Managing inventory and production capacity in start-up firms

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Abstract

Successful start-up firms make a valuable contribution to economic growth and development. Models that provide insight into the management of start-up firms are therefore important. We consider the problem of managing inventory and production capacity in a start-up manufacturing firm and argue that for such firms the objective of maximising the probability of the firm surviving is more appropriate than the more common objective of maximising profit. Using Markov decision process models, we characterise and compare the form of optimal policies under the two objectives. This analysis shows the importance of coordination in the management of inventory and production capacity. The analysis also reveals that a start-up firm seeking to maximise its chance of survival will often choose to keep production capacity significantly below the profit maximising level for a considerable time. This insight helps us to explain the seemingly cautious policies adopted by a real start-up manufacturing firm.
Keywords: start-up firms; inventory; production ramp-up; stochastic modelling; dynamic programming

1 Introduction

Start-up firms make an important contribution to the success of a country's economy by creating jobs and increasing competition and innovation. However such firms face a high risk of failure during the start-up phase. Hence there is a strong need for models that provide insight into the problems facing start-up firms and help to identify strategies that ensure the long-term survival of such firms. Two decisions that have a significant effect on the chance of long-term survival of a start-up manufacturing firm are the choice of initial production capacity and the subsequent production ramp-up (i.e. the increase of production capacity from its initial level). This paper investigates the characteristics of optimal policies for start-up manufacturing firms by modelling their production capacity decisions and inventory strategy during the start-up phase. Our model is based on the problem faced by a real start-up manufacturing firm and we explain how the model has provided important insights about this problem. The paper also examines how optimal strategies for start-up manufacturing firms differ from those for well-established firms.

Management science models of manufacturing problems almost always include the objective of optimising the cost or profit to the firm [Silver et al., 1998]. Archibald et al. [2002] suggest that such models are not suitable for start-up firms whose available capital is generally limited. They suggest that start-up firms should focus on maximising the probability of survival rather than optimising cost or profit. In this paper we develop a model of a start-up manufacturing firm that extends their model to include capacity planning decisions as well as inventory strategy. In the model a firm is said to have failed if, in any period, it has insufficient capital, including possible overdrafts and loans, available to meet its overhead costs. The overhead costs will typically include the recurrent cost of finance, employee wages, equipment lease charges and rent for premises. Due to the limited available capital, the optimal survival policy for a start-up firm may invest less capital in production capacity and inventory and so keep more capital in reserve to cover overhead costs in cases of low demand. In our model of a well-established manufacturing firm there is no constraint on available capital and so this will not be a consideration. We might therefore expect the optimal strategy for a start-up firm to be more cautious than that for a well-established firm.

There is little research on joint production and financial decisions for start-up manufacturing firms. Our previous work [Archibald et al., 2002, Possani et al., 2003, Thomas et al., 2003] uses a similar approach to that of this paper, but concentrates on inventory strategy assuming fixed production capacity for the entire lifetime of the firm. Betts and Johnston [2001] also focus on the inventory strategy of a manufacturing firm with limited capital. They compare the optimal strategy for their model with those of traditional modelling approaches to inventory management in a deterministic setting. Buzacott and Zhang [1998] use a linear programming model of a manufacturing firm to investigate inventory and borrowing strategies that maximise profit subject to a borrowing constraint. Unlike the problem analysed in this paper, the production capacity is fixed and the future demand for the manufactured product is assumed to be known. Cantamessa and Valentini [2000] use mixed-integer linear programming to find the optimal initial production capacity and inventory strategy for a manufacturing firm. Their model differs significantly from ours because it does not allow the firm any control over increases in production capacity. Their model also seeks to maximise profit, assumes future demand is known and assumes infinite borrowing. Terwiesch and Bohn [2001] develop a dynamic programming model of production ramp-up in which the firm can choose to lower production capacity in the short-term in order to free up time for training and so increase future production capacity. Unlike our model, the objective is to maximise profit and there is no capital constraint or uncertainty.

In section 2 we develop Markov decision process models of start-up and well-established manufacturing firms who must make decisions about capital investment in inventory and production capacity in the face of uncertain demand. In section 3 we derive properties of the optimal strategy for a start-up manufacturing firm with the objective of maximising its survival probability. In section 4 we characterise the optimal strategy for a well-established manufacturing firm under the objective of maximising profit. In section 5 we compare the forms of these optimal strategies. In section 6 we describe the application of the model to a situation facing a real start-up manufacturing firm. Finally in section 7 we present our conclusions.

2 Models of manufacturing firms

Consider a firm selling, at price S, one type of product that it manufactures to order from a component (or group of components) it purchases at cost C. The lead time for ordering components is L_q periods. The demand for the product each period is an independent identically distributed random variable. The maximum possible production capacity, and hence the maximum demand that can be satisfied in a period, is M. For $0 \leq d < M$, let $p(d)$ denote the probability that there is a demand for d items in a period and let $p(M)$ denote the probability that there is a demand for at least M items in a period. The production capacity is determined by equipment, number of staff and staff training. Any change to the production capacity (up or down) incurs a one-off charge of R per unit of production capacity and takes effect after L_r periods. Each period the firm has to meet a fixed overhead cost H plus a variable overhead cost of r per unit of production capacity. The one-off charge R covers, for example, the costs of acquiring or disposing of equipment and reorganising the workforce, while the recurrent cost r models changes in the cost of finance, labour etc. We will consider $L_q = L_r = 0$ and $L_q = L_r = 1$.

It is assumed that for an established firm, there is no practical constraint on the amount of capital available and that the objective is to maximise the long-run average profit per period. As the firm manufactures the product to order, an inventory of manufactured products will never be carried over from one period into the next. This situation may arise if, for example, storage of the product is impractical or the exact specification of the product is determined by the customer. Although there is no direct inventory cost in the model, it is assumed that the cost of capital is included in the overhead cost. In an earlier paper [Possani et al., 2003], the authors show that introducing a direct inventory cost to a simpler model does not alter the results, only the analysis. Hence the state of the firm at the start of a period is completely described by the number of components in stock and the production capacity. Each period the firm must decide the order quantity, k, and the new production capacity, j'. Standard results for average reward Markov decision processes [Puterman, 1994] can be applied as follows. Let g be the maximum average reward per period and let $v(i,j)$ be the bias term of starting with i components in stock and j units of production capacity. Define $a(i,k) = \begin{cases} i+k \text{ if } L_q = 0 \\ i \text{ if } L_q = 1 \end{cases}$ and $c(j,j') = \begin{cases} j' \text{ if } L_r = 0 \\ j \text{ if } L_r = 1 \end{cases}$. The optimality equation of the dynamic programming model of the firm under the above assumptions is as follows.

$$g + v(i,j) = \max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) \Big(\min(a(i,k), c(j,j'), d)S - kC - |j-j'|R \right. \\ \left. - H - c(j,j')r + v(i+k-\min(a(i,k), c(j,j'), d), j') \Big) \right\} \tag{1}$$

The model has finite state and action spaces ($0 \leq i \leq 2M$, $0 \leq k, j, j' \leq M$). Let $k_q(i,j)$ and $k_r(i,j)$ be the optimal order quantity and new production capacity respectively.

It is assumed that a start-up firm has a limited amount of capital available and has the objective of maximising its chance of long run survival. The state of the firm at the start of a period is described by the number of components in stock, the production capacity and the amount of available capital. Each period the firm must decide the order quantity, k, and the new production capacity, j'. Let $q(n,i,j,x)$ be the maximum probability that the firm survives for n periods given it currently has i components in stock, j units of production capacity and x units of available capital. Assuming that all revenues and costs can be expressed as multiples of a common unit, the problem can be formulated as a finite horizon Markov decision process with a countable state space and finite action space [Puterman, 1994]. For $n > 0$ and $x \geq 0$, the optimality equation of the dynamic programming model of the firm under the above assumptions is as follows.

$$q(n,i,j,x) = \max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) q(n-1, i+k-\min(a(i,k), c(j,j'), d), j', \right. \\ \left. x + \min(a(i,k), c(j,j'), d)S - kC - |j-j'|R - H - c(j,j')r) \right\} \tag{2}$$

We assume that the firm survives an interval of n periods if and only if the amount of available capital is non-negative at the start and end of every period in the interval. This is reflected in the boundary conditions $q(0,i,j,x) = 1$ if $x \geq 0$ and $q(n,i,j,x) = 0$ if $x < 0$. We are particularly interested in the $\lim_{n\to\infty} q(n,i,j,x)$ which can be interpreted as the probability that the firm survives in the long-run given that it currently has i components in stock, j units of production capacity and x units of available capital.

3 Properties of the survival model for a start-up firm

In this section we establish some important properties of the optimal survival strategy and the maximum survival probability for a start-up manufacturing firm under the assumptions of model (2) above. We first show that the survival probability $q(n,i,j,x)$ is monotonic in n, i and x but, due to the cost of removing or keeping additional capacity, not j.

Lemma 1

(i) $q(n+1,i,j,x) - q(n,i,j,x) \le 0$, i.e. $q(n,i,j,x)$ is non-increasing in n.

(ii) $q(n,i,j,x)$ is non-decreasing in i.

(iii) $q(n,i,j,x)$ is non-decreasing in x.

(iv) $q(n,i,j,x)$ is non-monotonic in j.

Proof

It is easy to see that properties (i), (ii) and (iii) hold for $n=0$. Assume these properties hold for n and use $\max_i\{a_i\} - \max_i\{b_i\} \le \max_i\{a_i - b_i\}$ to show that:

(i) $$q(n+2,i,j,x) - q(n+1,i,j,x) \le \max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) \left(q\Big(n+1, i+k-\min(a(i,k),c(j,j'),d), j', x + \min(a(i,k),c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big) - q\Big(n, i+k - \min(a(i,k),c(j,j'),d), j', x + \min(a(i,k),c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big)\right)\right\}$$
≤ 0 by inductive hypothesis for (i).

(ii) $$q(n+1,i,j,x) - q(n+1,i+1,j,x) \le \max_{k,j'} \left\{ \sum_{d=0}^{a(i,k)} p(d) \left(q\Big(n, i+k-\min(c(j,j'),d), j', x + \min(c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big) - q\Big(n, i+1+k-\min(c(j,j'),d), j', x + \min(c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big)\right) + \sum_{d=a(i+1,k)}^{M} p(d) \left(q\Big(n, i+k - \min(a(i,k),c(j,j')), j', x + \min(a(i,k),c(j,j'))S - kC - |j-j'|R - H - c(j,j')r\Big) - q\Big(n, i+1+k-\min(a(i+1,k),c(j,j')), j', x + \min(a(i+1,k),c(j,j'))S - kC - |j-j'|R - H - c(j,j')r\Big)\right)\right\}$$
≤ 0 by inductive hypothesis for (ii) and (iii).

(iii) $$q(n+1,i,j,x) - q(n+1,i,j,x+y) \le \max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) \left(q\Big(n, i+k-\min(a(i,k),c(j,j'),d), j', x + \min(a(i,k),c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big) - q\Big(n, i+k - \min(a(i,k),c(j,j'),d), j', x + y + \min(a(i,k),c(j,j'),d)S - kC - |j-j'|R - H - c(j,j')r\Big)\right)\right\} \le 0$$ by inductive hypothesis for (iii).

(iv) The fact that $q(n,i,j,x)$ is non-monotonic in j is verified by the following examples.

Assume that $H > S > r$, $R, r > 0$ and $p(0), p(1) > 0$.

From state $(1,1,0,H+r-S)$, it is impossible to survive if the demand in the next period is 0 or 1. Hence $q(1,1,0,H+r-S) \le 1-p(0)-p(1)$. However the decisions $k=0$ and $j'=1$ ensure survival from state $(1,1,1,H+r-S)$ provided the demand in the next period is not 0. Hence $q(1,1,1,H+r-S) \ge 1-p(0)$. Therefore $q(1,1,1,H+r-S) > q(1,1,0,H+r-S)$.

The decisions $k=0$ and $j'=0$ ensure survival from state $(1,0,0,H)$ regardless of the demand in the next period. Hence $q(1,0,0,H) = 1$. However from state $(1,0,1,H)$, it is impossible to survive if the demand in the next period is 0. Hence $q(1,0,1,H) \le 1-p(0)$. Therefore $q(1,0,1,H) < q(1,0,0,H)$.

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We are now in a position to describe some of the properties of the long-run survival probability of a start-up manufacturing firm.

Corollary 1

(i) $q(i, j, x) = \lim_{n\to\infty} q(n, i, j, x)$ exists.

(ii) $q(i, j, x)$ is non-decreasing in i.

(iii) $q(i, j, x)$ is non-decreasing in x.

Proof

(i) Since $q(n, i, j, x)$ is a probability, $0 \leq q(n, i, j, x) \leq 1$. Lemma 1 (i) shows that $q(n, i, j, x)$ is non-increasing in n. The result follows from the fact that bounded monotonic sequences converge.

(ii) & (iii) These follow immediately by taking limits in parts (ii) and (iii) of Lemma 1.

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We now describe conditions that are sufficient to ensure that the long-run survival probability of the firm is greater than zero. We also construct policies that provide the firm with a chance of long-run survival. We show that for any production capacity j satisfying the property that the expected revenue from sales each period (assuming unlimited inventory) is greater than the overhead costs each period, the policy of ordering up to $2j$ items and keeping production capacity at j units gives the firm a positive chance of survival.

Theorem 1

Assume that $\exists \tilde{\jmath}$ such that $(S-C)\sum_{d=0}^{M} p(d)\min(d, \tilde{\jmath}) > H + \tilde{\jmath}r$, since otherwise the firm can never be profitable in the long run. For all inventory levels i and production capacities j, $q(i, j, x) > 0$ for some finite x.

Proof

Consider the policy of ordering up to $2\tilde{\jmath}$ items each period and keeping the capacity fixed at $\tilde{\jmath}$. In the first period, the order cost is at most $2\tilde{\jmath}C$ and the overhead cost is at most $H + Mr$. These costs arise if the initial inventory level is zero, the initial production capacity is M and $L_r = 1$. Changing the production capacity to $\tilde{\jmath}$ initially will cost at most $\max(\tilde{\jmath}, M - \tilde{\jmath})R$. Hence $2\tilde{\jmath}C + \max(\tilde{\jmath}, M - \tilde{\jmath})R$ is sufficient capital to initiate the policy and a further $H + Mr$ units of capital are enough to ensure survival in the first period. After the first period, orders are only placed to replace items that have been sold and so can be financed from sales revenue. Also the overhead cost is fixed and equal to $H + \tilde{\jmath}r$. Now the argument of Archibald et al. [2002] can be used to show that if the initial capital is at least $2\tilde{\jmath}C + \max(\tilde{\jmath}, M - \tilde{\jmath})R + H + Mr$, $q(i, j, x) > 0$ for all i and j.

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Next we examine circumstances in which a start-up firm would prefer to have available capital rather than a greater investment in inventory or production capacity. When the lead times for orders and changes to production capacity are both zero, these results are straightforward because available capital can be exchanged immediately for inventory or production capacity. However when the lead times are non-zero and inventory or production capacity is scarce, the firm must consider the possibility of lost sales.

Lemma 2

(i) If $L_q = L_r = 0$, $q(i, j, x + C) \geq q(i + 1, j, x)$.

(ii) If $L_q = L_r = 0$, $q(i, j, x + R) \geq q(i, j + 1, x)$.

(iii) If $L_q = L_r = 1$ and $i \geq j$, $q(i, j, x + C) \geq q(i + 1, j, x)$.

(iv) If $L_q = L_r = 1$ and $i < j$, $q(i, j, x + S) \geq q(i + 1, j, x)$.

(v) If $L_q = L_r = 1$ and $i \leq j$, $q(i, j, x + R - r) \geq q(i, j + 1, x)$.

(vi) If $L_q = L_r = 1$ and $i > j$, $q(i, j, x + S + R - r) \geq q(i, j + 1, x)$.

Proof

(i) Suppose the optimal decisions in state $(i + 1, j, x)$ are to order k items and to set the production capacity to j' units. Consider the decisions to order $k + 1$ items and to set the production capacity to j' units in state $(i, j, x + C)$. Since these are feasible decisions in this state,

$$q(i, j, x + C) \geq \sum_{d=0}^{M} p(d) q\Big(i + k + 1 - \min(i + k + 1, j', d), j',$$
$$x + \min(i + k + 1, j', d)S - (k + 1 - 1)C - |j - j'|R - H - j'r\Big) = q(i + 1, j, x).$$

(ii) Suppose the optimal decisions in state $(i, j + 1, x)$ are to order k items and to set the production capacity to j' units. Since these are feasible decisions in state $(i, j, x + R)$,

$$q(i, j, x + R) \geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(i + k, j', d), j',$$
$$x + \min(i + k, j', d)S - kC + (1 - |j - j'|)R - H - j'r\Big) \geq q(i, j + 1, x)$$

where the second inequality follows from Corollary 1 (iii) and the fact that $|j - j'| \leq 1 + |j + 1 - j'|$.

(iii) Suppose the optimal decisions in state $(i + 1, j, x)$ are to order k items and to set the production capacity to j' units. Consider the decisions to order $k + 1$ items and to set the production capacity to j' units in state $(i, j, x + C)$. Since these are feasible decisions in this state,

$$q(i, j, x + C) \geq \sum_{d=0}^{M} p(d) q\Big(i + k + 1 - \min(j, d), j',$$
$$x + \min(j, d)S - (k + 1 - 1)C - |j - j'|R - H - jr\Big) = q(i + 1, j, x).$$

(iv) First prove by induction that $q(n, i, j, x + S) \geq q(n, i + 1, j, x)$ and then the result follows by taking the limit as $n \to \infty$. It is easy to see that $q(0, i, j, x + S) \geq q(0, i + 1, j, x)$. Assume the result holds for $n - 1$. Suppose the optimal decisions in state $(n, i + 1, j, x)$ are to order k items and to set the production capacity to j' units. Since these are feasible decisions in state $(n, i, j, x + S)$,

$$
\begin{aligned}
q(n,i,j,x+S) \geq \sum_{d=0}^{i} p(d) q\Big(n-1, i+k-d, j', x+(d+1)S - kC - |j-j'|R - H - jr\Big) \\
+ \sum_{d=i+1}^{M} p(d) q\Big(n-1, k, j', x+(i+1)S - kC - |j-j'|R - H - jr\Big) \\
\geq \sum_{d=0}^{i} p(d) q\Big(n-1, i+k+1-d, j', x+dS - kC - |j-j'|R - H - jr\Big) \\
+ \sum_{d=i+1}^{M} p(d) q\Big(n-1, k, j', x+(i+1)S - kC - |j-j'|R - H - jr\Big) = q(n, i+1, j, x)
\end{aligned}
$$

where the second inequality follows from the inductive hypothesis.

(v) Suppose the optimal decisions in state $(i, j+1, x)$ are to order k items and to set the production capacity to j' units. Since these are feasible decisions in state $(i, j, x+R-r)$,

$$
\begin{aligned}
q(i,j,x+R-r) \geq \sum_{d=0}^{M} p(d) q\Big(i+k-\min(i,d), j', \\
x + \min(i,d)S - kC + (1-|j-j'|)R - H - (j+1)r\Big) \geq q(i, j+1, x)
\end{aligned}
$$

where the second inequality follows from Corollary 1 (iii) and the fact that $|j-j'| \leq 1 + |j+1-j'|$.

(vi) Suppose the optimal decisions in state $(i, j+1, x)$ are to order k items and to set the production capacity to j' units. Since these are feasible decisions in state $(i, j, x+S+R-r)$,

$$
\begin{aligned}
& q(i,j,x+S+R-r) \\
& \geq \sum_{d=0}^{j} p(d) q\Big(i+k-d, j', x+(d+1)S - kC + (1-|j-j'|)R - H - (j+1)r\Big) \\
& \quad + \sum_{d=j+1}^{M} p(d) q\Big(i+k-j, j', x+(j+1)S - kC + (1-|j-j'|)R - H - (j+1)r\Big) \\
& \geq \sum_{d=0}^{j} p(d) q\Big(i+k-d, j', x+dS - kC - |j+1-j'|R - H - (j+1)r\Big) \\
& \quad + \sum_{d=j+1}^{M} p(d) q\Big(i+k-(j+1), j', x+(j+1)S - kC - |j+1-j'|R - H - (j+1)r\Big) \\
& = q(i, j+1, x)
\end{aligned}
$$

where the second inequality follows from Corollary 1 parts (ii) and (iii), and the fact that $|j-j'| \leq 1 + |j+1-j'|$.

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Lemma 3 provides insight about the structure of an optimal survival policy when the lead times for orders and changes to production capacity are both zero. We show that there exists an optimal survival policy that never places an order that would raise the inventory level above the production capacity for the period. This policy also has the property that in every period in which the production capacity is raised, the inventory level plus the order quantity for the period is at least as great as the new production capacity. Further if it is optimal to raise the production capacity to a level higher than the current inventory level, then it is optimal to order up to the new production capacity. When the lead times for orders and changes to production capacity are non-zero, less can be said about the structure of the optimal survival policy due to the uncertain demand during the lead time. Lemma 4 establishes properties of the optimal survival policy for the case in which the lead times are both one period. These results are considerably weaker than in the case of zero lead times.

Lemma 3

If $L_q = L_r = 0$ then in state (i, j, x) there exist optimal ordering and production capacity decisions k, j' satisfying:

(i) either $k = 0$ or $i + k \leq j'$;

(ii) either $j' \leq j$ or $i + k \geq j'$.

Proof

For state (i, j, x), let k be the smallest optimal order quantity and j' be the smallest optimal production capacity for order quantity k.

(i) Suppose that $k > 0$ and $i + k > j'$. Let $\delta = \min(k, i + k - j')$. The decision to order δ fewer items in state (i, j, x) is feasible but not optimal, so

$$\begin{aligned} q(i,j,x) &> \sum_{d=0}^{M} p(d) q\Big(i + k - \delta - \min(d, j'), j', \\ &\qquad x + \min(d, j')S - (k - \delta)C - |j - j'|R - H - j'r\Big) \\ &\geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(d, j'), j', \\ &\qquad x + \min(d, j')S - kC - |j - j'|R - H - j'r\Big) \text{ by Lemma 2 (i)} \\ &= q(i,j,x) \end{aligned}$$

This is a contradiction, so either $k = 0$ or $i + k \leq j'$.

(ii) Suppose that $j' > j$ and $i + k < j'$. Let $\delta = \min(j' - j, j' - i - k)$. The decision to set the production capacity to $j' - \delta$ units (with order quantity k) in state (i, j, x) is feasible but not optimal, so

$$\begin{aligned} q(i,j,x) &> \sum_{d=0}^{M} p(d) q\Big(i + k - \min(d, i + k), j' - \delta, x + \min(d, i + k)S \\ &\quad -kC - (j' - \delta - j)R - H - (j' - \delta)r\Big) \text{ since } j' - \delta \geq i + k \text{ and } j' - \delta \geq j \\ &\geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(d, i + k), j', \\ &\quad x + \min(d, i + k)S - kC - (j' - j)R - H - (j' - \delta)r\Big) \text{ by Lemma 2 (ii)} \\ &\geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(d, i + k), j', \\ &\quad x + \min(d, i + k)S - kC - (j' - j)R - H - j'r\Big) \text{ by Lemma 1 (iii)} \\ &= q(i,j,x) \end{aligned}$$

This is a contradiction, so either $j' \leq j$ or $i + k \geq j'$.

◇

Lemma 4

If $L_q = L_r = 1$ then in state (i, j, x) there exist optimal ordering and production capacity decisions k, j' satisfying:

(i) either $k = 0$ or $i + k \leq \min(i, j) + j'$;

(ii) either $j' \leq j$ or $i + k \geq j'$.

Proof

For state (i, j, x), let k be the smallest optimal order quantity and j' be the smallest optimal production capacity for order quantity k.

(i) Suppose that $k > 0$ and $i + k > \min(i, j) + j'$. Let $\delta = \min(k, i + k - \min(i, j) - j')$. The decision to order δ fewer items in state (i, j, x) is feasible but not optimal, so

$$\begin{aligned} q(i,j,x) &> \sum_{d=0}^{M} p(d) q\Big(i + k - \delta - \min(i,d,j), j', \\ &\quad x + \min(i,d,j)S - (k-\delta)C - |j - j'|R - H - jr\Big) \\ &\geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(i,d,j), j', x + \min(i,d,j)S - kC - |j-j'|R - H - jr\Big) \\ &\quad \text{by Lemma 2 (iii) since } i + k - \min(i,d,j) \geq i + k - \min(i,j) > j' \\ &= q(i,j,x) \end{aligned}$$

This is a contradiction, so either $k = 0$ or $i + k \leq \min(i, j) + j'$.

(ii) Suppose that $j' > j$ and $i + k < j'$. Let $\delta = \min(j' - j, j' - i - k)$. The decision to set the production capacity to $j' - \delta$ units (with order quantity k) in state (i, j, x) is feasible but not optimal, so

$$\begin{aligned} q(i,j,x) &> \sum_{d=0}^{M} p(d) q\Big(i + k - \min(i,d,j), j' - \delta, x + \min(i,d,j)S \\ &\quad -kC - (j' - \delta - j)R - H - jr\Big) \text{ since } j' - \delta \geq j \\ &\geq \sum_{d=0}^{M} p(d) q\Big(i + k - \min(i,d,j), j', x + \min(i,d,j)S - kC - (j' - j)R - H - jr + \delta r\Big) \\ &\quad \text{by Lemma 2 (v) since } i + k - \min(i,d,j) \leq i + k < j' \\ &\geq q(i,j,x) \text{ by Lemma 1 (iii)} \end{aligned}$$

This is a contradiction, so either $j' \leq j$ or $i + k \geq j'$.

$\diamond$

Theorem 2 demonstrates that a start-up manufacturing firm cannot afford to be too conservative with its ordering and production capacity decisions. We show that when the firm has no components in stock and zero production capacity, there is a minimum level to which the firm must raise the inventory and production capacity in order to have any chance of long-run survival.

Theorem 2

Define $\tilde{d}$ to be the largest integer less than $H/(S - C - r)$. If the optimal decisions in state $(0, 0, x)$ order $k \leq \tilde{d}$ items or set the production capacity to $j' \leq \tilde{d}$ units, then $q(0, 0, \tilde{x}) = 0$ $\forall \tilde{x} \leq x$.

Proof

Note that $S - C - r \geq 0$, since otherwise the firm can never be profitable in the long run. Let $\epsilon = H - (S - C - r)\tilde{d}$. Suppose there are no optimal decisions for state $(0, 0, x)$ that order more than $\tilde{d}$ items and set the production capacity to more than $\tilde{d}$ units.

Case 1: $L_q = L_r = 0$

For some order quantity k and production capacity j' satisfying $\min(k, j') \leq \tilde{d}$:

$$\begin{aligned}
q(0,0,x) &= \sum_{d=0}^{M} p(d) q\Big(k - \min(k, j', d), j', x + \min(k, j', d)S - kC - H - j'R - j'r\Big) \\
&\leq \sum_{d=0}^{M} p(d) q\Big(0, 0, x + \min(k, j', d)(S - C) - H - j'r\Big) \text{ by Lemma 2 (i) \& (ii)} \\
&\leq q\Big(0, 0, x + \min(k, j')(S - C - r) - H\Big) \text{ by Lemma 1 (iii)} \\
&\leq q\Big(0, 0, x + \tilde{d}(S - C - r) - H\Big) \text{ by Lemma 1 (iii)} \\
&= q(0, 0, x - \epsilon)
\end{aligned}$$

Assume the optimal decisions in state $(0, 0, x - \epsilon)$ are to order $k > \tilde{d}$ items and set the production capacity to $j' > \tilde{d}$ units.

$$\begin{aligned}
q(0,0,x) &> \sum_{d=0}^{M} p(d) q\Big(k - \min(k, j', d), j', x + \min(k, j', d)S - kC - H - j'R - j'r\Big) \\
&\geq \sum_{d=0}^{M} p(d) q\Big(k - \min(k, j', d), j', x - \epsilon + \min(k, j', d)S - kC - H - j'R - j'r\Big) \\
&= q(0, 0, x - \epsilon)
\end{aligned}$$

This contradicts $q(0, 0, x) \leq q(0, 0, x - \epsilon)$, so the optimal decisions in state $(0, 0, x - \epsilon)$ must either order $k \leq \tilde{d}$ or set the production capacity to $j' \leq \tilde{d}$ units.

Repeating this argument shows that $q(0, 0, x-\epsilon) \leq q(0, 0, x-2\epsilon) \leq \ldots \leq q(0, 0, x-y)$ where $y < 0$. Hence $q(0, 0, x) = 0$ and Lemma 2 (iii) implies that $q(0, 0, \tilde{x}) = 0 \; \forall \tilde{x} \leq x$.

Case 2: $L_q = L_r = 1$

For some order quantity k and production capacity j' satisfying $\min(k, j') \leq \tilde{d}$:

$$\begin{aligned}
q(0,0,x) &= q\Big(k, j', x - kC - H - j'R\Big) \\
&\leq q\Big(\min(k, j'), j', x - \min(k, j')C - H - j'R\Big) \text{ by Lemma 2 (iii)} \\
&\leq q\Big(0, j', x + \min(k, j')(S - C) - H - j'R\Big) \text{ by Lemma 2 (iv)} \\
&\leq q\Big(0, 0, x + \min(k, j')(S - C) - H - j'r)\Big) \text{ by Lemma 2 (v)} \\
&\leq q\Big(0, 0, x + \min(k, j')(S - C - r) - H\Big) \text{ by Lemma 1 (iii)} \\
&\leq q\Big(0, 0, x + \tilde{d}(S - C - r) - H\Big) \text{ by Lemma 1 (iii)} \\
&= q(0, 0, x - \epsilon)
\end{aligned}$$

Assume the optimal decisions in state $(0, 0, x - \epsilon)$ are to order $k > \tilde{d}$ items and set the production capacity to $j' > \tilde{d}$ units.

$$q(0,0,x) > q\Big(k, j', x - kC - H - j'R\Big) \geq q\Big(k, j', x - \epsilon - kC - H - j'R\Big) = q\Big(k, j', x - \epsilon)$$

This contradicts $q(0,0,x) \leq q(0,0,x-\epsilon)$, so the optimal decisions in state $(0,0,x-\epsilon)$ must either order $k \leq \tilde{d}$ or set the production capacity to $j' \leq \tilde{d}$ units.

Repeating this argument shows that $q(0,0,x-\epsilon) \leq q(0,0,x-2\epsilon) \leq \ldots \leq q(0,0,x-y)$ where $y < 0$. Hence $q(0,0,x) = 0$ and Lemma 2 (iii) implies that $q(0,0,\tilde{x}) = 0 \; \forall \tilde{x} \leq x$.

$\diamond$

Corollary 2

To have any chance of survival a firm with zero inventory and production capacity requires initial capital of at least:

(i) $\max\Big(0, H - (\tilde{d}+1)(S-C-r-R), H - M(S-C-r-R)\Big)$ if $L_q = L_r = 0$

(ii) $H + (\tilde{d}+1)(C+R)$ if $L_q = L_r = 1$

Proof

Theorem 2 shows that the ordering and production decisions in state $(0,0,x)$ must both be greater than $\tilde{d}$ for the firm to have any chance of survival. Therefore the largest profit that can be made in the first period when following a policy with a non-zero chance of survival is:

(i) $\max_{\tilde{d}<k\leq M}\{k(S-C-r-R)-H\}$ if $L_q = L_r = 0$

(ii) $-H - (\tilde{d}+1)(C+R)$ if $L_q = L_r = 1$

Hence result.

$\diamond$

4 Optimal strategy for a well-established firm

As discussed in section 2, we assume that the objective of an established manufacturing firm is to maximise the long-run average profit per period. In Theorem 3 we show that this is achieved by striking a balance between the marginal increases in the overhead costs per period and the expected profit from sales per period as the production capacity increases.

Theorem 3

Define $j^* = \min\left\{d^* | \sum_{d>d^*} (S-C)p(d) < r\right\}$. The optimal average reward, the optimal bias terms and an optimal policy are given by the following.

$$g = (S-C)\sum_{d=0}^{M} p(d)\min(j^*, d) - H - j^* r$$

$$v(i,j) = \begin{cases} iC - |j-j^*|R \text{ if } L_q = L_r = 0 \\ iC - |j-j^*|R + (j^*-j)r + (S-C)\sum_{d=0}^{M} p(d)\min(i,j,d) \text{ if } L_q = L_r = 1 \end{cases}$$

$$k_r(i,j) = j^*$$

$$k_q(i,j) = \begin{cases} \max(0, j^*-i) \text{ if } L_q = L_r = 0 \\ \max\Big(0, \min(j^*, j^*+j-i)\Big) \text{ if } L_q = L_r = 1 \end{cases}$$

Proof

It is easy to verify by substitution that the given values for g and $v(i,j)$ satisfy the optimality equations for the stated policies. Now apply policy improvement [Puterman, 1994] to verify that the policies are optimal for the two cases.

Case 1: $L_q = L_r = 0$

$$\max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) \left(\min(i+k, j', d)S - |j - j'|R - H - j'r + iC - \min(i+k, j', d)C - |j' - j^*|R \right) \right\}$$

$$= \quad iC - H + \max_{j'} \left\{ -|j - j'|R - |j' - j^*|R - j'r + (S-C) \max_{k} \left\{ \sum_{d=0}^{M} p(d) \min(i+k, j', d) \right\} \right\}$$

For $i + k < j'$, $\sum_{d=0}^{M} p(d) \min(i+k, j', d)$ increases with k while, for $i + k > j'$, it is independent of k. Hence $k = \min(0, j' - i)$ is an optimal order quantity. Suppose $j' = j^* + \delta$ where $\delta > 0$.

$$iC - H - |j - j^* - \delta|R - \delta R - (j^* + \delta)r + (S-C) \sum_{d=0}^{M} p(d) \min(j^* + \delta, d)$$

$$< \quad iC - H - |j - j^* - \delta|R - \delta R - j^* r + (S-C) \sum_{d=0}^{M} p(d) \min(j^*, d)$$

$$\text{since } \sum_{d>d^*} (S-C)p(d) < r \text{ for } d^* > j^*$$

$$\leq \quad iC - H - |j - j^*|R - j^* r + (S-C) \sum_{d=0}^{M} p(d) \min(j^*, d) \text{ since } |j - j^* - \delta| \geq |j - j^*| - \delta$$

Hence $j' \leq j^*$. Suppose now $j' = j^* - \delta$ where $\delta > 0$.

$$iC - H - |j - j^* + \delta|R - \delta R - (j^* - \delta)r + (S-C) \sum_{d=0}^{M} p(d) \min(j^* - \delta, d)$$

$$\leq \quad iC - H - |j - j^* + \delta|R - \delta R - j^* r + (S-C) \sum_{d=0}^{M} p(d) \min(j^*, d)$$

$$\text{since } \sum_{d>d^*} (S-C)p(d) \geq r \text{ for } d^* < j^*$$

$$\leq \quad iC - H - |j - j^*|R - j^* r + (S-C) \sum_{d=0}^{M} p(d) \min(j^*, d) \text{ since } |j - j^* + \delta| \geq |j - j^*| - \delta$$

Hence $j' \geq j^*$ and therefore j' must equal j^*. The maximising actions, $j' = j^*$ and $k = \max(0, j^* - i)$, correspond to the stated policy and therefore this policy is optimal.

Case 2: $L_q = L_r = 1$

$$\max_{k,j'} \left\{ \sum_{d=0}^{M} p(d) \left(\min(i,j,d)S - |j - j'|R - H - jr + iC - \min(i,j,d)C - |j' - j^*|R \right.\right.$$
$$\left.\left. + (j^* - j')r + (S-C) \sum_{s=0}^{M} p(s) \min\left(i + k - \min(i,j,d), j', s\right) \right) \right\}$$

$$= \quad iC - H + (j^* - j)r + (S-C) \sum_{d=0}^{M} p(d) \min(i,j,d) + \max_{j'} \left\{ -|j - j'|R - |j' - j^*|R - j'r \right.$$
$$\left. + (S-C) \max_{k} \left\{ \sum_{d=0}^{M} p(d) \sum_{s=0}^{M} p(s) \min\left(i + k - \min(i,j,d), j', s\right) \right\} \right\}$$

Any $k \geq 0$ satisfying $i + k - \min(i,j,d) \geq j'$ for all d maximises this expression with respect to the order decision. This simplifies to $k \geq 0$ when $i \geq j + j'$, $k \geq j + j' - i$ when $j + j' > i > j$ and $k \geq j'$ when $i \leq j$. Hence $k = \max(0, \min(j', j' + j - i))$ is an optimal order quantity. As $i + \max(0, \min(j', j' + j - i)) - \min(i,j,d) \geq j'$, the optimisation with respect to j' is exactly as for case 1. Hence the maximising actions correspond to the stated policy.

◇

5 Comparison of optimal strategies for start-up and well-established firms

Theorem 4

Assume that $L_q = L_r = 0$ and $q(i,j,x)$ is continuous and differentiable with respect to x in the interval $[X - H - M(C + R + r), \infty)$. The production capacity that maximises the survival probability in state $(0, 0, X)$ is no greater than the production capacity that maximises the expected profit.

Proof

Lemma 3 shows that there exist optimal ordering and production capacity decisions k, j' in state $(0, 0, X)$ satisfying $k = j'$. Let $Q^j(d)$ be the survival probability when decisions $k = j' = j$ are taken in state $(0, 0, X)$ and demand d occurs. It is easy to see from the optimality equation that $Q^j(d) = q\Big(j - \min(j, d), j, X + \min(j, d)S - H - j(C + R + r)\Big)$. If $d \le j$

$$\begin{aligned}
Q^j(d) - Q^{j+1}(d) &= q\Big(j - d, j, X + dS - H - j(C + R + r)\Big) \\
&\quad - q\Big(j + 1 - d, j + 1, X + dS - H - (j + 1)(C + R + r)\Big) \\
&\ge q\Big(j - d, j, X + dS - H - j(C + R + r)\Big) \\
&\quad - q\Big(j - d, j, X + dS - H - j(C + R + r) - r\Big) \text{ by Lemma 2 (i) \& (ii)} \\
&= r q'_x(j - d, j, \xi_d)
\end{aligned}$$

for some $\xi_d \in [X + dS - H - j(C + R + r) - r, X + dS - H - j(C + R + r)]$ where q'_x represents the derivative of q with respect to x. This result holds because q is continuous and differentiable with respect to x in this interval. If $d > j$

$$\begin{aligned}
Q^j(d) - Q^{j+1}(d) &= q\Big(0, j, X + jS - H - j(C + R + r)\Big) \\
&\quad - q\Big(0, j + 1, X + (j + 1)S - H - (j + 1)(C + R + r)\Big) \\
&\ge q\Big(0, j, X + jS - H - j(C + R + r)\Big) \\
&\quad - q\Big(0, j, X + jS - H - j(C + R + r) + S - C - r\Big) \text{ by Lemma 2 (ii)} \\
&= (r + C - S) q'_x(0, j, \xi_{j+1})
\end{aligned}$$

for some $\xi_{j+1} \in [X + jS - H - j(C + R + r), X + jS - H - j(C + R + r) + S - C - r]$.

By Corollary 1 (iii), $q(i,j,x)$ is non-decreasing in x, so $q'_x(i,j,x) \ge 0$ in the intervals in which the ξ_i values lie. Hence the difference in the expected survival probabilities

$$\sum_{d=0}^{M} p(d)\Big(Q^j(d) - Q^{j+1}(d)\Big) \ge \min_{0 \le i \le j+1} \{q'_x(0, j, \xi_{j+1})\} \left(r - (S - C) \sum_{d=j+1}^{M} p(d)\right) \ge 0$$

if $j \ge j^*$ from Theorem 3. Therefore increasing the production capacity beyond the level which maximises the expected profit does not increase the survival probability in state $(0, 0, X)$.

◇

The results of numerical experiments have shown that with the exception of an interval corresponding to low levels of capital, the maximum survival probability is continuous and differentiable with respect to the capital available. Hence Theorem 4 suggests that for sufficiently large levels of capital, the production capacity that maximises the survival probability is never greater than the profit maximising production capacity.

The following example is used to demonstrate that a similar result seems to hold when the lead times for ordering and production capacity decisions are one period. In the example, the

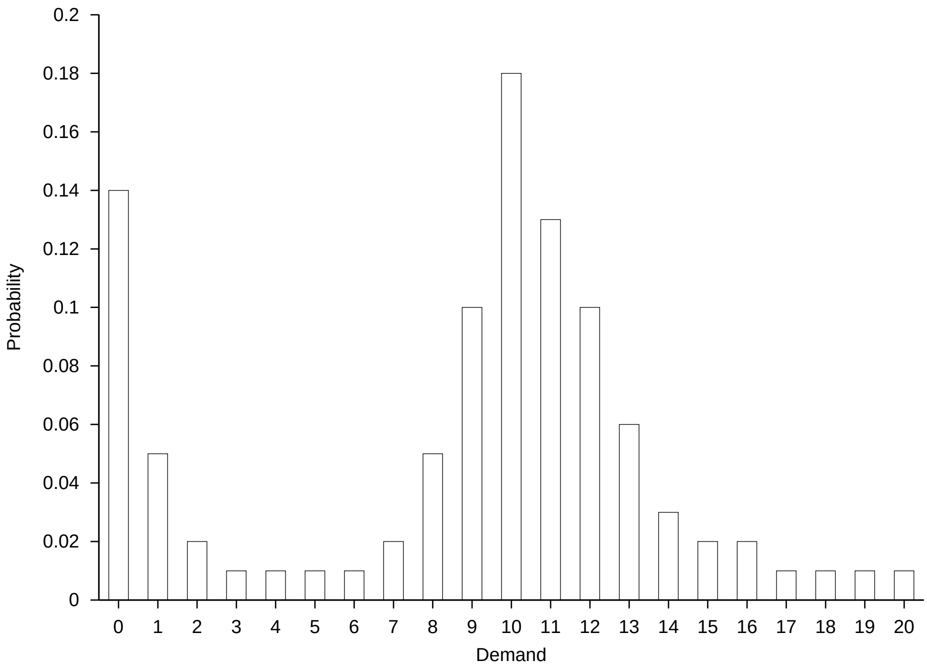

Figure 1: Weekly demand distribution

maximum possible production capacity $M = 12$, the fixed overhead cost $H = 5$, the variable overhead cost $r = 1$ per unit of production capacity, changes to production capacity cost $R = 2$ per unit, the unit cost of components $C = 3$ and the unit price of the product $S = 8$. The demand distribution is shown in figure 1 and is discussed further in section 6.

Figure 2 shows the maximum survival probability as a function of the capital available when the inventory level and production capacity are 0. When the capital available is less than 15 the firm has no chance of survival regardless of the decisions taken. For this example $H + (\tilde{d} + 1)(C + R) = 15$, so this illustrates that the bound in Corollary 2 can be tight. The maximum survival probability increases rapidly as the capital available increases from 15 to 35. After this interval the increase in the maximum survival probability is more gradual. When the capital available is greater than 68, the chance of the firm failing is less than 1 in 10,000.

These characteristics are typical of the numerical examples that we have examined. It is interesting that there appears to be a threshold value for the capital available beyond which the chance of survival is very good and below which the chance of survival is slim. During the interval of rapid increase in maximum survival probability, the optimal ordering and production capacity decisions often vary greatly and, for some cost configurations, even exceed the profit maximising levels. Generally the survival probability is highly sensitive to the decisions taken in these states and adopting a profit maximising strategy would be very likely to result in the failure of the firm.

Figure 3 shows the optimal survival policy as a function of the capital available when the inventory level and production capacity are 0. Lemma 4 shows that for all states $(0, 0, x)$ there exists an optimal survival policy which sets the order quantity and the production capacity to the same level. In figure 3 the solid line represents the maximum optimal level and the dotted line represents the minimum optimal level. When the capital available is between 15 and 182 the two lines coincide, indicating a unique optimal survival policy.

For this example there is always an optimal survival policy which sets the production capacity to a level below the optimal profit maximising production capacity, which from Theorem 3 equals 12. In other cases we have found this to be true for sufficiently large capital. This suggests that a result similar to Theorem 4 might hold for lead times of one period.

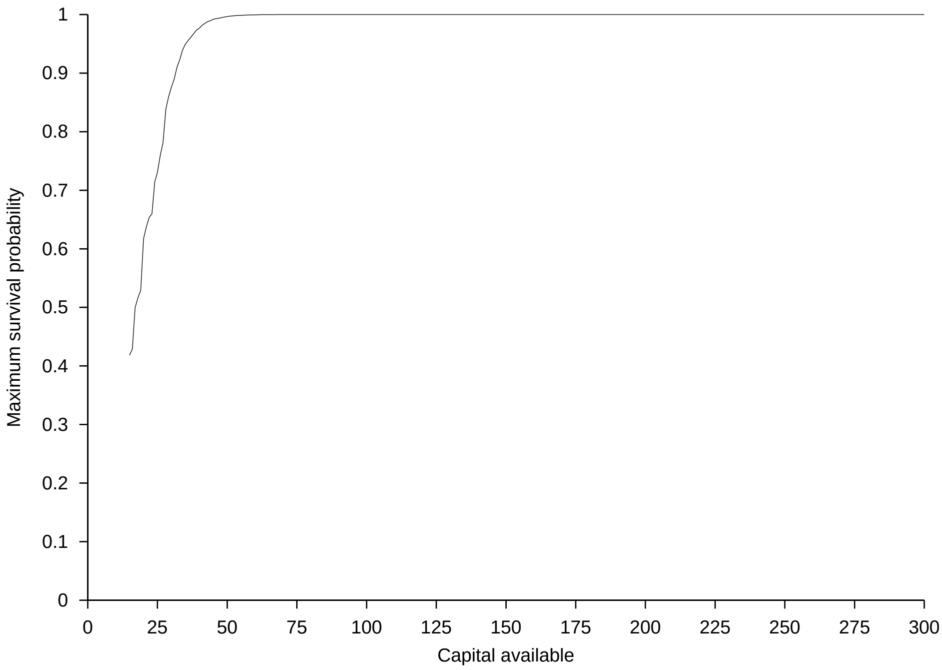

Figure 2: Maximum survival probability against capital available when inventory level and production capacity are both zero

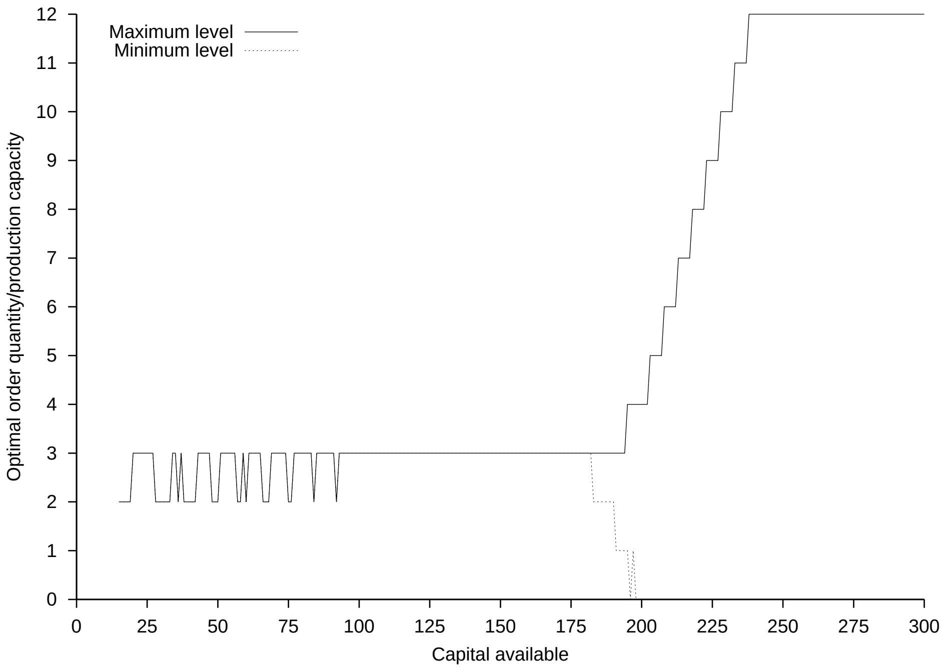

Figure 3: Optimal survival policy against capital available when inventory level and production capacity are both zero

Whenever the firm has a chance of survival (i.e. the capital available is at least 15), there exist optimal ordering and production capacity decisions greater than $\tilde{d} + 1 = 2$. The fact that there are instances where the unique optimal survival policy sets the order quantity and the production capacity equal to 2 illustrates that the bounds in Theorem 2 can be tight.

As the capital available increases, the optimal survival policy can be seen to pass through 5 distinct phases.

1. The capital available is insufficient to give the firm any chance of survival regardless of the decisions taken ($0 \leq x \leq 14$).
2. The optimal survival policy is unique and sensitive to changes in the capital available ($15 \leq x \leq 92$).
3. The optimal survival policy is unique and stable at a level that is considerably lower than the profit maximising production capacity ($93 \leq x \leq 182$).
4. The next period's decisions are becoming less important to the chance of survival and the optimal survival policy is no longer unique ($183 \leq x \leq 237$).
5. The next period's decisions have no significant effect on the chance of survival and the firm should be seeking to maximise expected profit ($x \geq 238$).

This behaviour is typical of the numerical examples we have examined.

Figure 4 shows the maximum survival probability as a function of the production capacity when the capital available is 30 and the inventory level is 0. This demonstrates that, as shown in Lemma 1 (iv) for the finite horizon model, the maximum survival probability is not monotonic in the production capacity.

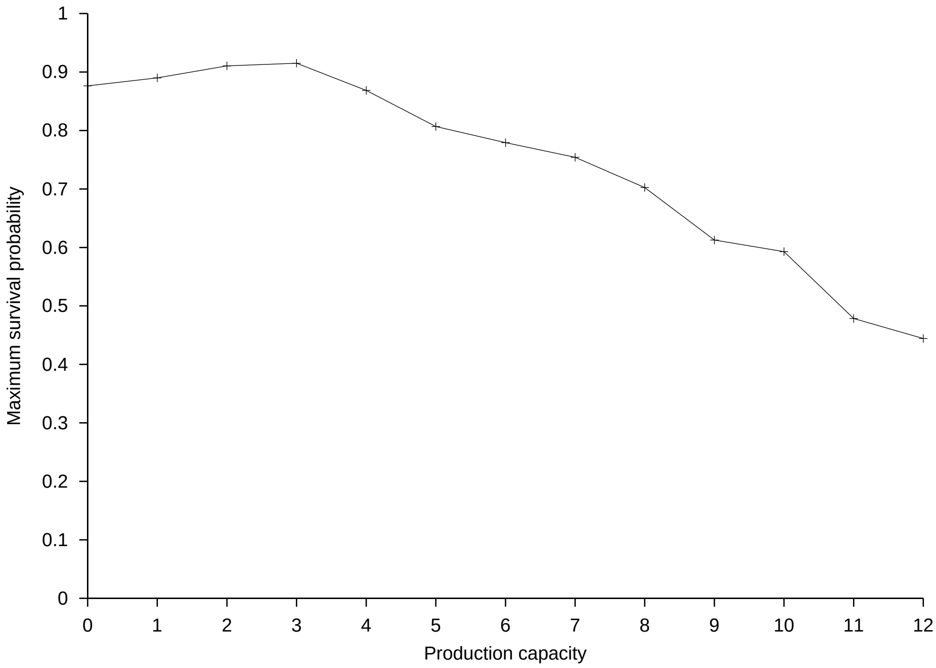

Figure 4: Maximum survival probability against production capacity when capital available is 30 and inventory level is zero

6 Application of model to a real start-up firm

Part of the motivation for developing and analysing the model in this paper was the case of a start-up manufacturing firm whose management believed that capacity expansion was too risky even though the firm often struggled to keep up with demand. Our aim was to investigate whether this attitude could be explained by the objective of maximising the chance of survival. The product manufactured by the case firm basically consists of a component housed in a cabinet. Due to the variety of possible sizes and finishes, the firm do not keep an inventory of products. The cabinet is made from materials that are generally readily available locally and so are not routinely kept in inventory. The component has to be imported and, while the delivery time is short compared to the manufacturing time, shipments are restricted to one per week. Hence a model of this firm would have the time period as a week and the lead times for ordering and production capacity decisions as zero ($L_q = L_r = 0$). The maximum possible production capacity $M = 12$. The demand distribution, shown in figure 1, is bimodal with peaks at 0 and 10 items. This models a lumpy pattern of demand in which periods tend either to be quite good or quite poor. The fixed overhead cost $H = 4$ and the variable overhead cost $r = 3$ per unit of production capacity. Adjusting the production capacity costs $R = 5$ per unit. The firm buys the component at unit cost $C = 9$ and sells the product at unit cost $S = 15$. (It is assumed that the cost of materials other than the component can be ignored.)

From Theorem 3 the profit maximising strategy is to have a production capacity of 10 and to order-up-to 10 items. Figure 5 shows the optimal survival production capacity as a function of the capital available when the production capacity and order quantity are both 3. In this case the minimum optimal order quantity is always 0 while the maximum optimal order quantity is always 3 less than the maximum optimal production capacity. When the available capital is below 498, the unique optimal survival policy is to do nothing. In fact when the production capacity is 3 and the inventory level does not exceed 3, the unique optimal survival policy for levels of capital between 20 and 497 is to leave the production capacity unchanged and to order-up-to 3 items. For capital above 497, this policy is still optimal but no longer unique. For capital below 20 the chance of survival is relatively low and the optimal survival policy may raise the production capacity and inventory level as high as 7. However these higher levels would only be temporary because, if the firm survives and therefore its capital recovers, the production capacity would be reduced again (see figure 6).

When the production capacity and inventory level are both 3, the expected profit per week is 1.76. Hence the width of the interval over which the policy described above is the unique optimal survival policy is equivalent to the expected profit from a period of more than 5 years. Hence the model demonstrates that if the objective is to maximise the chance of survival, the firm should operate with a production capacity well below the profit maximising production capacity for a considerable period of time. This is a possible explanation for the seemingly overly cautious policy used by the case firm.

Figure 6 shows the optimal survival production capacity as a function of the current production capacity when the capital available is 70 and the inventory level is 7. This shows that the result above is not simply due to the initial conditions and the relatively high cost of adjusting the production capacity. Even when the production capacity and inventory level are initially relatively high, the optimal survival policy chooses to reduce the production capacity to a seemingly conservative level. This is further illustrated in figure 7 which shows the optimal survival production capacity as a function of the capital available when the production capacity and the inventory level are both 10. We see that even when the production capacity and the inventory level are at the profit maximising level initially, the unique optimal survival policy is to reduce the production capacity to 4 when the capital available is between 102 and 430.

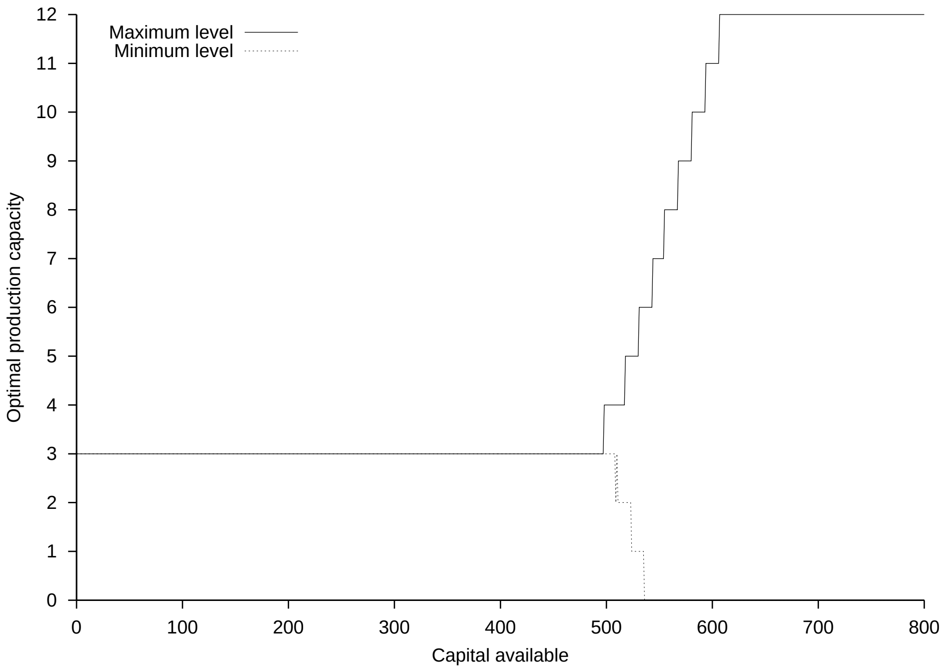

Figure 5: Optimal survival production capacity against capital available when production capacity and inventory level are both 3

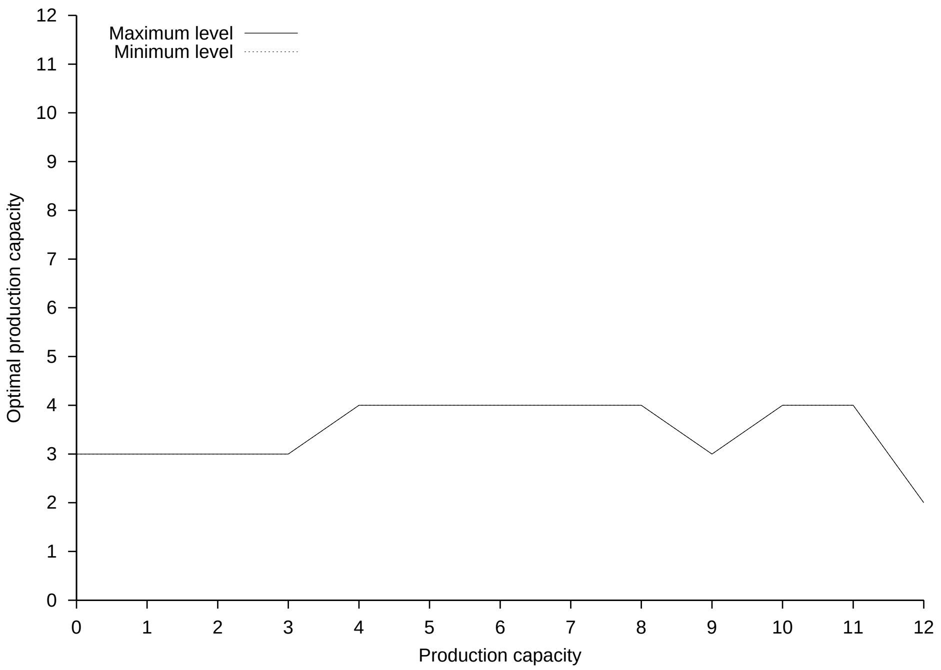

Figure 6: Optimal survival production capacity against current production capacity when capital available is 70 and inventory level is 7 (Note that the maximum and minimum optimal levels coincide at each point indicating unique optimal decisions)

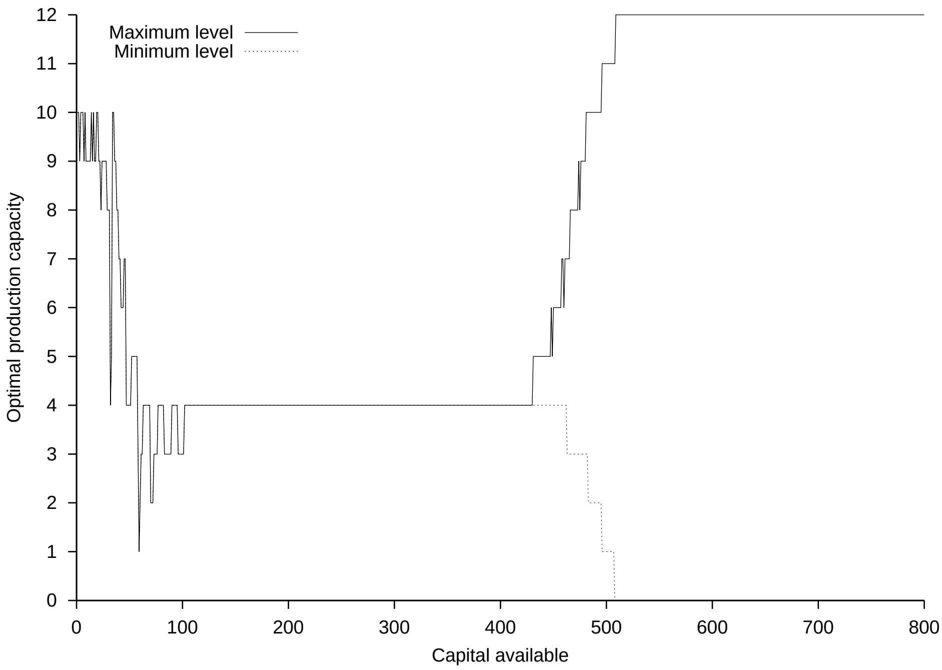

Figure 7: Optimal survival production capacity against capital available when production capacity and inventory level are both 10

7 Conclusions

We have presented a dynamic combined inventory and production model which determines what production capacity level a manufacturing firm should invest in and how many components it should order, in the situation where the demand for the product is varying. This is solved both under the maximising expected profit criterion and under the criterion of maximising the probability of the firm surviving in the long term, neither of which problem has been solved before. Whereas the former is an appropriate criterion for well established firms, we have argued that the latter is more appropriate for start-up firms, where the decisions are very dependent on the amount of capital available.

We have proved there are sensible interactions between the production and inventory decisions in the survival optimisation case, such as never ordering components so as to raise the inventory level above the production capacity, and if the production capacity is raised then we must raise the inventory level to this new production capacity. However it is not the case that the inventory and production levels are always set equal to one another.

We also investigate the relationship between the profit maximising strategy and the survival maximising decisions. We describe how as the capital increases, the survival optimising strategy goes though five phases. If the capital is below the level defined in Corollary 2, there is no chance of survival. Immediately above this level, the production and inventory decisions jump to levels which are dominated by short time survival considerations and so may not be monotonic in the capital available. With more capital, the policy becomes stable but at a level considerably below the profit maximising level. At some point there is sufficient capital available so that the next decision is not vital and so the optimal survival policy is not unique and eventually in the fifth phase, the next inventory and production level decisions have no discernible effect on the survival probability. At this point the firm should change to the profit maximising criterion.

Thus the paper does seem to explain why in the case study the firm believed it was better to operate for some time with a production level which was significantly below the profit maximising one. They were subjectively recognising that their survival was their most important objective, and were in the third phase of the optimal survival policy outlined above.

The model described could be reinterpreted as a marketing-production problem where one is interested in the mix of spending on advertising and on component inventory levels. One can think of production capacity as a limitation on the potential demand that can be turned into sales. So if one goes from production level j to production level j' one is paying a cost $R|j - j'|$ to increase the potential level of sales and an amount rj' to sustain the sales at that potential maximum level. One could think of advertising as doing the same sort of thing.

There is a potential demand available but in order to realise a maximum of j sales one has to sustain an advertising spend of rj. If one wants to increase the potential maximum sales level one has to develop further advertising at a cost of $R|j - j'|$ as well as then keeping the advertising spend at rj'. One might quibble about the extra cost R being involved if one wants to lower the advertising spend, but one could envisage penalty clauses in the contracts with the advertising media which would require payment if the advertising is cancelled. This is a simple model of the way advertising interacts with total sales, but it does allow one to investigate the impact that marketing may have on the survival potential of start-up firms.

We believe that this paper does contribute to an understanding of the operations management for start-up firms, by looking at the coordination needed between production and inventory decisions, and it does suggest ways of investigating what is the best strategic mix of investment in production capacity, component availability and marketing spend.

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