

**SENSORS FOR THE MEASUREMENT OF
SAND IN SUSPENSION**

by

R L SOULSBY

**A survey of requirements and possibilities
for development**

REPORT NO 27

1977



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FIGURES

ABSTRACT

A study is made of the relative feasibilities of a number of approaches to the problem of measuring sand in suspension, with a view to guiding instrument development within IOS(T). The known characteristics of sand in suspension are examined, leading to a formulation of the design requirements of a suitable instrument. A number of candidate techniques are discussed. No one instrument emerges as a best choice; the suitability of different techniques depends on the application required. The most generally promising candidates are a piezo-electric impact sensor, and an optical occultation system.

1. DEFINITION OF THE PROBLEM

1.1 Introduction

When in 1974 IOS reviewed its programme of research into sediment dynamics it was felt that the biggest obstacle to progress with coarse sediments was the lack of a satisfactory device for measuring the rate of transport of material either as bed-load or in suspension. At that time IOS was developing three techniques, all of which were at a fairly rudimentary stage. These were:

- (a) Sand impact counter for measuring sand moving in suspension (initial development by University of Swansea under an NERC contract);
- (b) A small acoustic transponder simulating a pebble and tracked by side-scan sonar for measuring gravel moving as bed-load (development by University of Southampton under an NERC contract);
- (c) An examination of the noise generated by coarse sediment in motion as bed-load to discover if it could give a useful measure of rate and type of sediment in transport.

In 1975 it was decided that preliminary research on these topics had reached a stage which warranted the initiation of a review of the whole problem, so that a long term programme of development could be formulated. However, no document of this sort can ever be the last word, and the author would welcome constructive comment or criticism.

The problems of measuring bed-load transport and suspended load transport can be considered separately, and this report will deal with instruments for measuring suspended load transport only. The measurement of bed-load transport is being studied concurrently by Dr A P Salkield of IOS(T) and will be the subject of a separate report.

For the purposes of this report sand grains will be taken as having a diameter, d , in the range $62.5\mu\text{m}$ to 2mm , and will be modelled as quartz spheres of density $\rho_s = 2.65 \times 10^3 \text{kg m}^{-3}$. They are taken to be suspended in water of density $\rho = 10^3 \text{kg m}^{-3}$ with a kinematic viscosity $\nu = 1.14 \times 10^{-6} \text{m}^2 \text{s}^{-1}$. Concentrations are given as parts per million (ppm) by weight.

1.2 The incidence of suspended sand

The demarcation line between bed-load and suspended load transport of sand is not clear, but a good definition is given by BAGNOLD (1966) as: 'the bed-load is that part of the load which is supported wholly by a solid-transmitted stress, and the suspended load is that part which is supported by a fluid-transmitted stress'. When the current velocity is small, but above a critical threshold value, the grains will move as bedload in a zone a few grain diameters in thickness. The mean velocity of the grains in the bedload layer is less than the local water velocity. If the current velocity is larger there will still be bed-load transport, but in addition, sand can be suspended in a considerably thicker zone and will travel at a velocity equal to the local stream velocity. Under these conditions the vast majority of the moving sediment will be carried as suspended load.

Suspended sand occurs over and around sandbanks in coastal waters due to the action of tidal currents or wave action. During storm conditions both agents combine to transport large quantities of sand. Sand suspensions have been observed acoustically above the crests of sandwaves, and are often visible there as upwelling patches of sandy water. In many high-energy estuaries there is an important fraction of suspended sand, chiefly the smaller grain-sizes. In addition sand is put into suspension by the action of waves on sandy beaches, sometimes considerably to seaward of the break point.

In order to define a design objective, attention will be focussed on suspensions in a basically uni-directional flow, such as rivers or tidal currents. Many of them, however, would be equally well suited to the more complicated case of oscillatory flow under waves.

1.3 Physical characteristics of suspended sand

The qualitative and quantitative properties of sand in suspension must now be studied, so as to be able to specify what will be required of the sensor.

Many areas of mobile sediment have been studied by IOS(T) using an underwater TV system, and it can invariably be seen that sand moves in suspension as

clouds and swirls of sizes typically 0.5m to a few metres. The current velocity fluctuations associated with these motions can already be measured to a time resolution of 0.2 sec and a length resolution of 0.1m, but there is no instrument for measuring the sediment characteristics to an equivalent resolution.

Sand will only start to move at all (initially as bed-load) if a threshold bed shear-stress, τ_o , is exceeded. This can alternatively be expressed in terms of the shear velocity, u_* , where $\tau_o = \rho u_*^2$. The most widely accepted criterion for motion is Shields' threshold curve (see, for example, GRAF, 1971), based mainly on laboratory experiments, which is shown in Fig 1 in a form suitable for quartz grains in water. The threshold shear velocity can be related to a threshold current velocity, U_{100} , at a height of 1m by assuming a logarithmic velocity profile in the bottom 1m, with a roughness length of 5mm, which is typical of a rippled sand bottom in a tidal stream. A scale for U_{100} is shown in Fig 1.

The threshold of suspension is much less well documented. BAGNOLD (1966) has suggested a curve of the form also shown in Fig 1 based on intuitive arguments. It is seen that the suspension threshold curve crosses the bed-load curve at a diameter of $200 \mu m$, implying that smaller grains go straight into suspension and do not move as bed-load, a fact which is observed in practice, although the limit appears to be nearer to $100 \mu m$. The minimum value of U_{100} at which suspension of sand can take place is about $0.2 m s^{-1}$.

As the grain diameter increases, the threshold value of U_{100} rises sharply, reaching a value of $1.6 m s^{-1}$ for a grain diameter of 1mm. Currents will only exceed this speed in a few localised areas such as the Portland Race and the Menai Straits. For most purposes we can say that the largest grain-size which can be held in suspension is 1 mm.

Once sand is in suspension it will assume a vertical concentration profile whose form will be dependent on grain-size and flow conditions. Knowledge of these profiles will help to determine the range of concentrations which the sensor will be called on to measure. A number of theoretical concentration distributions of varying sophistication have been suggested, but perhaps the most widely accepted is that due to ROUSE (1937). The diffusion equation with settling can be integrated, after fitting an eddy diffusivity distribution based on mixing length arguments, to give

$$\frac{C}{C_a} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^b \quad -1$$

where C is the concentration at height z, C_a is the concentration at a reference

height a , and h is the flow depth. For grains of settling velocity W_s , the exponent $b = \frac{W_s}{K u_*}$, where K is Karman's constant, usually taken as 0.4. Thus for very small grains (and thus small W_s) or a large shear velocity, b is small and the sediment concentration is relatively homogeneous throughout the depth, whereas large grains or a small shear velocity will give a large concentration gradient with most of the sediment confined to a layer near the bed.

Whilst this gives the general form of the profile, it does not give any information about actual concentrations, as the reference concentration is still unknown. A number of expressions for C_a have been derived, but none of these has as firm a basis as the profile expression, Eq 1. Consequently recourse must be made to measured data. Fig 2 shows data for the Thames estuary obtained by the Hydraulics Research Station, Wallingford.

Looking now at the range of concentrations likely to be met with in practice, the Thames Estuary data shows a range from about 100ppm to 1000ppm. Samples recently taken by IOS(T) in Start Bay, Devon, ranged from 3ppm to 4000ppm, and these latter values will be taken to include most concentrations likely to be encountered.

1.4 The design specification

We are now in a position to specify the limiting criteria of an ideal instrument.

At this stage the flow will be assumed to be uni-directional. It would in fact be sufficient in the first instance to have a device simply to indicate whether sand is present in suspension or not.

However, a more sophisticated instrument should measure the transport rate, or, as current velocity can be measured separately, the concentration and size distribution of sand in suspension. It should respond to grains in the size range 1mm down to $62.5 \mu m$, discriminating against smaller sizes, and allowing the transport of sand to be detected in the presence of a suspension of mud or organic matter. Concentrations may range from 3ppm to 4000ppm by weight. Current speeds of from $0.2 m s^{-1}$ to $1.6 m s^{-1}$ at 1m from the bottom must be catered for. The sensor should be able to resolve 0.2s in time and 0.1m in space. The complete instrument should have a sufficiently low power consumption that it can operate in a remote self-recording mode. It must be capable of working in fresh or salt water at depths of as much as 200m.

2. ANALYSIS OF POSSIBLE MEASURING TECHNIQUES

2.1 Existing instruments

Good reviews have been given by the INTER-AGENCY COMMITTEE ON WATER RESOURCES

(1963) and by GRAF (1971) of the instruments for measuring sediment transport available at the time of their writing. These mainly comprise sampling devices, which will be listed here:

- (a) Instantaneous bottle samplers - a sample of sediment-laden water, typically 0.5 - 5 litres, is trapped.
- (b) Integrating bottle samplers - designed to fill slowly so that the sample is more representative than an instantaneous sample.
- (c) Delft bottle - water flows through a nozzle into a chamber where the sediment settles out and is collected.
- (d) Pumped samplers - sediment-laden water is pumped onto the deck of a ship, where it can be filtered.

A number of continuous recording devices are also described, but none of these has become a widely accepted sensor for sediment specifically in the sand range. As all of these latter methods will be examined in detail in the following sections, they will not be described here.

2.2 Conductivity methods

The presence of suspended sand in sea water will decrease its measured bulk electrical conductivity. However, variations in conductivity due to temperature and salinity will in general mask any change attributable to the suspended sand; experiments within IOS(T) have shown that typically 1000ppm of sediment produces a change in conductivity equivalent to only a 0.01‰ change in salinity.

Another technique which is extensively used in the laboratory is the 'Coulter Counter' system. A suspension is drawn through a narrow orifice which has electrodes placed on either side. When a sand grain is partially blocking the orifice the electrical resistance is momentarily increased. A count of these events gives a measure of the concentration of the suspension. However, the difficulties associated with adapting the technique for use in the sea, such as pumping problems and blocking of the orifice, are sufficient to make it impracticable.

2.3 γ -Ray absorption

γ -ray absorption and back-scattering have been used successfully in IOS(T) and elsewhere in very dense suspensions of mud. However, the users state that the smallest concentration measurable with the device is 10,000ppm, which is well outside the present requirements.

2.4 Optical absorption

Optical extinction and scattering measurements have been used for some time as an indication of the concentration of suspended matter, although these techniques are most commonly used in suspensions of silt or clay - see for example, JONES and WILLS (1956) and THORN (1973). Instruments have also been designed for use in sand suspensions of high concentration (1,000ppm to 500,000ppm) by HOM-MA et al (1965), BHATTACHARYA et al (1969) and BRENNINKMEYER (1976), all of which have been used with some success.

The fractional transmittance of the light after passing through a length of suspension is $\exp(-\alpha x)$, where α is the beam transmittance coefficient. α is directly proportional to concentration, but is also a function of the wavelength of the light, and the grain diameter, as illustrated in Fig 3. This figure is drawn for green light of wavelength $0.5 \mu\text{m}$ and a sediment concentration of 1,000ppm. The dependence on grain diameter will cause difficulties if an instrument is used in suspended sediment of non-uniform diameter, and even in the case of uniform grain-size the instrument must be calibrated against known concentrations of the actual sediment being measured.

An even more serious difficulty is that α increases with decreasing grain diameter, so that a small concentration of suspended clay could mask a larger concentration of sand. To take an extreme case, 1ppm of $2 \mu\text{m}$ clay would give the same extinction as 1,000ppm of 1mm sand. Consequently, if this sort of instrument is used for measuring concentrations of suspended sand it must be used with extreme caution.

2.5 Optical occultation

An alternative optical technique is to exploit the shadowing of a light beam by individual sand-grains. The steady level of transmitted intensity, which may fluctuate slowly due to the presence of mud, is no longer of interest; instead the rate of partial or total occultation of the beam would be measured. This would give a measure of the number concentration of sand-size grains in suspension, the fraction of the beam shadowed indicating the grain-size. A similar technique is used for measuring raindrop concentration in the atmosphere.

A detailed study of this technique is given in Appendix A, the salient points being summarised below.

A suitable optical layout is shown schematically in Fig 4a. The following factors affect the size of beam chosen:

- (a) The fraction of the beam shadowed by a single grain must be large enough to be detected.
- (b) There must not be, on average, more than one grain in the beam at a time.
- (c) The duration of the occultation must be long enough to be detected.
- (d) The rate of occulting must not exceed that which can be counted electronically.
- (e) The duration of the occultation must be shorter than the waiting time between counts. This is equivalent to condition (b).

The most exacting restriction on the beam size is condition (b), which determines the measuring volume in terms of the number of grains per unit volume. To allow 10^4 grains/ml (eg 4,000 ppm of $62.5 \mu\text{m}$ sand) to be counted this condition indicates an optimum beam diameter of 0.2mm and a length of 3mm. This is very small, but might be achieved using fibre-optic techniques. In addition this would allow the measuring head to be placed well away from the body of the instrument.

However if conditions are relaxed to allow a maximum of only 100 grains/ml (eg 1100 ppm of $200 \mu\text{m}$ sand) the beam can be enlarged to 1.1mm diameter and 10mm length. This would then give scope for a further improvement. One design of rain-drop concentration sensor uses an annular beam, which has the advantage that the narrow cut part of the beam causes a signal dependence proportional directly to grain diameter, d , rather than to d^2 . An arrangement suitable for moderate concentrations (< 100 grains/ml) of grains larger than about $125 \mu\text{m}$ would be a 0.1mm wide annulus of diameter 4mm and length 10mm. The general layout, using fibre optics, is shown in Fig 4b.

An optical occultation system might best be built with alternative measuring heads for individual requirements attachable to a common package containing the light source, photocell and monitoring electronics.

2.6 Acoustic methods

The attenuation of the acoustic pressure of sound waves passing through a length x of a suspension of sand in water is given by $\exp(-\alpha_p x)$, where α_p is the acoustic pressure attenuation coefficient. As with optical attenuation, α_p is directly proportional to the concentration of suspended sediment, but is also a function of grain diameter and the frequency of the acoustic signal. The variation of α_p with grain diameter is shown for a number of frequencies in Fig 5, which is adapted from the work of BLUE and McLEROY (1968) and from HASLETT (1970).

The curves, which show some of the same characteristics as optical attenuation, exhibit two principal maxima, one in the viscous absorption region, and one at the upper end of the Rayleigh scattering region, both of which move towards smaller grain diameter as the frequency is increased. The frequency can be chosen so as to make a maximum occur at the centre of the sand size range. This will then effectively discriminate against grain sizes outside the sand range.

To get the viscous absorption maximum to occur in the middle of the sand size range ($250\ \mu\text{m}$) would require a frequency of 28 Hz. However, the wavelength would be impracticably large, and the attenuation is so small at this frequency that it would take a path-length of 2 km to give 1% attenuation in a 1,000 ppm suspension of $250\ \mu\text{m}$ sand.

Centring the Rayleigh limit maximum in the middle of the sand range is more promising, but the position of this maximum must be chosen so as to minimise the effect of the viscous absorption maximum, which now occurs in the mud size range. A frequency of 6 MHz has a maximum at $220\ \mu\text{m}$, and at this frequency a 1000 ppm suspension of $220\ \mu\text{m}$ sand would give 10% attenuation over a path-length of 11 mm. Assuming that equal concentrations of $5\ \mu\text{m}$ silt and $62.5\ \mu\text{m}$ sand are present the attenuation caused by the sand would be equal to the viscous absorption produced by the silt. This imposes a limitation on the use of this technique.

In low concentrations where attenuation is too small a scatterance meter might be more suitable.

2.7 Impact sensors

The high density of sand grains gives them an appreciable excess momentum over the surrounding water so that they would tend to strike a transducer placed in the stream rather than follow the path of the water particles round it. This will effectively discriminate against particles of both mud and organic matter, as these will not have sufficient inertia to be detected. The output will be a count of the frequency of impacts on a known area.

In principle the impact process transfers some of the momentum and energy of the sand grain to the sensor, which converts it into an electrical signal. A detailed analysis of some aspects of the process is given in Appendix B, and summarised here.

- (a) The output of the sensor is found to depend separately on both grain mass and velocity in the general case. However if the mass of the sensor is

much greater than the grain mass then the sensor output is directly proportional to the momentum of the grain.

- (b) The energy transferred from the grain to the sensor is maximised when the effective mass of the sensor is equal to the mass of the grain.
- (c) The trajectory of a grain suspended in water flowing onto a circular target is calculated. The analysis shows that not only is the momentum of the particle reduced from its free stream value, but that grains smaller than a critical size, which depends on free stream velocity and target diameter, will never reach the target. The largest target which can be struck by a $62.5\text{ }\mu\text{m}$ grain at a free stream velocity of 1.6 m s^{-1} has a diameter of 6.5 mm , and at 0.2 m s^{-1} this must be reduced to 0.8 mm .
- (d) The free stream momentum of a grain ranges from $7 \times 10^{-5}\text{ gm mm s}^{-1}$ for a $62.5\text{ }\mu\text{m}$ grain at a velocity of 0.2 m s^{-1} to 2 gm mm s^{-1} for a 1 mm grain at 1.6 m s^{-1} .
- (e) The maximum impact rate within the design specification will occur for $4,000\text{ ppm}$ of $62.5\text{ }\mu\text{m}$ sand moving at 1.6 m s^{-1} . This will give 6.4×10^5 impacts per second on a target of diameter 6.5 mm .
- (f) The shape of the sensor must be chosen by trial and error to minimise the possibility of multiple impacts by a single grain.
- (g) The effect of pressure fluctuations on the sensor due to either turbulence or acoustic ambient noise is shown to be small provided that the resonant frequency of the sensor is greater than about 20 kHz .
- (h) Six different types of transducer have been considered, four of which were discarded as having no particular advantage over the remaining two. The most promising techniques are:

piezo-electric - a ceramic disc which is water-proofed and struck directly by the sand grain

capacitative - the sand grain strikes a diaphragm which forms one plate of a capacitor.

Points (a) and (c) together show that a sensor of this type cannot easily give a measure of the grain mass, so that the best that can be obtained is a number concentration. Points (b), (c) and (g) all indicate that the sensor must be very small. The restrictions imposed by points (b), (c), (d) and (e) would all be eased if the minimum grain-size to be detected were increased to, say, $125\text{ }\mu\text{m}$, which would also ensure better discrimination against silt.

3. CONCLUSIONS

3.1 Comparison of the possible measuring techniques

It is, perhaps, not surprising that none of the systems considered fulfils all the design requirements. The evaluation of the sensors has not revealed any one sensor as being outstandingly more suitable than all the others, but rather has shown that some systems are better suited to a particular set of conditions than others. Only once a particular application has been decided on (eg, grains larger than $200\mu\text{m}$ in suspension under waves) can the most promising sensor be chosen.

Some of the systems which were evaluated can be immediately discarded, for reasons given in the individual assessments. Thus conductivity methods, γ -ray methods and acoustic viscous absorption are all non-starters.

For work in specific areas where it is known that there is no mud in suspension an optical extinction method would be able to indicate the presence of suspended sand. If, in addition, the grain-size is known to be uniform, an instrument of this type could give a measure of concentration, after in-situ calibration. The simplicity of this kind of instrument makes it well worth consideration, but it would be misleading if used in areas which do not meet these restrictions.

This leaves optical occultation (OO), acoustic Rayleigh limit scattering (ARS), impact piezo-electric (IPE) and impact capacitative (IC) sensors as the possible alternatives.

If a simple instrument is required, capable only of indicating whether or not there is sand in suspension, then IPE or IC transducers coupled to a simple detector would be most suitable.

All the systems are capable of measuring number concentrations of grains, given some sort of counter for the OO, IPE and IC systems.

If grain-size distribution is required the most suitable system is OO. To measure grain-size with ARS a frequency scanning system might be possible, but this would be excessively complicated both in design and interpretation. For the impact sensors it is possible in principle to analytically obtain a sort of transfer function to take account of the slowing up of the grains approaching the transducer face. The independently measured mean stream velocity can then be inserted into calculated grain momentum to obtain the grain mass and hence its size. However in practice it is felt that this would be unworkable.

For use in oscillating flows under waves, OO and ARS are more directly applicable than impact methods, as an omnidirectional impact sensor would be difficult to construct.

Deployment very near to the sea bed is easiest with OO, IPE and IC.

All the systems would be much easier to design if it were unnecessary to measure large concentrations of the smallest grains. In particular if this requirement were relaxed the OO and impact sensors could be made a more manageable size, and would not have to cope with such high count rates.

Only the impact sensors are completely proof against the presence of very large concentrations of mud. The OO system would have its light extinguished, and the ARS might experience interference from viscous absorption.

If there is a large quantity of broken-up weed, and similar organic detritus, present, there could be severe difficulties with OO and ARS.

The IPE and IC detectors both have advantages. A capacitative system has a low mass, comparable with that of a sand-grain, for efficient energy transfer. A piezo-electric crystal has a larger mass, but also a higher spring constant giving very high resonant frequencies and consequently good signal-to-noise resolution.

As regards suitability for remote sensing, both OO and ARS have a fairly heavy power consumption. That for OO might be reduced by the use of light-emitting diodes, which would also obviate bulb replacement. However, marine fouling of the optical surfaces could present a further problem. For this sort of work impact sensors seem to be more suitable.

3.2 Recommendations for sensor development

As a guide to the most generally useful lines to take, it is suggested that as an initial 'yes-no' type of instrument, work should proceed with an impact sensor. This could consist of a 2mm diameter ceramic disc, or a condenser microphone of similar diameter, waterproofed and mounted in the flow. The output could be amplified and displayed directly.

At the same time work might commence on an optical occultation system. Alternative measuring heads could be made for use in different conditions, using a tungsten filament light source and a photo-cell, the light being carried down to the measuring volume and the beam formed, with optical fibres. A cylindrical beam of length 10mm and diameter 1.1mm would make a suitable trial beam. For high concentrations of small grains a smaller beam of length 3mm and diameter 0.2mm is needed. If lower concentrations of larger particles are likely to be encountered an annular beam 10mm long of diameter 4mm and thickness 0.1mm could be made. This arrangement would also make grain-size distributions, in principle, measurable.

Any of the systems outlined above would require careful calibration in a laboratory flume.

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APPENDIX A. STUDY OF OPTICAL OCCULTATION SYSTEM

A.1 Size of signal

We will initially consider a cylindrical beam of length l and diameter D (see Fig 4a). The cross-sectional area of the beam is $\pi D^2/4$ and the area obscured by a grain of diameter d is $K\pi d^2/4$ provided this is less than $\pi D^2/4$. Thus the size of the spike in the intensity caused by the sand grain relative to the unobscured intensity is

$$\frac{Kd^2}{D^2} \text{ for } d < \frac{D}{\sqrt{K}}$$

or $1 \text{ for } d \geq \frac{D}{\sqrt{K}}$

-A1

The effective area coefficient, K , will be assumed to take the value 2 for grains in the sand size range (BURT, 1955), although the value may fall as low as 1 for an imperfectly collimated beam of finite width. The relative size of the spike is shown in Table A1 for a range of grain-sizes and beam diameters.

$\frac{D}{d}$	0.2	2	20mm
62.5	0.2	2×10^{-3}	2×10^{-5}
250	1	.03	3×10^{-4}
1000 μm	1	0.5	5×10^{-3}

TABLE A1. SIZE OF SPIKE RELATIVE TO UNOBSURED INTENSITY

It appears that the beam must be no wider than 2mm diameter if the smallest grains are to produce a detectable signal.

A.2 Measuring volume

If individual grains are to be counted the optical measuring volume must be restricted so that it contains no more than one grain at any instant.

The number of grains per unit volume, C_3 , is given by

$$C_3 = \frac{6\rho C}{\pi \rho_s d^3}$$

-A2

where ρ is the density of water, and ρ_s the density of sand.

Typical values are tabulated in Table A2.

$\frac{d}{C}$	62.5	250	1000 μm
3	10	0.14	.002
30	100	1.4	.02
300	10^3	14	.22
4000 ppm	1.2×10^4	180	2.9

TABLE A2. NUMBER CONCENTRATION, C_3 (grains/ml)

The number of particles in the beam is then $\pi D^2 l C_3 / 4$, so that the criterion is

$$\pi D^2 l C_3 / 4 < 1$$

-A3

The beam length satisfying this is shown in Table A3 for different beam radii, and typical number concentrations selected from Table A2.

$\frac{C_3}{D}$	1	100	10^4 grains/ml
0.2	3.2×10^4	320	3.2
2	320	3.2	.032
20mm	3.2	.032	3.2×10^{-4}

TABLE A3. BEAM LENGTH IN mm

Thus if the instrument is to detect number concentrations of 10^4 grains/ml the beam diameter must be reduced to about 0.2mm with a beam length of 3mm.

A.3 Duration and rate

Another important factor is the duration of the signal spike caused by an individual grain. This will depend on the speed at which the grain is carried through the beam, and the combined width of the beam and the grain. The duration is given by $(D + d)/U$ and is tabulated in Table A4.

$D + d \backslash C_3$	0.2	1.0	1.6 m s^{-1}
0.1	0.5	0.1	.06
1	5	1	0.6
10mm	50	10	6

TABLE A4. DURATION OF SPIKE (msec)

These values seem quite acceptable for electronic processing.

The number of spikes occurring every second is given by the rate at which grains cross the diameter-plane of the optical volume. Taking the beam geometry mentioned above (3mm long by 0.2mm diam.), some typical results for interception rate are given in Table A5.

$U \backslash C_3$	0.2	1.0	1.6 m s^{-1}
1	0.12	0.6	1
100	12	60	100
10^4 grains/ml	1.2×10^3	6×10^3	10^4

TABLE A5. INTERCEPTION RATE (counts/sec)

Only the fastest of these rates would present any difficulty in counting.

The most significant result of this analysis is the very small size of the optical measuring volume. This need not necessarily cause insuperable problems since a fibre-optic system could carry the measuring volume well away from the flow disturbance caused by the body of the instrument. Consequently this type of instrument could give measurements very close to the bed, and would be well suited to oscillating flows.

APPENDIX B. STUDY OF IMPACT SENSORS

B.1 Energy exchange between a sand grain and a transducer

The impact of a sand grain on a transducer is modelled as a point mass, m , striking a simple harmonic oscillator (SHO) of mass M and spring constant k (Fig 6). The point mass has a velocity V just before impact, and immediately after the impact a velocity V' . The mass part of the SHO has velocity V .

In order to calculate the amplitude of oscillation of the SHO it is necessary to apply conservation of energy and momentum. This is not immediately allowable, as the system is not closed as long as the spring of the SHO is backed by a finite mass (conservation of momentum of the SHO does not hold). However, the results obtained here can be more rigorously shown to be the limiting case of a closed system. Thus conservation of momentum is applied in the usual manner for impacts in the absence of external forces (valid as long as the duration of the impact is short compared with the period of the SHO).

$$mV = mV' + MV \quad -B1$$

If no energy is lost during the impact, then

$$\frac{1}{2}mV^2 = \frac{1}{2}mV'^2 + \frac{1}{2}MV^2 \quad -B2$$

These equations are combined to give V , which is equal to the amplitude, U_0 , of the oscillating velocity.

$$U_0 = V = \frac{2mV}{M+m} \quad -B3$$

The angular frequency, ω , of resonance of the SHO is given by

$$\omega = \sqrt{\frac{k}{M}} \quad -B4$$

The amplitude of the displacement of M is

$$a = \frac{2mV}{\omega(m+M)} \quad -B5$$

as $U_0 = \omega a$.

The energy which the oscillator extracts from the sand grain is equal to the kinetic energy of the oscillator as it passes the rest position,

$$\begin{aligned} E_{SHO} &= \frac{1}{2}MU_0^2 \\ &= \frac{2Mm^2V^2}{(M+m)^2} \end{aligned} \quad -B6$$

Differentiation with respect to M shows that this is maximised for $M = m$. It is unlikely that a target of mass equal to a sand-grain will be feasible, but the smaller M can be made the better.

B.2 Range of momenta and impact rates

More practically we will have $M \gg m$, in which case the amplitude and velocity of the oscillator are proportional to the momentum of the grain. When the flow is not impeded by the sensor, the momentum of the grain is given by $\frac{\pi}{6} \rho_s d^3 U$; typical values are shown in Table B1.

$d \backslash U$	0.2	1.0	1.6 m s ⁻¹
62.5	7×10^{-5}	3×10^{-4}	5×10^{-4}
250	4×10^{-3}	.02	.03
1000 μm	0.3	1.4	2

TABLE B1. MOMENTUM OF GRAIN (gm.mm. s⁻¹)

To gain some idea of the area of target that should be exposed to the flow the impact rate onto a surface of area A has been calculated. This rate is given by $C_3 U A$ where C_3 is the number concentration (see Table A2), and is tabulated in Table B2.

$C_3 U \backslash A$	1	10	100 mm ²
0.5	0.5	5	50
100	100	10^3	10^4
2×10^4	2×10^4	2×10^5	2×10^6

grains mm⁻² s⁻¹

TABLE B2. IMPACT RATE (counts/sec)

The extreme figures ($>10^5$) may be unacceptable. However, these represent large concentrations of fine grains striking a large target. There is a certain amount of self-consistency here, because, as shown below, the smallest grains will be swept past a large target completely.

B.3 Deceleration of a sand grain in the flow in front of a transducer face.

We will next investigate the behaviour of a sand grain which is being carried by a current to strike a transducer. Upstream of the transducer the flow will be retarded. However, the inertia of the grain will carry it through this near-stagnant zone, so that it may still strike the transducer, but with a reduced velocity.

In this context the sand grain is modelled by a quartz sphere of diameter d , mass $\frac{1}{6}\rho_s\pi d^3$ and velocity $\dot{\xi}$, being carried uniformly by a flow of free-stream velocity U and kinematic viscosity ν . The actual transducer face might be almost any shape, but it is supposed that the details of the flow some way upstream of the transducer will be relatively independent of shape, and that anyway the variations will not alter the final results by as much as an order of a magnitude. For the same sort of reason the problem is taken to be two-dimensional, with x along the flow, as this simplifies the analysis. The model is illustrated in Fig 7.

The first task is to calculate the flow. Consider initially an inviscid flow, $u(x)$ confined to the left half-plane, and symmetrical about $y = 0$, this being the stagnation stream-line. The irrotational equations of motion for a stream function ψ gives

$$\nabla^2\psi = 0 \quad - B7$$

with solution $\psi = ax^2 + by^2$ - B8

which already satisfies the conditions of no flow normal to the transducer face, $x = 0$, and no flow normal to the stagnation streamline $y = 0$. Inserting the further condition that at some point $x = -X$ the current is given by $u(-X, 0) = (U, 0)$ gives the flow field

$$u = -\frac{Ux}{X} \quad - B9$$

The sand grain is now put into this flow on the x -axis at a position $(\xi, 0)$ and is taken to have the stream velocity U at $\xi = -X$. The grain is supposed to be small enough to experience Stokes drag $= 3\pi\rho\nu d(u(\xi) - \dot{\xi})$ and all other forces are ignored. Then the motion of the grain is given by

$$\left(\frac{1}{6}\rho_s\pi d^3\right)\ddot{\xi} = 3\pi\rho\nu d(u(\xi) - \dot{\xi}) \quad - B10$$

or $\gamma\ddot{\xi} = u(\xi) - \dot{\xi}$ writing $\gamma = \frac{\rho_s d^2}{18\rho\nu}$

Thus the equation to be solved is

$$\gamma\ddot{\xi} + \dot{\xi} + \frac{U}{X}\xi = 0 \quad - B11$$

This has solution $\xi = A_1 \exp(-p_1 t) + A_2 \exp(-p_2 t)$ - B12

where
$$p_1 = \frac{1 + (1 - 4U\gamma/X)^{1/2}}{2\gamma}, \quad p_2 = \frac{1 - (1 - 4U\gamma/X)^{1/2}}{2\gamma}$$

Introduce the boundary conditions $\xi = -X, \dot{\xi} = U$ at $t = 0$

$$\xi = - \frac{(U - X p_2) \exp(-p_1 t)}{p_1 - p_2} + \frac{(U - X p_1) \exp(-p_2 t)}{p_1 - p_2} \quad - B13$$

We now examine the form of this solution. $\Re(p_1)$ and $\Re(p_2)$ are always positive, so $\xi \rightarrow 0$ as $t \rightarrow \infty$ for all values of U, X and γ (ie the grain will always strike the transducer ultimately). However, p_1 and p_2 may or may not contain an imaginary part.

Case I. $4U\gamma/X < 1$. p_1 and p_2 real. The grain tends asymptotically to $\xi = 0$, but does not cross it (Fig 8).

Case II. $4U\gamma/X > 1$. p_1 and p_2 contain an imaginary part. The solution represents a decaying oscillation about $x = 0$. This occurs because the flow solution, eqn. B9, does not contain the information that the grain cannot pass through the transducer face, but does mirror the flow about $x = 0$.

Only in case II does the grain reach the transducer in a finite time and with a finite velocity.

Returning to the flow, we can now improve on the infinite half-plane assumption. Noting that eqn. B8 has symmetry about $y = \pm x$ (for $b = 0$), it seems reasonable to suppose that a disturbance to the flow at $y = B$ would be reflected in a disturbance at $x = -B$. Thus the departure from the flow, eqn. B8, occurring at the edge of the transducer will correspond to a disturbance the same distance upstream. It is further assumed that this upstream disturbance is the reversion to unobstructed flow. Thus the flow is taken to be roughly represented by eqn. B8 inside a semi-circle of radius X equal to the radius of the transducer face, and by the unobstructed flow outside this (Fig 7).

This gives then the condition that the grain should strike the transducer as

$$\frac{4U\gamma}{X} \equiv \frac{2\rho_s U d^2}{9\rho_v X} > 1 \quad - B14$$

where U is the unobstructed stream velocity and X is the radius of a flat circular transducer face.

The interpretation of case I is that in practice turbulent fluctuations in the y -direction will carry the grain away from the transducer long before it

can strike it. This analysis also holds for grains a short distance off-axis, as the u-component of velocity is independent of y.

The minimum diameter grain which can reach the transducer is thus

$$d_{\min} = \left(\frac{q \rho_v X}{2 U \rho_s} \right)^{1/2} \quad - B15$$

Typical values for d_{\min} are shown in Table B3.

$\begin{array}{c} U \\ X \end{array}$	0.2	1.0	1.6 m s ⁻¹
0.5	70	30	25
2	140	70	50
10	310	150	110
40mm	620	300	210

TABLE B3 MINIMUM DIAMETER (μm) OF GRAIN TO STRIKE A TARGET OF RADIUS X

These values are the mathematical limiting case. In practice the size will be rather bigger, but even the values shown indicate that the target must be really quite small (certainly less than 4mm diameter) if it is to detect the smallest sand grains at all.

One problem which may occur is multiple impacts. A particle which has struck the target will bounce back into the flow and may then make a second impact, though at a much reduced momentum. At the moment the only solution seems to be careful choice of the geometry of the sensor.

B.4 Signal-to-Noise Ratio

Impacts are not the only external forces acting on the target, however. The most important sources of extraneous signals are turbulent pressure fluctuations, and acoustic ambient noise. These both have the same effect on a detector much smaller than one wavelength, and can be represented by a pressure at the detector, $p = p_0 \exp(i\Omega t)$.

The detector being considered is modelled by a simple harmonic oscillator as described in Section B.1. If the front face is circular of radius X, the pressure represents a forcing function of amplitude $p_0 \pi X^2$. The amplitude of oscillation which this will induce in the SHO is

$$a = \frac{p_0 \pi X^2}{M |\omega^2 - \Omega^2|} \quad \text{where } \omega = (k/M)^{1/2} \text{ is the resonant frequency,}$$

or for $\Omega \ll \omega$, $a = p_0 \pi X^2 / M \omega^2$ - B16

The energy which is transferred to the oscillator is

$$\begin{aligned} E_{\text{pressure}} &= \frac{1}{2} M a^2 \Omega^2 \\ &= \frac{p_o^2 \Omega^2 \pi^2 X^4}{2 M \omega^4} \end{aligned} \quad - \text{B17}$$

It was shown in Section B.1 that the energy transferred by the impact of a grain of mass m and velocity v is

$$\begin{aligned} E_{\text{impact}} &= \frac{2 M m^2 v^2}{(M+m)^2} \\ \text{of for } M \gg m, &= \frac{2 m^2 v^2}{M} \end{aligned} \quad - \text{B18}$$

Thus the ratio of signal (impact) to noise (pressure) is

$$\frac{E_{\text{impact}}}{E_{\text{pressure}}} = \frac{\omega^4}{\pi^2 X^4} \cdot \frac{4 m^2 v^2}{p_o^2 \Omega^2} \quad - \text{B19}$$

It is useful to write

$$\frac{E_{\text{impact}}}{E_{\text{pressure}}} = \frac{\omega^4}{\pi^2 X^4} \cdot \Phi \quad - \text{B20}$$

where $\Phi \equiv \frac{4 m^2 v^2}{p_o^2 \Omega^2}$ is due to the external forces, and $\frac{\omega^4}{\pi^2 X^4}$ represents the characteristics of the detector.

Taking first the turbulent pressure fluctuations, we will consider a typical value of 0.1 Pa over a frequency range of 1 Hz centred on 1 Hz (as measured in a tidal stream of speed 0.5 ms^{-1} , 0.18m above a sand bed in 12 m of water), compared with the impact of a $250 \mu\text{m}$ grain in a 0.5 ms^{-1} flow. The ratio Φ works out to be 1.2×10^{-7} . To achieve a final signal to noise ratio of $\gg 1$ with a target of radius 5.6 mm would need the resonant frequency of the oscillator to be $\gg 8.6 \text{ Hz}$.

For the case of acoustic ambient noise a typical value of 1.1 Pa over a range of 100 Hz (representative of a Wind Force 4 over shallow water (Knudsen et al, 1948)) centred on 100 Hz will be compared with the same impact. This gives Φ the value 9.8×10^{-14} , so that a resonant frequency of $\gg 284 \text{ Hz}$ for a target of radius 5.6 mm is required. The same calculation for a Wind

Force 8 taking noise at 10^4 Hz gives $\Phi = 9.2 \times 10^{-17}$ and a frequency $\gg 1626$ Hz. None of these frequencies is likely to be restrictive.

B.5 Detectors

So far the only process which has been considered is that of transferring energy from the sand-grain to an oscillator. This mechanical energy must then be converted to an electrical signal. A number of detectors could be used, namely (a) optical (a light beam deflected by a mirror on the back of a diaphragm); (b) magnetostrictive (the target could consist of a thimble-shaped piece of nickel wound with wire); (c) moving coil (the coil would be attached to a diaphragm); (d) piezo-electric (a ceramic disc could either back a diaphragm, or be struck directly); (e) capacitative (a diaphragm forms one plate of a condenser); and (f) resistive (this is slightly different and will be described separately).

Most of these use a diaphragm as the oscillator, though (b) and (d) use direct compression of the nickel or ceramic. The effect of one impact must be detected quickly to make way for the next impact, so that a low Q system is desirable (almost critically damped, preferably). As already discussed the frontal area must be small in order to reduce the maximum impact rate, to allow the smallest grains to strike the face, and to increase the signal-to-noise ratio. In addition the mass must be small in order to give the maximum energy transfer on impact, and the resonant frequency high (spring constant large) to satisfy the signal-to-noise requirements. Considerations of the sheer physical and engineering difficulties involved in obtaining small mass and area, and high frequency are sufficient reason to rule out all but the piezo-electric and capacitative transducers.

The resistive approach turns to advantage a handicap observed by McQUIVEY (1973) in the use of hot-film flow-meters in sediment-laden flows. It was noted that, at times, the output of the sensor was interrupted by spikes. These were attributed to sand grains hitting the metal film, compressing it and momentarily increasing its resistance. The grains were measured by McQuivey as having a median diameter of $200 \mu\text{m}$ and were part of a suspension of concentration 6000 ppm. A detector constructed along these lines might not have a very long life due to abrasion; the hot film probes are quoted as lasting about 15 hours. The physics of the effect is too complicated to produce an analysis here, but the idea might be worth experimenting with.

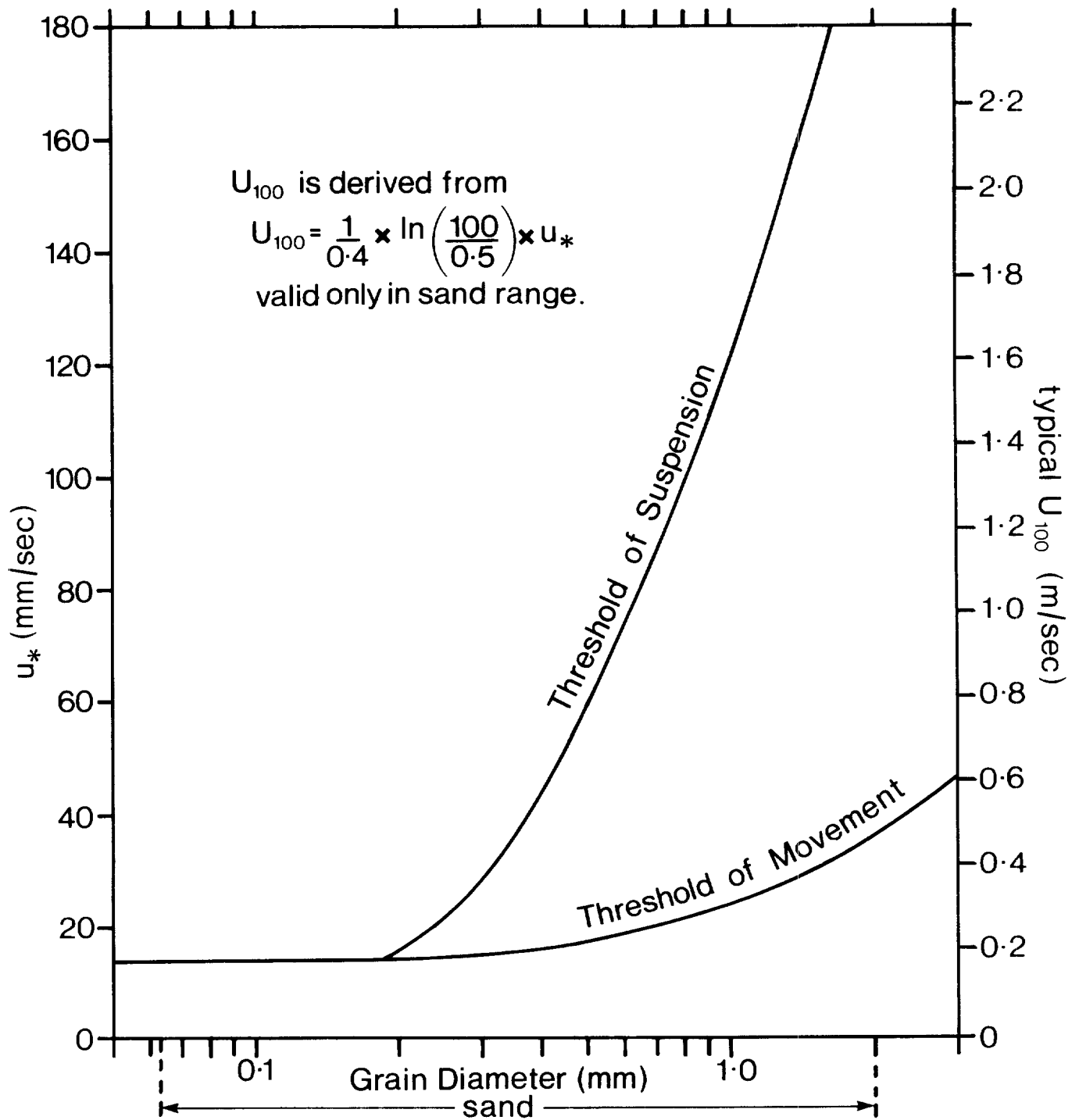


Fig.1

Threshold values of shear velocity for bed-load movement, and for suspension. Values for U_{100} shown assume a logarithmic velocity with a roughness length of 5mm.

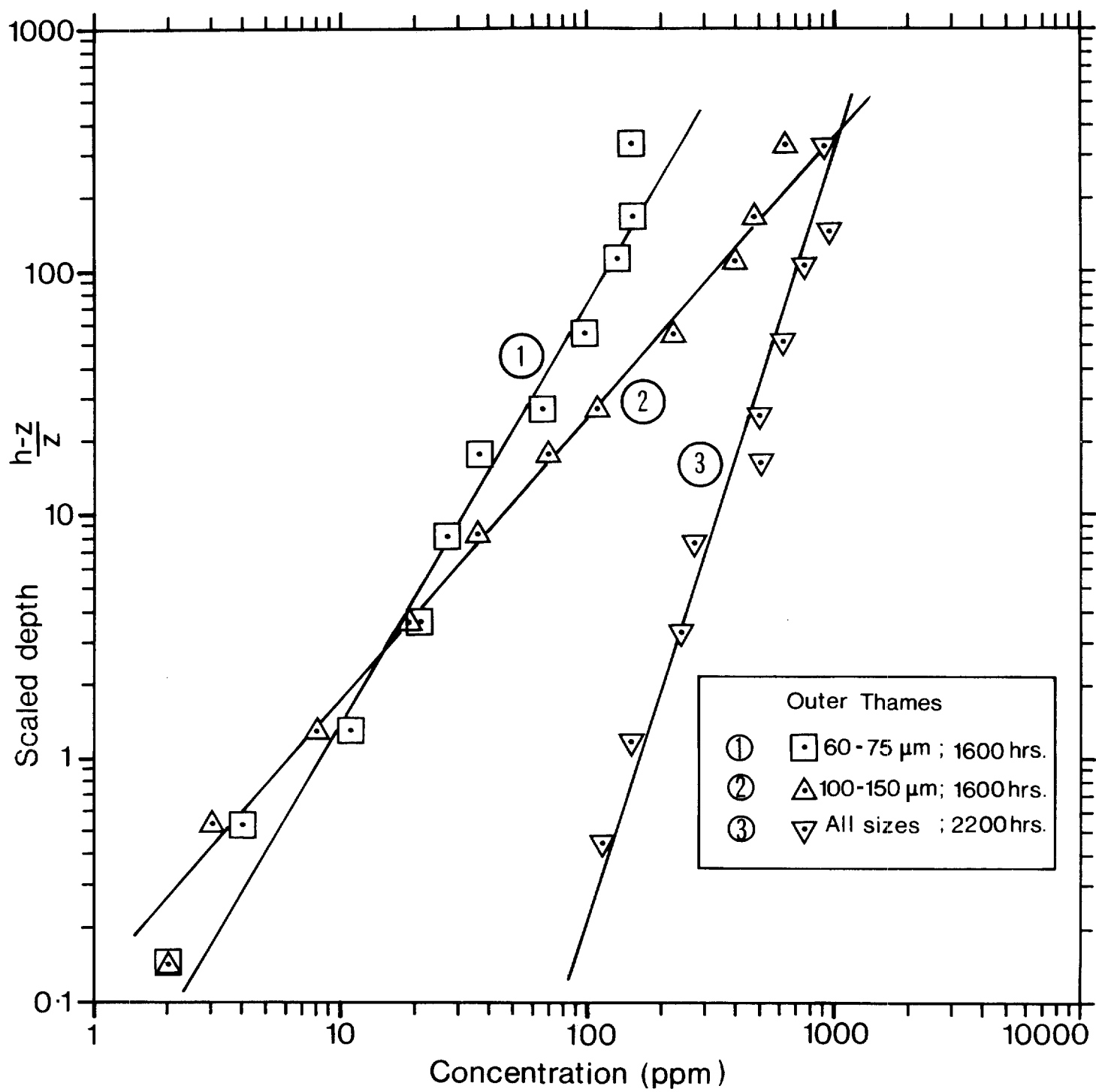


Fig.2

Concentration profiles for the Outer Thames

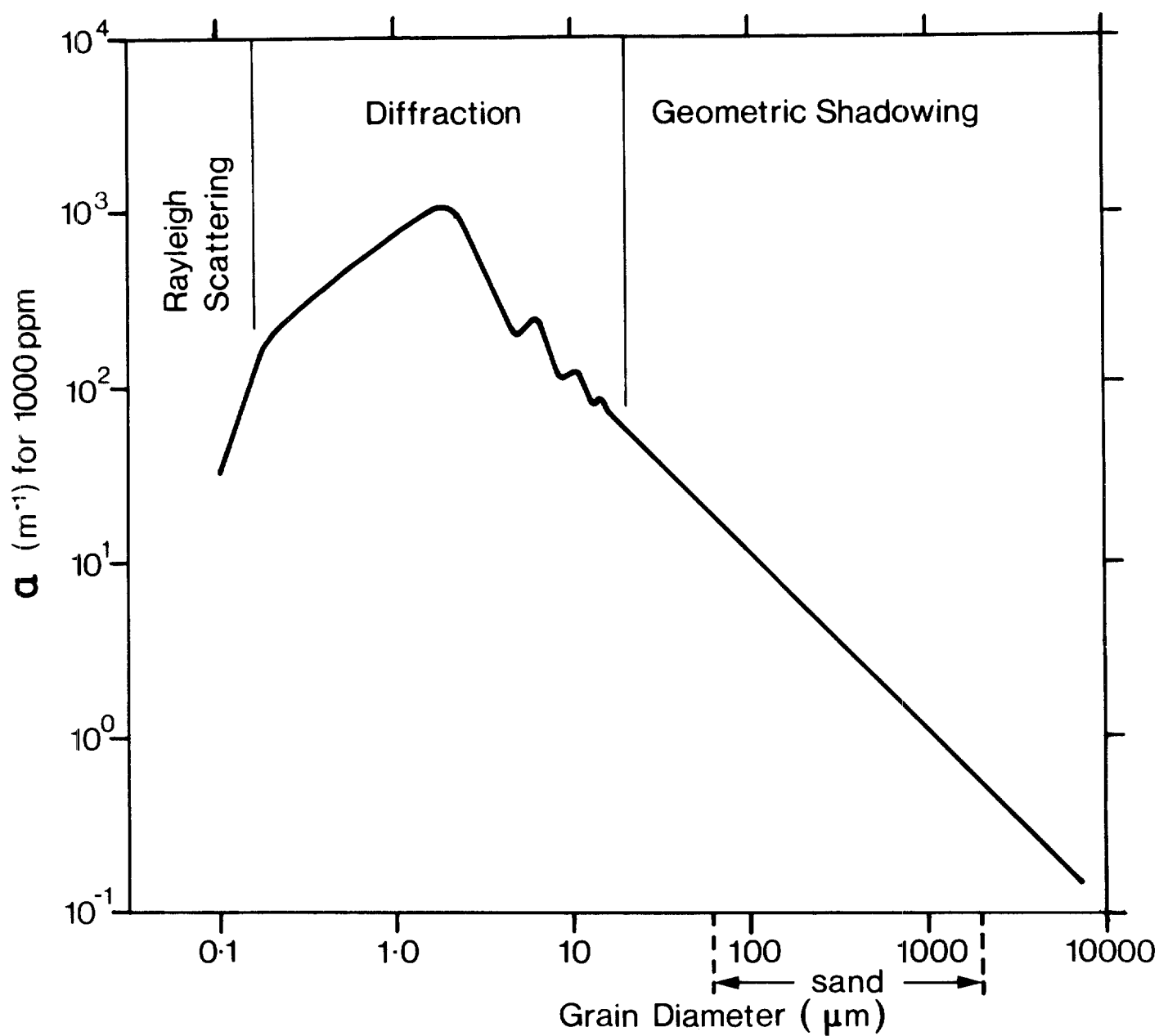


Fig. 3

The beam transmittance coefficient, α , for light of wavelength $0.5 \mu\text{m}$, passing through a 1000 ppm suspension.

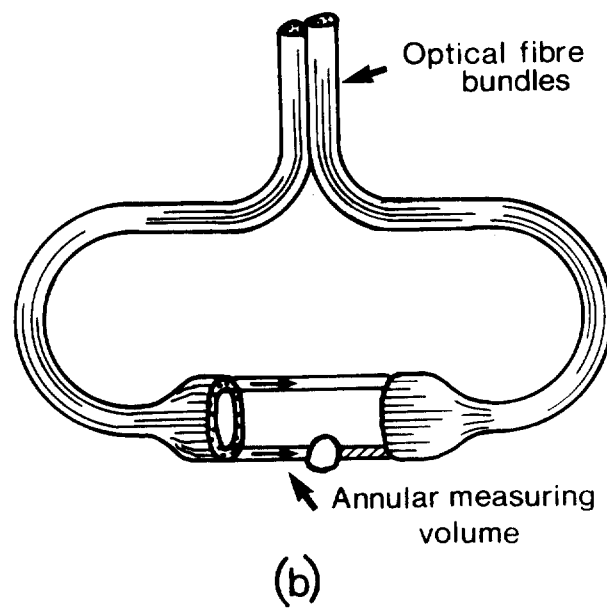
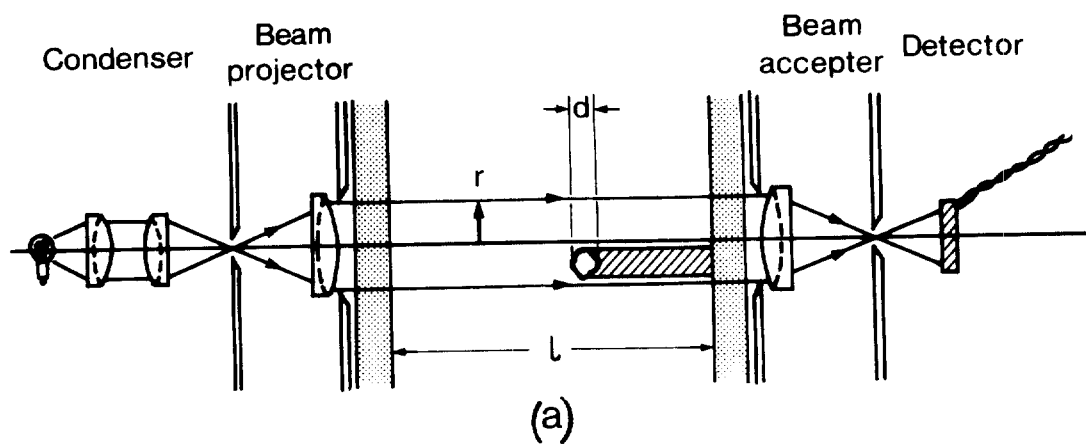


Fig. 4

Layouts for optical occultation systems: (a) cylindrical beam;
(b) annular beam.

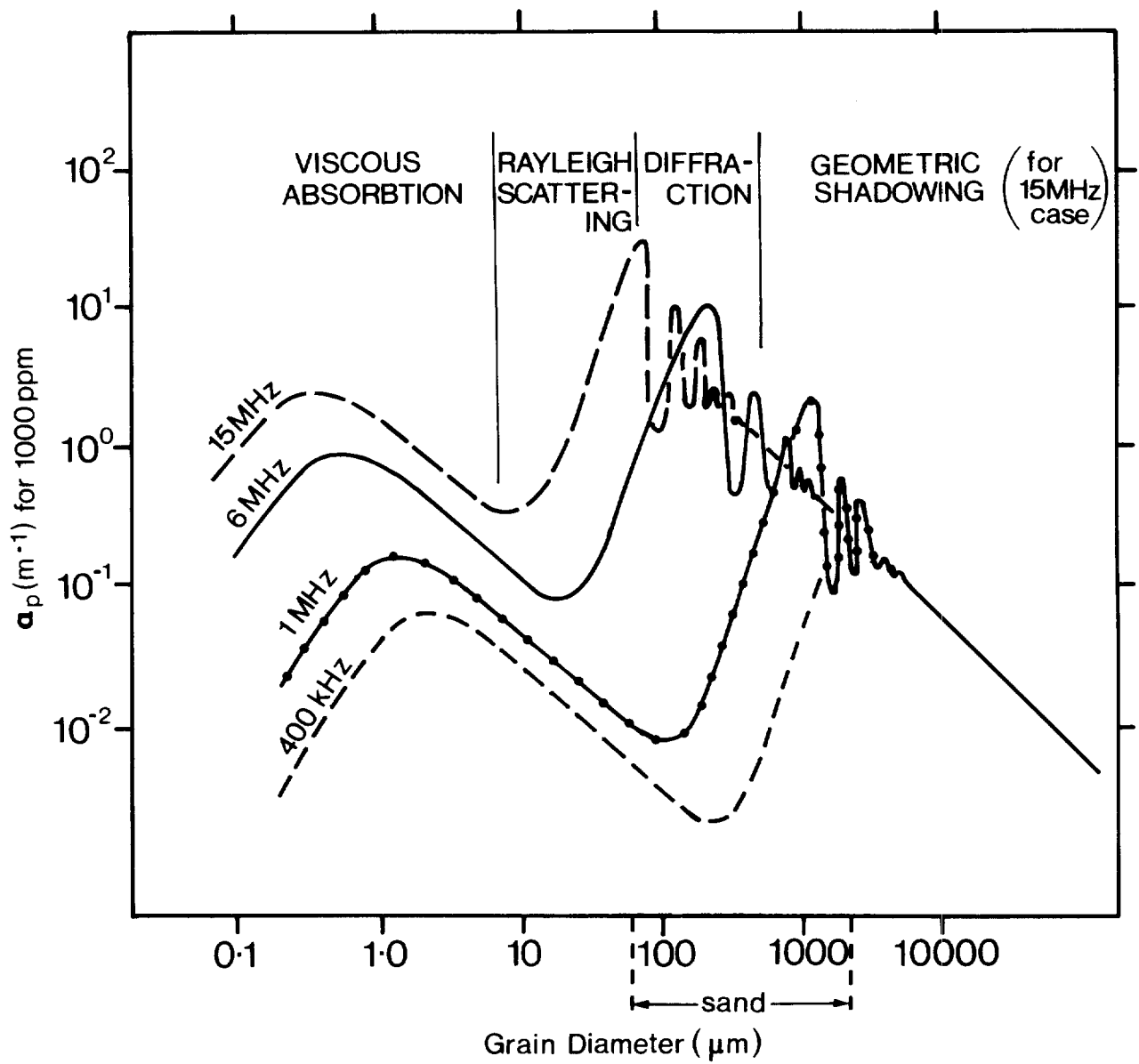


Fig.5

The pressure attenuation coefficient, α_p , for sound of frequencies 400 kHz to 15 MHz passing through a 1000 ppm suspension.

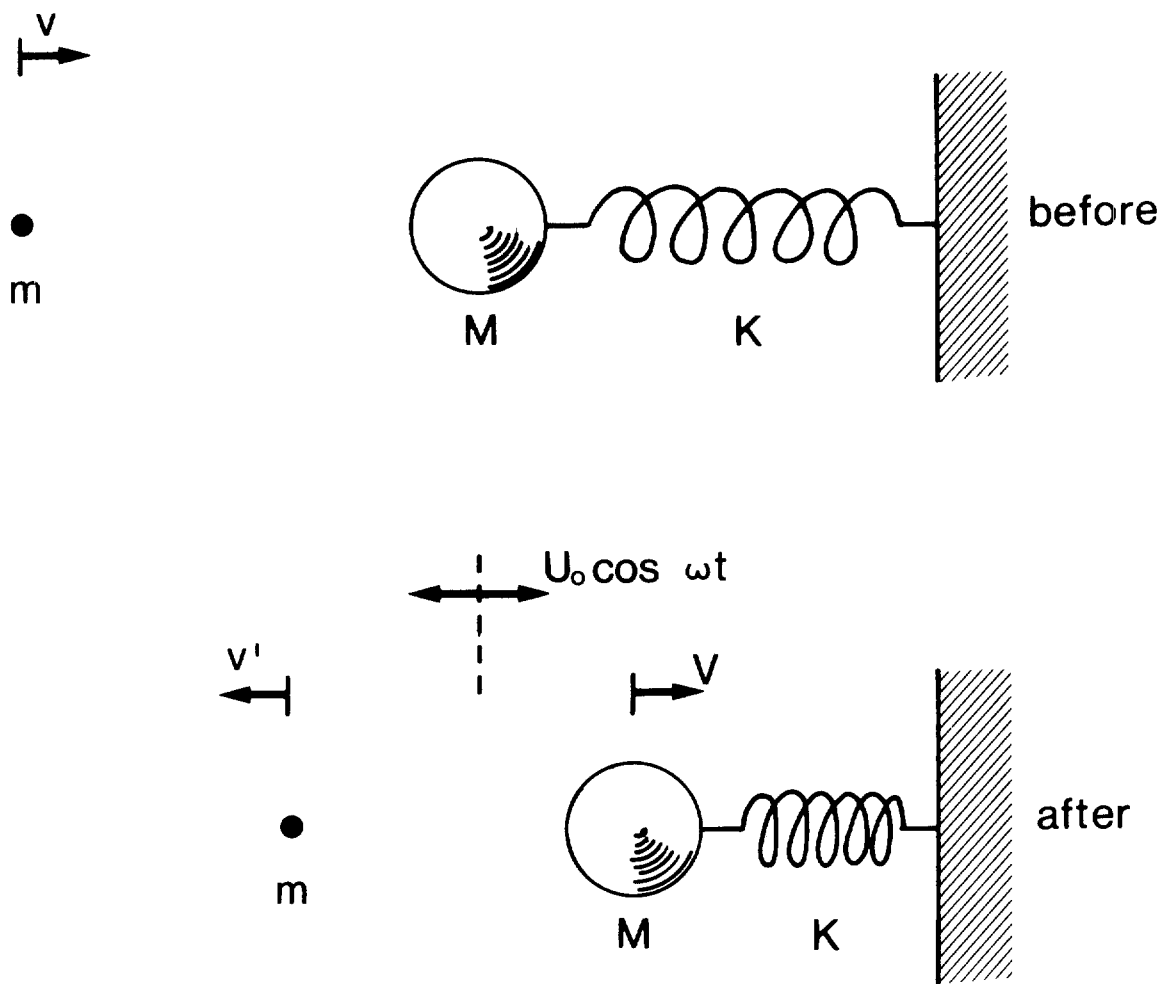


Fig.6

A sand grain striking a transducer, modelled as a point mass striking a simple harmonic oscillator.

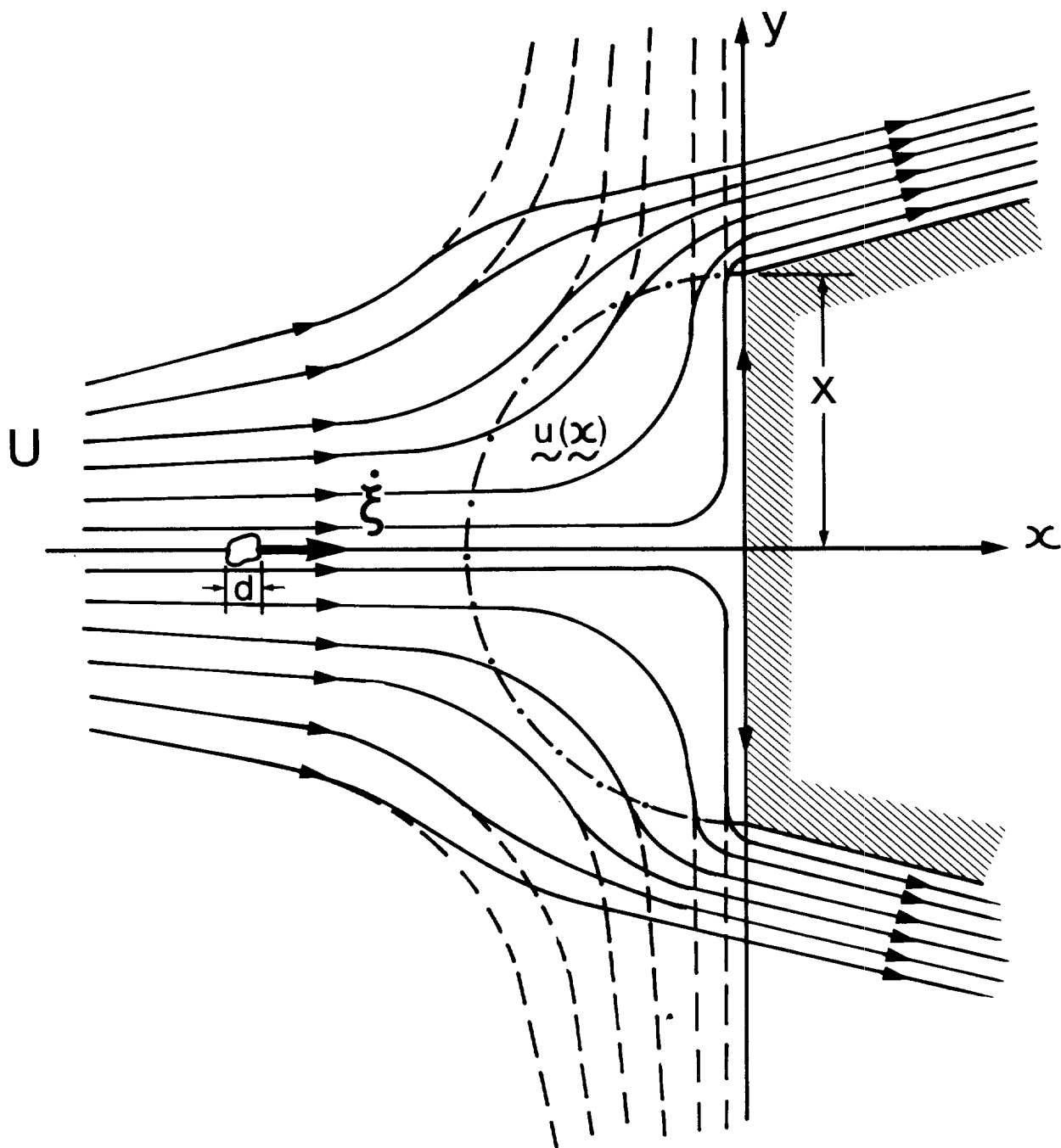


Fig. 7

The flow round a transducer head, with a sand grain moving on-axis.
 The chain-line semi-circle indicates the limit of validity of the
 hyperbolic solution of the flow.

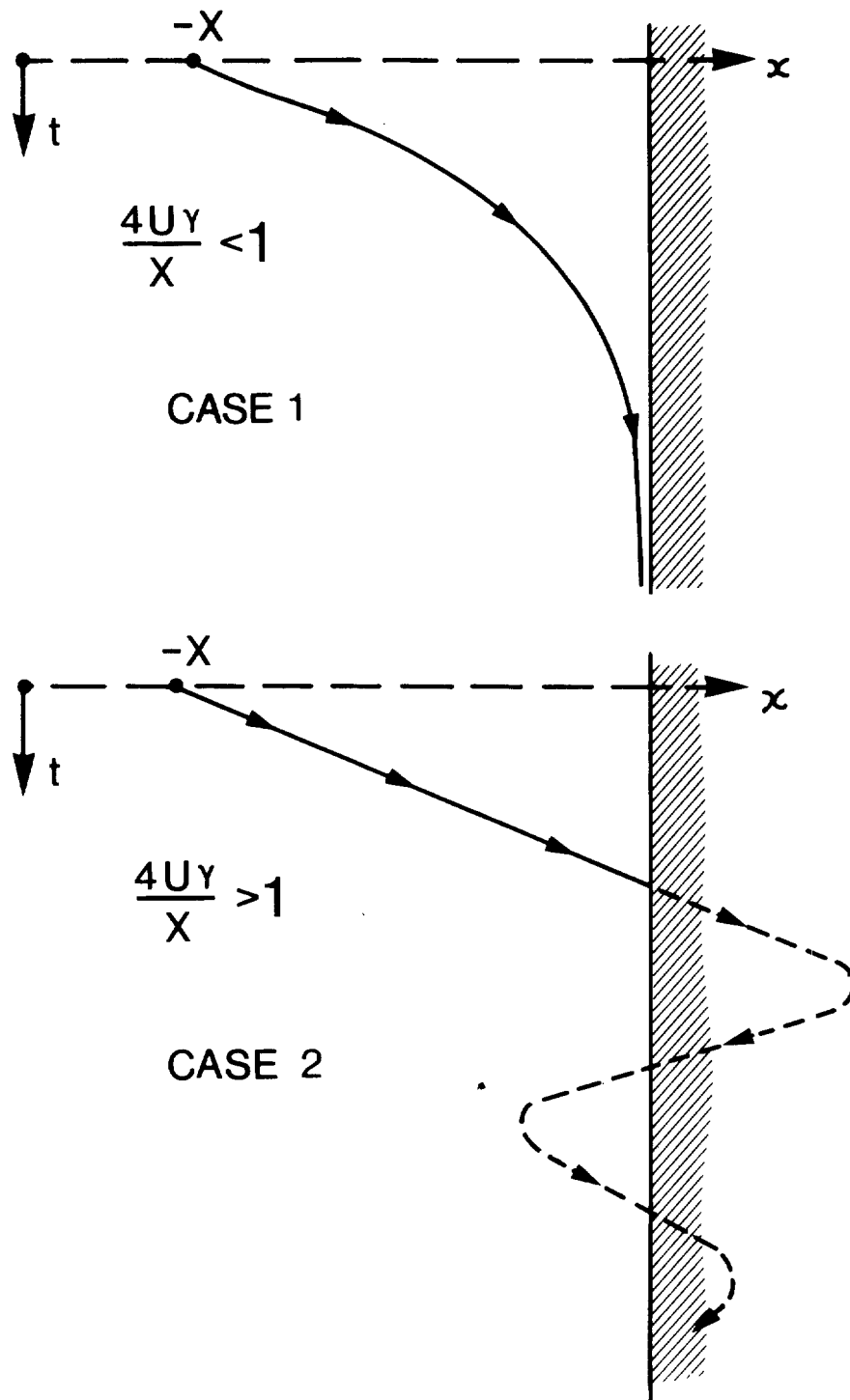


Fig. 8

The two forms of solution obtained for a sand grain in the flow round a transducer. In Case 1 the grain does not strike the transducer within a finite time. In Case 2 the grain strikes in a finite time, and with a finite momentum.