MINDING YOUR PS AND CS: SUBJECTING KNOWLEDGE TO THE PRACTICALITIES OF TEACHING GEOMETRY AND PROBABILITY

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The review of the implementation of the National Numeracy Strategy by Ofsted (Nov. 2003) has highlighted weak subject knowledge as a consistent feature in unsatisfactory teaching. This study looks at the subject knowledge of generalist primary trainees in the areas of geometry and probability and their ability to apply their knowledge to problem solving tasks. The study goes on the raise the question, 'is a profound understanding of fundamental mathematics (PUFM)(Ma, 1999) possible in generalist teachers?'

INTRODUCTION

This study looks at pre-service primary teachers' subject knowledge of geometry and probability, both procedural and conceptual, within the practicalities of teaching. It aims to identify the nature of the connections between subject confidence, competence and classroom practice in these areas of mathematics. The study considers areas of mathematics that trainees encounter less frequently and more periodically, requiring them to use and apply mathematics knowledge with which they may be less familiar. Both geometry and probability are topics that need to be delivered statutorily, but which tend to be taught in discrete units rather than as a continuous strand, this might have particular consequences for the nature of subject knowledge.

This study looks at the geometric and probability knowledge that the trainees identify and use. Building on the theoretical definitions of knowledge, it identifies evidence of how trainee teachers use their mathematics subject knowledge to underpin their teaching. It considers what knowledge they are using to inform their planning, teaching and assessment of mathematics. It compares their own subject matter knowledge confidence and competence with their pedagogical content knowledge.

METHODOLOGY

370 trainees across two HE institutions (PG and GTP routes) and one SCITT will be surveyed. The methodology comprises the following activities:

- Subject knowledge in geometry and probability is assessed;
- Confidence is self-audited and graded;
- The ability to apply this knowledge to problems solving tasks is assessed and coded;
- Lesson plans are analysed and the subject knowledge identified is encoded;
- Trainees are videoed teaching geometry and/or probability;

- Videos are analysed in order to identify and encode the subject knowledge that is demonstrated, both spontaneously and within planned situations;
- Quality grades are allocated to videoed lessons;
- Relationships between the variables and the impact on teacher effectiveness is identified.

MATHEMATICAL KNOWLEDGE

Within primary ITE it is fairly straightforward to develop and resource activities for trainees that allow them to develop and evidence their procedural understanding of mathematics. Undoubtedly they need to be able to apply mathematical procedures effectively in order to support their development as teachers of primary mathematics, however, although this procedural knowledge is clearly necessary it is not sufficient.

Eddie Gray's (1997) work with children looked at distinguishing between processes and procedures in mathematics and is equally applicable to ITE trainees learning as it is to other learners. Processes do not include any implication that they are carried out in a unique manner. For example, the 'process of addition' might use a mental method, might use counting on a number line or a formal written algorithm, but the method used is not implied within the process. Conversely 'procedure' describes a specific algorithm. Gray believes that learners who interpret processes only as procedures make mathematics harder for themselves. However, if they do not restrict their understanding of processes to procedures, then they are able to see processes as flexible precepts. The divergence between the two he refers to as the 'proceptual divide'. He argues that the difference between success and failure lies in the difference between the use of precepts and procedures.

It would be easy to assume that the mathematical knowledge necessary to teach effectively is a grasp of the content as flexible precepts. However it is now recognised as being a more complex issue than this. Shulman (1986) used the term *pedagogical content knowledge* (PCK) to represent a blend of content and 'ways of transforming that content in terms of its teachability'. In mathematics PCK includes forms of representation of concepts, useful analogies, examples, demonstrations, and so on that can help to make mathematical ideas comprehensible to others.

It could be argued that Shulman's model may be too simplistic, however it has proved useful in distinguishing the two relevant knowledge domains; *subject matter knowledge* (which includes key facts, concepts, principles, and explanatory frameworks of a discipline, as well as the rules of evidence used to guide inquiry in the field), and *pedagogical content knowledge* (which consists of an understanding of how to present specific topics in ways appropriate to the students being taught). Shulman's *pedagogical content knowledge* is further support by Cochran et al (1993) with their model of *pedagogical content knowing*. This phrase describes the knowledge of the subject matter, the learners and the environmental context of the learning.

A further extension of this has been proposed by Ma (1999), who proposed the term *profound understanding of fundamental mathematics* (PUFM). PUFM refers to the depth, breadth, and thoroughness of the knowledge that is required to be an accomplished teacher of primary mathematics. One example she explores is that of division of fractions. She suggests that a profound understanding of this concept results in a teacher not only being able to calculate an answer to a problem (1¾ divided by ½), but also being able to suggest story problems to represent the meaning. Being able to identify an appropriate context to model the different division structures was a clear indication of the depth of teachers' understanding. Ma suggested that teachers with PUFM make connections between mathematical concepts and procedures from the simple to the complex, appreciate different facets of an idea and various approaches to a solution, are particularly aware of the simple but powerful foundational concepts and principles of mathematics (such as equality), and are knowledgeable about the whole primary mathematics curriculum, not just the content of a particular age level.

This study considers Ma's research that compared specialist Chinese elementary teachers with generalist American elementary teachers. Whilst accepting that the teachers in China had not studied for such a long period, it could be argued that being a specialist should give rise to a PUFM more than for a generalist teacher. Is it therefore reasonable to seek a PUFM from generalist teachers? It would be very desirable, but is it feasible?

This study proposes therefore that as a minimum exit requirement, generalist primary trainees should be able to demonstrate a *Sufficient Understanding of Fundamental Mathematics* (SUFM).

SUFM

The proposal that trainees should possess a SUFM gives rise to the obvious question, 'What is sufficient?' It could be proposed that in light of the *Adding It Up: Helping Children Learn Mathematics* (2001) report from the USA, that the definition of mathematical proficiency for students contained there could be taken as mathematical sufficiency in trainees.

- *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *Procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence* – ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning* – capacity for logical thought, reflection, explanation and justification

- *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

(page 115)

As well as identifying these components of mathematical proficiency, the report also highlights how the quality of mathematics teaching is dependent on teachers application of both their knowledge of the mathematical content and their knowledge of the learners. This is very clearly reflected in Ofsted's findings in their 2003 report into the implementation of the National Numeracy Strategy,

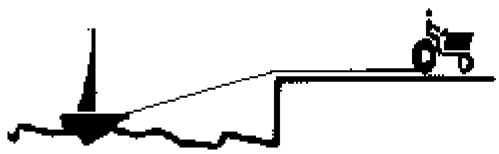
> Weak subject knowledge is a consistent common feature in unsatisfactory teaching, restricting teachers' ability to respond effectively to pupils' difficulties and to make connections with other learning. It also affects the quality of planning and assessment. (p.6)

PROBLEM SOLVING

This study is still in the early stages of implementation. Trainees geometric and probability knowledge has been tested, they have self-audited their confidence in these areas and have completed problem solving questions which have been coded.

Within geometry, trainees were asked the following two questions, taken from 1000 problems website:

1) Imagine you are on a jetty, and you are pulling in a boat that is floating on the water some way away. The rope comes up over the edge of the jetty, and lies along the jetty as you pull it in.

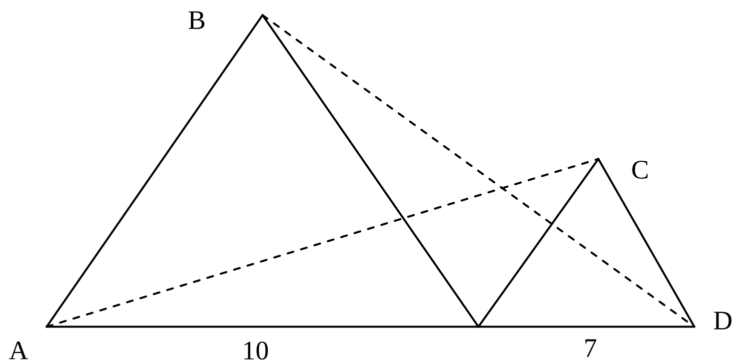


As you pull in 10 metres of rope the boat clearly moves in too, but does it move

a) exactly 10 metres b) more than 10 metres c) less than 10 metres

Explain your reasoning

2) Two equilateral triangles, of lengths 10 and 7 respectively, are drawn with their bases touching and in line.



Lines are drawn to connect the tops of each to the furthest corner of the other.

Explain clearly why these two lines, AC and BD are of equal length.

CONCLUSIONS

The data gathered to date leads to the following conclusions:

Question 1

- Trainees who score well on standard geometry tests do no better than trainees who score badly on standard geometry tests.
- Trainees who had high self-audit scores in geometry are no better than unconfident trainees.
- Trainees who score well on standard geometry tests tend to use Pythagoras' Theorem to attempt to solve the problem.

Question 2

- Trainees who score well on standard geometry tests do better than trainees who score badly on standard geometry tests.
- Trainees who score well on standard geometry tests tend to use Pythagoras' Theorem.
- Trainees who had high self-audit scores in geometry do significantly better than unconfident trainees.

These are conclusions drawn at an early stage in the study. The next stage involves sampling trainees' lesson plans for geometry and probability in order to identify the subject knowledge that is planned for. Then trainees will be videoed teaching geometry or probability in order to identify the subject knowledge that is underpinning their teaching, both that which has been planned for and also that which is used spontaneously in interactions with the learners. All of this data should support a more 'sufficient' definition of mathematical 'sufficiency'.

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