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THE DESIGN OF GEOMETRY TEACHING:

LEARNING FROM THE GEOMETRY TEXTBOOKS OF GODFREY AND SIDDONS

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Deciding how to teach geometry remains a demanding task with one of major arguments being about how to combine the intuitive and deductive aspects of geometry into an effective teaching design. In order to try to obtain an insight into tackling this issue, this paper reports an analysis of innovative geometry textbooks which were published in the early part of the 20th Century, a time when significant efforts were being made to improve the teaching and learning of geometry. The analysis suggests that the notion of the 'geometrical eye', *the ability to see geometrical properties detach themselves from a figure, might be a potent tool for building effectively on geometrical intuition so as to provide a bridge into deductive geometry.*

INTRODUCTION

The design of the teaching of geometry remains controversial (Villani; 1998). One major problem may be due to the dual nature of geometry; geometry is both a theoretical domain and perhaps the most concrete, reality-linked part of mathematics. In fact, a major argument in the teaching of geometry is what extent, and how we should combine the intuitive and the deductive aspects of geometry. To tackle this issue, this paper examines the design of the geometry textbooks in the early 20th Century in England, a time when significant efforts were being made to improve the teaching and learning of geometry (Howson; 1982, pp. 141-168). The main focus is the design of the textbooks written by C. Godfrey and A. W. Siddons, two leading reformers at that time. In particular, we discuss how these authors attempted to combine experimental/intuitive and deductive approaches in their books.

THEORETICAL CONSIDERATIONS

Before examining the content of the textbooks of Godfrey and Siddons, we introduce some theoretical considerations regarding the teaching and learning of geometry. Theoretical work concerned with the teaching and learning of geometrical ideas has concentrated on investigating the developments of cognition. One of the most well-known works is by van Heile who suggests that learners advance through levels of thought in geometry, characterised as visual, descriptive, abstract/relational, and formal deduction (see, van Hiele 1986). Another useful theoretical idea, due to Fischbein, is that geometrical figures have a unique feature, described by Fischbein and Nachlieli's notion of *figural concept*, in that 'geometrical figures are characterized by both conceptual and sensorial properties' (Fischbein and Nachlieli; 1998, p. 1193). As these authors argue, successful reasoning in geometry may be related to the harmony between figural and conceptual constrains.

Geometry is also an area of mathematics in which intuition is frequently mentioned. Views vary, however, about the role and nature of geometrical intuition, and how it might help or hinder the learning of geometry (and other areas of mathematics). Whereas Piaget or van Hiele gave intuition a relatively minor role in the latter stages of learning, Geometers, nevertheless, tend to recognise the importance of geometrical intuition (see, for example, Hilbert; 1932, Atiyah; 2001). Fischbein also stated;

> The interactions and conflicts between the formal, the algorithmic, and the intuitive components of a mathematical activity are very complex and usually not easily identified or understood. (Fischbein; 1994, p. 244)

The question that these considerations highlight is how to resolve these apparently opposing positions, if this is possible. To address this question, and attempt to illuminate the relationship between practical and deductive geometry, we examine a time in mathematics education when these issues were being seriously tackled.

METHOD

The methodological approach is documentary analysis (Jupp and Norris, 1993) enhanced by the methodology for textbook analysis proposed by Schubring (1987). The documents analysed as part of the project reported in this paper include:

- the two main Euclidean-style textbooks by Godfrey and Siddons, *Elementary Geometry* (Godfrey and Siddons, 1903) and *A Shorter Geometry* (Godfrey and Siddons, 1912);
- their articles published in the journal *Mathematical Gazette*;
- a collection of educational writing published in the 1930s (Godfrey and Siddons, 1931);
- a memoir published in the 1950s (Siddons, 1952).

The analysis conducted for the study focused on the design of the textbooks, in particular the relationship between experimental and deductive geometry, and on their pedagogical purpose gleaned from documents written by Godfrey and Siddons.

THE DESIGN OF THE EXPERIMENTAL TASKS IN THE TEXTBOOKS BY GODFREY AND SIDDONS

In the textbooks of Godfrey and Siddons, the experimental tasks, drawing, measurement, and experiments, can be found not only in the introductory stages, but also in the deductive stages (for further details, see Fujita; 2001a). In this section, we focus on those in the deductive stages. In both *Elementary Geometry* and *A Shorter Geometry*, most of the experimental tasks were located immediately preceding the theorems that were being introduced. The purpose of these tasks was to lead students to discover various geometrical properties. In addition to this use of exercises, some of them would seem to enhance the visualisation of various theorems, constructions, and theoretical exercises. For example, the exercises before theorem 2 in Book II, 'Triangles on the same base and between the same parallels (or, of the same altitude) are equivalent', were as follows (Godfrey and Siddons; 1912, p. 120);

¶ **Exercise 698.** Draw an acute-angled triangle and draw the three altitudes. (Freehand.)

¶ **Exercise 699.** Repeat Ex. 698 for a right-angled triangle. (Freehand.)

¶ **Exercise 700.** Repeat Ex. 698 for an obtuse-angled triangle. (Freehand.)

¶ **Exercise 701.** In what case are two of the altitudes of a triangle equal?

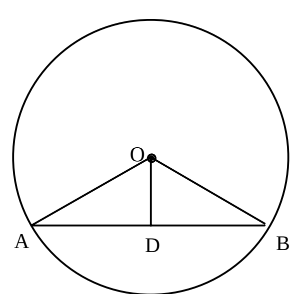
These exercises would encourage the students to pay attention to the height of triangles, which would be important to understand the theorem above (the symbol ¶ was used in the textbooks to indicate that discussion between the teacher and students would be appropriate). Here is another example. Before theorem 1 in Book III, 'A straight line, drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord', the following exercises were studied in *A Shorter Geometry* (Godfrey and Siddons, 1912, pp. 151-2),

¶ **Exercise 877.** Draw a circle of about 3 in. radius, draw freehand a set of parallel chords (about 6), bisect each chord by eye. What is the locus of the mid-points of the chord?

¶ **Exercise 878.** Draw a circle and a diameter. This is an axis of symmetry. Mark four pairs of corresponding points. Is there any case in which a pair of corresponding points coincide? (*Freehand.*)

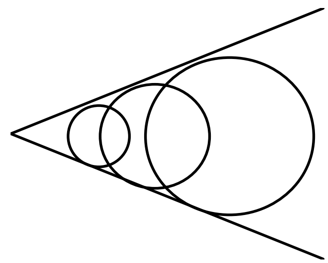
¶ **Exercise 879.** What axes of symmetry has (i) a sector, (ii) a segment, (iii) an arc, of a circle?

These exercises would encourage students to become aware of the symmetry of the circle as well as leading them to discover the theorem. Also, to prove this, it should be shown that the triangles OAD and OBD were congruent, and the exercises would help students to see the congruency of the triangles.



A further example is that before the construction of an inscribed circle in a given triangle, the following exercise is studied (Godfrey and Siddons; 1912, p. 171);

¶ **Exercise** 974. What is the locus of the centres of circles touching two lines which cross an angle of 60°. Draw a number of such circles.



This would help the understanding of the proof of this construction, but also solving the following theoretical exercise, 'Prove that the bisectors of the three angles of a triangle meet in a point' (Godfrey and Siddons; 1912, p. 172).

PEDAGOGICAL BACKGROUND OF GODFREY AND SIDDONS

Let us discuss further the pedagogical background to the design of these exercises. Godfrey was explicit that mathematics cannot be undertaken by logic alone (Godfrey; 1910, p. 197). He wrote that another important 'power' is necessary for solving mathematical problems. This he called *geometrical power*, which he defined as "the power we exercise when we solve a rider [a difficult geometrical problem requiring the use of several pieces of theoretical knowledge]" (Godfrey; 1910, p. 197). To develop this *geometrical power*, Godfrey suggested that it was essential to train what he called the *geometrical eye*. This, Godfrey defines as 'the power of seeing geometrical properties detach themselves from a figure'. In other words, Godfrey argued that we would not solve geometrical problems unless we could create proper geometrical images in the mind. Recent educators have also discussed similar aspects, 'mathematisation', 'the mental process which produce mathematics' (Wheeler; 2001, p. 50), and Godfrey's geometrical eye can be considered the primitive and narrower concepts of this 'mathematisation'.

Godfrey particularly discussed that it was experimental tasks that would make possible the training of the 'geometrical eye' at any stage in learning geometry (Godfrey; 1910, p. 197). Thus, as we have seen in above, their textbooks included the experimental exercises which would enhance students' intuitive ability, even in the deductive stages. The design of the experimental tasks in their textbooks can be summarised as follows;

- the experimental exercises were carefully chosen and designed leading to showing and requiring a proof
- using this design the aim of Godfrey and Siddons was to develop what they called the 'geometrical eye'.

CONCLUDING COMMENTS

A major improvement in geometry pedagogy would be to improve on calls to develop geometrical intuition by linking more directly with geometrical theory. This would entail developing pedagogical methods that mean that a deductive and an intuitive approach are mutually reinforcing when solving geometrical problems (see, Jones 1998). This paper argues that Godfrey's notion of the *geometrical eye* might be a potent tool for building effectively on geometrical intuition. As we have shown through our analysis, Godfrey and Siddons considered that practical and deductive geometry should be combined in the latter in the teaching of geometry. Godfrey considered that the *geometrical eye* would be essential for successfully solving geometrical problems, and that it should be trained by practical tasks at all stages of geometry. We consider the concepts of the geometrical eye can be used as a pedagogical tool which enable to bridge experimental and deductive geometry. Future research should be concerned with the theoretical development of this concept as well as empirical examinations whether successful reasoning in geometry would be related to it.

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