. Por todo ello, se hace necesario precisar nuestra interpretación del procesamiento visual necesario para la resolución de problemas matemáticos que implican fenómenos visuales. Procesamiento visual implica la habilidad para manipular mentalmente y transformar representaciones visuales e imagería visual. Este escrito describe tres elementos del proceso de visualización: visualización simple, visualización como lectura de información visual y procesamiento visual. Se presentan ejemplos de cómo la utilización de un paquete integrado de geometría dinámica del tipo Cabri-Géomètre necesita de la visualización y contribuye al desarrollo de la misma en los tres sentidos mencionados.

Visual representations in mathematics provide essential experience of abstract mathematical objects and concepts. While for most of the twentieth century the mathematical literature has been predominantly algebraic, the advent of powerful computer graphics packages has coincided with renewed interest in new forms of visual representation. The prediction is that such computer graphics technology will have a significant positive influence on the progress of mathematics (National Research Council 1990, Science and Engineering Research Council 1991).

Nowhere is this more keenly felt than in geometry which is no longer simply the mathematical description of certain aspects of physical space. As geometry evolves to encompass the understanding of diverse visual phenomena, we need to be clear about what we mean by the visual processing necessary to solve mathematical problems involving visual phenomena. Within the school context, the increasing use of dynamic geometry packages gives us the opportunity to describe how the use of such packages involves various aspects of mathematical visualisation.

Our starting point is the idea that geometrical thought is the interaction between external representations and geometrical knowledge where mental images have an important role to play. We also know that visual processing in mathematics involves the ability to mentally manipulate and transform visual representations (Bishop 1983). In this paper we describe three different elements of the visualisation process and illustrate how the use of a dynamic geometry package such as *Cabri-Géomètre* both needs and contributes to developing visualisation in all these three senses.

Visualisation in Geometry

In order to describe these three different elements of the visualisation process, we first need to establish the meaning of visualisation and then consider the nature of the computer environment. Zimmermann and Cunningham (1991 p3) argue that in mathematical visualisation, the main interest is in "the student's ability to draw an appropriate diagram (with pencil and paper, or, in some cases, with a computer) to represent a mathematical concept or problem and to use a diagram to achieve understanding, and as an aid in problem solving". However, such a definition overlooks the notion of visual processing outlined by Bishop (1983) which has proved useful when describing the solving of geometrical problems (Gorgorió to appear). Thus, in the domain of geometry, visualisation involves both the ability to draw an appropriate diagram *and* the mental manipulation of geometrical images.

Next we need to consider significant aspects of the dynamic geometry environment. For the user, the important aspects of such an environment are:

- the ability to *directly* manipulate geometrical objects. The drawing on the screen can be manipulated by means of the mouse. Objects can be 'dragged' while, all the time, all the geometric properties used to construct the figure are preserved.

- the ability to *define relationships* between objects and to explore graphically the implications. This is perhaps the most powerful feature of a dynamic geometry package.

It is important, here, to clarify the importance of distinguishing between what Parzysz (1988) refers to as *drawing* and *figure*. Laborde (1993 p 49) makes clear the distinction in the following way: "*drawing* refers to the material entity while *figure* refers to a theoretical object". In terms of a dynamic geometry package, a *drawing* can be a juxtaposition of geometrical objects resembling closely the intended construction. In contrast, a *figure* additionally captures the geometric relationships between the objects used in the construction. In such a way, the figure is invariant when any basic object used in the construction is dragged (see Holzl 1995).

With the above discussion in mind, we can consider the term visualisation in three different senses: *crude visualisation, visualisation as the reading of visual information,* and *visual processing*:

- crude visualisation is seeing a diagram and being able to interpret its technical rules or constraints.
- *visualisation as the reading of visual information* means interpreting the geometrical relationships present.
- *visual processing* involves the ability to mentally manipulate and transform visual representations and imagery.

Given that the dynamic geometry package mediates the problem solving process and thus is more of a context for problem-solving rather than merely a tool, we describe how the use of a dynamic geometry package such as *Cabri-Géomètre* both needs and contributes to developing visualisation in all the above three senses.

Elements of the Visualisation Process

crude visualisation

As outlined above, crude visualisation is seeing a diagram and being able to interpret its technical rules or constraints. For example, two 12 year old students, H and R, were asked to construct the figure shown in below (Jones 1995).

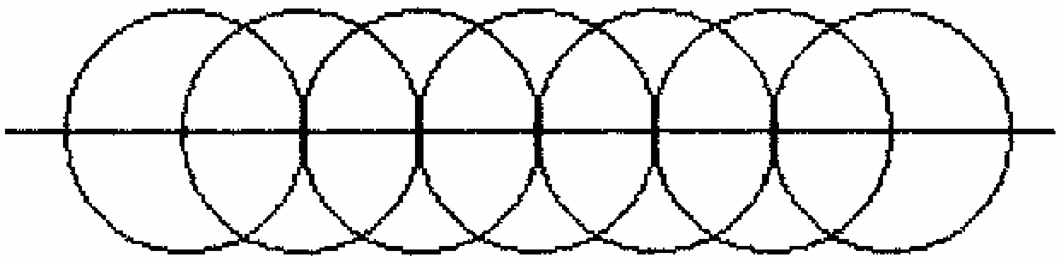

Figure 1

H and R discuss the pattern and begin constructing it using basic circles starting from the left. They form three basic circles of approximately the correct size and drag them into approximately the correct orientation. This demonstrates that the students are able to interpret *Cabri*'s technical rules to produce a drawing but this drawing does not embody the geometrical relationship present. With three circles in place, the teacher invites the students to drag one of the circles away. One of the students immediately exclaims:

R: So, you can mess it up!

The teacher then reminds the students that their task is to construct the figure so that it could not be "messed up" (a term suggested by Healy *et al* 1994 to refer to whether a figure could be dragged to see if it became unrecognisable).

visualisation as the reading of visual information

With experience of using a dynamic geometry package with appropriate tasks and input from the teacher, students learn that figures need to be interpreted. The above two students, having used *basic circle* and failed, then try using *circle by centre and radius point*. They begin by constructing two circles such that the radius point of the first circle is the *centre* of the second circle. This illustrates that H and R have begun to develop some understanding of the relationship between the objects they need to construct.

In a subsequent lesson, the same two students are constructing the pattern of circles shown in figure 2. They confidently create two circles that can not be "messed up" but, rather than using a *point of intersection* (in *Cabri I*) they use *point on object* to create the centre of the third circle.

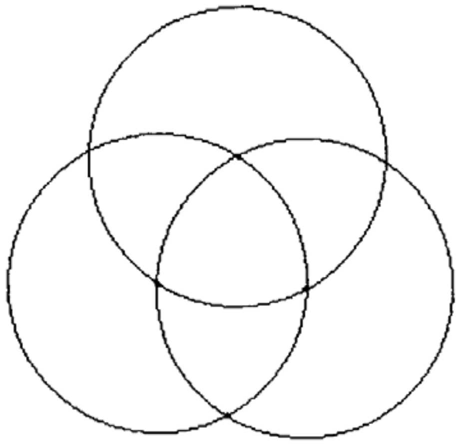

Figure 2

As a result, their final construction could still be “messed up” albeit not drastically. The students are aware that something about their construction is unsatisfactory and after being reminded of their previous work, they correctly chose *point of intersection* and complete the figure.

Using *Cabri*, the students may be applying the appropriate geometric ideas but if these ideas are not logically connected then the figure does not remain invariant when dragged. Thus using *Cabri* helps to connect what is visually true with what is logically correct. This aids the dialetic between figures and concepts (see Fischbein 1993).

visual processing

Visual processing involves the ability to mentally manipulate and transform visual representations and imagery. It is well-established that, in developing visual processing strategies, students benefit from appropriate experience of working with models and other forms of practical apparatus in geometry (Gorgorió 1995).

The following example illustrates how working with *Cabri* also develops developing visual processing strategies. A group of four 14 year-old students successfully construct the diagram shown in figure 2. They are asked to construct figure 3.

The students recognise the equilateral triangle formed in the centre of the diagram by the intersection of the three circles. They know it is an equilateral triangle because the sides, if drawn, would each be radii of equal-sized circles. By erasing the circles in figure 2, and using perpendicular bisectors, they find the point where the three circles in figure 3 intercept. With that they have solved the problem. This is an example of the mental manipulation of a visual representation.

Our experience suggests that using a dynamic geometry package may be beneficial in developing visual processing ability

- in students who find it difficult to mentally manipulate and transform visual representations
- in situations where it becomes impossible to imagine the figure
- when a large number of figures are required to solve a problem

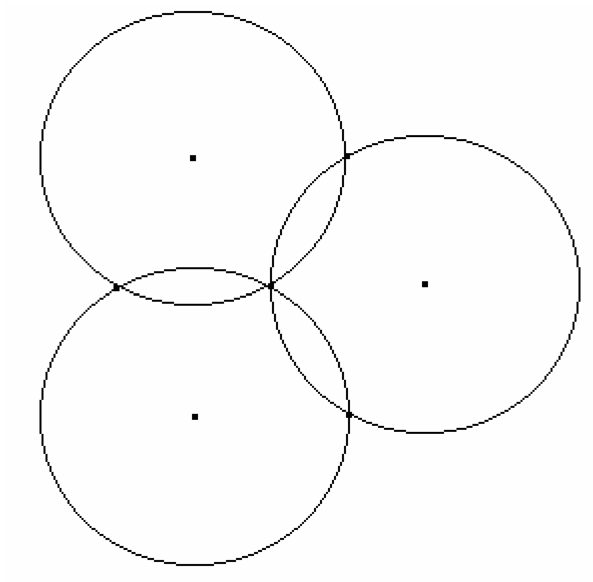

Figure 3

Conclusions

There is a risk, of course, that using *Cabri* might discourage students from anticipating what is going to happen with a construction. However, when involved in a construction, a student can have an intuition of what is needed but disregard it *before* constructing because the student realises through imagining that this action would not work. In this way, *Cabri* can help to develop visual processing.

When using a dynamic geometry package such as *Cabri*, the element of establishing conjectures in geometry is essentially visual. *Cabri* facilitates the visualisation process, facilitates the conjecturing and, hence, helps to implement the students' geometric knowledge. Given that geometry now encompasses the understanding of visual phenomena, there are many new geometries that need to be included in the school curriculum. As this paper has argued, the use of a dynamic geometry package can support the development of important visualisation skills necessary for the understanding of visual phenomena.

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