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THE DISTRIBUTION OF THE M_2 OCEAN TIDE IN THE VICINITY OF THE BAY OF FUNDY – FROM A GLOBAL NUMERICAL MODEL BY W. ZAHEL

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NATURAL ENVIRONMENT RESEARCH COUNCIL

INSTITUTE OF OCEANOGRAPHIC SCIENCES

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Institute of Oceanographic Sciences
Bidston Observatory
Birkenhead
Merseyside L43 7RA

ABSTRACT

Using the results of a global numerical tidal model by W. Zahel, co-phase and co-amplitude charts for the M_2 tide are drawn for a region of shelf and ocean lying off the eastern coasts of the United States and Canada in the general vicinity of the Bay of Fundy.

Tidal values from the model are compared with corresponding values derived from coastal measurements. It is found that there are significant discrepancies between theory and observation which are attributed to the coarseness of the grid network of the model.

Starting from the tidal charts obtained here, a new numerical modelling programme is proposed to determine the M_2 tides in the Bay of Fundy and the effects of cross-channel barriers on those tides. The construction of a large numerical model of the Bay of Fundy region, including shelf and ocean areas, is suggested. Ultimately, it is envisaged that this model would be patched into a global tide calculation.

CONTENTS

INTRODUCTION

Dr. W. Zahel, of the Institut für Meereskunde in the University of Hamburg, has kindly sent us values of the M_2 tide, derived from his 1^o global numerical model (ZAHEL 1976), which refer to a region of shelf and ocean lying off the eastern coasts of the United States and Canada in the general vicinity of the Bay of Fundy. On the basis of these values, we have drawn co-phase and co-amplitude charts for M_2 in this region (figures 1 and 2).

The charts are of interest in connection with the possible future construction of a new detailed numerical model of the M_2 tide in the Bay of Fundy, incorporating a wide area of the neighbouring shelf and ocean. GREENBERG (1975) has formulated a tidal model of the Bay including the Gulf of Maine, extending seawards to the edge of the continental shelf. We now envisage a considerably larger model than his, including much of the ocean region mapped out in figures 1 and 2. The information in these figures might provide first estimates of the tides to be applied along the open boundaries of the future model. Subsequently, the procedure would be to adjust the boundary tides progressively, through a series of numerical experiments, until the model finally gave results in satisfactory agreement with all relevant onshore and possibly offshore tidal observations. This kind of optimisation technique was employed by Greenberg in the work already quoted.

Having thus set up and verified an M_2 tide model of the Bay of Fundy, the Gulf of Maine and a large section of the neighbouring shelf and ocean, the next step would be to patch this system into a global M_2 tide calculation. Tides in the Bay of Fundy would then be derived numerically as a response to the tide-generating forces. In such circumstances, studies of barrier effects on the resonant M_2 tidal regime in the Bay would not involve the uncertainties, encountered when using models of limited geographical extent, in prescribing open-boundary tidal conditions capable of allowing for barrier influence (HEAPS and GREENBERG 1974).

Further to drawing the tidal charts for M_2, with the intention of providing some check on their accuracy we have, for the region of the Bay of Fundy under consideration, compared appropriate amplitudes and phases from the 1^o global model with corresponding amplitudes and phases derived from coastal tide gauge measurements. These comparisons show significant discrepancies between theory and observation which have to be expected in view of the coarseness of the grid network of the model and its poor resolution of the coastal shelf areas. Notwithstanding this limitation, the tidal distributions presented here form a useful starting point for the proposed new modelling programme outlined above. Combined with possible future deep-sea tidal measurements, the distributions might well be improved and thereby made more relevant to the purposes on hand.

THE CO-PHASE AND CO-AMPLITUDE CHARTS

Surface elevation is given by

$$\zeta = \zeta_1 \cos \sigma t + \zeta_2 \sin \sigma t$$

where σ denotes the speed of M_2 and t the time measured from the Moon's passage through Greenwich. Values of both ζ_1 and ζ_2 were available from Dr. Zahel for points on a grid formed by latitudes at intervals of 1° between 35.5° and 90.0° N and longitudes at intervals of 1° between 56° and 88° W. Phase G and amplitude H, such that

$$\zeta = H \cos(\sigma t - G)$$

were calculated for each grid point using

$$G = \tan^{-1}(\zeta_2/\zeta_1) \quad , \quad H = (\zeta_1^2 + \zeta_2^2)^{1/2} \quad .$$

Contours of G and H, constructed on the basis of linear interpolation between grid-point values, are shown in figures 1 and 2 respectively. The land boundary of Zahel's model is marked out by a thick line.

The main general feature of the co-phase chart (figure 1) is a region of low phase, i.e. early high water, centred off the coast of Nova Scotia. From here there is a progressive increase in phase on passing into the ocean area and the phase lines furthermost out presumably originate from the central Atlantic amphidrome. Thus, the high water moving outwards from Nova Scotia joins with the high water of the North Atlantic system, the latter rotating anticlockwise, sweeping southwards off the eastern seaboard of North America. According to co-tidal charts derived from coastal observations (FARQUHARSON 1962) the amphidrome in the Gulf of St. Lawrence is not degenerate as in figure 1 but is located within the Gulf. Hence, in the region of the Gulf, including the entrance area between Cape Breton Island and Newfoundland, the contour pattern in figure 1 cannot be regarded as anything more than very approximate, giving only a general idea of the tidal propagation. Indeed one may anticipate an eventual revision of the contours there. Further south, the phase lines show high water travelling southwards, and westwards on to the coast of the United States. On the continental shelf the lines are closer together, showing a slower movement of the tidal wave towards the coast in the shallower water. The Bay of Fundy is very poorly represented in the model and the phase values there do not reach anywhere near their highest observed values (DOHLER 1966).

The co-amplitude chart (figure 2) indicates, over a wide area, a rising amplitude in moving shorewards from the ocean on to the shelf. However, the highest values of H in the Gulf of Maine and the Bay of Fundy area fall well below those attained in reality, i.e. the full build-up of the tides in that area is far from achieved in the model. The co-amplitude contours in the Gulf of St. Lawrence compare quite favourably with those derived from coastal observations. Figure 4 shows the variation in amplitude, also phase, along the section AB from shelf to ocean drawn in figure 3. Noticeably, the rate of change of amplitude with distance measured along this line is greater on the shelf than in the ocean area; the minimum of phase on the line corresponds to the leading edge of the tongue of tidal high water which spreads to the south-west from the Nova Scotian shelf (see figure 1).

Figure 5 shows the locations of coastal tide gauge stations, numbered 1 to 35; also depth contours 100, 1000 and 2000 fathoms. A list of the stations, with corresponding values of amplitude H and phase G , derived from observations at each station, is given in table 1. For each station, table 1 also gives the H and G values taken from the nearest grid point at Zahel's model. Figure 6 compares these amplitude and phase values as obtained, on the one hand, from observations and, on the other hand, from the model. Observed and model amplitudes agree satisfactorily : apart from at stations 10 to 18, lying along the Gulf of Maine - Bay of Fundy coastline, where Zahel's values are much smaller than those observed - as expected from his model's poor representation of the coastal configuration of this region. Observed and model phases are in fairly close agreement between stations 27 to 35 at the entrance to the Gulf of St. Lawrence, but elsewhere there are significant discrepancies. Thus, the model tides are in advance of the observed by an average of about 50° at stations 1 to 9, by about 140° at stations 10 to 15, and by about 40° at stations 19 to 26. Phases from the model lie between 300° and 360°, and several other global tide calculations appear to give this same result for the section of North American coastal area under consideration (HENDERSHOTT 1973).

Figures 7 and 8 compare contours of phase and amplitude in the Gulf of Maine - Bay of Fundy region as derived, on the one hand, by GREENBERG (1975) and, on the other hand, from the present values provided by Zahel. Greenberg's results, which are expected to reflect fairly accurately the real situation - since they come from a detailed adjusted tidal model of the region, show considerably greater variation in both phase and amplitude. It seems clear that the network of the global model is far too coarse for that model to give a full reproduction of the actual phase and amplitude changes within this area.

CONCLUDING REMARKS

The diagrams presented in this paper are intended to give a first estimate of the nature of the M_2 ocean tide distribution in the neighbourhood of the Bay of Fundy. The preparation of the diagrams was motivated by interest in knowing how the tidal resonance in the Bay, associated with M_2, develops from the oceanic oscillation outside. Thus, the basic problem is one of determining the transition of a tidal regime from deep-sea to coastal waters, a matter already considered for the area off the California coast by MUNK, SNODGRASS and WIMBUSH (1970). In the case of the Bay of Fundy, the interest in the problem is heightened by proposals for the construction of barriers, for tidal power schemes, in the headwaters. The modification of the tides in the Bay by such barriers may be affected significantly by a coupling between the tidal motion on the shelf and that in the ocean, and therefore tidal information is required over a wide area well beyond the confines of the Bay itself. Just how wide an area should be considered is not known at present, but it has to be recognised that, ultimately, a completely satisfactory result may only be achieved by considering the entire world oceans. This may be regarded as a puristic point of view.

It must be emphasised that we have illustrated Dr. Zahel's results, not our own. We are grateful to him for making them available to us. Our contribution is restricted to a graphical interpretation of numerical values coming from his global tide model, comments on the diagrams thus obtained, and a discussion of the diagrams in relation to the problem of tidal barriers in the Bay of Fundy. Results from the 1^{o} global model may be expected to indicate the broad features of the tides in the shelf-ocean region connecting with the Bay, but the mesh employed is certainly too coarse to account for the important changes in amplitude and phase which occur in the Bay itself. The failure to reproduce the full resonance in the Bay might significantly affect the energy flux between shelf and ocean and the cotidal pattern in the ocean as well as on the shelf (PROUDMAN 1941, WEBB 1976). For this reason, at least, there is some uncertainty about the tidal patterns here obtained which really only represent a start in the procedure of building up a coherent picture of the tides of the coastal and deep-sea waters of the region. The marked discrepancy in phase at the coast between observational and model values, noted earlier - see figure 6, is another indication that the present results should be regarded as tentative. Observations from a modest deployment of off-shore tide gauges might confirm or possibly lead to an adjustment of the patterns, leading to increased overall knowledge. For example, the variations of amplitude and phase along AB (figures 3 and 4) might be tested by tidal observations at positions along or near to this line. Figure 5 indicates seamounts in the ocean area which might prove useful for this purpose.

The figures in this contribution have been drawn by Mr. R. A. Smith.

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Table 1. Values of amplitude H, in cms, and phase G, in degrees, along the east coast of the United States and Canada (a) from I.H.B. lists and (b) from Zahel's 1^o global model.

		I.H.B.		Zahel	
		H	G	H	G
1.	Virginia Beach	47.98	4.5	56.8	322.9
2.	Cape Charles	34.08	44.9	58.0	322.5
3.	Atlantic City	58.77	355.0	64.8	326.6
4.	Sandy Hook	65.56	7.0	71.5	332.2
5.	Montauk L.I.	28.32	46.0	69.2	324.6
6.	New London	35.54	58.2	69.2	324.6
7.	Providence R.I.	61.45	9.0	66.0	323.1
8.	Newport	51.33	2.0	66.0	323.1
9.	Woods Hole	23.13	33.0	73.1	329.0
10.	Boston	134.48	111.0	82.2	327.1
11.	Portsmouth	120.95	110.0	82.2	327.1
12.	Portland	132.95	104.0	91.3	328.7
13.	Bar Harbour	154.93	93.0	86.7	320.6
14.	Welshpool	259.70	100.0	97.1	325.9
15.	St. John	303.25	97.8	98.5	326.7
16.	Meteghan	202.70	72.0	98.5	326.7
17.	Yarmouth	148.16	63.3	81.7	317.9
18.	Clarks Harbour	107.90	27.0	81.7	317.9
19.	Lockeport	69.80	359.0	69.7	306.0
20.	Liverpool	64.60	353.0	69.7	306.0
21.	Halifax	63.09	350.0	54.0	307.4
22.	Sheet Harbour	61.60	347.0	54.0	307.4
23.	Sonora	61.90	351.0	50.1	307.0
24.	Port Bickerton	59.40	344.0	50.1	307.0
25.	Canso	58.80	347.0	26.7	311.8
26.	Sable Island	56.40	352.0	44.4	311.8
27.	Louisberg	50.30	344.0	32.3	318.8
28.	Sydney	4.60	352.0	27.9	349.0
29.	Dingwall	27.70	357.0	27.9	349.0
30.	St. Paul Is.	30.82	5.4	27.9	27.5
31.	Cheticamp	21.90	2.0	8.0	21.1
32.	Port aux Basques	44.41	12.5	46.7	359.8
33.	Pushthrough	59.40	352.0	55.1	345.3
34.	Harbour Breton	62.20	357.0	55.1	345.3
35.	St. Pierre (St. Pierre & Miquelon Is.)	60.30	348.0	55.5	323.9

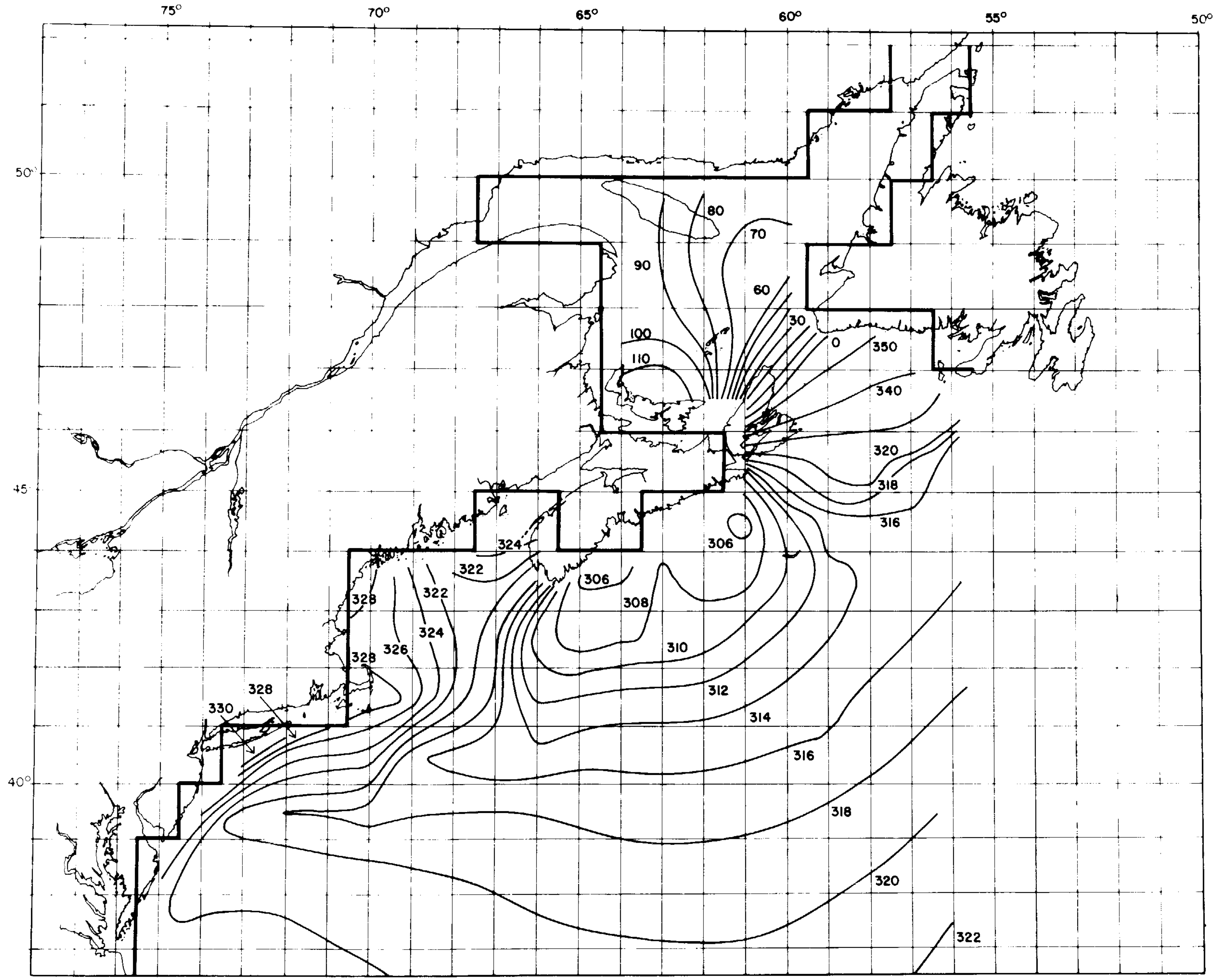

Figure I Cotidal chart : phase in degrees relative to the Moon's transit in Greenwich

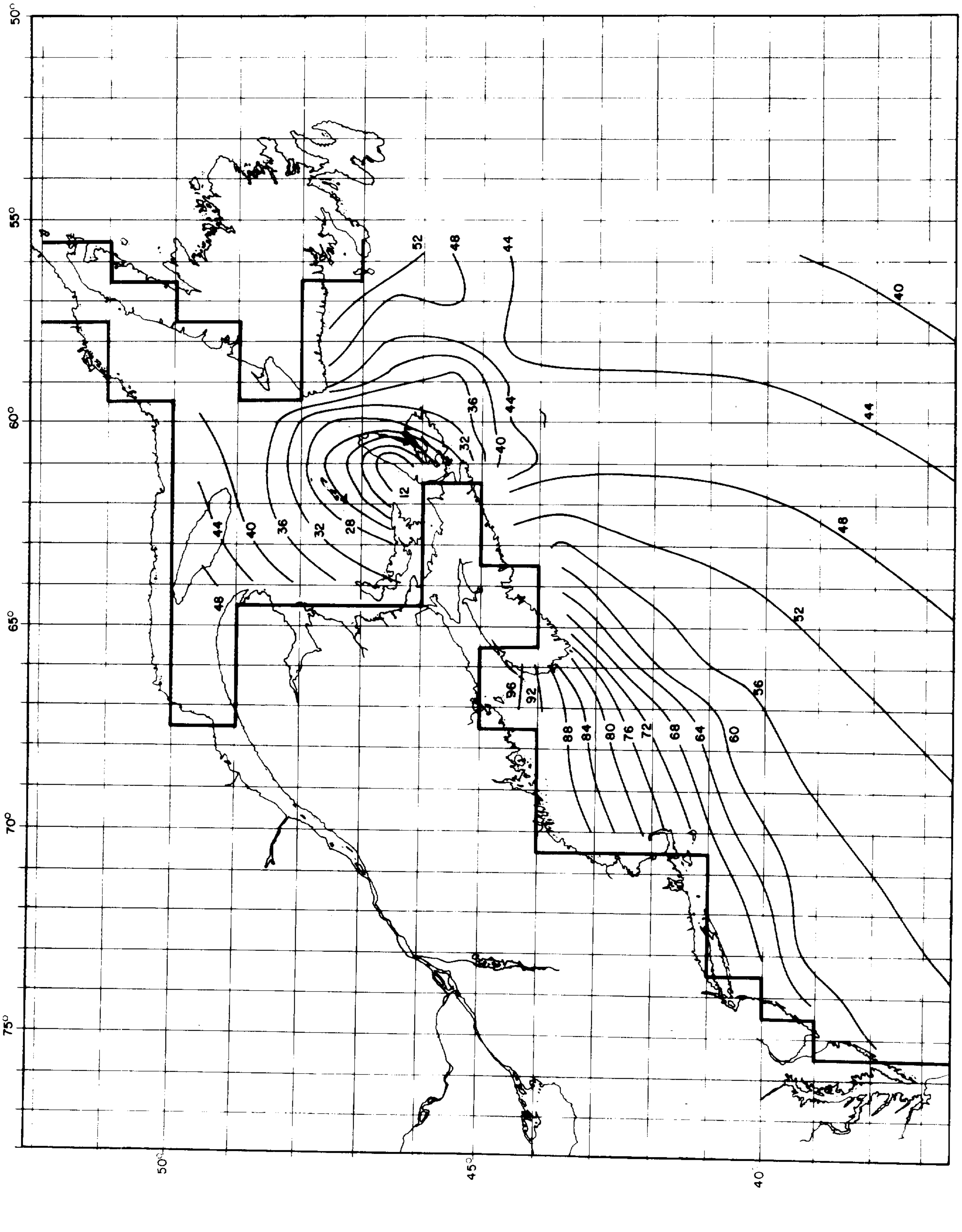

Figure 2 Amplitude in cms

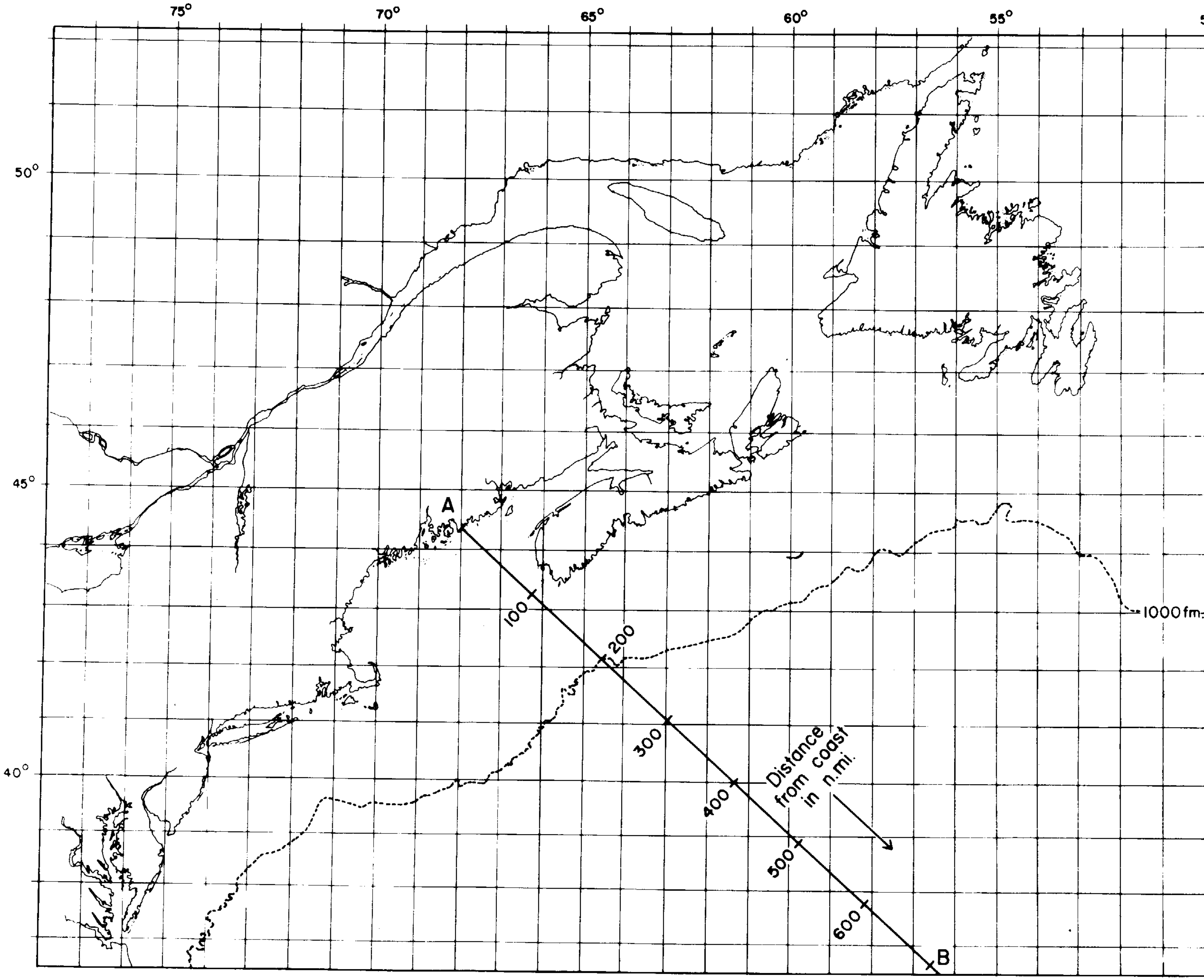

Figure 3 Section AB from shelf to ocean

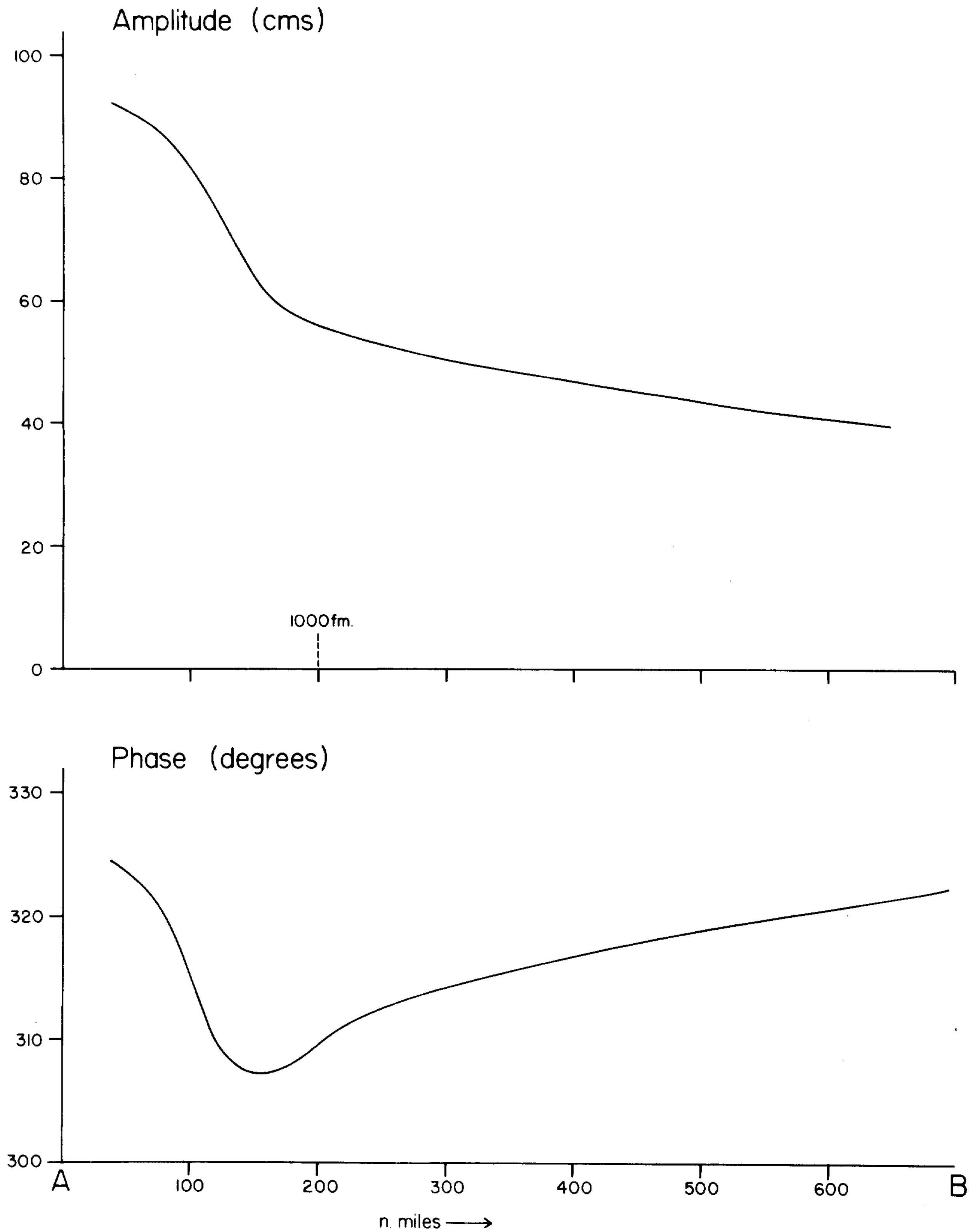

Figure 4 Variations of amplitude and phase along section AB (figure 3)

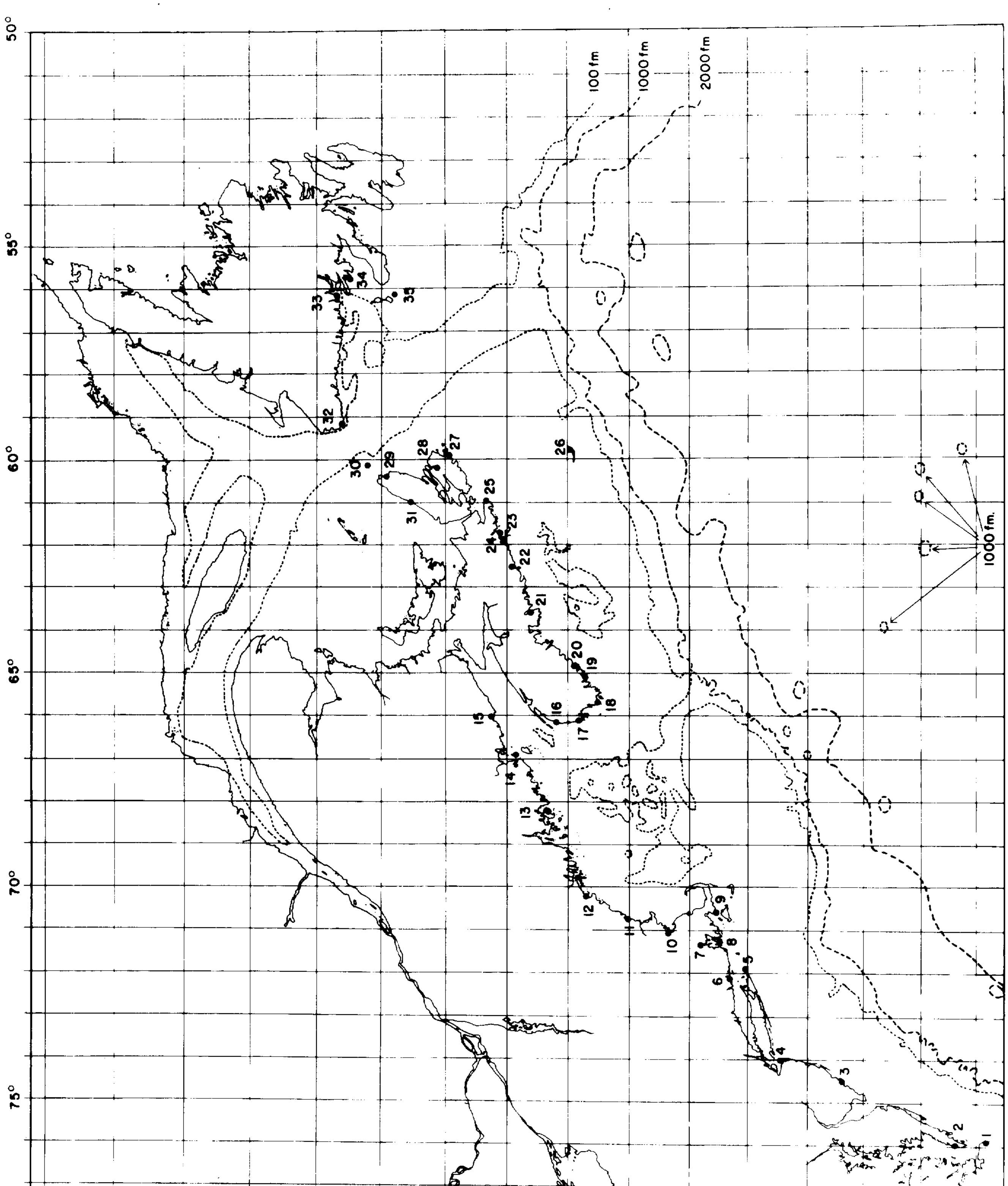

Figure 5 Tidal Stations and Depth Contours

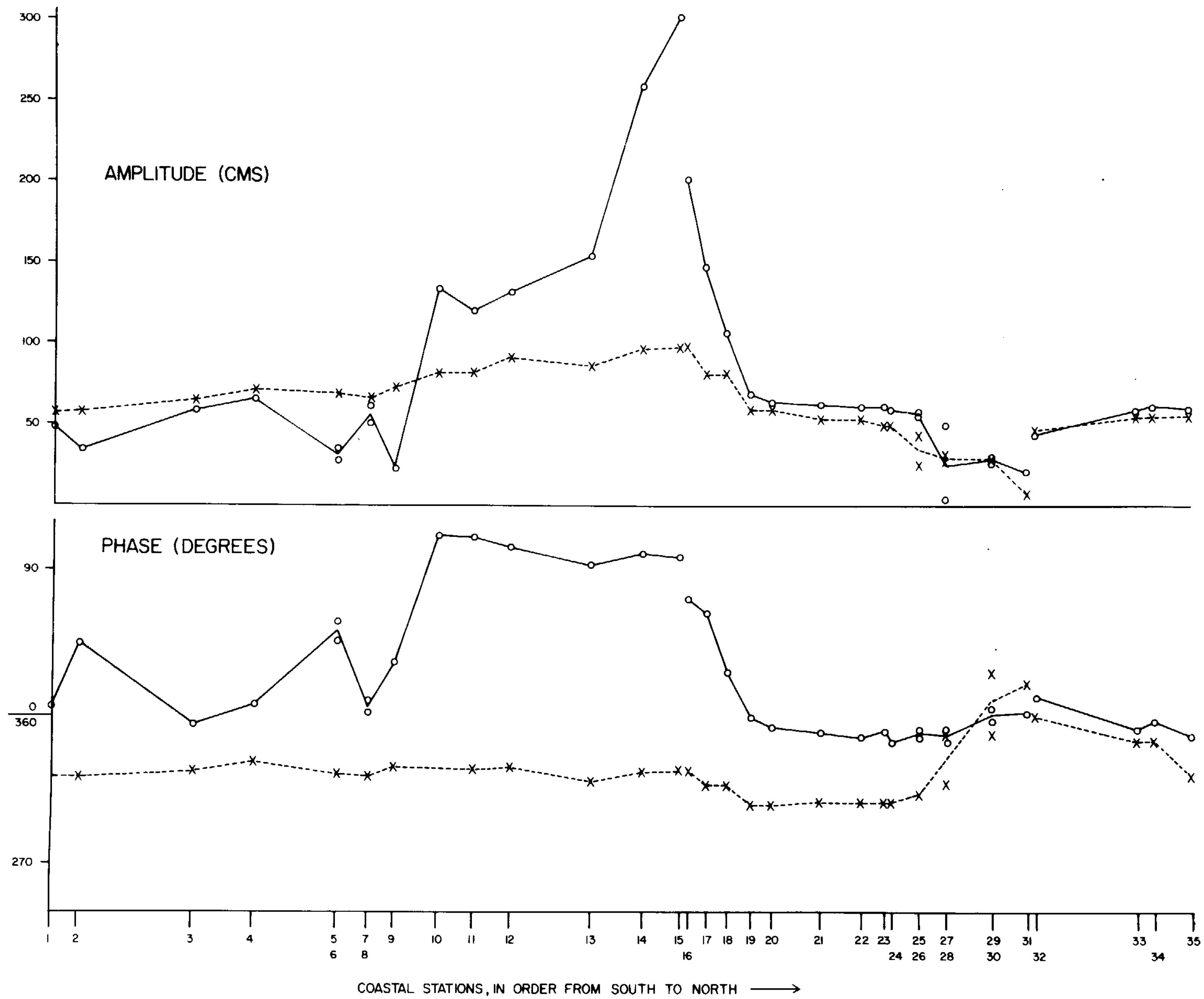

FIGURE 6 AMPLITUDE AND PHASE FROM OBSERVATIONS AT COASTAL STATIONS (---x--x--x--)
COMPARED WITH CORRESPONDING VALUES FROM ZAHEL'S MODEL (—o—o—o—)

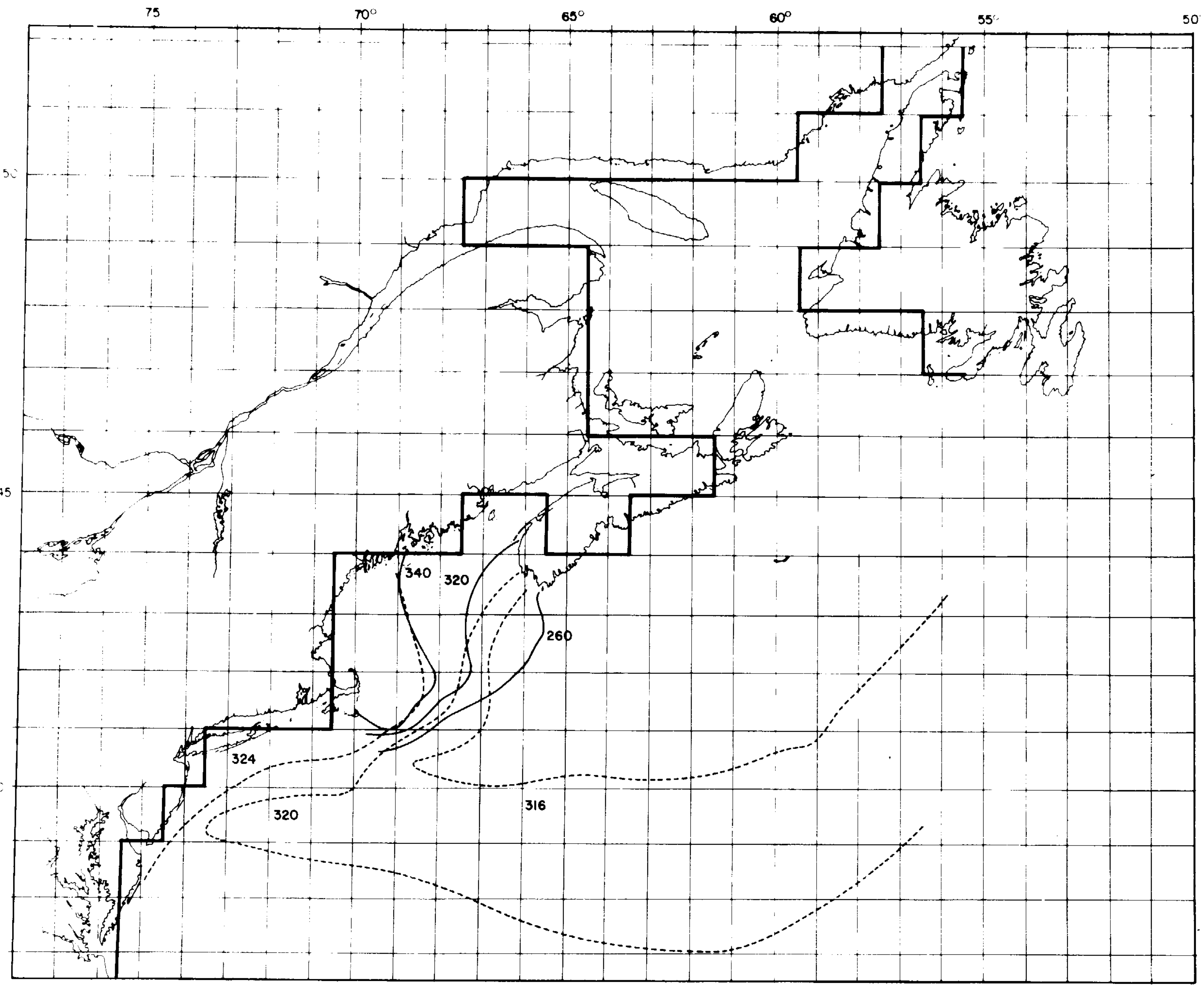

Figure 7 Phases: ——— from Greenberg (260,320,340 degrees)
-------- from Zahel (316,320,324 degrees)

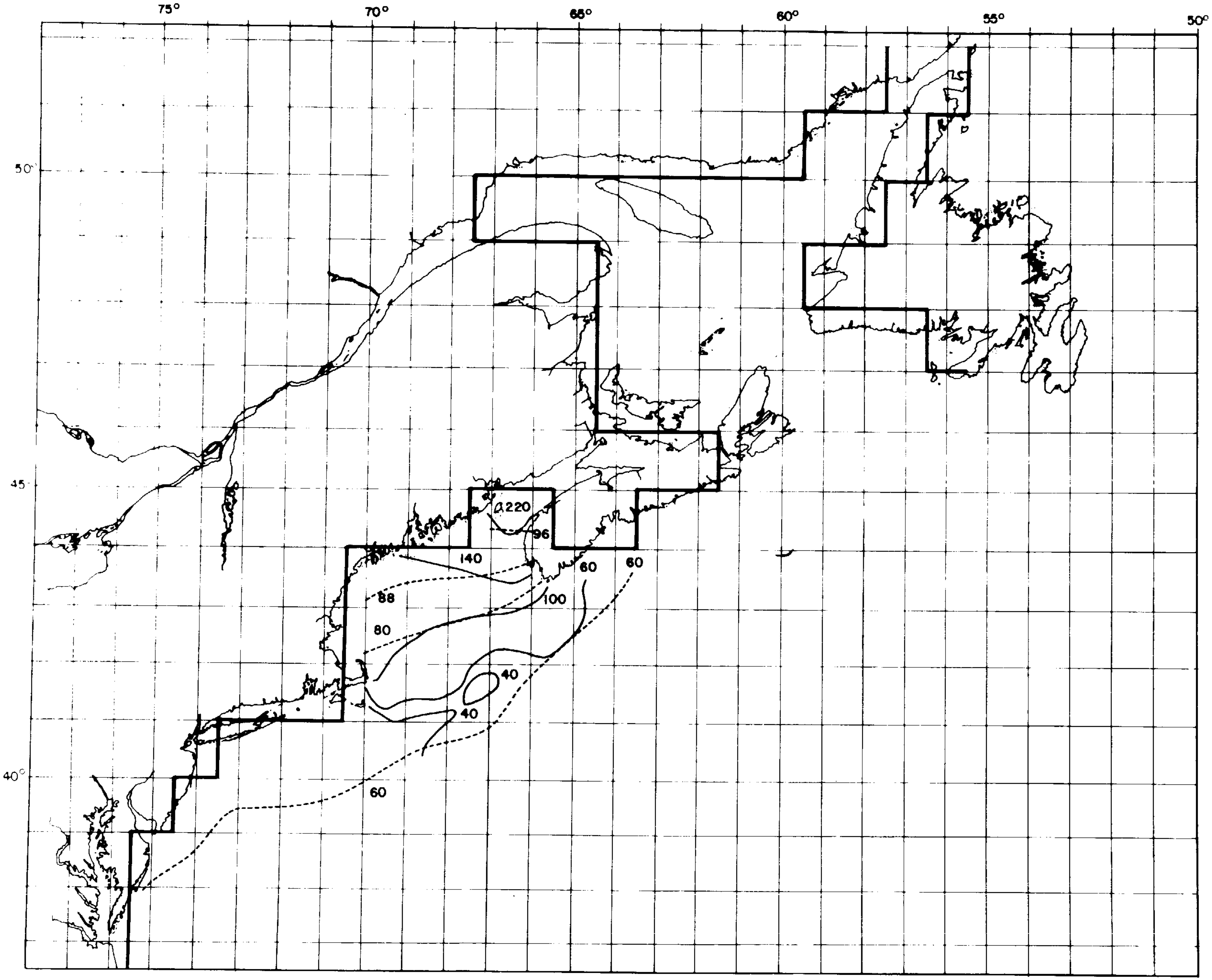

Figure 8 Amplitudes : ——— from Greenberg (40,60,100,140,220 cms)
--------- from Zahel (60,80,88,96 cms)