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The Process of Re-designing the Geometry Curriculum: the case of the Mathematical Association in England in the early 20th Century

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This paper examines a key period of change in geometry teaching in England. Our focus is the character and nature of the recommendations of the geometry report of the UK Mathematical Association in 1902. We analyse historical documents of the Mathematical Association using a theoretical framework developed from Cooper's model. Our analysis shows that the character and recommendations of the Mathematical Association report was influenced by various factors including: that the Mathematical Association members still respected the traditional Euclidean approach to geometry as a basis for school geometry; that the academic and power resources available to the Mathematical Association at the time were not sufficient for a complete change from the traditional approach; that conflicts between the various members of the Mathematical Association prevented a complete consensus; and that the climate outside the teaching committee of the Mathematical Association was not ready for radical reform at that time.

Introduction

In the history of mathematics education in secondary schools in England, one of the major events, occurred in and around 1900; it came to be known as the 'Perry movement' (Price, 2001, p. 217). The main argument at that time concerned the use of geometry textbooks based on a rigid following of the Books of Euclidean, such as those edited by Potts (1845) or Todhunter (1862). The Mathematical Association (MA), which was originally founded in 1871 as the Association for the Improvement of Geometrical Teaching (AIGT), acted for the reform of the teaching of geometry, and published an important geometry report in 1902 (MA report 1902). This report can be considered as an important document in the history of the teaching of geometry, given its prominence in various articles and studies; examples include Godfrey (1920, p. 20), Department of Education and Science (DES) (1958, p. 9), Howson (1982, p. 149), Price (1994, p. 56).

Yet some of the commentary on the MA report has suggested that it was rather conservative in that it could be considered to be quite cautious and favouring tradition and gradual change. For example, Godfrey, a prominent member of the MA at the time stated that 'the M.A. published a report on Geometry teaching; a conservative report, as it was considered impracticable to secure the abolition of the sequence' (Godfrey, 1920, p. 20). Much later a UK Government report commented that 'this body [the Teaching Committee of the MA] despaired of abolishing Euclid as an examination textbook and concentrated on less sweeping changes' (DES, 1958, p. 9).

In this paper, we examine why the MA report 1902 can be seen as a modest reform, something which has yet to be answered in historical studies. To address the issue of this paper, we employ an historical case study approach by using a theoretical model proposed by Cooper (1985) to analyse historical documents which recorded discussions leading up to the MA report 1902 including the unpublished Minute book of the teaching committee of the MA found in the MA archive at Leicester, UK.

Analysing the process of the reform in mathematics education

The development of the teaching of mathematics is always accompanied by designing and redesigning syllabi, contents of subject-matter, textbooks, and so on. Such changes may be partially as a result of the development of teaching and learning theories, teaching methods, technology, social demands, and so on. In this paper, we refer to this process as 'reform'.

Reforms usually involve not just a few individuals, but various people and organisations from both inside and outside the subject (Griffiths and Howson, 1974, p. 135). As Marsh states "Proposals for curriculum reform come from various sources including: teachers, teacher unions, policy-makers, academics, politicians, media and pressure groups" (Marsh, 1997, p. 211). As we describe in the next section, various people were involved in the process of the reform of the teaching of geometry in England in the late and early 20th centuries.

The work of Cooper (1985) reveals why and how in the 1950-60s a traditional approach in mathematics in schools was replaced by content based on 'modern mathematics'. During these decades, Cooper found that the nature of the 'mathematics' to be taught in schools was discussed by people from inside and outside 'mathematical communities', and that several projects were founded to replace the 'traditional' mathematics curriculum. Of these projects, the School

Mathematics Project (SMP), of the University of Southampton (UK) was particularly successful. Cooper concludes that "SMP's 'success' relative to such project as MME (Midlands Mathematical Experiment) must be understood, at least partially, in terms of the differential availability of such resources as status, academic legitimacy and finance, ..." (p. 265).

Cooper's study reveals what factors we should examine to understand the complex process of reform. In terms of the 1902 MA report, first, it should be appreciated that various opinions were expressed in the process of drawing up the report between the members of the MA. Therefore, it is reasonable to see that the MA report would reflect the various opinions of the different members, even though the foci would be rather narrow since it only concerned the teaching of geometry. Secondly, we should consider the 'power' of each opinion, and, as in Cooper's model, it can be understood as the availability of 'resources' which include not only money, but also academic authority, books, people, time, research evidence, and so on. Therefore, the locations of reformers also have to be examined, because they can be seen as a factor contributing to the possibility of access to 'resources' (for example, it might be true that teachers from prominent private schools such as Harrow or Winchester could access universities more easily than other teachers). Furthermore, interactions between members, particularly conflicts, are also important, because, in Cooper's model, such interactions relate to "changes in what counts as school mathematics" (Cooper, 1985, p. 31). As such, it can be inferred that some disagreements between members of the MA did exist in discussions.

In summary, our analysis focuses on the discussions of the teaching committee of the MA from following points of view:

- Can the MA report be seen as a collection of various ideas by the different members?
- What 'resources' did the member use to justify their proposal, and what strategies did the members use to obtain resources?
- What conflicts existed between the MA members, and what was the relationship between the teaching committee and outside interested parties?

In the next section, before analysing the discussions recorded in the minutes of the MA, we provide a short account of the teaching of geometry in the late 19th and early 20th centuries in England in order to give background about what issues were being discussed at that time. Following this, an

analysis is presented of the discussions recorded in the minutes of the MA that led to the drawing up the MA report 1902, utilising Cooper's theoretical framework.

The reform of the teaching of geometry in the late 19th century in England

In the late 19th century in England, Todhunter was a Cambridge mathematician and prominent textbook writer (see Barrow-Green, 2001). He wrote that 'In England the text-book of Geometry consists of the Elements of Euclid; for nearly every official programme of instruction or examination explicitly includes some portion of this work' (Todhunter, 1862, p. vii). At the time, the value of the teaching of geometry was that it was considered to train students' ability in logical reasoning (Howson, 1982, p. 131) such that "every Gentleman should know Greek thought" (Griffiths, 1998, p. 195). The problem was, however, that the direct teaching of strict Euclidean-style geometry was not altogether successful.

In 1871, Association for the Improvement of Geometrical Teaching (AIGT) was founded by University mathematicians and teachers from private schools to improve the teaching of geometry. In 1875, the MA's *Syllabus of Plane Geometry* was published, and approved by the British Association for the Advancement of Science (the BAAS) in 1876 (AIGT, 1877, p. 11). In 1877, the AIGT circulated the syllabus to Examination Boards at Universities including Oxford, Cambridge, London, Durham, and the Department of Science and Art (AIGT, 1878, pp. 18-21). However, English universities, in particular the University of Cambridge, were not favour of the AIGT's syllabus. Similarly, the University of Durham reported significantly that it could do "nothing until it saw a textbook based upon the syllabus" (Brock; 1975, p. 28). Such comments from universities may have made the AIGT change its activities and begin work on the publication of a textbook of its own.

In 1884 and 1886, the AIGT edited and published a geometry textbook entitled *The Elements of Plane Geometry* which included proofs of the theorems contained in the syllabus of 1875. Howson related that this textbook "whilst retaining Euclid's overall sequence, rearranged theorems within allied groups and supplied new proofs", whilst being "without doubt, one of the dreariest books the present author has ever seen" (Howson, 1973, p. 158). In 1887, the AIGT sent this textbook to both Oxford and Cambridge Universities. Again, the universities only agreed that "proofs other than Euclid could be used providing the Euclidean order was not violated" (Siddons,

1936), i.e. the answer from the Universities was very modest (the comments from Cambridge and Oxford are provided at the end of the part I of the MA's *The Elements of Plane Geometry*, 1903 edition).As such, and as has already been pointed out by Brock and Price, the efforts of the AIGT failed to change the teaching of geometry (see, for instance, Brock, 1975, p. 29). Little progress was seen until the beginning of 20th Century.

Perry's address in 1901 and the reform by the MA

Whereas the reforms of the late 19th Century emerged within mathematical communities, the early 20th Century reform in England was begun by J. Perry, Professor of Engineering at the Royal College of Science, with his address entitled *The Teaching of Mathematics* given at the British Association for the Advancement of Science (BAAS) meeting in Glasgow on 13th September 1901 (see Perry, 1902). In his speech, Perry denounced contemporary mathematics teaching in England from an engineer's point of view. With regard to the teaching of geometry, he questioned the educational value of Euclidean geometry for all students, and emphasised the value of the introduction of experimental tasks in the early stages (see Perry, 1902, pp. 158-81). After Perry's speech, various opinions from people from inside and outside mathematics were voiced and argued: examples include the debate by the BAAS (1901), a letter from school masters from private schools, later known as 'the letter of the 22 schoolmasters' (Godfrey and Siddons *et al*, 1902), and the annual meeting of the Mathematical Association in 1902 (note that the A.I.G.T. changed its name to the Mathematical Association, MA, in 1897). In what follows, we focus on the Annual Meeting of the MA in 1902, which was, according to Siddons (1952, pp. 153-155), one of the "main causes of the appointment of the first Teaching Committee" of the MA.

The MA, numbering about 300 members at that time (Price, 1994, p. 64) was described by Godfrey as having "awoke as one out of sleep, thanks to Perry" (Godfrey, 1906, p. 76). The Annual Meeting of the MA was held at King's College, London, on Saturday, January 18th 1902 (MA, 1902a, pp. 129-143). At this meeting, first, the Chairman (Minchin) declared the object of the meeting to be the reform of geometry teaching (MA, 1902a, p. 129). Lodge then read his paper *Reform in the Teaching of Mathematics*, in which he pointed out the problems of the teaching of geometry in England as well as giving his suggestions for improvement. In his paper, Lodge identified that the main problem was caused by "a fixed ancient model" based on the teaching of

traditional Euclidean-style geometry (MA, 1902a, p. 129). Then he outlined his suggestions to reform the teaching of geometry, referring to French textbooks; his suggestions included: the introduction of practical work, the rearrangement of the order of theorems in Euclid, the introduction of hypothetical constructions, the teaching of proportions, the introduction of algebra, and so on (MA, 1902a, pp. 130-131). Following this, the other MA members at the meeting reacted to Lodge's suggestions.

Most of the members seemed to recognise that the traditional style of geometry teaching was the main cause of the problem. In particular, the members considered the greatest problem to be the strict allegiance to the order of theorems in Euclid, primarily because there was no alternative but to expect students to memorise the particular order. To overcome this problem, an introductory course was suggested comprising practical work in the early stages, with the idea that this would enable students to grasp important geometrical facts. Secondly, the members considered that a rearrangement of theorems was necessary. Lodge introduced some ideas and Hill also suggested that the order be rearranged in 'a more natural order'. Godfrey briefly stated his idea of the rearrangement of Euclid's *Elements* such that " [Euclid] Book II. were taken after [Euclid] Book III" (MA, 1902a, p. 140). However no other members offered significant opinions. In fact, Lodge said "the whole subject of rearrangement is too vast to be treated in the course of a paper - it must be settled by a committee" (MA, 1902a, p. 131). Accordingly, tackling these problems became the focus for subsequent meetings of the MA. In the next section we examine how the ideas discussed in the annual meeting led to the proposals contained in the MA geometry report published later in 1902.

The geometry report of the MA published in 1902

Soon after the Annual Meeting, the first Teaching Committee of the MA was established (with the chair being taken by Lodge). There were 26 members of the Committee, mainly from private schools (the members' locations, see Appendix 1). This Committee published two reports in 1902: the geometry report in May (MA, 1902b, pp. 168-172), and the algebra and arithmetic report in July.

In the geometry report, the teaching of geometry was divided into two stages: the introductory and experimental course, and the deductive course (MA, 1902b, pp. 168-172). In the

first stage (the introductory and experimental course), it was suggested that "a first introduction to Geometry should not be formal but experimental, with use of instruments and numerical measurements and calculations" (MA, 1902b, p. 168). In the second stage, the formal course was divided into (i) theorems and (ii) constructions. In general, it was suggested that the related theorems be associated together and the definitions "should not be taught *en bloc* at the beginning of each book, but that each definition should be introduced when required" (MA, 1902b, pp. 168-9). The recommended use of Euclid's order was as follows: Euclid Book I → Euclid Book III to proposition 32 inclusive → Euclid Book II → Euclid Book III proposition 35 to the end → Euclid Book IV. In addition to this, the importance was highlighted of the use of 'riders' (theoretical exercises) in the teaching of geometry. Having made proposals in general, detailed suggestions included 61 suggestions about axioms and definitions, order of theorems, omission of theorems, notation of theorems, methods of proof, and introduction of new theorems. These suggestions can be summarised as follows:

- Suggestions were mainly concerned with the methods of proof, e.g. "That [Euclid Book I proposition] 8 be proved by placing the triangles in opposition";
- This report specified which of Euclid's propositions should be omitted, e.g. "That [Euclid Book I proposition] 7 be omitted";
- Only one suggestion, "That [Euclid Book I propositions] 13, 14, 15 be taken first", was proposed as to the detailed order of theorems. There was no specific order of theorems which the Committee thought appropriate;
- It was suggested that algebra be introduced: "That illustration from Algebra ought to be given where such is possible";
- It was suggested that the theory of proportion be dealt with only in commensurable magnitudes.

A specific detailed order of theorems was not proposed in this report, even though this issue was central to the Annual Meeting of the MA earlier in 1902. Furthermore, the geometry report stated that "it is not proposed to interfere with the logical order of Euclid's series of theorems; - in other words, it is not proposed to introduce any order of theorems that would render invalid Euclid's proof of any proposition" (MA, 1902b, p. 168). That is to say, the conclusion was very modest with regard to the order of theorems.

Now, we focus on why the members of the MA Teaching Committee reached such modest conclusions. In particular, it is key interest that the geometry report did not propose a new order of theorems (remember Lodge's word in the earlier Annual Meeting). To address this issue, we analyse the discussions held by the teaching committee of the MA (members' name with their locations are listed in table 1 in the Appendix) leading up to the geometry report of 1902. The source of data is the book of minutes of the teaching committee of the MA (unpublished), in which is recorded the discussions of the meetings of the committee. This book of minutes, covering five meeting from February 15th to March 22nd 1902, can found in the archive of the MA's library in Leicester, UK. In the next sections, we analyse the meetings in terms of the theoretical framework outlined in the early part of this paper.

Discussions of the teaching committee of the MA in 1902

First meeting

Place and time: King's College, London, on Saturday, February 15th, at 3:00.

Members: Professors Minchin and Lodge, Messrs Godfrey, Garstang, Saunder, Holmes, Barnard, Dyer, Pendlebury, Dr Macaulay, Messrs Roseveare, Rumsey, Hogg, F. W. Hill and Siddons.

Secretary: Minchin

In this meeting, first, the chair and secretary were decided: Minchin and Siddons were voted, respectively. It was also decided to write invitation letters to the headmasters of Marlborough, Haileybury, Cheltenham, Chifton, Westminster and Shrewsbury. Sub-committees, which would consider the draft of the report on geometry and algebra, were decided. As the members of the geometry subcommittee, the following members were elected: Lodge, Godfrey, Barnard and Rumsey; and the following became members of the algebra and arithmetic subcommittee: Garstang, Tuckey, F. W. Hill, Roseveare and Dyer.

Second meeting

Place and time: King's College, London, on Saturday, March 1st, at 3:00 pm.

Members: Professors Minchin and Lodge, Messrs Godfrey, Barnard, Sherwood, Romsey, Dyer, Garstang, Hawkins, Baker, Hogg, Saunder, Playne. Drury, Tuckey, Holmes, Dr Macaulay, Messrs Roseveare, Rumsey, Saunders and Siddons.

Secretary: Minchin

In this meeting, the following five proposals and recommendations for the teaching of the theorems in Euclid's Book I were discussed in the first draft of the report, drawn up by the geometry sub-committee.

1. That the first introduction to Geometry should not be formal but experimental, with use of instruments, and numerical measurements and calculations.
2. That Private Schools in their entrance examinations should set a fair proportion of questions requiring the use of instruments, and the obtaining of numerical results from numerical data by measurements from accurately drawn figures; and that in their entrance scholarship examinations the same principle should be recognized.
3. That in formal Geometry, constructions should not form part of the logical course on theorems, but in proving theorems, hypothetical constructions be permitted.
4. That examination papers in Elementary Geometry, such as University Local Examinations, the Examinations, Oxford Responsions (the first of the three examinations once required for an academic degree at the University of Oxford), and the Cambridge Previous Examination, ought to contain some questions requiring the practical use of instruments.
5. That such examinations should contain only a small amount of book-work, questions being set about propositions, and that some ability to solve 'riders' (theoretical exercises) should be needful to secure a pass.

The third proposal was referred back to the sub-committee, and the fifth was withdrawn. A few amendments were made by the members of the committee as to the suggestions of Book I, e.g. that "29 and 30 be proved from Playfair's axiom" was amended to "Playfair's axiom is preferable to Euclid's 12th axiom" (carried by votes 12 to 5). The following proposals were also made by individuals:

- Saunder proposed that "these words be added to §4 of the geometry report 'The proportion of such questions should not exceed 1:4."', but this was not seconded.
- Macaulay proposed that "it is desirable that the axioms, definitions and postulates be revised by the subcommittee", and Playne seconded this. This motion was rejected by 8 votes to 9.
- Playne proposed that "after §5 the rest of the geometry report be rejected", but this was not seconded.

Finally, the following motion was carried unanimously at the end of the meeting: 'this committee does not propose to interfere with the logical order of Euclid's theorems; so long as this is retained the actual order and number is immaterial.'

Third meeting

Place and time: King's College, London, on Saturday, March 15th, at 3:00 pm.

Members: Professors Minchin and Lodge, Messrs Playne, Sherwood, Saunder, Dr Macaulay, Messrs Baker, Barnard, Dyer, Gerrans, Roserveare, Rumsey, Holmes, Garstang, Godfrey, F. W. Hill and Siddons.

Secretary: Minchin

In this meeting, first, the following people were unanimously elected members of this committee: H. T. Gerrans of Oxford, Hudson of King's College, and E. T. Wittaker, St. Paul's School. Then, Gerrans read to the members the future regulation for the Oxford Local Examination in 1903, which stated that 'Any solution which shows an accurate method of geometrical reasoning will be accepted', i.e. it was decided that other proofs than Euclid's would be accepted.

Then, the draft of the report was considered. The main focus of discussion in this meeting was suggestions as to the theorems in Euclid Books I, II, and III, for example:

- The proposal 'introduction of algebraic methods of proof in book II' was rejected by 6 votes to 9, but it was decided that comments about this might be considered at the next meeting.
- It was decided to retain Euclid Book III 9, although its omission had been considered in the first draft.

- The proposal that 'Euclid book III 9 be taken as a corollary to Euclid book III. 1' was received 8 votes in favour and 8 against, but the chairman, Minchin, gave his casting vote in favour of it.
- The proposal that 'Euclid Book III 7-8 be omitted' was rejected by 4 votes to 7, and finally decided that 'the last parts of 7 and 8 be omitted'.

When we refer to the second draft, almost all proposals were carried unanimously. The few that were not included, e.g. proposal II 5, "That connected theorems should be as far as possible be associated together in the pupil's mind", was carried by 9 votes to 3, the omission of Euclid Book III 4 and 11 was carried by 8 votes to 4, and 10 votes to 1 respectively, and the limit definition of a tangent was carried by 13 votes to 1.

Fourth meeting

Place and time: King's College, London, on Saturday, March 22nd, at 3:00 pm.

Members: Prof. Minchin (in the chair), Lodge and Hill, Messrs Garstang, Rumsey, Gerrans, Rosenveare, Barnard, Dyer, F. W. Hill, Saunder, Dr Macaulay, Messrs Playne, Sherwood, Baker, Hogg, Godfrey and Siddons.

Secretary: Minchin

First, the comments from Cambridge Local Examinations, and Civil Service Commission were announced; the former stated that they "would be glad to consider the suggestions made by the committee' and the latter stated that 'in Geometry the demonstrations of sequence of propositions need not be those of Euclid".

Then the draft of the geometry report was considered. First, the algebraical method which remained unsolved from the previous meeting, was discussed. Lodge proposed that "an Algebraical treatment be allowed in [Euclid] Book II except in Prop. 1, Euclid proof being there retained so as to establish rigidly a geometrical Analogue of distributive law", and Rumsey seconded it. Gerstang proposed an amendment recommendation that "after proof 7 [of Euclid Book II], algebraic methods of proof be allowed with a special view to proofs 12 & 13 [Euclid Book II]". Rosenveare seconded the amendment, but this was rejected by 3 votes to 15. The original motion was also rejected by 3 votes to 15. Hill proposed that Euclid Book IV. 10 and 11 be omitted, and

this was seconded by Saunder. The motion was carried by votes 9 votes to 6, and 8 votes to 5 respectively. Other proposals were not decided unanimously, e.g. the definition of a square was carried by 15 votes to 2, the order of the first three books in Euclid was carried by 11 votes to 4.

Fifth meeting

Place: King's College, London, on Saturday, May 10th, at 3:00 pm.

Members: Prof. Minchin (in the chair), Prof. Lodge, Messrs Sherwood, Tuckey, F. W. Hill, Roserveare, Dyer, Macaulay, Garstang, Baker, Saunder, Lyna,. Barnard, Rumsey, Godfrey and Siddons.

Secretary: Lodge

In this meeting, the recommendations as to Euclid Book VI, i.e. the applications of the theory of proportion (the similarity of figures), were mainly discussed, First, they discussed Macaulay's following proposals in sections 47 and 48 in the draft:

> The committee suggests
>
> 1. That the study of Book VI should be preceded by
>
> (a) A theory of measurement of lengths of lines and areas of rectangles for cases in which the lines and the sides of the rectangles are commensurable
>
> (b) An algebraical treatment of ratio and proportion for commensurables.
>
> 2. That a first course on Book VI should be for commensurable only and that an ordinary school course should not be required to include more than this.
>
> 3. That the course on commensurables should be followed by a general theory of ratio and proportion, including incommensurables, either on the lines of Euclid Book IV or some more modern arrangement.

After considerable discussions, recommendations agreed were decided as follows, i.e. part of his proposals was rejected by the members, in particular the third recommendation (MA, 1902b, p. 171):

> 47. The committee suggest that the study of Book VI. should be preceded by
>
> (1) A theory of measurement of lengths of lines and areas of rectangles for cases in which the lines and the sides of the rectangles are commensurable, (Cf. § 27.)

(2) An algebraical treatment of ratio and proportion for commensurables.

48. That an ordinary school course should not be required to included incommensurables; - in other words, that in such a course all magnitudes of the same kind should be treated as commensurable.

Then, the members discussed the proposal by Macaulay: "That in the opinion of the Committee the course in commensurables might with advantage be followed, in the case of advanced students, by a general theory etc". Garstang seconded but the motion was rejected by 6 votes to 9. After several changes and additions had been made in Euclid Book VI, finally Lodge stated that the whole Report be passed, and Godfrey seconded. The motion was carried unanimously.

The process of the redesigning geometry curriculum – the case of the MA

Having described the discussions leading up to the geometry report of the MA, we now analyse the process of the drawing up the report in more detail in terms of the framework for analysis outlined in the early part of this paper.

The geometry report of the MA as a collection of various ideas

As we have seen in the previous section, various ideas, such as the omission of theorems, and the methods of proofs were discussed in the meetings. Furthermore, practical work or the introduction of algebra, which were not included in Euclid, were proposed. These conclusions were not reached by only one person: all proposals needed to be seconded and carried by other members' votes. Therefore, in brief, it can be said that the report reflects a compromise of the members' opinions. However, it should be noted that the weight of the committee's different conclusions was not equal. The conclusion that the teaching of geometry should be based on Euclid was still very powerful in this meeting: this report confirmed that the members would not violate the logical order of theorems in Euclid. On the other hand, the introduction of the algebra, though discussed during several meetings, remained an ambiguous, and therefore weak conclusion.

Members' locations

We must pay attention to the locations of the members, i.e. the committee members were teachers in prominent private schools and university mathematicians, who had thus all taught and respected

Euclid's *Elements*. When we consider this, it is reasonable to say that it would be difficult for them to do away completely with traditional Euclidean style of geometry teaching in secondary schools. That the statements 'this committee does not propose to interfere with the logical order of Euclid's theorems' was carried unanimously clearly supports this point of view. The locations of the members therefore suggested their conclusions might not be radical.

Power against traditional Euclidean style of geometry teaching

From inside and outside the teaching committee, the main resources, which might be used to be against with traditional Euclidean style of geometry teaching are considered as follows: Perry's address in Glasgow, 'the letter of the 22 schoolmasters', and the information of the examination boards of universities. Of these resources, the most important one would be the information from universities, because the universities had strong control over the syllabi and textbooks in the teaching of geometry at that time. The members definitely knew how important the university examinations were, and hence they approached H. T. Gerrans, Secretary of the Oxford Local Examination Delegacy to join them. In fact, as we have seen, they obtained the information that concessions had been made by the university examiners, thanks to pressure from earlier reformers, such as Perry, or 22 schools maters, i.e. any proof would now be accepted in geometry examinations. Thus, the members could recommend different methods of proofs of Euclid's propositions. Nevertheless, the detailed revised order of theorems, or the strong advocacy of the use of algebra in geometry were not proposed in the final draft, although, in particular, the detailed order was included in the first drafts of the report. It seemed that the members did not have enough resources to give weight to these radical ideas in geometry. The members probably did not make full use of all the resources at their disposal. In the minute book, there is no evidence that they referred to the textbooks or syllabi in existence in France or other countries, in which the theorems were arranged differently from Euclid, or the introduction of the algebraical method in the teaching of geometry.

Internal interaction

The disagreements and conflicts between members, i.e., the internal interactions, can be recognised in discussions, and it can be considered that they affected the final decisions of the report. For

example, the detailed revised order was omitted in the final draft of the report. The issue of the order of theorems were very controversial in the meetings: whereas the order of the first three books was proposed Book I 32 → Book III 32 → Book I 33 to end→ Book II → Book III 35 to end, it was amended Book I → Book III 32 → Book II, Book IV 35 to end, and still four people opposed this decision on 22nd March. Another example of disagreement was, again, the introduction of algebra. The final recommendation as to algebra is rather ambiguous in the geometry report of the MA: '42. That illustration from Algebra ought to be given where such is possible' (MA, 1902b, p. 170). As we have seen, the methods of algebra caused discussions and conflicts in the meetings (see the descriptions in the second and third meetings) with the result that the only conclusion they could reach was such an ambiguous suggestion. Furthermore, even the omission of certain theorems caused controversy in this meeting, and it can be inferred that these conflicts wasted time (one of the 'resources') in discussions. In fact, Siddons stated that "the standard order had not been sufficiently discussed" (Siddons, 1902, p. 253)

External interaction

As the external interactions, it can be considered that the climate of the reform at that time was ready for the complete abolishment of traditional Euclidean style. In particular, the issue of the order of theorems could be a cause of controversy. For example, whereas Perry severely attacked such geometry teaching in his address in Glasgow in 1901, Forsyth, Lamb, and Larmor voiced unsatisfactory comments on Perry's opinions (Perry, 1902; Howson, 1982, p. 149). 'The letter of the 22 schoolmasters' stating that 'it may be felt convenient to retain Euclid' (Godfrey and Siddons *et al*, 1902, p. 258) also showed the modest attitude to reform. In 1902, before the privateation of the geometry report of the MA, Lodge proposed a detailed revised order of theorems in Book I of the *Elements*, and suggested that the order be rearranged from angles, parallel lines, congruent triangles to inequalities of triangles (Lodge, 1902, p. 534). This caused immediate responses from W. C. Fletcher, E. T. Dixon, T. Petch, R. B. Hayward, and G. H. Bryan in 1902 in *Nature.* Some of them agreed with Lodge's order, and while others proposed different orders or disagreed with it. If even the revised ordering of Euclid's Book I was controversial, what would happen if the committee members advocated a detailed revised new order for the whole work, Euclid Book I, II,

III, IV and VI? The members must have been aware of the fairly anxious climate towards change, and considered that radical reform would cause confusion among teachers.

Conclusion

In this paper, we have focused on the geometry report of the MA, which was one of the major achievements in the early 20th Century in the history of mathematics education. Although it was considered that Euclid was no longer suitable as a textbook in secondary schools and the necessity of rearranging the order of theorems was advocated, the recommendations of the report were quite modest. To examine the reasons for this, we analysed (using a theoretical framework developed from Cooper's model) the discussions leading up to the report through the minutes of the teaching committee of the MA in 1902.

In summary, the causes of the rather conservative character of the report can be stated as follows: because of the nature of the members' locations, the members still respected Euclid as a basis of school geometry; the 'resources' available to the MA committee (status, academic legitimacy and finance) were not sufficient to devise the complete replacement of the traditional Euclidean approach, but they were enough for at least the recommendation covering different methods of proof; the conflicts between the members prevented a complete consensus; and the climate outside the teaching committee was not ready for radical reform at that time. Hence, the 1902 MA geometry report can be described as a compromise by the members of the MA's teaching committee.

These results suggest that we must be aware of the facts that various people are likely to be involved in subject reform, and if we want to make a successful reform, we have to consider various reformers' locations, the availability of 'resources' and interactions. This is likely to still be the case and is applicable to the interpretation and analysis of current curricula in schools. As such, we could enhance our understanding of the curriculum by looking at, for example, who was involved in the decision-making process behind it, what and whose were the strongest opinions, or what resources did they use to advocate their ideas.

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Appendix

Table A.1. List of the members of the MA Teaching Committee in 1902

Members	Locations
Mr. J. Fretcher	Moulton
M. J. M. Hill	University College
A. Lodge	Royal Indian Engineering College
G. M. Minchin	Royal Indian Engineering College
W. M. Baker (joined from 2nd meeting)	Cheltenham College
S. Barnard	Rugby School
H. D. Drury	Marlborough College
J. M. Dyer (joined from 2nd meeting)	Eton College
T. J. Garstang	Bedales School
H. T. Gerrans (joined from 3rd meeting)	Secretary of the Oxford Local Examination Delegacy
C. Godfrey	Winchester College
W. J. Greenstreet	Marling School
C. Hawkins (joined from 2nd meeting)	Haileybury College
F. W. Hill	City of London School
R. W. Hogg	Christ's Hospital
H. T. Holmes	Merchant Taylors' School
Prof. Hudson (joined from 3rd meeting)	King's College
E. M. Langley	Bedford Modern School
C. C. Lynam (joined from 4th meeting)	Oxford Preparatory School
C. Pendlebury	St. Paul School
H. C. Playne (joined from 2nd meeting)	Clifton College
W. N. Roseveare	Harrow School
C. A. Rumsey	Dulwich College
S. A. Saunder	Wellington College
H. A. Saunders (joined from 2nd meeting)	Hailerybury College
E. C. Sherwood (joined from 2nd meeting)	Westminster School
A. W. Siddons (secretary)	Harrow School
C. O. Tuckey	Charterhouse School
E. T. Whittaker (joined from 3rd meeting)	Trinity College, Cambridge
Dr. F. S. Macaulay	St. Paul's School