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Statistical Energy Analysis and Finite Elements

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SEA and FEA have become established engineering methods for noise and vibration analysis. They are contrasting approaches targeted at the high and low frequency ends of the analysis spectrum. This paper concerns the use of FE methods in SEA. Attention is focussed on two areas. The first concerns the use of FE models to predict SEA parameters, such as coupling loss factors. The second area concerns the coupling of subsystems described by FE and SEA models. This is an important "mid-frequency" vibration issue, and arises, for example, when stiff, low mode-count subsystems are connected to flexible, high mode-count subsystems. The different subsystems are suited to different modelling methods, but coupling the models is problematical.

1. INTRODUCTION

Statistical Energy Analysis (SEA) [1] and Finite Element Analysis (FEA) [2] have become established engineering methods for noise and vibration analysis. They are contrasting approaches, aimed in principle at high and low frequency applications respectively. In general, FEA adopts a deterministic analysis to produce "exact" predictions of a structure's response assuming structural parameters are known precisely, although methods exist for including (normally small) uncertainties (e.g. [3]). FEA gives a detailed model and detailed response predictions, such as frequency response functions (FRFs), with the modes of the structure typically being found first. SEA, on the other hand, provides a broad, approximate model for the behaviour of a built-up structure comprising assembled subsystems. The response is described in terms of time, frequency and space-averaged energy within each subsystem, these averages implicitly also being averages taken over an ensemble of systems with widely varying properties. In principle, FEA is a low frequency method which encounters difficulties as frequency increases due to the increasing size of the FE model and due to the increasing sensitivity of the response to uncertainties in the properties of the system being analysed. On the other hand, SEA is a high frequency technique that averages out the detailed properties of the structure and hence averages out the details of the response.

Although inhabiting opposite ends of the analysis spectrum, both in terms of frequency (and number of modes) and parametric uncertainty, there are areas of overlap, two of which are discussed in this paper. The first concerns the use of FE models to predict SEA parameters, such as coupling loss factors (CLFs), and to develop SEA-like energy models of systems. The second area concerns the coupling of FEA and SEA models.

2. SEA MODELS FROM FEA

The SEA equations relate ensemble average subsystem input powers **P** and energies **E** by

$$\mathbf{P} = \omega \mathbf{L}\mathbf{E}; \quad L_{ij} = -\eta_{ji}; \quad L_{ii} = \eta_i + \sum \eta_{ij} \qquad (1)$$

where **L** is a matrix of damping loss factors η_i and CLFs η_{ij} for subsystems *i* and *j*. FEA of two subsystems can be used to estimate the SEA parameters (e.g. [4-9]), with results being used in SEA models of the larger assembled structure, and to explore the validity and accuracy of the SEA equations [8,9]. (Indeed, FEA of just one subsystem can be used to estimate its modal density by counting natural frequencies.) Numerical experiments are performed and the power injection method applied: a modal analysis of the system is performed; the subsystems are excited one at a time; the forced response calculated; the frequency average input powers and subsystem energies determined. The SEA equations can then be written in terms of these frequency averages as

$$\begin{bmatrix} P_{in,1}^{(1)} & 0 \\ 0 & P_{in,2}^{(2)} \end{bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} \begin{bmatrix} E_1^{(1)} & E_1^{(2)} \\ E_2^{(1)} & E_2^{(2)} \end{bmatrix} \tag{2}$$

where the superscripts (1) and (2) identify the subsystem being excited. Therefore

$$\begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} P_{in,1}^{(1)} & 0 \\ 0 & P_{in,2}^{(2)} \end{bmatrix} \begin{bmatrix} E_1^{(1)} & E_1^{(2)} \\ E_2^{(1)} & E_2^{(2)} \end{bmatrix}^{-1} \tag{3}$$

although more cumbersome formulations have appeared in the literature.
Many applications of FEA to the estimation of SEA parameters have been reported, and there is insufficient space to review these here. However, some observations are appropriate. First, SEA is not an exact theory, but one that involves a number of assumptions and approximations. The SEA equations apply to ensemble averages (although the ensemble is rarely defined in theoretical studies) but in numerical studies they are applied to the frequency average response of a single system - an ergodic assumption is made, with ensemble and (broadband) frequency averages being assumed equal. In the SEA equations the excitation is "rain-on-the-roof" and the response quantity is subsystem energy. In most FE studies the excitation is applied at only a few points and the response found at only a few points, and averages are then taken: these will differ from the true averages. However, this spatial averaging need not be applied: the mass and stiffness matrices are required to determine the modal properties and can also be used to perform true spatial averaging [7], but the software available may not allow direct access to these matrices. Finally, there is a finite number of modes in a given band: this gives inherent variability in the responses of individual systems.

3. COUPLING SEA AND FEA MODELS

Structures often comprise subsystems with quite different dynamic characteristics. Some subsystems may support long wavelength motion, have low modal density, large dynamic stiffness and be well-defined, while others may have high modal density, short wavelengths, small dynamic stiffness and be poorly defined. Examples include beam-stiffened plates or components that support both bending and in-plane vibrations. Modelling the vibrations of such structures is a mid-frequency problem that poses real challenges: the short wavelength, high mode-count subsystems are amenable to SEA rather than FEA, while the converse is true for the long wavelength subsystems, and the computational cost of FEA of the whole structure is prohibitive. A hybrid approach is thus required, typically by coupling an FE model of the deterministic subsystem to a statistical, SEA-like model of the uncertain subsystem.

This section concerns various approaches to this problem of coupling FE and SEA models. Reference is made to the case of a beam-stiffened plate, with excitation applied to the beam (the source structure) and energy flowing to the plate (the receiver). It is natural to describe the behaviour of the source in terms of FRFs when loaded by the panel and to predict the net power transmitted to the receiver, which forms the input power for the SEA-like part of the structure. The approaches differ in the assumptions and approximations made, and whether the approximate description of the receiver is developed in terms of waves or modes. The detailed behaviour of the uncertain receiver is of course unknown (e.g. exact natural frequencies and mode shapes), but some gross features are known (e.g. modal density etc) so that some approximate or statistical description is necessary.

3.1 FEA, uncertainty and model reduction

Although a full FEA of the structure is generally not feasible, some methods are worth mentioning in passing. Stochastic FE methods [3] can accommodate small levels of uncertainty, the uncertain parameters being meshed in a manner similar to the response field. Various techniques exist to reduce the number of degrees of freedom (DOFs) of a numerical model. The most well known include Guyan reduction and component mode synthesis (CMS) using free or fixed interface modes, together with attachment or constraint modes [10]. Dynamic reduction may be of value, where only those modes in a frequency band are retained, the others being approximated by stiffness and mass residuals. A further possibility is loaded-interface CMS. Here, it would be natural to approximate the uncertain receiver as if it were infinite, since it is known that [11] the infinite structure approximates the finite structure, especially as its modal overlap increases or when frequency averages are taken. The receiver then loads the FE model of the source structure at their interface. Finally, uncertainty can be included in the component modal properties themselves [12], which substantially reduces size and cost.

3.2 Wave approaches

Wave methods can be used to develop an approximate model of the receiver. The simplest approach is to approximate the receiver as if it were infinite and to use a FRF-based substructuring approach. A continuous interface is discretised into a series of point connections. The input and transfer FRFs of an infinite plate [11] and the uncoupled modes of the beam are coupled in the frequency domain. One disadvantage is that many coupling DOFs may be required: the discrete points should typically be at most one quarter of a wavelength apart, and this wavelength is of course relatively small compared to the length of the beam.

The locally reacting impedance method [13,14] recognises that the uncoupled beam wavenumber $k_b = \sqrt[4]{m_b \omega^2 / EI}$ is usually substantially smaller than the plate wavenumber $k_p = \sqrt[4]{m_p \omega^2 / D}$. Here m_b, m_p, EI and D are the mass per unit length or area and bending stiffnesses of the beam and plate. Suppose that, when coupled, there is a strongly excited response component in the beam with a wavenumber k_b'. Wave motion in the plate will have this trace wavenumber along the beam and hence a relatively very large wavenumber component perpendicular to the beam.

The wavefield in the plate can be approximated as propagating perpendicular to the beam. The plate then appears to load the beam with a locally reacting impedance given by

$$Z \approx \frac{2m_p}{k_p}(1+i) = \frac{\lambda_p m_p}{\pi}(1+i) \tag{4}$$

where λ_p is the plate wavelength. The plate therefore adds mass and damping to the beam, the mass per unit length being that contained within a strip of width λ_p/π, and the damping corresponding to the energy radiated into the plate. This loading can then be included in a modal (FE) model of the source structure. The restrictions of this approach are that the source structure must be somewhat uniform and that there must be a large enough stiffness and wavenumber mismatch between source and receiver. The method can capture some detail of the receiver such as the presence of a boundary relatively close to the source.

A final approach [15] couples a modal model of a straight, uniform source to a wave model of the receiver using a Fourier transform method with approximations. The results reduce to those of the locally reacting impedance method as the source/receiver mismatch increases.

3.3 Modal approaches

The short wavelength subsystem can equally be described in terms of its modes, although a statistical description is required because these modes cannot be calculated deterministically. In the 'Resound' approach [16,17] the DOFs of the structure are partitioned into sets of DOFs $\mathbf{q}_g$ and $\mathbf{q}_l$, which are associated with global and local basis functions respectively. These are typically the modes of the deterministic and uncertain parts of the structure and correspond to the long wavelength, global modes and short wavelength, local modes. The equations of motion are then written in terms of dynamic stiffness matrices **D** as

$$\begin{bmatrix} \mathbf{D}_{gg} & \mathbf{D}_{gl} \\ \mathbf{D}_{lg} & \mathbf{D}_{ll} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_g \\ \mathbf{q}_l \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_g \\ \mathbf{f}_l \end{Bmatrix} \tag{5}$$

where **f** are the corresponding generalised forces. The local DOFs are then in essence mass-reduced, resulting in the equations

$$\left(\mathbf{D}_{gg} - \Delta\mathbf{D}_{gg}\right)\mathbf{q}_g = \mathbf{f}_g - \Delta\mathbf{f}_g; \quad \mathbf{D}_{ll}\mathbf{q}_l = \mathbf{f}_l - \Delta\mathbf{f}_l \tag{6}$$

The first is solved deterministically (to give the response in terms of loaded global modes) and the second forms an SEA model for the uncertain subsystems. The rationale for the mass reduction is that the flexible subsystem is mass-controlled for any of the wavenumber components in the long-wavelength structure that may be strongly excited. The net effect of the local modes is then to add mass and damping to the global modes.

The perturbations $\Delta\mathbf{D}$ and $\Delta\mathbf{f}$ in equation (6) can be written explicitly in terms of the global and local modal properties. The statistical distribution of the local modes is then assumed to be such that they are uniformly probable in frequency and uniformly distributed in wavenumber space, with the dispersion properties of the equivalent infinite structure determining the wavenumber domain appropriate to a given frequency band.

In the mode based approach of [18] modal descriptions of each subsystem are coupled by decomposing the interface forces and displacements into a set of basis functions. These would usually be the unloaded mode shapes of the beam, found from FE. The modes of the flexible subsystem are described statistically by a simple standing wave model - in principle a similar description to that used in [16]. For large enough dynamic mismatch of the dynamic properties the main effect of the flexible receiver is to add damping and mass to the unloaded beam modes, as seen in [13-15], with the coupling terms between these modes being small.

In the fuzzy structure approach [19-21] a deterministic 'master' structure (the beam) is coupled to a set of 'fuzzy' attachments. Each member of the fuzzy set is a single DOF oscillator (a mode of the plate). The properties of the fuzzy set are described statistically as a continuum of oscillators whose masses and natural frequencies are distributed over frequency. One can interpret the system as being the beam attached to a continuum of vibration absorbers, each of which has an infinitesimal mass. The response of the master is found by averaging the effects of the fuzzy oscillators. The fuzzy attachments add damping to the master structure. The added loss factor depends on the mass of the fuzzy attachments and is given by $\eta_{fuzzy} = m_p \lambda_p / m_b \pi$ for a beam/plate system. The fact that the added damping is independent of the damping of the fuzzy oscillators is not as surprising as it may seem at first sight. Suppose each member of the fuzzy set has a loss factor η. At any frequency, the responses of the resonant members of the fuzzy set are proportional to η^{-1} and thus so, too, are the forces they apply to the master. However, the number of fuzzy members excited at resonance is proportional to the bandwidth (which is proportional to η) and hence the net force from the fuzzy set is independent of η. The conclusion is not valid for zero damping, when the steady state behaviour is never reached.

While fuzzy structure theory gives a simple description, there may be difficulties in finding the properties of the fuzzy set from those of a continuous receiver such as a plate. This is particularly true if there is a plate boundary close to the beam, so that the fuzzy mass distribution with frequency may be difficult to determine.

4. CONCLUDING REMARKS

This paper concerned two areas in which FE and SEA methods can be combined. FEA of two subsystems can be used to estimate the SEA parameters. The estimates may be biased and variability arises from frequency averaging, finite mode count effects and from averaging over a finite number of excitation and response points. Ideally FE should involve ensemble averaging, but such averaging has rarely been attempted and raises issues of computational cost and how the ensemble is to be defined. One possibility is the component modal method of [12], which is computationally very efficient. Another problem is that the system may be strongly coupled, so that the CLFs will depend on damping and, in the built-up structure, there may be non-zero indirect CLFs. FEA of just two subsystems will only reveal this if the CLFs are estimated for a range of damping loss factors to determine whether they are dependent on damping.

The various methods for coupling FE and SEA subsystems have different accuracy, data requirements and computational cost. The most general, and most costly, are the mode-based methods. They require assumptions concerning the modal statistics of the receiver. If the modal overlap of the receiver is large enough, the detailed natural frequency distributions are unlikely to be important. However, under some circumstances one could envisage secular effects concerning mode shape statistics being important, and a mid-frequency approach should be able to capture these. One example is where the beam is applied (almost) parallel to, and a few plate wavelengths from, a plate edge. Fluctuations in the beam/plate interface forces would be expected on frequency scales inversely proportional to the time it takes waves to travel from beam to plate edge, and these scales could be very much larger than the mean modal spacing. Such fluctuations are easily accommodated in the wave approach of [13,14] and in the modal approaches of [16,18] if suitable mode shape statistics are assumed.

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