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British Society for Research into Learning Mathematics
Geometry Working Group

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TEACHER KNOWLEDGE AND PROFESSIONAL DEVELOPMENT IN GEOMETRY

A report based on the meeting at the University of Loughborough, 6th May 2000

by

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The successful teaching of geometry depends on teachers knowing a good deal of geometry and how to teach it effectively. This report provides a review of what is known about teacher knowledge in geometry, how the knowledge develops and how this knowledge development can be supported by professional development .The available evidence suggests that attention could usefully be paid both to the initial and continuing education of teachers of mathematics in terms of their background and understanding of geometry.

INTRODUCTION

Geometry (in its widest sense) is one of the most interesting areas of mathematics to teach. It has a host of interesting problems and surprising theorems that are open to many different approaches (see, for example, Clausen-May, Jones, McLean and Rollands, 2000). Geometry has a long history, intimately connected with the development of mathematics. It is an integral part of our cultural experience, being a vital component of numerous aspects of life from architecture to design (in all its manifestations). What is more, geometry appeals to our visual, aesthetic and intuitive senses.

These aspects and considerations also tend to make geometry a demanding part of mathematics to teach well. Teaching geometry effectively involves, amongst other things, appreciating the history and cultural context of geometry, knowing how to recognise interesting geometrical problems and theorems, understanding the many and varied uses to which geometry is put, and incorporating all these things into the practice of teaching in the classroom (see Jones, 2001).

These demands are considerable and are exacerbated by several, not unrelated, issues. First, the crowded school mathematics curriculum often leaves insufficient room for geometry, a phenomenon that can be traced back to the 1960s when algebra began to dominate the curriculum. More recently this lack of curriculum time for geometry has been exacerbated by a substantial increase in the coverage of statistics and, especially of late in the UK, a major focus on numeracy, both of which developments have tended to deflect even more attention away from geometry. Second, the amount of geometry that is known has grown considerably since the end of the 19th century. It is possible today to classify more than 50 geometries (see: Malkevitch 1991). This richness of modern geometry creates a fundamental problem for curriculum designers in terms of deciding what geometry should be included in the mathematics curriculum at any particular level. Yet, if anything, the amount of geometry *decreases* as one moves into the mathematics curriculum for 16-19 years olds and can disappear completely when one examines the mathematics offered at University level. This leads to a third major problem. Teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school, and possible little even then.

This latter problem has been recognised for some time. It is worth reflecting on the words of A. L. O'Toole, written in the context of college geometry, in 1941: “It is a shame that higher institutions that prepare secondary school teachers to send them back to the high schools knowing no more about the subject [geometry] they are to teach than when they left high school. In fact, they know less, for they have several years to forget what they once knew”.

The intention in this report is to review what is known about teacher knowledge in geometry, how it develops and how it can be supported by professional development.

THE NATURE AND ROLE OF TEACHER KNOWLEDGE

The mathematical knowledge necessary to teach ‘effectively’ is recognised as being a more complex issue than simply requiring a grasp of mathematics content or subject knowledge (Ball, 1990, Fennema and Franke, 1992). The term *pedagogical content knowledge* (PCK) was first employed by Shulman (1986) to depict a blend of content and ‘ways of transforming that content in terms of its teachability’. For mathematics, as for any given subject area, PCK includes forms of representation of concepts, useful analogies, examples, demonstrations, and so on, that can help to make mathematical ideas comprehensible to others.

Shulman’s model may be too simplistic (in that, for instance, it does not distinguish between the nature of different school subjects, nor between the academic subject, in this case mathematics, and the school subject) and several modifications have been suggested (see, for instance, Cochran, DeRuiter and King, 1993). Nevertheless, it has proved useful to distinguish the two relevant knowledge domains; *subject matter knowledge* (which includes key facts, concepts, principles, and explanatory frameworks of a discipline, as well as the rules of evidence used to guide inquiry in the field), and *pedagogical content knowledge* (which consists of an understanding of how to present specific topics in ways appropriate to the students being taught).

A range of studies has analysed teachers' mathematical knowledge, both subject knowledge and PCK, within particular topic areas. For example, Lampert (1986) looked at multiplication, while Even (1993) focused on teachers' conceptions of functions. Only a very few studies have looked at teachers' knowledge of geometry. These are outlines in the next section.

TEACHER KNOWLEDGE OF GEOMETRY

Lampert (1988) looked at how teachers' thinking about their students' thinking about geometry was effected by the use of new teaching tools, in this case a piece of educational software called the Geometric Supposer. Chinnappan, Nason and Lawson (1996) worked with pre-service teachers and examined their pedagogical and content knowledge of trigonometry and various aspects of plane geometry. Swafford, Jones and Thornton (1997) examined the effect of enhancing teachers' knowledge of geometry, and how it can be taught, on what the teachers taught and how they taught it. Leikin, Berman and Zaslavsky (2000) looked at how teaching about symmetry led to deeper understanding for the pre-service teachers involved.

This evidence from the research literature suggests that many beginning teachers of mathematics, and, possibly, a range of experienced teachers, do not possess the sorts of repertoires of subject-matter knowledge and pedagogical content knowledge of geometry which would enable them to teach geometrical topics as well as they might. This indicates that attention could be usefully paid both to the initial education of teachers of mathematics and to their continuing professional development.

INITIAL AND CONTINUING TEACHER EDUCATION

While the training curriculum specified in the UK for the initial training of both primary and secondary teachers contains very little in the way of geometry and how it should be taught, the complexity of the issues means that there is a certain lack of consensus about what geometry can and should be contained within such courses. Grover and Connor (2000) found that across more than 100 courses in the USA, no "typical" geometry course was evident.

The US Conference Board for the Mathematical Sciences (CBMS) (2000) has been examining the issue of what kind of geometry mathematics teachers need knowledge of in order to be well-prepared to teach. This includes:

- Mastery of core concepts and principles of Euclidean geometry in the plane and space,
- Understanding of the nature of axiomatic reasoning and the role that it has played in the development of mathematics and facility in fundamental proof strategies,
- Understanding and skill in use of a variety of methods for studying geometric problems-including synthetic, transformation, coordinate, and vector strategies,

- Understanding of trigonometry from a geometric perspective and skill in use of trigonometric relationships to solve problems,
- Knowledge of some significant modern aspects of geometry like tiling, computer graphics, robotics, fractals, and spatial visualization,
- Ability to use computer-based dynamic drawing tools to conduct geometric investigations emphasizing visualization, pattern recognition, and conjecturing.

While noting that a high school teacher who has some familiarity with aspects of modern geometry, such as tiling and fractals, and with applications such as computer graphics and robotics, will convey a richer view of the subject to students, the CBMS report points out that fitting all of those topics into one college geometry course that also treats Euclidean geometry and axiomatics in-depth runs a clear risk of covering ground without developing depth of understanding. In terms of continuing professional development, the study by Swafford, Jones and Thornton and the one by Jacobson and Lehrer (2000), both point to teachers' eliciting more sustained and elaborate patterns of classroom conversations about geometry when the teachers have enhanced knowledge oft typical milestones and trajectories in children's reasoning about space and geometry.

CONCLUDING COMMENTS

It is clear that mathematics teachers need to have a *deep* understanding of the geometry that is appropriate for school mathematics if they are going to teach it well. To a large extent in the UK this is built up during undergraduate courses. For the majority of secondary teachers of mathematics this occurs *before* their graduate teaching course. Thus, University mathematics departments need to develop carefully thought out courses about school geometry that will explicitly prepare future primary, middle, and high school teachers of mathematics to offer high quality teaching in their classes.

While subject matter "matters", deciding what subject matter, for whom, and in what depth, is a substantial challenge. Yet it is also clear that it iss not just the mathematics that is important. Knowing mathematics does not ensure the effectiveness of prospective and serving teachers. How they come to know their mathematics matters as well.

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BSRLM Geometry Working Group

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions that could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

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