



The Estimation of Place-to-Place Migration Flows Using an Alternative Log-Linear Parameter Coding Scheme

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ALTERNATIVE LOG-LINEAR PARAMETER CODING SCHEME

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ABSTRACT

The log-linear model, with an alternative parameter coding scheme, is used in this paper to obtain estimates of place-to-place migration flows in situations where the data are inadequate or missing. The alternative parameter coding scheme is particularly useful in constructing the origin-destination interaction structure. To illustrate the method, two empirical examples are presented. The first demonstrates the effectiveness of the methodology by estimating known migration flows between states in the Western region of the United States during the 1985-1990 period. The second example focuses on estimating international migration flows in the Northern region of Europe during the 1999-2000 period where the data are incomplete. Both examples demonstrate the usefulness and generality of this particular method for estimating migration flows.

Key words: migration analysis, migration estimation, log-linear models, categorical data analysis, missing data, parameter interpretation

THE ESTIMATION OF PLACE-TO-PLACE MIGRATION FLOWS USING AN ALTERNATIVE LOG-LINEAR PARAMETER CODING SCHEME

1. INTRODUCTION

Estimates of place-to-place migration flows provide national and regional governments with the means to improve their planning policies directed at supplying particular social services or at continuing, increasing, or decreasing levels of interregional or international migration. Furthermore, our understanding of how or why populations change requires relatively accurate estimates of migration flows. Without these, the ability to predict or attempt to control that change is limited. The purpose of this paper is to illustrate a methodology for obtaining estimates of place-to-place migration flows for a variety of data situations. This work follows some recent developments on the identification and description of migration spatial structures in terms of categorical log-linear and logit model parameters (Rogers et al. 2002; Rogers, Willekens and Raymer 2001, 2002, 2003). In particular, this research provides an alternative parameter interpretation of the log-linear model, which is useful for guiding the modeling of the interactions between origins and destinations of migration flows --- the key element required for more accurate predictions.

Models for estimating migration flows are necessary because the data are often inadequate or missing (Willekens, 1999). For example, the sample size of the survey used to obtain the migration statistics may have been too small for the level of detail in the analysis. This tends to cause unexpected irregularities in the data. The level of demographic, socioeconomic, or spatial detail required for a particular study may not have been collected. The survey question about migration might not fit the research question regarding migration.

There may be situations in which the required migration data are available but cannot be considered reliable, such as emigration data provided by sending countries. And, there may be significant non-responses in the survey. Aside from collecting the missing data, the solutions to the above problems include using ancillary data, smoothing the data, or estimating the missing data.

For the estimation of migration flows, gravity and spatial interaction models are the most commonly applied because they are still considered to be the best models for accurately predicting *aggregate-level* migration flows (Fotheringham, Brunsdon and Charlton 2000:217). This is true despite the known behavioral inadequacies of these models (see, e.g., Sheppard 1979). The gravity model is a relatively simple model which includes the factors of population sizes of the origin and destination regions, the distance between them, and some measure of competition or attractiveness (Lowry 1966:7). The spatial interaction model is essentially a statistical form of the gravity model. Wilson's (1971) introduction of "families of spatial interaction models" using entropy-maximization techniques was a key turning point in the modeling of spatial patterns of migration. Soon after, this framework was applied for use in modeling migration flows (i.e., Plane 1981; Willekens 1977, 1980; Willekens, Por and Raquillet 1981).

Willekens (1983) demonstrated that the log-linear model could be used to model spatial interaction patterns. In general, log-linear models are used to model contingency tables. A migration flow table can be considered a two-way (i.e., origin by destination) contingency table, where the cells represent counts of migrants. The advantage of the log-linear model over the general spatial interaction model is that it has a well-formed theory and methods, associated in the framework of contingency-table analysis or discrete multivariate

analysis (Willekens 1999). In addition to log-linear models of migration estimation, similar work has been carried out in the name of Poisson regression (Flowerdew 1991; Flowerdew and Aitkin 1982; Flowerdew and Lovett 1988).

Iterative proportional fitting (Deming and Stephan 1940) is another technique that has been used to obtain, or update tables of, place-to-place migration flows (e.g., Nair 1985; Rees and Duke-Williams 1997; Willekens 1982; Willekens, Por and Raquillet 1981). More recently, Schoen and Jonsson (2003) introduced an alternative form of the iterative fitting procedure, termed Relative State Attraction, to estimate multistate transition rates. This technique assumes that the marginal totals of a migration flow table are available. The flows are estimated by adjusting, through iteration, a corresponding table of numbers, for instance, a historical set of place-to-place migration flows, to fit the given marginal totals of the table. Note, for a two-way contingency table, the iterative proportional fitting procedure produces the same results as a log-linear main effects model with an offset.

In the next section, a place-to-place migration flows situation containing inadequate and missing data is set out, namely international migration flows between countries in Northern Europe during the 1999-2000 time period. This is followed by a presentation of a general methodology for estimating missing or inadequate migration flow data. Then, two empirical demonstrations are put forward. The first, in Section 4, illustrates the effectiveness of the methodology by modeling known migration flows between states in the Western region of the United States during the 1985-1990 period. The second example, in Section 5, applies the methodology to estimate the unknown or inadequate European migration data mentioned above. The purpose of both examples is to show that a single methodology can be used to estimate both internal and international migration flows.

2. THE PROBLEM: INADEQUATE AND MISSING INTERNATIONAL MIGRATION FLOW DATA IN THE NORTHERN REGION OF EUROPE

Because of differences in data availability, quality, and measurement, no consistent set of migration flow estimates exist between the countries in Europe. At best, net migration and some general directions of the migration patterns are known. The countries in the Western and Northern regions of Europe tend to have better (or excellent) migration statistics, whereas in the Eastern and Southern regions, much of the patterns are largely unknown. As such, reports on the patterns of migration in Europe have largely relied on known patterns (e.g., de Beer and van Wissen 1999; Eurostat 1999; Massey et al. 1998; Salt 1996, 2001). This paper begins to address the problem of missing and inconsistent migration flow data in Europe by focusing on the international migration patterns between countries in the Northern region of Europe during the 1999-2000 period. For most of these countries, international migration is the most important factor of demographic change. In fact, several now have proportions of foreign populations similar to the “traditional” immigration countries of the United States, Australia, and Canada (Massey et al. 1998).

The European migration data set out in this article comes from the *Eurostat NewCronos* database (as of February 2003). Eurostat is considered Europe’s main statistical agency responsible for collecting and storing macroeconomic and social statistical data. For international migration patterns in Europe, there are two major agencies that provide an international database of migration statistics (King 2002:101; Salt 2001): Eurostat and the Organization for Economic Co-Operation and Development (OECD). Both agencies have large databases (i.e., *NewCronos* and *SOPEMI*, respectively) and produce annual reports of international migration flows (e.g., Eurostat 2000; SOPEMI 2003). The data gathered by

these two agencies are similar to each other in that they obtain their migration data by sending out questionnaires to the statistical agencies in each of the countries representing Europe and elsewhere (for the case of OECD). The statistical agencies then report back their observed or estimated numbers of migrants for certain specified time periods. Neither Eurostat or OECD alters these data --- they simply report them as given. As the numbers come from various statistical agencies with varying methods of data collection and definitions, some inconsistencies naturally result.

One set of international migration flows, obtained from the available *immigration* flow data provided by Eurostat, are set out in Table 1. Here, we see that Denmark, Finland, Iceland, Norway, and Sweden provide origin-destination-specific data. Ireland and the United Kingdom provide some detailed information, but not for all origins and destinations. Estonia, Latvia, and Lithuania provide no immigration data at all. In total, we have information on 51 out of a possible 90 country-to-country flows (or 57 percent of the information available). Similarly, an additional migration flow table can be produced from the available *emigration* data, also provided by Eurostat. These flows are set out in Table 2.

For comparison purposes, consider the migration flows between Scandinavian countries (i.e., Denmark, Iceland, Norway, and Sweden) in Tables 1 and 2. These numbers are relatively similar to each other. For example, according to the *immigration* flow table (i.e., Table 1), there were 3,188 migrants from Denmark to Norway. This number is much like the one found in the *emigration* flow table (i.e., 3,141; see Table 2). Next, consider the Norway to United Kingdom migration flow. In this case, two very different numbers arise in Tables 1 and 2. In Table 1, there are 3,188 persons who migrated from Norway to the United Kingdom, whereas in Table 2, there are 1,735 persons. The former number comes from the

United Kingdom government. The latter comes from the Norwegian government. This is just one example of inconsistency. There are many more to be found between the two tables. Much of this inconsistency is likely due to different data collection systems (e.g., survey-based versus registration-based) and to the timing of the collection (Poulain 1994).

Table 1. International migration flows based on available immigration data for the Northern region of Europe, 1999-2000

Country of Origin	Country of Destination										Total
	Den.	Est.	Fin.	Ice.	Ire.	Lat.	Lith.	Nor.	Swed.	U.K.	
Denmark			355	1,446				2,734	2,194	2,025	
Estonia	257		784	6				85	262		
Finland	448			46				1,380	3,647	1,556	
Iceland	1,267		54					463	384		
Ireland	266		41	6				73	199		
Latvia	376		44	7				116	169		
Lithuania	499		21	33				104	111	149	
Norway	3,188		955	602					5,496	3,814	
Sweden	2,298		3,229	572				6,044		1,539	
U.K.	3,965		586	167	21,611			2,014	2,447		
Total	12,564		6,069	2,885				13,013	14,909		



 Non-Migrants
  Not Available

Table 2. International migration flows based on available emigration data for the Northern region of Europe, 1999-2000

Country of Origin	Country of Destination										Total
	Den.	Est.	Fin.	Ice.	Ire.	Lat.	Lith.	Nor.	Swed.	U.K.	
Denmark		232	392	1,422	368	322	325	2,786	2,295	4,291	12,433
Estonia											
Finland	415	264		43	101	22	10	1,383	3,695	941	6,874
Iceland	1,327	3	60		15	0	5	492	406	201	2,509
Ireland										10,188	
Latvia											
Lithuania											
Norway	3,141	40	978	606	77	59	40		5,523	1,735	12,199
Sweden	2,196	71	3,178	548	251	33	19	5,912		3,281	15,489
U.K.	1,831		4,849				220	2,164	4,352		
Total											



Non-Migrants
 Not Available

Despite the known inconsistencies, the immigration flow (Table 1) and emigration flow (Table 2) data are combined in Table 3 with the assumption that some data is better than no data. By combining both tables of flows, 74 of the 90 flows are now (in some way) accounted for. Now, the only missing data are the migration flows between Estonia, Ireland, Latvia, Lithuania, and the United Kingdom. Preference for the numbers in Table 3 was given to the immigration data. Numbers from the emigration table (Table 2) were used only when numbers from the immigration table (Table 1) were not available. The immigration data are generally considered more reliable because immigrants are present in the country providing the data. Emigration data, on the other hand, is based on either respondents reporting back to their country of origin, or from statements on intended destinations. In the latter case, it is not possible to verify whether or not these migrants actually migrated to where they said they would. However, in some cases it could be argued that the emigration data from countries

with strong registration systems might be more accurate than immigration data based from either weak registration systems or surveys (Poulain 1999).

Table 3. International migration flows based on available immigration and emigration data for the Northern region of Europe, 1999-2000*

Country of Origin	Country of Destination										Total
	Den.	Est.	Fin.	Ice.	Ire.	Lat.	Lith.	Nor.	Swed.	U.K.	
Denmark		232	355	1,446	368	322	325	2,734	2,194	2,025	10,001
Estonia	257		784	6				85	262		
Finland	448	264		46	101	22	10	1,380	3,647	1,556	7,474
Iceland	1,267	3	54		15	0	5	463	384	201	2,392
Ireland	266		41	6				73	199		
Latvia	376		44	7				116	169		
Lithuania	499		21	33				104	111	149	
Norway	3,188	40	955	602	77	59	40		5,496	3,814	14,271
Sweden	2,298	71	3,229	572	251	33	19	6,044		1,539	14,056
U.K.	3,965		586	167	21,611		220	2,014	2,447		
Total	12,564		6,069	2,885				13,013	14,909		

 Non-Migrants
  Not Available

* Preference given to immigration data.

Finally, the numbers in the above table are considered *in this paper* to be the best available information for international migration between countries in Northern Europe during the 1999-2000 period. This information is used later in this paper to obtain a complete and consistent set of estimated migration flows between the ten countries in Northern Europe. But first, in the next section, a proposed model solution is presented for obtaining these numbers.

3. A PROPOSED MODEL SOLUTION

This section consists of two parts. In Section 3.1, the log-linear model, with an alternative parameter coding scheme, is put forward for the purpose of obtaining estimates of place-to-place migration flows in situations where the data are inadequate or missing. The alternative parameter coding scheme is particularly useful in constructing the origin-destination interaction structure, which is then incorporated into the “offset” (discussed below). In Section 3.2, the model is tested on known internal migration flows between states in the United States West region.

3.1 Modeling the Spatial Patterns of Migration

The modeling framework described in this paper focuses on one stage in the modeling process of migration. There are three distinct stages of modeling aggregate levels of migration (Raymer 2003). The first stage focuses on estimating the *gross flows* of migration (i.e., in-migration and out-migration or immigration and emigration) using ordinary least squares regression (e.g., Cadwallader 1992). These estimates form the marginal totals of a migration flow table. The second stage, which is the topic of this paper, focuses on estimating the *spatial patterns* using spatial interaction or log-linear models (e.g., Willekens 1983). The third stage focuses on estimating the *age patterns* using, for example, multiexponential model migration schedules (Rogers and Castro 1981). The three stages are hierarchical and, ideally, should be constrained so that the estimates are consistent with the net migration obtained via the demographic accounting model (e.g., Siegel and Hamilton 1952). This paper assumes that the gross flows of in-migration and out-migration (i.e., the marginal totals of a migration flow table) are known or have already been estimated.

The models set out in this paper estimate the migration flows between origins i and destinations j . The counts of origin-destination-specific migrants are denoted n_{ij} . When $i = j$, the persons are said to be “stayers”. Stayers are non-migrants or persons who may have migrated out and back within the time interval. The numbers of persons living in a particular place at the beginning of the time interval are denoted by n_{i+} . The numbers of persons living in a particular place at the end of the time interval are denoted n_{+j} .

3.1.1 Describing the Spatial Patterns of Migration

Place-to-place migration flows are often set out in two-way (i.e., origin by destination) contingency tables. These migration flow tables can be disaggregated into four separate components (Rogers et al. 2002): an *overall* component representing the level of migration, an *origin* component representing the relative “pushes” from each region, a *destination* component representing the relative “pulls” to each region, and an *origin-destination interaction* component representing the physical or social distance between places not explained by the other three components. The motivation for this paper comes from this approach to examining migration flows.

A model that is very useful for describing the spatial patterns of migration is the log-linear model. This model, in multiplicative form, is specified as:

$$\hat{n}_{ij} = \tau \tau_i^O \tau_j^D \tau_{ij}^{OD} \quad (1)$$

where \hat{n}_{ij} is the predicted migration flow in cell ij . The parameters of the model (τ s) have superscripts O and D denoting origin i and destination j , respectively. The overall effect is

denoted by ϑ , the origin main effect is denoted by τ_i^O , the destination main effect is denoted by τ_j^D , and the origin-destination interaction effect is denoted by τ_{ij}^{OD} . The log-linear model typically assumes a Poisson probability distribution because its properties are relatively simple and well known and, like counts, the outcomes can only be positive. Note, another distribution that is often used to model counts is the negative binomial regression model. Unlike the Poisson regression model, this model allows the variance to exceed the conditional mean. The parameters of this model are estimated using maximum likelihood methods. Finally, log-linear models and Poisson regression models are analogous when the data are categorical.

Because different statistical packages use different reference coding schemes, it is important to know how the parameters are interpreted for purposes of analysis and estimation. The coding scheme should be based on the research question. The parameters of the log-linear model in its multiplicative form are interpreted as odds and odds ratios with reference to some category. The two most popular reference category coding schemes are the cornered-effect coding scheme and the geometric mean coding (effect coding) scheme. A third coding scheme is presented that is not in the literature but is believed by the author of this paper to be more intuitive and practical. This coding scheme is called the *total sum reference coding scheme*. Cornered-effect coding implies that the parameters are interpreted with reference to a single cell in the table. Most standard statistical software packages use this coding scheme as the default. For example, *SPSS* uses a last reference category coding scheme, whereas *Stata* uses a first reference category coding scheme. Geometric mean coding implies that the parameters are interpreted with reference to the overall geometric mean of

values in the table. Finally, the total sum reference coding scheme implies that the parameters are interpreted with reference to the overall total migration level (i.e., n_{++}) associated with the migration flow table.

When *cornered-effect coding* (e.g., the last reference category) is used, the main effects correspond to the reference category (cell) only. This scheme is not as intuitive for the analysis of migration flows because often researchers are interested in overall patterns, not patterns related to a specific region. Also, for both the first category and last category coding schemes, migrants are compared to non-migrants. This causes parameter interpretation to be particularly confusing.

The effects of the *geometric mean coding* model are expressed in terms of ratios between geometric means. Note, for a detailed description, refer to Willekens (1983). For this coding scheme, any increase or decrease to any one cell value (e.g., in the diagonal) would alter all the parameter values, because the relative differences between the cells would change. The main effects and interaction effects would stay the same only if one divided all the cells by the same number (i.e., by 1000, by 2000, by 1000000, etc.). In that case, only the overall effect would change. The advantage to this parameter coding scheme is interpretation. One is basically comparing odds and odds ratios of averages.

For the model that uses the *total sum reference category coding* scheme, the overall main effect is equal to $\tau = \sum_{ij} n_{ij}$, the main effects are equal to $\tau_i^O = \frac{1}{\tau} \left[\sum_j n_{ij} \right]$ and $\tau_j^D = \frac{1}{\tau} \left[\sum_i n_{ij} \right]$, and the interaction effects are equal to $\tau_{ij}^{OD} = n_{ij} / \tau \tau_i^O \tau_j^D$. The constraints of the model are $\sum_i \tau_i^O = \sum_j \tau_j^D = 1$ and $\tau_i^O \sum_j \tau_{ij}^{OD} = \tau_j^D \sum_i \tau_{ij}^{OD} = 1$. The overall effect is simply

the overall level of the migration flow table. The origin effect is the proportion of all out-migrants from origin i . Likewise, the destination effect is the proportion of all in-migrants to destination j . Multiply these three effects together, and one gets an expected migration flow which uses information obtained from the marginal totals, not the cell values. Finally, the interaction effect essentially measures the strength of connection between two places (i.e., observed divided by expected flows).

The total reference category coding scheme has some clear advantages over the other coding schemes. This coding scheme relies entirely on the marginal totals, which is important since one is more likely to have the marginal totals than the origin-destination-specific flows. Also, the interaction effect parameters are simpler to interpret. They represent the ratio of observed migration flows to expected migration flows, where the latter is obtained by multiplying an overall level, a proportion observed in the row-sum marginal total, and a proportion observed in the column-sum marginal total. This method of coding is very useful for explaining migration flow tables and also for estimating a set of interaction effects when the data are incomplete, as is demonstrated later on in this paper. The main disadvantage of the total sum reference category coding scheme is that it is not an option in standard statistical packages, so one is forced to translate between different coding schemes. However, the results produced from the total reference category coding scheme are the same as the results produced by either geometric mean or cornered-effect coding schemes; the difference lies in the interpretation of the parameters and in finding a logical way to estimate migration flows based on limited information.

3.1.2 Models for Imposing Spatial Structures of Migration

If the data are incomplete, auxiliary information may be used to predict migration flows (see, for example, Rogers, Willekens and Raymer 2003). Let n_{ij}^* denote a hypothetical migration flow table. The migration flow table for the current period may be predicted on the basis of, for example, information regarding the aggregate number of migrants living in regions i and j at the beginning and end of the time interval, n_{i+} and n_{+j} , respectively, and some information on the interaction between places represented by n_{ij}^* . The model then is:

$$\hat{n}_{ij} = n_{ij}^* \tau_i^O \tau_j^D. \quad (2)$$

The result of the above model is a migration flow table that exhibits the level of a current period, but adopts the structure of the offset, n_{ij}^* .

Contingency table analysis of migration flow data is generally difficult because of the fact that most people stay in their region of residence during the interval being studied. This results in much larger values in the diagonal elements than in the off-diagonal elements of a two-way contingency table. Because the diagonal values are so much larger, they tend to dominate the analysis and estimation process. To deal with this imbalance, one must either control for the diagonal elements, e.g., with structural zeros, or use models that separate the diagonal effects from the off-diagonal effects, e.g., with generation and distribution components (Rogers, Willekens and Raymer 2001). The modeling strategy in this paper adopts the former --- structural zeros and the method of offsets --- because it is more

straightforward. Note, a main effects log-linear model *without* an offset predicts the non-migrants (i.e., diagonal values).

The iterative proportional fitting procedure produces the same results as those produced by the log-linear model with an offset. In this case, the marginal totals are given (the n_{ij} 's are assumed to be unavailable) and the offset includes ones in the off-diagonals and zeros in the diagonal. The iterative procedure works by adjusting the offset to fit the marginal totals of the observed table. First, the values in the offset are proportionally forced to fit the row sums and, in the next iteration, they are proportionally forced to fit the column sums. By including zeros in the offset, the predicted values for those cells are ensured to equal zero in each of the iterations. The iteration process continues until the predicted migration data satisfy both the row and column marginal totals.

Finally, the observed data are perfectly predicted if one applies an offset that is obtained by dividing the observed values by the corresponding set of predicted values obtained using a main effects model with an offset that includes zeros in the diagonals and ones in the off-diagonals (as described above). Knowing this is a very important start because, with the total sum reference category coding scheme, we now have a simple platform to apply any available information, qualitative or quantitative, to improve our estimates *provided that the marginal total information is available*.

3.2 Testing the Model Solution: Estimating Interstate Migration in the U.S. West

The observed internal migration flow patterns between states in the West region of the United States during 1985 to 1990 are estimated in this subsection. For this illustration, the answers are known. The migration data come from a special 1990 census tabulation that represented the full sample of the long form respondents (U.S. Census Bureau 1993). Table 4

contains the observed migration flows between the thirteen states in the West region and the “Rest of U.S.”. The numbers of non-migrants, persons who out-migrated and returned within the time interval, and those who immigrated or emigrated are excluded. In total, there were 7.6 million migrants between these regions with California and Rest of the U.S. being the dominant regions (i.e., 61 percent of all out migration came from these regions and 56 percent of all migrants went to these two regions). Aside from the flows entering and leaving the two bigger regions, there were relatively large flows (over 20,000) from Colorado and New Mexico to Arizona, Arizona to Nevada, Washington to Oregon, and Alaska, Colorado, Idaho, Montana, and Oregon to Washington.

3.2.1 Analyzing the Observed to Expected Ratios of Migration Flows

The ratios of observed to expected flows of migration between states in the West and the rest of the United States are set out in Table 5. Note the expected flows were obtained using a main effects log-linear model with an offset (see Equation 2) that contains zeros in the diagonal and ones in the off-diagonals. These ratios provide information on the relative strength of connections --- values greater than one state that the observed flows are greater than the expected flows, values less than one state that the observed flows are less than the expected flows, and values close to one state that the observed flows are nearly the same as the expected flows.

Table 4. Interstate migration in the U.S. West, 1985-1990

State of Residence, 1985	State of Residence, 1990														Rest of U.S.	Total
AK	AZ	CA	CO	HI	ID	MT	NV	NM	OR	UT	WA	WY				
AK		5,273	23,124	4,000	3,977	3,856	1,661	2,687	1,402	14,034	1,880	30,180	532	61,484	154,090	
AZ	2,753		109,134	19,962	3,988	4,832	2,680	20,645	15,003	11,466	12,297	18,421	1,671	210,792	433,644	
CA	11,493	136,465		62,397	44,862	26,887	11,990	110,985	27,437	128,797	38,356	155,394	5,114	1,041,070	1,801,247	
CO	4,219	38,514	99,461		5,105	5,584	4,765	14,751	15,207	11,093	11,932	20,715	8,492	303,874	543,712	
HI	1,990	4,720	54,098	4,868		1,028	541	3,994	1,314	5,210	1,737	11,885	198	95,626	187,209	
ID	2,490	7,561	24,342	4,400	1,220		4,581	8,403	1,769	17,133	18,455	31,111	2,160	33,496	157,121	
MT	2,434	7,448	16,442	6,471	1,057	7,265		5,423	1,666	8,350	3,491	24,817	6,044	46,219	137,127	
NV	1,030	10,934	51,894	4,261	1,534	4,215	1,295		1,972	7,762	6,744	7,337	869	54,220	154,067	
NM	1,451	23,080	29,828	15,604	1,191	1,259	1,200	7,050		3,068	3,718	5,885	1,037	109,847	204,218	
OR	6,830	11,268	80,550	6,145	3,910	11,925	3,353	8,223	1,648		5,298	78,006	1,057	62,662	280,875	
UT	1,450	18,018	51,265	12,398	2,252	11,837	2,635	16,874	3,501	7,387		13,508	4,349	67,759	213,233	
WA	12,003	17,255	99,980	10,611	8,538	19,696	7,876	7,560	2,989	59,020	7,685		1,547	155,126	409,886	
WY	1,110	8,092	12,762	14,482	520	3,741	5,674	6,744	2,724	3,606	7,678	5,438		46,408	118,979	
Rest U.S.	56,352	361,193	1,321,953	300,115	88,799	35,417	36,272	113,580	116,129	86,521	57,800	223,459	29,216		2,826,806	
Total	105,605	649,821	1,974,833	465,714	166,953	137,542	84,523	326,919	192,761	363,447	177,071	626,156	62,286	2,288,583	7,622,214	

Table 5. Ratios of observed to expected interstate migration flows in the U.S. West, 1985-1990

Origin	Destination													Rest of U.S.
	AK	AZ	CA	CO	HI	ID	MT	NV	NM	OR	UT	WA	WY	
AK	0.000	0.515	0.598	0.539	1.554	1.834	1.289	0.538	0.473	2.493	0.691	3.067	0.561	0.950
AZ	0.572	0.000	0.946	0.903	0.523	0.771	0.697	1.386	1.700	0.683	1.515	0.628	0.591	1.093
CA	0.463	0.866	0.000	0.546	1.138	0.831	0.604	1.443	0.602	1.486	0.915	1.026	0.350	1.045
CO	0.713	1.026	0.701	0.000	0.544	0.724	1.008	0.805	1.401	0.537	1.195	0.574	2.442	1.281
HI	1.009	0.377	1.144	0.537	0.000	0.400	0.343	0.654	0.363	0.757	0.522	0.988	0.171	1.209
ID	1.508	0.722	0.615	0.580	0.466	0.000	3.473	1.644	0.584	2.976	6.627	3.090	2.227	0.506
MT	1.699	0.819	0.479	0.982	0.465	3.891	0.000	1.222	0.633	1.670	1.444	2.840	7.178	0.805
NV	0.624	1.044	1.312	0.562	0.586	1.960	0.983	0.000	0.651	1.349	2.423	0.729	0.896	0.820
NM	0.672	1.686	0.577	1.573	0.348	0.448	0.696	1.055	0.000	0.408	1.021	0.447	0.818	1.270
OR	2.262	0.588	1.112	0.443	0.816	3.030	1.390	0.879	0.297	0.000	1.040	4.235	0.595	0.518
UT	0.645	1.262	0.951	1.199	0.631	4.039	1.466	2.422	0.848	0.941	0.000	0.985	3.290	0.751
WA	2.647	0.600	0.920	0.509	1.187	3.333	2.174	0.538	0.359	3.731	1.005	0.000	0.580	0.853
WY	0.894	1.028	0.429	2.539	0.264	2.314	5.726	1.755	1.197	0.833	3.668	0.719	0.000	0.933
Rest of U.S.	1.124	1.136	1.100	1.302	1.117	0.542	0.906	0.732	1.263	0.495	0.684	0.731	0.992	0.000

Consider, for example, the California to Arizona observed to expected ratio of 0.866. This value states, controlling for the relative shares of in-migration and out-migration and zeros for non-migrants, that the observed flow is 87 percent of the expected flow. This ratio can then be compared, for example, to the Idaho to Utah migration flow, which has a ratio of 6.627. In this case, the observed flow of 18,455 is almost seven times greater than the expected flow of 2,785. The expected flow between these two states was small because both Idaho and Utah sent and received relatively few migrants in comparison to other states. As another example, consider the value of 0.546 for the ratio corresponding to the California to Colorado migration flow. This ratio tells us that the observed flow of 62,397 migrants was a little more than half the expected flow of 114,280 migrants.

Two overall patterns are found in the ratios of observed to expected migration flows set out in Table 5. The first is that proximity to other states is important. For example, Idaho exhibits ratios with values much greater than one with its neighboring states of Montana, Nevada, Oregon, Utah and Wyoming. This is true for both Idaho's in-migration and out-migration. The exception is the interactions with California, which were less than one. The second general pattern found is that states with smaller population sizes tend to have more variation in their ratios than states with larger populations. For example, consider the ratios for migration flows associated with Wyoming, which range from 0.171 (Hawaii to Wyoming) to 7.178 (Montana to Wyoming), whereas for the Rest of U.S., the range was 0.495 (Rest of U.S. to Oregon) to 1.302 (Rest of U.S. to Colorado).

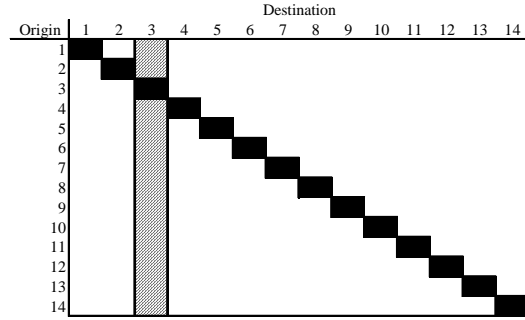
The expected values used in the denominator to calculate the ratios set out in Table 5 are necessary when one wants to control for the diagonal in the modeling of migration flows. For estimation purposes, this represents the baseline and the key to including hypothetical

relationships or partial information. When all of the observed to expected ratios are used as an offset in a log-linear main effects model, the observed values are perfectly predicted. When no interaction information is known, one is forced to use zeros in the diagonal and ones in the off-diagonals. In between these two situations (i.e., all information and no information) are a multitude of options.

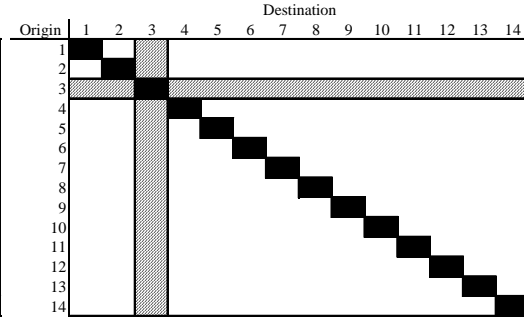
3.2.2 Missing Data Scenarios

The diagrams set out in Figure 1 represent four partial data scenarios of interstate migration in the West. The data scenario presented in Model 1 includes California's observed in-migration flows. Model 2 includes both in-migration and out-migration flows for California. Model 3 includes in-migration and out-migration flows for the states of Alaska, Arizona, California, and Colorado. And Model 4 includes in-migration and out-migration for all states, except Hawaii, Idaho, Montana, and Washington. For each of these scenarios, an offset was constructed that included zeros in the diagonal, the observed to expected ratios for the available data, and ones for all other interactions.

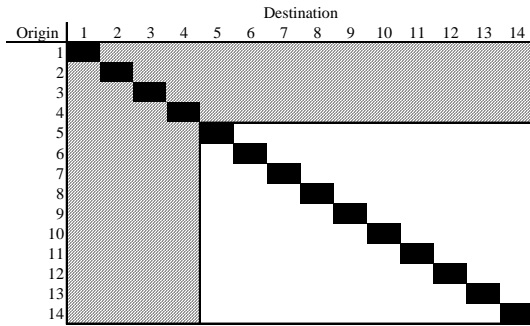
Model 1



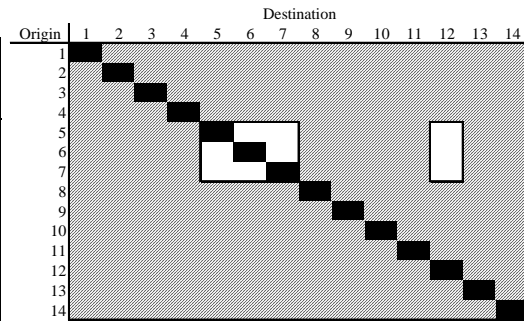
Model 2



Model 3



Model 4



Zero
 One
 Observed interaction

Figure 1. Four partial data scenarios of interstate migration in the West (1=AK, 2=AZ, 3=CA, 4=CO, 5=HI, 6=ID, 7=MT, 8=NV, 9=NM, 10=OR, 11=UT, 12=WA, 13=WY, and 14=Rest of U.S.)

A contiguity offset was also constructed to represent neighboring states in the West. (Note, AK's neighbours are HI, ID, MT, OR, and WA and HI's neighbours are AK, CA, OR, and WA.) When the values in the contiguity matrix were regressed against the ratios of observed to expected migration flows (see Table 5), a significant relationship was found with an R^2 of 0.264. The bivariate regression equation expressing this relationship was:

$$\ln(Y_i) = -0.065 + 0.432X_1$$

where Y_i represented the ratio of observed to expected migration and X_1 the contiguity matrix. When two states were neighbors (i.e., $X_1 = 1$), the ratio was predicted to be 1.443. When two states were not neighbors (i.e., $X_1 = 0$), the predicted ratio was 0.937. This information can be included along with available information to better estimate the missing data. Of course, in a missing data situation, the above ratios would not be known. To make the scenarios more realistic, four regressions were run based on the amount of information available. For Model 1, the predicted ratio for neighbors was 1.080 and 1.010 for non-neighbors. For Model 2, the predicted ratio for neighbors was 1.153 and 0.996 for non-neighbors. For Model 3, the predicted ratio for neighbors was 1.289 and 0.988 for non-neighbors. And, for Model 4, the predicted ratio for neighbors was 1.415 and 0.931 for non-neighbors. These predicted ratios, along with the available observed ratios, were included in an offset to estimate the missing data for the four model scenarios (Figure 1).

The offsets created from the four available data scenarios and from the contiguity matrix were used to predict the observed migration flows in the West region. The coefficient of determination ($R^2 = \sum (\hat{n}_{ij} - \bar{n})^2 / \sum (n_{ij} - \bar{n})^2$), chi-square ($\chi^2 = \sum (n_{ij} - \hat{n}_{ij})^2 / \hat{n}_{ij}$), and likelihood-ratio ($G^2 = 2 \sum n_{ij} \log(n_{ij} / \hat{n}_{ij})$) goodness-of-fit indicators of these predictions are set out in Table 6 below. As expected, the results improved as more information was added. The predictions started out with a log-linear main effects model. This model produced a χ^2 of 2,273,507. The model fit improved substantially to 1,304,928 when structural zeros in the diagonal were included using an offset. When the offset also included contiguity (i.e., 1.443 for neighbors and 0.937 for non-neighbors), the χ^2 statistic dropped to 899,604. For the four available data scenarios, the χ^2 statistics dropped from 1,157,407 (Model 1) to 73,460

(Model 4). When contiguity was included with the four available data scenarios, the χ^2 statistic dropped from 1,061,575 (Model 1 with contiguity) to 68,185 (Model 4 with contiguity).

Table 6. Comparison of different log-linear models used to predict interstate migration flows in the U.S. West, 1985-1990

Model	R^2	χ^2	G^2
Main Effects	0.94407	2,273,507	2,091,602
Diagonal	0.97999	1,304,928	443,859
Contiguity	0.98248	899,604	313,040
Model 1	0.98351	1,157,407	397,847
Model 2	0.98878	1,036,878	341,745
Model 3	0.99124	791,182	252,664
Model 4	0.99967	73,460	24,277
Model 1 with Contiguity	0.98443	1,061,575	369,335
Model 2 with Contiguity	0.99039	862,362	294,044
Model 3 with Contiguity	0.99264	594,663	203,133
Model 4 with Contiguity	0.99968	68,185	24,177

The goodness-of-fit values set out in Table 6 show how the overall model fits improve with added information. To examine some particular origin-destination-specific migration flows, consider the comparison of five model scenarios the set out in Figure 2, which includes just the in-migration patterns to Idaho --- a state in which no observed migration data were included in any of the scenarios. Model 0 assumes no available information and includes an offset with zeros in the diagonal and ones in the off-diagonals. Models 1, 2, 3, and 4 corresponds to the four scenarios set out in Figure 1, where the available observed to expected ratios are included along with the predicted ratios based on contiguity. In Figure 2,

there are some interesting patterns. First, for flows from Hawaii, Montana, and Washington, none of the models performed that well. For the other ten states, the predicted values were very close to the observed values by Model 4.

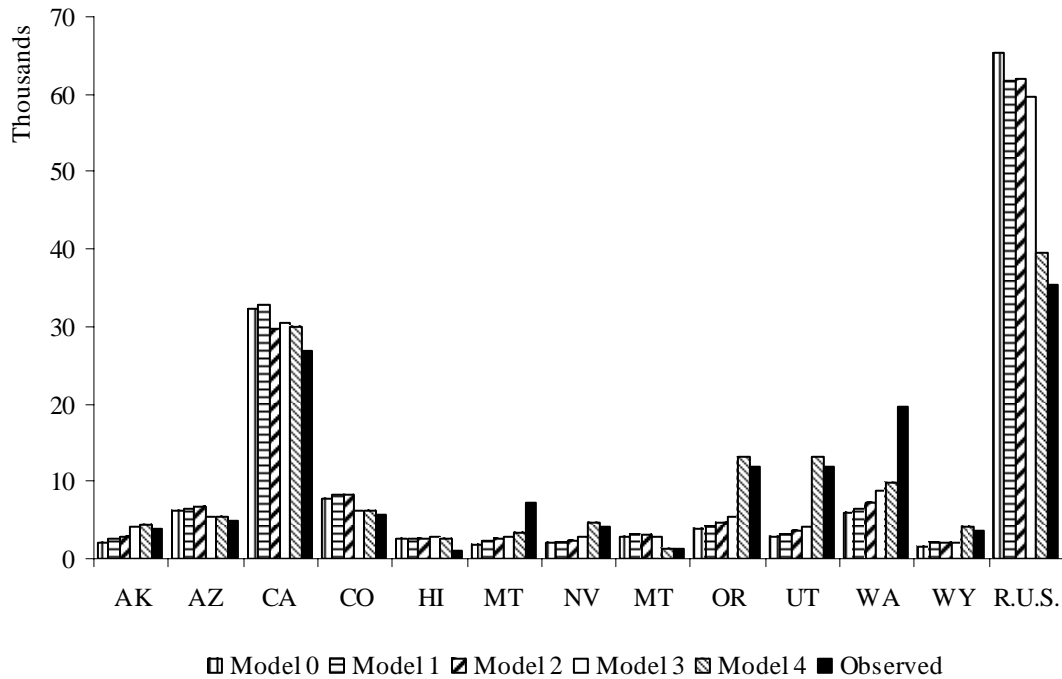


Figure 2. Comparison of predicted and observed migration flows to Idaho: Model 0 (no data available), Model 1 (California in-migration data available), Model 2 (California in- and out-migration available), Model 3 (Alaska, Arizona, California, and Colorado in- and out-migration data available), and Model 5 (all data available, except Hawaii, Idaho, Montana, and Washington in- and out-migration).

To summarize, the origin-destination-specific migration flows between states in the U.S. West region were modeled effectively using a log-linear main effects model with an offset. The key to the modeling strategy involved creating a good offset. Here, several available data situations were examined. When more migration flow information was added

to the offset, the predictions improved. In addition, contiguity was shown to explain about 25 percent of the spatial interaction.

4. ESTIMATING INTERNATIONAL MIGRATION FLOWS IN THE NORTHERN REGION OF EUROPE

In this section, the reported international migration flows obtained from both the immigration and emigration flow tables (set out in Table 3) are used to obtain estimates for all origin-destination-specific migration flows between countries in the Northern region of Europe during the 1999-2000 time period. The modeling process is the same as the one described in the previous section --- the log-linear model with an offset.

The first step in the estimation process was to obtain a consistent set of marginal totals of the international migration flow table. This process involved forcing the gross flows of immigration and emigration to correspond with the demographic accounting equation. The demographic accounting equation for this situation states that the population in 1999 plus natural increase plus net migration equaled the population in 2000. The populations in 1999 and 2000, natural increase 1999-2000, and net migration 1999-2000 were available from the Eurostat NewCronos database for all countries in Northern Europe and formed the basic constraints of the modeling process. The result was that immigration levels from Northern European countries, with the exception of Finland, were reduced. The corresponding emigration levels, with the exception of Norway and Sweden, were increased. Once the numbers were consistent with each country's reported sources of population growth, ordinary least squares regression techniques were then used to obtain estimates of the missing marginal totals. Here, the natural logarithm of population size, per capita income, and age

were used as key explanatory factors. The dependent variable was the natural logarithm of emigration. This estimation process required two steps. The first estimated the gross flow of emigration for each country (immigration was obtained as a residual, i.e., $\text{immigration} = \text{net migration} + \text{emigration}$). The second step estimated the portions of the immigration and emigration flows corresponding to the Northern region. These regressions were based on the reported data. Finally, the estimated parameters were used to obtain estimates of the missing gross migration flows. For more detailed information on the particulars of this estimation process, refer to Raymer (2003).

The gross flow estimates of immigration and emigration are set out in the marginal totals of Table 7 (i.e., row-sums and column-sums, respectively). The missing marginal totals for Estonia, Ireland, Latvia, Lithuania, and the United Kingdom were estimated based on the information from the Northern European countries that did provide information. Since the focus of this paper is on the place-to-place estimation of migration flows, we (for the moment) assume that these marginal estimates are reasonable and accurate. The expected international migration flows, included within the marginal totals of Table 7, were calculated based on the marginal total information. Specifically, the iterative proportional fitting procedure was used to force a table of ones (in the off-diagonals) and zeros (in the diagonal) to fit the estimated marginal totals of the migration flow table.

Table 7. Expected international migration in Northern Europe, 1999-2000

Country of Origin	Country of Destination										Total
	Denmark	Estonia	Finland	Iceland	Ireland	Latvia	Lith.	Norway	Sweden	U.K.	
Denmark	0	170	1,032	350	2,887	244	958	1,698	1,643	2,153	11,136
Estonia	407	0	264	89	737	62	245	434	420	550	3,208
Finland	1,211	129	0	266	2,191	185	727	1,289	1,247	1,633	8,877
Iceland	317	34	205	0	574	49	190	337	326	428	2,460
Ireland	1,513	161	978	332	0	232	908	1,610	1,558	2,041	9,333
Latvia	630	67	407	138	1,140	0	378	671	649	850	4,930
Lithuania	1,250	133	808	274	2,262	191	0	1,330	1,287	1,686	9,222
Norway	1,600	171	1,035	351	2,896	245	961	0	1,648	2,159	11,068
Sweden	1,632	174	1,056	358	2,954	250	980	1,738	0	2,203	11,345
U.K.	3,052	326	1,974	670	5,523	467	1,833	3,249	3,143	0	20,237
Total	11,612	1,366	7,760	2,828	21,164	1,925	7,181	12,355	11,922	13,703	91,816

The “observed” to expected migration flow ratios are set out in Table 8 below. Note the reported international migration flows (Table 3) were proportionally adjusted to fit the adjusted column marginal totals in Table 7 *before* calculating the ratios. Several cells in Table 8 contain values of one. Since there were no observed values for these cells, the initial assumption was that the observed values were equal to the expected values. To improve upon this assumption, a simple regression equation based on the contiguity of countries (same as in Section 3.2.2) was used to estimate the missing ratios. Note, aside from countries that shared a border, the Scandinavian countries (i.e., Denmark, Iceland, Norway, and Sweden), and the countries of the former Soviet Union (i.e., Estonia, Latvia, and Lithuania) were considered to be neighbors. For neighboring countries, the regression model predicted a ratio of 2.1199. For non-neighboring countries, the predicted ratios were 0.2621. This regression model had an R^2 of 0.346 and was significant at the 0.05 level.

Table 8. Ratios of observed to expected international migration flows in Northern Europe, 1999-2000

Country of Origin	Country of Destination									
	Denmark	Estonia	Finland	Iceland	Ireland	Latvia	Lith.	Norway	Sweden	U.K.
Denmark	0.0000	1.5175	0.4398	4.0495	0.1203	1.4681	0.3777	1.5284	1.0676	1.3883
Estonia	0.5830	0.0000	3.8037	0.0658	1.0000	1.0000	1.0000	0.1861	0.4992	1.0000
Finland	0.3420	2.4276	0.0000	0.1698	0.0435	0.1410	0.0163	1.0168	2.3390	1.4060
Iceland	3.6946	0.0912	0.3368	0.0000	0.0247	1.0000	0.0270	1.3030	0.9406	0.6937
Ireland	0.1625	1.0000	0.0536	0.0177	0.0000	1.0000	1.0000	0.0430	0.1021	1.0000
Latvia	0.5517	1.0000	0.1381	0.0497	1.0000	0.0000	1.0000	0.1643	0.2083	1.0000
Lithuania	0.3691	1.0000	0.0332	0.1180	1.0000	1.0000	0.0000	0.0742	0.0690	0.1304
Norway	1.8410	0.1817	1.1796	1.6807	0.0251	0.1868	0.0323	0.0000	2.6661	2.6068
Sweden	1.3011	0.3290	3.9101	1.5656	0.0802	0.1066	0.0156	3.3025	0.0000	1.0313
U.K.	1.2007	1.0000	0.3795	0.2445	3.6931	1.0000	0.0783	0.5886	0.6225	0.0000

The predicted migration flows based on the estimated marginal totals (Table 7), the ratios of observed to expected flows (Table 8), and the estimated ratios based contiguity (discussed in the previous paragraph) are set out in Table 9 below. For the most part, the results appear reasonable. The only flows that may be unreasonable are the ones from Lithuania. Here, the gross flow of emigration to Northern European countries (i.e., 9,221) was most likely overestimated. This was carried out in the first stage in the modeling process --- the estimation of the marginal totals or gross flows of migration.

Table 9. Predicted international migration flows in Northern Europe, 1999-2000

Country of Origin	Country of Destination										Total
	Denmark	Estonia	Finland	Iceland	Ireland	Latvia	Lith.	Norway	Sweden	U.K.	
Denmark	0	168	634	1,590	577	303	1,033	3,226	1,876	1,729	11,136
Estonia	222	0	1,121	5	257	89	1,186	80	179	67	3,206
Finland	610	257	0	64	200	28	43	2,057	3,940	1,679	8,878
Iceland	1,313	2	93	0	23	10	14	525	315	165	2,460
Ireland	689	66	176	16	0	123	1,633	207	409	6,014	9,333
Latvia	529	121	102	10	647	0	2,986	179	188	168	4,930
Lithuania	2,011	686	140	136	3,680	1,280	0	458	355	475	9,221
Norway	2,712	16	1,345	522	95	30	70	0	3,706	2,569	11,065
Sweden	1,578	24	3,671	401	251	14	28	4,542	0	837	11,346
U.K.	1,948	25	477	84	15,435	47	187	1,082	953	0	20,238
Total	11,612	1,365	7,759	2,828	21,165	1,924	7,180	12,356	11,921	13,703	91,813

To summarize, the estimates set out in Table 9 are the result of the combination of several pieces of information. The log-linear model presented in this paper is flexible enough to deal with wide variety of place-to-place migration situations. With regard to international migration flows in Europe, there are several possible avenues for improving the estimates presented above, such as focusing on those countries with “good” data (i.e., registration-based) consistent first and then focusing on countries with relatively poor or questionable data and then those without data. Also, additional variables could be incorporated to estimate the interaction between countries. Proximity is just one factor. Other contiguity factors could include language and European Union membership, for example. Finally, it needs to be stressed that, in order to accurately model place-to-place migration flows, one needs accurate estimates of the marginal totals of a migration flow table or the gross flows of immigration and emigration for a particular migration system.

5. DISCUSSION

This paper has focused on the second stage of the estimation process and used the categorical log-linear model as the modeling platform. The main effects version of this model with an offset was used to predict the migration flows. The main effects portion ensured that the predicted values fitted the observed or estimated marginal totals. The offset portion permitted some form of spatial interaction to be included, which for the examples used in this paper, involved incorporating the available ratios of observed-to-expected migration flows and contiguity.

By considering a different interpretation of the log-linear parameters, a more straightforward and logical solution to the estimation of origin-destination-specific migration flows was revealed. Traditionally, the parameters were interpreted in terms of geometric means or by reference to a particular category (e.g., Rogers et al. 2002; Rogers, Willekens and Raymer 2001, 2002, 2003; Willekens 1983). With an offset that includes structural zeros, the parameters of the first or last reference coding scheme are not interpretable because the reference cell is zero. With geometric mean coding, the interpretation also is not straightforward. The interpretation of the parameters associated with both the cornered effect and effect coding schemes rely on the cell values themselves. The total sum reference category coding scheme, on the other hand, does not. It relies on the total sum and the proportional levels represented in the row and column marginal totals. This is more useful when estimating migration flows, because one often has the marginal totals but not the cell values. The parameters of the model in this case can be used to improve the estimates.

Another development important to the estimation of origin-destination-specific flows was the identification of the offset required to perfectly predict the observed data. By

knowing that this offset represented the observed-to-expected ratios of origin-destination-specific migration flows, the focus could then be directed at the modeling of the spatial interaction. For example, the interaction parameters representing contiguity were included to improve the estimates. This identification also allowed for an easy method to convert available data into interaction data that again improved the migration flow estimates.

Migration is generally viewed as a complex phenomena that is difficult to model or incorporate into population projection models (Smith, Tayman and Swanson 2001; van der Gaag and van Wissen 1999). Unfortunately for those researchers in the area of population change, it is often the most important factor. This means it is no longer acceptable to simply ignore migration because the data are not available or are not reliable. If feasible, the missing or inadequate migration data should be collected. However, for situations in which the collection of the missing or inadequate migration data is not feasible, then an estimation procedure should be adopted and, if necessary, improved upon. The methodology provided in this paper suggests one possible avenue that is relatively robust and is based on relatively few auxiliary data.

In conclusion, estimates of place-to-place migration are needed to improve the understanding of population change, to provide more accurate population projections, and to design more effective migration policies. Toward that end, this paper has shown that is possible to produce reasonable estimates of migration flows based on partial information. The research described herein provides new insights and improves the state-of-the-art for estimating both internal and international migration. There are many areas where such research is needed. For example, estimates of place-to-place migration patterns could help organizations in the less developed areas of the world to construct policies that deal with

either rapid population growth or persistent decline. Historians could use this research to identify the sources of population redistribution. And, local administrators could use these methods to help predict how many jobs, schools, or hospitals their areas might require in the near future.

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