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A Quasi-Mode Interpretation of Radiation Modes in Long-Period Fiber Gratings

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ABSTRACT

We have found an efficient approximation based on a quasi-mode method to analyze radiation-mode coupling in long-period fiber grating devices, and which was deduced by consideration of the best confinement condition of radiation-mode spectra inside an optical fiber. A single-mode representation via the quasi-mode method enables one to solve the radiation-mode-coupling problems analytically, which is the advantage of the method. The numerical accuracy is found to quite acceptable in comparison with the conventional method of the Green-function representation of radiation-modes via numerical computation of the integral. Utilizing the proposed method, we present several numerical examples for radiation-mode coupling problems both in uniform and non-uniform LPFGs.

INDEX TERMS

Optical fiber, mode theory, radiation mode, mode coupling, fiber grating

I. INTRODUCTION

Optical fiber communication systems demand the use of passive devices of even more flexible and controllable characteristics. Thus, the development of reliable passive devices which can be electrically controlled or tuned has been an important on-going issue in recent years [1-6]. Attempts to develop controllable long-period fiber grating (LPFG) devices are of particular relation to this category [1-5]. LPFGs are useful for broadband rejecting or comb filtering devices in optical fiber communication systems [7]. Frequently, they are also used as fiber-optic sensors to measure physical parameters such as temperature, tension, chemical composition, etc. [8]. The characteristics of an LPFG are defined by the periodic mode coupling between a fundamental core mode and a co-propagating cladding mode. Normally, the surrounding material (the outer cladding) of an LPFG has a lower refractive index than that of the inner cladding. Thus, inner cladding modes are maintained by total internal reflection (TIR) at the boundary of the inner/outer cladding. However, in many attempts so far to realize controllable LPFG devices, the active materials (such as liquid crystals and electro-optic polymers [9-11]) used as the surrounding outer cladding material have had a higher refractive index than that of silica. With a higher surrounding index, the inner cladding modes experience Fresnel reflection (FR) rather than TIR at the inner/outer cladding boundary, and are thus no longer guided [12]. However, if the refractive index difference between the claddings is made sufficiently high ($>10^{-2}$), the strength of the FR increases enough to support and guide cladding modes over a distance of a few centimetres. Thus, an LPFG structure with an outer cladding of a higher-refractive index is still of great interest [4], [5], [9-11].

Modes supported by FR are usually expressed in terms of radiation modes, because of their outward radiating nature [12]. Sometimes, they are also regarded as leaky modes, from the viewpoint of the loss of the power propagation via outward radiation from the fiber [9], [12]. A simplified approach to describe leaky cladding-mode propagation in an LPFG has previously been developed [9], in which the leaky modes in the fiber were determined by means of a ray model in a one-dimensional waveguide. The propagation losses of the leaky modes were approximated by assuming an effective reflectivity of the waveguide walls. This is a simple and time-efficient method. However, it is based on a simplified one-dimensional ray model, and hence, it is restricted in terms of determining accurate coupling or propagation constants for the leaky cladding modes.

Recently, a numerical approach to the analysis of radiation-mode coupling in an LPFG has been presented [10], [11], in which the whole radiation modes were represented in a Green's function form. The complete set of coupled equations associated with the coupling between a fundamental mode and a co-propagating cladding radiation mode was solved numerically. This approach, although capable of accurate results, requires successive numerical integrations to evaluate the Green's function and which is computationally intensive and thus not very efficient. An approximated approach using perturbation theory was also provided by the author; however, it was restricted to a very simple case. Although the numerical approach gives good accuracy, it is depended on a suitable choice of the numerical integration routine and has drawbacks, such as difficulty in obtaining physical insight and calculation time. Thus, a simplified but reliable approach is necessary to compensate the drawbacks of the numerical method.

The analysis of radiation-mode coupling has been being discussed for a long time [12]. While the most approaches have been based on the numerical integration method using the Green's function representation, and there has been little effort to obtain a reliable approximation to the radiation-mode representation in order to achieve improved computation times. Here we propose an alternative approach to the radiation-mode coupling method and which can be called an approximation by a quasi-mode interpretation of radiation modes. We deduce a set of radiation modes that allow the highest probability of quasi-guidance inside a fiber, and from which we readily obtain its effective propagation constant and effective loss. We regard this leaky modal set as a quasi-mode. As a consequence, the whole set of radiation modes that should be represented by an integral form of infinite Green's functions is reduced to a single quasi-mode. This allows an analytical solution to the mode-coupling problem. Thus, intensive numerical integration is no longer required. A detailed discussion of this approach and several numerical examples analyzed by the proposed method follow this section. (It might prove helpful to reader to refer to the Appendices prior to the next section.)

II. RADIATION-MODE CHARACTERISTICS AND ITS QUASI-MODE INTERPRETATION

If the outer cladding of a fiber has a higher refractive index than that of silica, as depicted in Fig. 1, the inner cladding modes can be expressed in terms of radiation modes that are supported by FR rather than TIR at the inner/outer cladding boundary [10-12]. (At the moment, we do not discuss the fundamental mode that exists in the core region.) In fact, the entire power spectra by the radiation modes can be represented by a continuum [12]. From a theoretical point of view, the total power flow into the entire space should be conserved; however, if we restrict our consideration to the power flow close to the fiber, the flux inside the section of the fiber considered tends to decrease with the propagation length. This is because the light waves lose their power towards free space as they bounce off the inner/outer cladding boundary. Thus, they would be regarded as leaky modes [12]. Even though radiation-mode spectra should be represented by a continuum, an individual spectrum varies with its modal characteristics. That is, if we assume that every component of a radiation-mode set carries the same power, namely, 1 W in entire space, the power density concentrated inside the fiber is dependent on the individual modal characteristics. For some radiation modes, the confinement to the fiber is well supported in comparison with others. This provides a clue to using a quasi-mode representation of the radiation mode.

In the regime of a quasi-mode representation, the best confinement condition is regarded as the modal condition that leads to the most constructive interference inside the fiber. For example, the coefficient A_r^2 for the first-kind Bessel function that expresses the electric- or magnetic-field strength in the core region is shown in Fig. 2 (see Appendix A for the radiation-mode representation). The positions of effective indices for guided cladding-modes with an outer cladding of air are also indicated by reversed triangles. There are several interesting characteristics. First, the coefficient has repeated peaks in terms of the effective index (i.e., the propagation constant). This feature hints it might be possible to consider the propagation condition that allows a localized peak of the coefficient, as a quasi-mode condition, because the peak coefficient leads to the best local confinement inside the fiber. (The power flow inside the fiber increases with the strength of A_r^2.) The repeating peaks are nearly coincident with the guided-mode behaviour, as shown in the example. Thus, the peak position in terms of the effective index can be regarded as a quasi-modal index. The second thing to note is that the width of the local lobes might be related to the behaviour of the quasi-mode through the propagation, since it intuitively looks like a mode-spectral width. Actually, the width is dependent on the difference of the refractive indices between the inner and outer claddings (Δn_{23}) when we consider a single lobe placed at a certain position. (The spectral widths of the lobes also vary with the quasi-modal order.) It tends to increase as the index difference Δn_{23} decreases. One can then assume that the modal behaviour of the radiation-mode spectra inside a single lobe is represented by two parameters, the center propagation constant and the spectral width. With the aid of this concept, we separate the continuum of the radiation-mode spectra into individual quasi-mode sets that include a single guided mode like peak. This is a heuristic introduction to the quasi-mode interpretation of radiation modes. In the following, the detailed mathematical derivation of this approach will be discussed.

Let us assume that a continuum of radiation-mode power spectra can be separated into intervals which contain an individual localized peak. The continuum within the interval is arbitrary at the moment. We assume that one of the propagation constants for the peaks of the localized spectra is given by β_{r0}. The variation of the propagation constant of the radiation mode will be analyzed by the amount that deviates from the principal value of β_{r0}. Following the derivation of radiation modes [12], one can represent a radiation-mode set by a closed form

$$\begin{pmatrix} \boldsymbol{E}_r \\ \boldsymbol{H}_r \end{pmatrix} = \begin{pmatrix} \int u_r(\gamma)\boldsymbol{e}_r(x,y,\gamma)\exp(-i\gamma z)d\gamma \\ \int u_r(\gamma)\boldsymbol{h}_r(x,y,\gamma)\exp(-i\gamma z)d\gamma \end{pmatrix} \cdot \exp(-i\beta_{r0}z), \qquad (1)$$

where $\boldsymbol{e}_r(x,y,\gamma)$ and $\boldsymbol{h}_r(x,y,\gamma)$ are normalized electric- and magnetic-field vectors that lead to a power flow of 1 W in the z-direction, and γ is the amount that the propagation constant deviates from β_{r0}. In addition, $u_r(\gamma)$ is the complex-field amplitude for each radiation mode and remains constant for a fixed γ during the propagation assuming a uniform medium. From the orthogonality relation between the radiation modes [12], one can obtain the whole power flow carried by the radiation-mode set through all space and we can assume it to be 1 W, as follows (see Appendix A and refer to [12]):

$$P_{r,\infty} = \int |u_r(\gamma)|^2 d\gamma \equiv 1, \qquad (2)$$

which is derived by doing the surface integral of the Poynting vector on the entire surface transverse to the z-direction. The radiation modes seem that they never lose their power during their propagation in space, i.e., the power flow is independent of z. However, to an observer who stays inside the fiber, it would seem that a proportion of them gradually escape from the waveguide and never return to it, because the light wave loses power as it suffers FR at the inner/outer cladding boundary. In other words, they can be regarded as leaky modes, if we restrict the area of the power flow to a finite dimension of interest [9], [12] which is close to the idea we experience in practice. Thus, an alternative leaky quasi-mode representation is also possible.

Then, the power flow inside a finite area S_0 carried by the radiation modes at $z = 0$ is given by

$$\begin{aligned} P_{r,S_0}(0) &= \frac{1}{2}\mathrm{Re}\left\{\int_{S_0} \boldsymbol{E}_r \times \boldsymbol{H}_r^* \cdot \hat{z}\, dxdy\right\}\Bigg|_{z=0} \\ &= \mathrm{Re}\left\{ \int u_r(\gamma)u_r^*(\gamma')\frac{1}{2}\int_{S_0} \boldsymbol{e}_r(x,y,\gamma)\times \boldsymbol{h}_r^*(x,y,\gamma')\cdot \hat{z}\, dxdy\, d\gamma d\gamma' \right\}, \end{aligned} \tag{3}$$

where 'Re' denotes the real part of the argument. The surface integral including the normalized Poynting vector would lead to a Dirac delta function, if one took the area S_0 as the infinite space (see Appendix A); however, in the case in which we consider the integral inside a finite area, for example, inside the core, it depends on the products of the normalized $\boldsymbol{e}_r(x,y,\gamma)$ and $\boldsymbol{h}_r(x,y,\gamma)$, both of which are proportional to the coefficient A_r. Thus, the integral value on the right-hand side of (3) can approximately be expressed in terms of the product of the coefficients A_r's, i.e.,

$$\frac{1}{2}\int_{S_0} \boldsymbol{e}_r(x,y,\gamma)\times \boldsymbol{h}_r^*(x,y,\gamma')\cdot \hat{z}\, dxdy \sim F(\gamma)F^*(\gamma'), \tag{4}$$

where $F(\gamma)$ is a normalized function for $A_r(\gamma)$ that leads to unity at $\gamma = 0$, i.e., at the peak position of $A_r(\gamma)$. A numerical example is shown in Fig. 3, in which the exact value of the integral on the left-hand side of (4) and its approximated function on the right-hand side of (4) are shown at the same time. The approximation is in good agreement with the exact value of the integral if Δn_{23} is sufficiently large (>10^{-2}). With the aid of the approximation (4), we obtain the power flow inside the finite area S_0 carried by the radiation modes as

$$\begin{aligned} P_{r,S_0}(0) &\approx q_0 \int u_r(\gamma)u_r^*(\gamma')F(\gamma)F^*(\gamma')d\gamma d\gamma' \\ &= q_0\left|\int u_r(\gamma)F(\gamma)d\gamma\right|^2, \end{aligned} \tag{5}$$

where q_0 is a constant of proportionality. Actually, $u_r(\gamma)$, which is not determined as yet, is an arbitrary function based on the variety of radiation-mode spectra, since we assume that the continuum of the radiation-mode spectra is arbitrary from the beginning of this section. However, $u_r(\gamma)$ can be determined so that it should lead to maximum power flow inside S_0, i.e., allows the best confinement of the radiation-mode set inside the fiber, which leads to the quasi-mode condition. To obtain the maximum value of (5), the complex amplitude function $u_r(\gamma)$ should be proportional to the complex conjugate of $F(\gamma)$, according to the well-known Cauchy-Schwarz inequality (see Appendix B), i.e.,

$$u_r(\gamma) = u_{r0}F^*(\gamma), \tag{6}$$

where u_{r0} is a proportionality constant. Thus, the maximum initial power flow inside S_0 becomes

$$P_{r,S_0}(0) \equiv q_0 |u_{r0}|^2 \left| \int |F(\gamma)|^2 \, d\gamma \right|^2. \tag{7}$$

Similarly, one can derive the power flow through the *z*-direction, just by adding the complex phase factor of an exponential function of *z*, as follows:

$$P_{r,S_0}(z) \equiv q_0 |u_{r0}|^2 \left| \int |F(\gamma)|^2 \exp(-i\gamma z) \, d\gamma \right|^2. \tag{8}$$

Finally, we arrive at the power attenuation relation of

$$\frac{P_{r,S_0}(z)}{P_{r,S_0}(0)} = \frac{\left| \int |F(\gamma)|^2 \exp(-i\gamma z) \, d\gamma \right|^2}{\left| \int |F(\gamma)|^2 \, d\gamma \right|^2}. \tag{9}$$

At this moment, let us discuss the features of the function $|F(\gamma)|^2$. It follows a well-defined Lorentzian shape if the difference in the refractive indices between the inner and outer claddings, i.e., Δn_{23}, is sufficiently large, as shown in Fig. 4. Thus, it is also possible to approximate it to

$$|F(\gamma)|^2 \approx \frac{1}{1 + (\gamma/\alpha)^2}, \tag{11}$$

where α is the half width at the half maximum of $|F(\gamma)|^2$. (One can also expect the Lorentzian-like shape without the graphical assistance of Fig. 4, since the Fourier transform of a Lorentzian function leads to an exponentially decaying function [12]. If the mode were a well-defined leaky mode, an exponential attenuation of the power should be expected.) Substituting (11) into (9), the power attenuation relation is simplified to

$$\frac{P_{r,S_0}(z)}{P_{r,S_0}(0)} = \exp(-2\alpha z), \tag{12}$$

on condition of $\alpha z \geq 0$. The power decays exponentially with respect to the propagation length. The radiation-mode set that has the highest probability of existence is like an exponentially decaying mode. In other words, it can be represented by a leaky quasi-mode that has its fundamental propagation constant of β_{r0} and an attenuation constant of α. As was introduced at the beginning of this section, all information concerning the radiation modes inside a certain spectral interval is defined by its peak position and width, and which enables one to avoid the time-consuming calculation of all radiation modes.

Following this idea, we introduce a quasi-mode in place of the radiation mode as follows:

$$\begin{aligned} \boldsymbol{E}_q &\equiv \int u_r(\gamma)\boldsymbol{e}_r(x,y,\gamma)d\gamma \cdot \exp(-i\beta_{r0}z - \alpha z) \\ &\equiv \boldsymbol{e}_q(x,y)\exp(-i\beta_q z) \end{aligned} \tag{13}$$

where $\boldsymbol{e}_q(x,y)$ is a newly defined normalized electric-field vector that leads to 1-W power flow at $z = 0$. It is noteworthy that the z-dependence of the field function is extracted from the integral. From power normalization, one can obtain the coefficient for u_{r0} of (6) as

$$|u_{r0}| = \left(\frac{1}{\pi\alpha}\right)^{\frac{1}{2}}. \tag{14}$$

In addition, one can determine the quasi-mode by finding the propagation constant which leads to the localized maximum value of A_r^2 and its half, i.e., β_{r0} and α, in an appropriate interval of the spectra. The characteristic equation that leads to β_{r0} is given by the consideration that the derivative of A_r^2 vanishes, i.e.,

$$\frac{dA_{\gamma}^{2}}{d\beta_{r}}=0, \tag{15}$$

or

$$\frac{d}{d\beta_{r}}(C_{r1}^{2}+C_{r2}^{2})=0. \tag{16}$$

where the parameters can be found in Appendix A.

As a numerical example, the loss constant α is shown in Fig. 5 for several lowest order modes of the quasi-cladding modes, in which the axes are shown by logarithmic scales. One can see that the trend of variation of α is quite linear with respect to the inner/outer cladding index difference, Δn_{23}, when they are shown on logarithmic scales. From Fig. 5(a), $\log\alpha$ is proportional to $-0.5\cdot\log\Delta n_{23}$ approximately. In addition, the pivot points that are enclosed by an ellipse of inset of Fig. 5(a) are also plotted in terms of the mode order in Fig. 5(b). Similarly, one can find out that $\log\alpha$ is proportional to $2\cdot\log m$, where m is the mode ordering number of a quasi-cladding mode $Q-LP_{0m}^{(cl)}$. Thus, one can obtain an interesting empirical relation for α as

$$\alpha\approx g_{0}\frac{m^{2}}{\sqrt{\Delta n_{23}}}, \tag{17}$$

where g_0 is a proportional constant. One can compare this to a numerical example that was carried out by a leaky-mode approach in a one-dimensional waveguide [12].

III. NUMERICAL EXAMPLES OF QUASI-CLADDING-MODE INTERPRETATION: RADIATION-MODE COUPLING IN LPFGS

In this section we analyze radiation-mode couplings in LPFGs by utilizing the quasi-cladding-mode method derived in the previous section. We will try both methods, i.e., the numerical method following Ref. [11] and the proposed quasi-mode method. We will compare results each other for several examples of LPFGs. In the case of the numerical method, we use the fourth-order Runge-Kutta method aided with a numerical integration.

We assume that an induced index change, i.e., a permittivity change in the core is described by

$$\Delta\varepsilon = \Delta\varepsilon_\sigma + \Delta\varepsilon_\xi \cos(\frac{2\pi}{\Lambda} z), \tag{18}$$

where $\Delta\varepsilon_\sigma$ the dc part of the permittivity change that can be eliminated by adding it to the initial permittivity of the core, $\Delta\varepsilon_\xi$ is the amplitude of the alternating part, and Λ is the nominal period. With the aid of the quasi-cladding-mode representation of radiation modes, one can take the perturbed field as

$$\boldsymbol{E}' = a_f(z)\boldsymbol{e}_f(x,y)\exp(-i\beta_f z) + a_q(z)\boldsymbol{e}_q(x,y)\exp(-i\beta_q z), \tag{17}$$

where $a_f(z)$ and $a_q(z)$ are the slowly varying amplitudes of the modal fields for a fundamental core mode and a co-propagating quasi-cladding mode. The other parameters are the same as shown in the previous section. According to the coupled-mode theory for the coupling between the fundamental core mode and the quasi-cladding mode [8], one can obtain the final expression of the coupled equations as

$$\frac{da_f(z)}{dz} = -i\kappa_\xi a_q(z)\exp(+i\Delta\beta z), \tag{18}$$

$$\frac{da_q(z)}{dz} = -i\kappa_\xi^* a_f(z)\exp(-i\Delta\beta z), \tag{19}$$

where

$$\Delta\beta = \beta_f - \beta_q - \frac{2\pi}{\Lambda}, \tag{20}$$

$$\kappa_\xi = \frac{\omega}{4}\int \boldsymbol{e}_q^*(x,y)\cdot\Delta\varepsilon_\xi \boldsymbol{e}_f(x,y)dxdy. \tag{21}$$

To calculate (21), one can also utilize an approximation rule related to $F(\gamma)$. As was discussed in (4) and Fig. 3, the approximation also holds for the overlap integral of (21) inside the core between the quasi-cladding-mode and the fixed fundamental mode, i.e.,

$$\int e_r^*(x,y,\gamma)\cdot\Delta\varepsilon_\xi e_f(x,y)dxdy \sim F(\gamma). \tag{22}$$

Thus, substituting (22) into (21), one can obtain a simplified form of

$$\kappa_\xi = \frac{\omega\sqrt{\pi\alpha}}{4}\int e_r^*(x,y,0)\cdot\Delta\varepsilon_\xi e_f(x,y)dxdy, \tag{23}$$

with which one needs to calculate the overlap integral only for the case of $\gamma = 0$, i.e., for the case of β_{r0}.

The next steps to get a final solution are so straightforward that we skip them (these can be found in Ref. [8]). For the case of a non-uniform grating analysis, one can also utilize the transfer matrix method or the discretization method [8], [13], [14].

The numerical parameters of the LPFGs considered in the following are summarized at Table 1. An example of a uniform LPFG is shown in Fig. 6, in which (a) is by the numerical method and (b) by the quasi-mode method, respectively. We consider the transmission spectra on varying the inner/outer cladding index differences. The resonant mode coupling with the given grating period occurs between the fundamental core mode $LP_{01}^{(co)}$ and the quasi-cladding mode $Q\text{-}LP_{04}$. Both results are in a good agreement. Even though there is small difference in the fringe spectra, it seems negligible inside the spectral region of interest.

Furthermore we consider a non-uniform LPFG, i.e., an apodized grating as shown in Fig. 7, in which the apodizing function is assumed as a symmetric Gaussian one. At the edges of the grating, the induced index modulation depth is reduced to 25 % of the maximum value. The other conditions are the same as in Fig. 6. In this calculation we utilize the transfer matrix method to analyze the non-uniformity [8], [13]. The results by the quasi-mode method are still in good agreement with those by the numerical method. From the viewpoint of numerical accuracy, the validity of the quasi-mode model is quite acceptable, and the calculation time is much shorter.

In fact, the accuracy of the quasi-mode method is dependent on the accuracy of the Lorentzian fitting to the spectral amplitude $u_r(\gamma)$ (see (11)). As the refractive index of the surrounding material decreases or the mode order increases, the localized lobe of A_r^2 will be broadened, as shown in Fig. 1. As the width of the lobe increases, the Lorentzian fitting curve gives rise to some errors in the wings. This may lead to an error in the analysis. However, as verified in the numerical examples, the numerical accuracy of the quasi-mode model is quite acceptable for the case of the normal condition, namely, $\Delta n_{23} > 10^{-4}$.

IV. CONCLUSION

We have derived a quasi-mode method to analyze the radiation-mode couplings in LPFG devices. Deducing a quasi-mode condition on which the confinement of radiation-mode spectra is maximized, we found a quasi-mode model suitable to describe the radiation-mode coupling. The detailed mathematics to obtain the effective propagation constant and attenuation constant was also discussed. Because the quasi-mode method leads to a single-mode representation rather than an integral form of radiation modes, it enables one to solve the mode-coupling problems analytically. Utilizing the derived quasi-mode method, we presented several numerical examples for the radiation-mode coupling problems both in uniform and non-uniform LPFGs. We found that the numerical accuracy provided by the quasi-mode method was quite acceptable in comparison with that obtained by the numerical method. The quasi-mode method is not restricted to the analysis of LFFGs. This can be applied to general problems of radiation-mode coupling, such as radiation-mode couplings in short-period fiber gratings, bend-induced mode couplings, etc. We expect this method will prove to be an efficient way to obtain the analysis of radiation-mode coupling in optical fibers.

APPENDIX A

Using the weakly guiding mode approximation [12], one can obtain the electric field of a linearly polarized radiation mode as follows:

$$E_r = A_r U_l(r,\phi)\exp(-i\beta_r z) \tag{A1}$$

with

$$U_l(r,\phi) = \cos(l\phi)\begin{cases} J_l(\sigma r) & r \le a \\ B_{r1}J_l(\rho r) + B_{r2}Y_l(\rho r) & a < r \le b \\ C_{r1}J_l(\eta r) + C_{r2}Y_l(\eta r) & b < r \end{cases} \tag{A2}$$

$$B_{r1} = \frac{\pi a}{2}\{\sigma J_{l+1}(\sigma a)Y_l(\sigma a) - \rho J_l(\sigma a)Y_{l+1}(\sigma a)\}, \tag{A3}$$

$$B_{r1} = -\frac{\pi a}{2}\{\sigma J_{l+1}(\sigma a)J_l(\sigma a) - \rho J_l(\sigma a)J_{l+1}(\sigma a)\}, \tag{A4}$$

$$C_{r1} = \frac{\pi b}{2}\{\rho Q_{l+1}(\rho a)Y_l(\eta a) - \eta Q_l(\rho a)Y_{l+1}(\eta a)\}, \tag{A5}$$

$$C_{r2} = -\frac{\pi b}{2}\{\rho Q_{l+1}(\rho a)J_l(\eta a) - \eta Q_l(\rho a)J_{l+1}(\eta a)\}, \tag{A6}$$

$$Q_l(\rho r) \equiv B_{r1}J_l(\rho r) + B_{r2}Y_l(\rho r), \tag{A7}$$

$$\sigma^2 = k_0^2 n_1^2 - \beta_r^2, \tag{A8}$$

$$\rho^2 = k_0^2 n_2^2 - \beta_r^2, \tag{A9}$$

$$\eta^2 = k_0^2 n_3^2 - \beta_r^2, \tag{A10}$$

where k_0 is the propagation constant in vacuum, a and b are the radii of the core and the cladding, n_1, n_2, and, n_3 are the refractive indices of the core, the inner cladding, and the outer cladding, and β_r is the effective propagation constant in the z direction. Similar expressions can be found in Refs. [11] and [12].

If one assumes that $\boldsymbol{e}_r(x,y,\beta_r)$ and $\boldsymbol{h}_r(x,y,\beta_r)$ are the normalized electric- and magnetic-field vectors that lead to a power flow of 1 W in the z-direction, the following orthogonal relation is approximately valid [12]

$$\frac{1}{2}\mathrm{Re}\left\{\int_{\infty} \boldsymbol{e}_r(x,y,\beta_r)\times \boldsymbol{h}_r^*(x,y,\beta_r')\cdot \hat{z}\, dxdy\right\} = \delta(\beta_r - \beta_r'). \tag{A11}$$

Thus, one can obtain the field amplitude of the normalized electric field through (A1) and (A11) as follows:

$$A_r^2 = \frac{2k_0\sqrt{\mu_0/\varepsilon_0}}{v_l \pi (C_{r1}^2 + C_{r2}^2)} \tag{A12}$$

with

$$v_l = \begin{cases} 2 & \text{for } l = 0 \\ 1 & \text{for } l \neq 0 \end{cases} \tag{A13}$$

and ε_0 and μ_0 are the permittivity and the permeability in vacuum. It is noteworthy that the appearances of (A11) and (A12) are different from those in Refs. [11], [12], because we express the radiation mode field in terms of a different notation. We are representing the cladding fields with the first and second kind of Bessel functions rather than with the Hankel functions, and are using the integration variable of β_r to obtain the orthogonal relation of (A11).

Note that A_r is a function of β_r, i.e., C_{r1} and C_{r2} vary with β_r though (A5)-(A10). In addition, we also use the following notation to express A_r as

$$A_r(\gamma) \equiv A_{r0}F(\gamma), \tag{A14}$$

where γ is the amount of the propagation constant deviated from β_{r0} that is a center propagation constant, and $F(\gamma)$ is a normalized function that leads to unity at $\gamma = 0$, i.e., at the peak position of $A_r(\gamma)$.

Appendix B

The Cauchy-Schwarz inequality for integrals is given by [15]

$$\left|\int f(x)g(x)dx\right|^2 \leq \int\left|f(x)\right|^2 dx \int\left|g(x)\right|^2 dx. \tag{B1}$$

The equality holds if and only if $f(x) = c_f g^*(x)$ where c_f is a proportional constant.

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