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TRAINEE PRIMARY TEACHERS' KNOWLEDGE OF GEOMETRY FOR TEACHING

A report based on the meeting at the University of Bristol, 18th November 2002

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One outcome of the implementation of the (UK) National Numeracy Strategy at the primary school level is the privileging of the teaching and learning of number. Yet, as the recent Royal Society report on geometry stresses, it is important to begin the developing of spatial thinking and reasoning at this level. This report reviews what trainee primary teachers might need to know about geometry in order to teach the geometry component of the mathematics curriculum effectively and confidently. Some initial findings are given from research which suggests that, in the UK, geometry is the area of mathematics in which trainees perform most poorly in initial baseline tests and have the least confidence to teach. Hence it is the area in which trainees need to make most progress if they are to gain qualified teacher status

INTRODUCTION

The (UK) National Numeracy Strategy, as might be inferred from its title, tends, whether explicitly or not, to privilege the teaching and learning of number. For example, the Numeracy Task Force chose to define numeracy at Key Stages 1 and 2 as "a proficiency that involves a confidence and competence with numbers and measures" (Numeracy Task Force, 1998, para 15). The official report from which this extract is taken goes on to say that "Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams, charts and tables". The only mention of geometry in the report is in the section on 'mathematics across the curriculum' (note that here the term numeracy is not used) when it is noted that "in art, children can look at the properties of shapes and patterns" (*ibid*, para 133).

Yet, of course, there has to be some geometry in the numeracy framework for primary schools because the (UK) national curriculum includes some geometry at that level. Such a curricular design is in line with the recent report on the teaching

and learning of geometry (Royal Society and Joint Mathematical Council, 2001), which, although it focuses on geometry in secondary schools and beyond, emphasises how important it is to begin the developing of spatial thinking and reasoning at the primary level.

Thus are raised questions about what primary teachers might need to know about geometry in order to teach the geometry component of the mathematics curriculum at that level effectively and confidently. This, in turn, raises questions about the knowledge that trainee primary teachers have of geometry and of how it might be measured, developed, enhanced, and so on.

THE NATURE AND ROLE OF TEACHER KNOWLEDGE

The mathematical knowledge necessary to teach 'effectively' is recognised as being a more complex issue than simply requiring a grasp of mathematics content or subject knowledge (Ball, 1990; Fennema and Franke, 1992). The term *pedagogical content knowledge* (PCK) was first employed by Shulman (1986) to depict a blend of content and 'ways of transforming that content in terms of its teachability'. For mathematics, as for any given subject area, PCK includes forms of representation of concepts, useful analogies, examples, demonstrations, and so on, that can help to make mathematical ideas comprehensible to others.

Shulman's model may be too simplistic (in that, for instance, it does not distinguish between the nature of different school subjects, nor between the academic subject, in this case mathematics, and the school subject) and several modifications have been suggested (see, for instance, Cochran, DeRuiter and King, 1993). Nevertheless, it has proved useful to distinguish the two relevant knowledge domains; *subject matter knowledge* (which includes key facts, concepts, principles, and explanatory frameworks of a discipline, as well as the rules of evidence used to guide inquiry in the field), and *pedagogical content knowledge* (which consists of an understanding of how to present specific topics in ways appropriate to the students being taught).

More recently, Ma (1999) coined the term *profound understanding of fundamental mathematics* (PUFM) to refer to the depth, breadth, and thoroughness of the knowledge that is required to be an accomplished teacher of primary mathematics. According to Ma, teachers with PUFM make connections between mathematical concepts and procedures from the simple to the complex, appreciate different facets of an idea and various approaches to a solution, are particularly aware of the simple but powerful foundational concepts and principles of mathematics (such as equality), and are knowledgeable about the whole primary mathematics curriculum, not just the content of a particular age level.

PRIMARY TEACHERS AND GEOMETRY

A previous report from the BSRLM Geometry Working Group (Jones, 2000) sets out to review what is known about teacher knowledge in geometry, how the knowledge develops and how this knowledge development can be supported by

professional development. The review concludes that the available evidence suggests that attention could usefully be paid both to the initial and continuing education of teachers of mathematics in terms of their background and understanding of geometry as the successful teaching of geometry depends on teachers knowing a good deal of geometry and how to teach it effectively.

The review found that much of the research on teachers' knowledge of geometry was carried out in high schools (and beyond). There appears to be little research on primary teachers' knowledge of geometry. The situation is the same in the case of trainee primary teachers' knowledge of geometry.

This means that the question of what trainee primary teachers' need to know about geometry in order to teach the geometry component of the mathematics curriculum effectively and confidently is, at the moment, an open question. The UK Government regulations, contained in Circular 4/98 *Teaching: High Status, High Standards* (DfEE, 1998) specifies a body of mathematical subject knowledge in Annex D. This annex includes a section on geometry (referred to as "Shape and Space" in order to be consistent with the National Curriculum) as follows:

- co-ordinates in 2-D,
- 2-D transformations,
- congruence and similarity,
- constructions,
- Pythagoras,
- area formulae,
- surface area and volume of prisms,
- the properties of 3-D shapes.

It is worth comparing this with a recent US report (Conference Board for the Mathematical Sciences, 2000) on the mathematics to be included in the training of primary teachers in the US. This reports suggests that such training should include, for prospective elementary school teachers:

- Visualization skills
- Basic shapes
- Technical vocabulary and understanding the role of mathematical definition
- The process of measurement
- Length, area, and volume

For prospective middle-grade teachers, the list was extended to include:

- Common two- and three-dimensional shapes
- Making conjectures about geometric shapes and then proving or disproving

- Transformations
- Similar figures
- Ability to visualize and solve problems
- Connect geometry to other mathematical topics, and to nature and art
- Understand the common forms of measurement and use measurement techniques and formulas

While there is some overlap in the approaches to specifying the knowledge of geometry that primary teachers need, it is interesting to note that the US specification includes some mathematics processes (understanding the role of mathematical definition; making conjectures and then proving or disproving; the ability to visualize and solve problems; the connections between mathematics and other subjects; and so on) in addition to the content knowledge prevalent in the UK specification. In the UK, from September 2002, a new specification supersedes Circular 4/98 and is entitled *Qualifying to Teach: Professional Standards for the Award of Qualified Teacher Status* (TTA, 2002). These new requirements for initial teacher education no longer specify a statutory body of mathematical subject knowledge, either content or process. Rather, guidance on the requirements for such knowledge is contained in the non-statutory 'handbook', which, at the time of writing, has yet to be published.

RESEARCHING PRIMARY TEACHERS AND GEOMETRY

In order to monitor trainee progress towards acquiring subject knowledge in geometry, a process of testing and self-auditing has been implemented within a number of institutions providing initial teacher education. Initial findings from one particular institution, gathered over four years, indicate that geometry is the area of mathematics in which trainees perform most poorly in initial baseline tests. Their personal confidence in teaching geometry, indicated within a self-audit, is also low. Their knowledge of geometric vocabulary is particularly weak. Particular difficulties arise in answering questions requiring calculation of area, surface area and volume, including those that do not require the recall of specific formulae. Many trainees are unable to record transformations in four quadrants using Cartesian coordinates.

This weakness within geometry has been found in trainees across all the subject specialisms, including those training as mathematics specialists. The mathematics specialists tend to score higher than the other specialists but still identify geometry as an area of weakness both in the baseline test and initial self-audit. The summative test in mathematics clearly shows that the vast majority of trainees make substantial gains in the subject knowledge of geometry during their training (which includes self-evaluation, target-setting and supported self-study).

Trainees in this particular institution are also graded during school-based training on several fronts: their ability to apply their mathematical subject knowledge and pedagogical content knowledge to their teaching; their planning for mathematics

teaching and learning; their teaching of mathematics; their classroom management within mathematics sessions; and their ability to monitor, assess and record the mathematical attainment of the learners. Both the mid-year teaching grades and the end-of-year grades indicate that trainees with a low personal confidence in geometry, as indicated in their self-audits and tests, are more likely to achieve lower teaching grades for mathematics than those with a higher confidence and competence. These preliminary findings accords with the findings of Rowland *et al* (2000) who looked more broadly at mathematical subject knowledge.

CONCLUDING COMMENTS

Despite the (UK) numeracy strategy, with its apparent privileging of number of over areas of mathematics, geometry remains of crucial important in mathematics education. As the great (UK) mathematician Michael Atiyah recently wrote:

> *spatial intuition or spatial perception is an enormously powerful tool and that is why geometry is actually such a powerful part of mathematics - not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition. Our intuition is our most powerful tool...*
>
> Atiyah, 2001

While, as a previous geometry working group report (Jones, 2000) concludes, subject matter 'matters', deciding what subject matter, for whom, and in what depth, is a substantial challenge. Yet it is also clear that it is not just the mathematics that is important. Knowing mathematics does not ensure the effectiveness of prospective and serving teachers. How they come to know their mathematics matters as well. Thus, further research is going to need to focus in part on the nature of the geometry subject knowledge of trainee teachers. Just as there seem to some key concepts in numeracy that trainees need to be confident about so there are in geometry. This leads to the idea of subject knowledge for the teacher of geometry and how we are able to articulate what this means.

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BSRLM GEOMETRY WORKING GROUP

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions that could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

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