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THE MOVEMENT OF NON-COHESIVE SEDIMENT BY SURFACE WATER WAVES

A G Davies and R H Wilkinson

Part I

Literature Survey

REPORT No 45

1977

NATURAL ENVIRONMENT RESEARCH COUNCIL

INSTITUTE OF OCEANOGRAPHIC SCIENCES

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CONTENTS

ABSTRACT

The passage of progressive surface waves in shallow water may produce sediment transport as either bed load or suspended load. The oscillatory motion induced near the bed may also lead to the formation of sand ripples. This report consists of a literature survey in which both the fluid flow and sediment transport aspects of these problems are discussed. Experimental work and theoretical developments are examined in assessing the degree of understanding of the various topics into which the problem can be subdivided.

It appears that the threshold of sediment motion occurs under similar conditions to the transition from laminar to turbulent flow, though the 'turbulence' that is produced seems to be of a more regular nature than usual. It is uncertain whether the ripple height or the grain diameter is the length relevant to these considerations. However, it is evident that if an eddy viscosity approach is used to model the turbulent flow, the effective viscosity used must be both height and time dependent for instantaneous predictions of parameters of importance to be obtained. Full examination of the boundary layer requires a comparison with an irrotational model; this does not exist for flow over a rippled bed.

The phase angle of the incipient motion in relation to the fluid velocity is important in the study of the threshold and duration of motion per wave cycle. Laboratory tests with monochromatic waves do not consider this aspect, concentrating only on the critical wave's amplitude and period. This, coupled with the natural variability in the waves, makes comparison with field observations difficult.

The effects of the permeability of the bed appear to be of second order except in extreme conditions, but as such may still influence residual sediment motion.

CHAPTER 1

INTRODUCTION

The passage of progressive surface waves over the continental shelf gives rise to bottom velocities of the same order of magnitude as those required to move sediment (DRAPER (1967)). For ocean swell in the offshore zone, there is evidence to suggest that inviscid linear wave theory adequately describes the flow field between the water surface and a level fairly close to the seabed (DRAPER (1957), LUKASIK and GROSCH (1963)). Near the bed where the boundary influences the flow, oscillatory boundary layertheory must be used to predict the velocity profile, the bed shear stress and the associated energy dissipation in the fluid. Although adequate for laminar flows, this theory is not yet well developed for turbulent flow conditions over the rough beds which are commonly found in the sea. The frictional dissipation which occurs in the boundary layer may be associated with sediment movement, and the development of a variety of bed forms. In addition, the granular nature of the seabed means that the bed is permeable to a degree which is dependent upon the grain size. However, in Chapter 4, the effects of permeability are shown to be of only secondary importance in the problem.

Although the oscillatory flow in the offshore zone is one of considerable complexity, the nature of the boundary layer and the mechanics of any resulting sediment transport are capable of approximate theoretical or semi-empirical description. However, within the surf zone following the breaking of the waves, linear theory is no longer capable of describing the wave motion and, up to the present, only crude approximations relating wave energy flux to sediment flux have been developed (eg IPPEN (1966), SILVESTER and MOGRIDGE (1970)). Such approximations as do exist for the calculation of, for example, the littoral drift, take little account of the mechanics of sediment transport. In this report, we restrict our attention to aspects of the problem arising in the offshore zone.

1.1 The boundary layer problem

Many studies have attempted to elucidate particular aspects of the flow field under progressive waves, such as the resultant mass transport and the wave friction factor. Also the characteristics of such features of the granular bed as the ripple length and height have been examined. The studies have been both theoretical and experimental, although few of the experiments have been performed in the field. Leaving aside such second order effects as the mass transport and considering the first order problem of the oscillatory flow in the layer adjacent to the bed, we can identify initially the main areas of importance in the study.

In the first place, we must consider whether linear potential theory is adequate in providing a description of flow conditions outside the boundary layer. Since the usual formulation of small amplitude wave theory applies to waves travelling over an impermeable flat horizontal bed, it is important to develop an understanding of the changes which arise in the inviscid solution when the bed is rippled. The presence of ripples implies that the bed is in some sense 'rough', and it is necessary to consider how easy it is to classify beds as 'smooth' or 'rough' in terms both of ripples and of the individual grains of which the bed is composed. Turning next to the flow in the layer adjacent to the bed, we must ask whether the usual boundary layer equations can be applied in the cases of both rough and permeable beds. Furthermore, we must ask whether the dissipative flow in this zone is laminar or turbulent, and whether we can identify a transition condition between these flow states. This problem is not quite as straightforward as it might appear on account of the definition of the term 'turbulence' in the present context; perhaps the expression 'instability giving rise to momentum transfer at a rate greater than would result from purely molecular processes' is better here, since the term 'turbulence' usually implies a particular behaviour of the kinetic energy of the flow in the frequency domain. At transition, some experimental workers have noted a band of turbulence some small distance above the bed in an otherwise laminar flow (eg CARSTENS and NEILSON (1967)), while others have written of phenomena which they attribute to vortices being shed from the bed and convected up into the main flow (SLEATH (1976), KELLER and SLEATH (1976)). It is possible that the latter phenomenon may be explained in terms of unsteady boundary layer separation, although separation in an unsteady non-uniform flow is a far more difficult concept to define and interpret than in the steady flow case (WILLIAMS and JOHNSON (1974)). There is no doubt that fully laminar and fully turbulent oscillatory boundary layers can exist in extreme cases, but the point of transition has yet to be fully understood and clearly defined. Furthermore the thickness of the boundary layer is critically dependent upon whether or not transition has occurred.

There are a number of important additional questions. If a boundary layer is known to be fully turbulent (rough turbulent), we can ask whether any of the existing turbulent oscillatory boundary layer theories or empirical representations can be adopted with confidence. Furthermore, we must consider whether the velocity amplitude and phase relationships in these theories are sufficiently accurate to draw any detailed conclusions about instantaneous velocity profiles in the boundary layer and about instantaneous values of the bed shear stress. Phase differences in velocities at different heights above the bed are to be expected in turbulent boundary layers, although these may not be similar to those existing in the laminar flow case. If eddy viscosity representations provide a realistic picture of the momentum transfer process in turbulent boundary layers, we must consider whether such representations should be functions of the depth only, or functions of both depth and phase in the wave cycle. All of the questions posed so far have been the subject of theoretical and laboratory investigations. How closely the laboratory studies come to scaling the phenomena as they arise in nature is clearly a matter of the utmost importance.

1.2 The sediment transport problem

The presence of an oscillatory boundary layer implies energy loss from the progressive wave train on the water surface. For the case of a flat horizontal permeable bed, this energy loss occurs by bottom friction and, to a lesser extent, bottom percolation. With these energy losses may be associated sediment motion at the bed, and also in suspension in sufficiently active flows. To first order, sediment transport under waves consists of a symmetrical to- and fro- motion of grains in each wave cycle, the grains being in motion for only that part of each cycle during which the threshold conditions for motion are exceeded. Although experimental and semi-theoretical studies of sediment motion in oscillatory flows have been undertaken by several authors, there seems to be only a limited measure of agreement in their findings. Also, unfortunately, some of the experimental studies have employed techniques which may cast a certain amount of doubt on the findings presented (see section 3.3 (ii)). We therefore have to ask whether the existing approaches are capable of defining realistic conditions for the threshold of motion and, further, whether or not the amount of sediment in motion can be quantified by these means. The latter part is no doubt the most difficult of the entire study.

A symmetrical to- and fro- motion of grains does not of itself provide any overall movement of sediment in a wave cycle. Thus, to predict the net sediment transport under waves, it is generally necessary to look to the second-order mass transport and any other non-zero steady terms. Then, by noting the sediment mobility resulting from the first-order oscillation, it becomes possible to estimate, at least qualitatively at present, the net motion. This approach is likely to be of value in the general problem involving a train of progressive waves superimposed on a tidal stream. Given the present state of knowledge in the field of study it would be unwise to tackle such a complicated problem as this at once. Therefore we concentrate in this report on the progress so far made towards an understanding of the first order problems of boundary layer dynamics and any resulting sediment transport, and, to a lesser extent, on the secondary problem of ripple formation in oscillatory flows.

CHAPTER 2 FLUID FLOW ASPECTS

2.1 The free stream flow

Small amplitude wave theory has been adopted by many authors as an adequate description of the flow field under progressive waves outside the boundary layer. However this theory has seldom been checked under an ocean swell in shallow water.

Basically the theory predicts that a progressive wave of amplitude a and wavelength λ, travelling in the x-direction in water of constant depth h, is associated with a velocity potential ϕ :

$$\phi = \frac{ga}{\omega} \cdot \frac{\cosh kz}{\cosh kh} \cos (kx - \omega t) \qquad (1)$$

where $k = 2\pi/\lambda$ = wave number, g is the acceleration of gravity, t is the time, the z-axis is measured positive vertically upwards from the bed and ω, the wave frequency, is given by

$$\omega^2 = gk \tanh kh \qquad (2)$$

The velocity component in the x-direction, $U\,(= -\partial\phi/\partial x)$, can be shown to be in phase with the pressure p, while the component in the z-direction, $W\,(= -\frac{\partial\phi}{\partial z})$, is in quadrature with p. At the bed $z = 0$,

$$U = \frac{a\omega}{\sinh kh} \sin (kx - \omega t)\,, \quad W = 0 \quad \text{at } z = 0 \qquad (3)$$

from Eqs (1) and (2).

One of the few attempts to check in the field some of the basic phase and velocity amplitude relationships in this theory, has been that of LUKASIK and GROSCH (1963). Having measured the pressure at the bed with an absolute pressure gauge, and the velocity at various heights above the bed with a thermistor device, they have used spectral analysis methods to test for the existence of a zone of potential flow above the bed by making a comparison with the theory. For waves having periods of 8 and 13 seconds, they have demonstrated the in-phase behaviour of the velocity and pressure at a height of 38 cms above the bed, and have verified the predictions of the theory for the relative magnitudes of these quantities. In the layer between 38 cms height and the bed itself, the existence of a laminar boundary layer (having the character of the shear wave solution described in the next section) has been demonstrated by the departure of the observations from the predictions of the potential theory. It is reasonable to assume that the readings were obtained in the presence of a certain amount of background turbulence, even though the principal feature of the flow field was the wave action. The results indicate that the main properties of surface waves are modified very little by some turbulence in the body of the fluid, where potential theory can be applied.

The small amplitude theory has been developed for a flat bed, and departures from Eq (1) may be expected to arise if the bed is rippled significantly. A number of authors have pointed this out (eg KENNEDY and LOCHER (1972)), but no theoretical developments on this aspect of the problem appear yet to have been made. As we argue later, the uncertainty about the effect of ripples on the potential flow means that the boundary layer cannot easily be identified in the usual way, that is by identifying the layer in which departures from potential theory arise. (LUKASIK and GROSCH (1963) made some of their readings above a hard sand bed with 2cm high ripples, but this was sufficiently flat apparently for the theory to still apply.) Essentially, two situations can arise when the bed is rippled: firstly, a fluid oscillation in which no separation occurs over the ripples and secondly, in more vigorous conditions, an oscillation in which separation occurs in the lee of the ripples and which is associated with the formation of vortices. Such vortical activity has been noted in the laboratory by a number of experimental workers (eg HORIKAWA and WATANABE (1970) and CARSTENS and NEILSON (1967)) and is discussed in the later sections. Although vortices are modelled in hydrodynamics as potential flow features, Carstens and Neilson are correct in pointing out the lack of a mechanism for their generation in an irrotational flow model. Nevertheless certain theoretical advances appear to be possible and are very much needed in determining the character of the potential flow over a rippled bed. In conclusion, given this proviso concerning a rippled bed and another concerning possible small effects arising from the permeability of a sand bed, we may accept that linear theory is adequate to first order outside the boundary layer.

2.2 The laminar boundary layer

At the bottom flow boundary, which we will assume initially to be smooth and impermeable, the fluid velocity must be zero. In place of (3) from potential theory, we require in practice

$$U = 0 , \quad W = 0 \qquad \text{at} \qquad z = 0 \tag{4}$$

Thus an oscillatory boundary layer of some sort must exist, and this layer is normally assumed to be sufficiently thin for (3) to be rewritten as an outer boundary condition, as follows:

$$U_\infty = \frac{a\omega}{\sinh kh} \sin(kx - \omega t) \quad \text{at} \quad z = \delta \tag{5}$$

where δ is the thickness of the boundary layer. The total water depth does not play a role in the processes within the frictional layer ($0 < z < \delta$) if $\delta < h$, although in the extreme case of a shallow water wave δ may be limited by the depth h. The boundary layer being thin it is assumed that the free stream pressure gradients are impressed through it and, therefore, that the pressure distribution within the boundary layer is the same as for the corresponding region of potential flow. On this basis, the linearised equation of motion in the x-direction

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(\nu \frac{\partial U}{\partial z}\right) \tag{6}$$

where ν is the molecular viscosity, can be solved to give to first order

$$U = U_\infty\Big|_{max} \left\{ \sin(kx - \omega t) - e^{-\beta z} \sin(kx - \omega t - \beta z) \right\} \tag{7}$$

where $\beta = \sqrt{\omega / 2\nu}$ is called the 'shear wave number' and $U_\infty\Big|_{max} = \frac{a\omega}{\sinh kh}$. Equation (7) is the well-known shear wave solution for laminar flow (eg LONGUET-HIGGINS (1958)), in which both phase and velocity amplitude are functions of distance from the bed. It can be simply shown that the bottom stress $\tau_0 = \rho\nu \frac{\partial U}{\partial z}\Big|_{z=0}$, leads U_∞ in phase by $\pi/4$. The principal features of the first order oscillation (with $x = 0$ in Eq (7) are shown in Fig 2.1. In this figure, taken from LAMB (1932), the curves represent the successive forms assumed by the same line of particles, at intervals of one tenth of a wave period.

The solution (7) has been verified experimentally over a smooth bed by SLEATH (1970b). However, in the case of a rough sand bed, significant departures have been found. Sleath explains these in terms of the formation of vortices in the lee of individual sand grains and the consequent change in the pattern of momentum transfer in the boundary layer as a whole. The principal effects of vortex formation are seen to be an increase in the boundary layer thickness, and a reduction in the phase lead of the velocity in the immediate vicinity of the bed over the free stream velocity. To account for the 'sheet' of vortices at the level of the bed, Sleath has suggested a relaxation of the 'no slip' condition at the bottom boundary (Eq (4)).

The role of the advective non-linear terms in the equation of motion has been examined by GROSCH (1962), who has sought an exact solution of the laminar boundary layer equation by setting up an initial value problem in which $U_\infty = 0$ if $(kx - \omega t) > 0$. The power series solution developed has much in common with classical steady boundary layer analysis, and has been compared by Grosch with the theoretical solution for the comparable linearized initial value problem. Comparisons of velocity profiles and bottom stresses suggest that if the parameter $\alpha = \frac{ak}{\sinh kh} << 1$ or if $|kx - \omega t| \approx 0$, linear theory is valid. Otherwise the non-linear terms play a significant role, the flow being substantially different from that predicted by linear theory, not only in amplitude but also in phase. In particular, the stress is shown to be underestimated by linear theory, so too are the velocity gradients close to the bed.

The retention of the non-linear terms also leads to solutions containing small second-order time independent drift or mass transport velocities. This aspect of the problem has been the subject of many theoretical treatments. LONGUET-HIGGINS (1958) and JOHNS (1970) have examined the case of small amplitude waves over a flat bed, and the former paper contains a simple physical description of the motion.

SLEATH (1972) has extended this analytical work to finite amplitude waves arguing that the higher order terms produce significant effects in mass transport velocities. SLEATH (1974) has also studied the problem of the mass transport over a rough bed, and has shown that the effect of bed roughness is to produce an increase in the mass transport velocity, except for very long waves for which a decrease may result. Also, using a numerical model, SLEATH (1974b) has shown the very different results that can be expected when the roughness elements of the bed are firstly small, and secondly large compared with the boundary layer thickness. These aspects will not be treated further here.

2.3 Boundary layer thickness, transition to turbulence and bed roughness

These three topics are so interrelated, and definitions of one so dependent upon those of the other two, that they will be dealt with together.

In general, if a solid boundary is introduced into a flow field, the boundary layer is defined as the zone in which the previously undisturbed flow is affected in some way. The thickness of the boundary layer will depend on which property of the flow is used in its definition, although this will not cause much variation in practice. In laminar wave flows of periods around 10 secs, the thickness is about 12 mm (ie the wavelength of the vertically propagating shear wave). In 'turbulent' flows, JONSSON (1963) and JONSSON and CARLSEN (1976) have found the thickness to be about 6 cms, with a wave period of the order 8 secs.

The standard steady definitions of displacement, momentum and 99% U_∞ thickness (SCHLICHTING (1968)) do not hold for laminar oscillatory flows, as the nature of the laminar solution (Eq (7)) causes the resulting integrals to become infinite and even negative at certain stages in the wave cycle. In fact, at certain phases, the velocity, mass flow and momentum flux can be greater than, or in the opposite direction to, the free stream. Thus alternative definitions have made use of the amplitude of the shear wave. For example LI (1954) has defined the upper edge of the boundary layer as the level at which the velocity amplitude is within 1% of the free stream velocity; this occurs at $4.6/\beta$ from the bed. Other possibilities involve arbitrary fractions of the shear wavelength, for example $3/\beta$ (LUKASIK and GROSCH (1963)).

As there is yet no generally accepted form for the 'mean' velocity profile in turbulent oscillatory flow following the breakdown of Eq (7) at transition, there are similarly no definitions of boundary layer thickness under these conditions. Different workers in developing turbulent solutions have used different definitions. For instance, JOHNS (1975) defines the wave boundary layer as the region close to the bottom in which a logarithmic velocity profile exists. This boundary layer thickness is shown to be periodic with the wave period, and Johns' numerical results suggest a typical value of 3 mm. On the other hand, KAJIURA (1968) has developed a time averaged model of the turbulent flow in the wave boundary layer, in which his definition of thickness is independent of phase angle. In particular, he has used a wave displacement thickness, δ^*, where

$$\delta^* \bar{U}_\infty \Big|_{max} = \text{Amplitude} \int_0^\delta \left(\bar{U}_\infty - \bar{U} \right) dz$$

Another model of the turbulent wave boundary layer from which boundary layer thickness can be derived has been proposed by JONSSON (1963) and JONSSON and CARLSEN (1976). They have defined the thickness of the boundary layer as the distance between a zero datum level and the level at which the velocity first reaches the free stream value, and they have used this definition in a semi-theoretical model of the flow. In addition, they have calculated the boundary layer thickness from experimental velocity observations made with a micro-propellor in an oscillatory water tunnel; in the experiments the bed roughness was rather large, and the boundary layer was found to be about 6 cms. thick. However there is a lack of clarity in their model as to whether the boundary layer thickness is dependent upon phase in the wave cycle, and whether there is even a basic consistency between model and experiment in this matter.

One of the crucial factors influencing the transition to turbulence in the boundary layer is the ratio of boundary layer thickness to the size of the bed roughness elements. For a steady unidirectional flow, the transition to turbulence over a flat plate is typified by constant values of such Reynolds numbers as $U_\infty x/\nu$ and $U_\infty \delta/\nu$ (where x is the distance from the leading edge and δ is the thickness of the laminar boundary layer). Attempts have been made to find similar Reynolds numbers for an oscillatory boundary layer. In this case, the parameters governing the flow over a rough bed are:

$$U_\infty\Big|_{max} ,\ A ,\ \rho_f ,\ \mu \text{ and } k$$

where $U_\infty\Big|_{max} = \frac{A\omega}{2}$, μ is the dynamic viscosity and k is the roughness size. Clearly the parameters $U_\infty\Big|_{max}$ and A specify only a simple harmonic motion. General unsteadiness in the flow can be approached either by means of a Fourier synthesis of the motion or by the specification of the derivatives of the fluid velocity field. In the comparatively simple case of a sinusoidal motion, what is sought is a 'critical wave' cycle for transition. In practice, however, it is possible that portions of this critical cycle will be on either side of the transition point. This makes the precise definition of conditions for transition to turbulence difficult. Application of Buckingham's π- Theorem reveals that there must be two dimensionless groups that typify the flow, one possible combination being $U_\infty|_{max} k/\nu$ and k/A . (Alternative forms for the Reynolds number are shown in Table2.1.) Any non-dimensional dependent variable of the flow can then be specified as a function of these two groups. In the present problem, a variable must be selected that can be used to typify the transition from laminar to turbulent flow.

One of the fundamental features of a turbulent flow is that, under similar mean flow conditions at a given point, momentum is transferred from regions of high to low bulk velocity at a greater rate than in laminar flow. In the latter case, the rate of momentum transfer (which is proportional to the shear stress on the plane normal to the direction, say z , in which momentum is being transferred) is such that

$$\tau_f = \mu \frac{\partial U}{\partial z}$$

Here the molecular viscosity μ is the relevant momentum diffusion coefficient. Turbulent mixing effectively increases this coefficient, and the increased momentum transfer rate can be written such that the turbulent shear stress, τ_t, is given by

$$\tau_t = (\mu + \varepsilon_m) \frac{\partial \bar{U}}{\partial z}$$

This technique was first used by BOUSSINESQ - see SCHLICHTING (1968). The turbulent momentum diffusion coefficient ε_m (or effective dynamic viscosity) is a property of the flow and of position in the flow, whereas the laminar coefficient μ is a property of the fluid and is thus a constant. If the diffusion coefficient ($\mu + \varepsilon_m$) is non-dimensionalised with respect to the molecular viscosity, then transition to turbulence can be defined as occurring when the non-dimensional momentum transfer coefficient $K_\nu = \left(\frac{\mu + \varepsilon_m}{\mu}\right)$ departs significantly from unity.

The dimensional analysis above has shown that K_ν is a function of $\frac{\bar{U}_\infty|_{max} k}{\nu}$ and k/A, and so contours of constant K_ν can be drawn in the plane of these two variables. If the onset of turbulence at a particular point is defined by K_ν taking a critical value, $K_{\nu crit}$, the contour corresponding to this will show the critical Reynolds number $\bar{U}_\infty|_{max} k/\nu$ for transition to turbulence as a function of k/A. (We recall here that we are attempting to define a critical cycle, even though the state of turbulence may vary during this period. The average value of K_ν over the cycle will depend very much on the measurement technique used.) If we assume that K_ν is given by

$$K_\nu = c \left(\frac{\bar{U}_\infty|_{max} k}{\nu}\right)^a \left(\frac{k}{A}\right)^b$$

it is possible to draw analogies between steady and sinusoidal flow, since it is normally supposed that 'hydraulically smooth' and 'rough turbulent' flows are typified by being independent of bed roughness and molecular viscosity respectively (YALIN (1972)). Thus, if we let $a = -b$, we obtain an equation relating to the hydraulically smooth flow condition:

$$K_\nu = c \left(\frac{\bar{U}_\infty|_{max} A}{\nu}\right)^a \tag{8}$$

and similarly, if we let $a = 0$ we obtain for the hydraulically rough condition:

$$K_\nu = c \left(\frac{k}{A}\right)^b \tag{9}$$

Consider a graph of κ against $\frac{\bar{U}_\infty|_{max} k}{\nu}$ for several values of relative roughness (Fig 2.2). Low values of Reynolds number will correspond to $\kappa = 1$ where the flow is laminar. Departing smoothly from this line, at different $Re = \frac{\bar{U}_\infty|_{max} k}{\nu}$ for different k/A, will be curves given by Eq (8). At high values of Reynolds number, Eq (9) will hold, and thus the curves will be parallel with the horizontal axis. Between the smooth and rough conditions will be a transition region in which the curves are of unknown form. Initially, we can argue that if the curves increase monotonically between the laminar and the smooth turbulent zones as depicted, the index 'a' must be > 1. This results in the curves of increasing k/A being displaced as shown, from Eq (8). If also the curves of constant k/A do not intersect in the transition region, then from Eq (9) it can be seen that $b < 0$. The above information when transferred to the $(\bar{U}_\infty|_{max} k/\nu,\ k/A)$ plane gives contours of constant κ, as shown in Fig 2.3.

Although the property κ of the flow cannot be measured directly, any other non-dimensional variable will have the same general behaviour as κ in terms of the dimensional arguments presented above. HINZE (1959) states that the transfer of mass and momentum appear to be approximately analogous, and JOBSON and SAYRE (1970) show that the diffusion coefficient for dye ε_D has a value within 3 % of the momentum diffusion coefficient ε_m. Flow visualisation experiments can be used to detect large changes in ε_D, but the subjectivity involved will inevitably result in a rather vague criterion unless curves comparable to those in Fig 2.2 are steep when κ is increasing. Similarly, the non-dimensional bed shear stress can be used (eg JONSSON (1966)) in the formulation of a transition criterion.

Following guide lines from the steady flow case, experimental workers have attempted to identify the magnitude of a suitable Reynolds number and the size of the bed roughness, at the transition points between laminar, smooth and rough turbulent flows. The thin oscillatory boundary layer has been studied mostly by flow visualization techniques. Typically, dye has been introduced into the boundary layer, and some flow parameter increased until transition has been detected visually. Unfortunately, the results from such experiments have been rather ambiguous, different workers interpreting different phenomena as transition, and their descriptions of the critical flow phenomena observed varying somewhat. For example, CARSTENS et al (1969) describe how, in the transitional range, vortices were formed about $\frac{1}{2}$ cm from the bed (with axes parallel to the bed and perpendicular to the direction of motion) at about the time of flow reversal, and how these persisted for an increasing portion of the cycle as fully turbulent conditions were approached. LI (1954) however, simply recalls a disordered mixing. Something similar to Carstens' vortex formation is referred to by VINCENT (1957) who observed 'tongues or flames' of dye being ejected from the bed a little way into the flow at transition to turbulence, in a model wave flume. This has also been the experience of SLEATH, who has observed 'flames of dye' in a wave flume (1970b) and over oscillating rough plates ((1975) and KEILLER and SLEATH (1976)). In addition, the same flow patterns have been said to have been found in a numerical model of viscous oscillating flow over a rough bed (1974c).

Thus there appears to be some uncertainty about the type of turbulence that occurs after transition. That there is an increase in momentum transfer in the 'turbulent' regime is evident, but whether this is due to 'normal' turbulence (ie a random eddying motion) or an organised motion at particular points of the wave cycle is debatable. The experimental and numerical evidence of Sleath supports this latter view. He interprets the motion as involving the formation of discrete vortices behind ripples or individual grains under vigorous flow conditions, with upward velocities occurring at certain phase angles in the wave cycle. These effects are repeatable between cycles, and SLEATH (1974c) argues that, as such, are part of a laminar flow process. It is interesting to note that this phenomenon may not be wholly unconnected with that described by WILLIAMS and JOHNSON (1974). They have shown analytically that in an unsteady flow, decelerating uniformly in both space and time, an increase in the vertical velocity, W, is found in the boundary layer as separation is approached. They point out that, in unsteady flow, the criterion for boundary layer separation of a vanishing shear stress at the wall (which is normally used in steady flow) is no longer valid. They suggest that the unsteady separation point is characterised by (i) the simultaneous vanishing of shear and velocity at a point in the boundary layer seen by an observer moving with the separation point, and (ii) a singular behaviour of the boundary layer equations at this point. A wave boundary layer flow involving periodic separation has been suggested by EAGLESON (1959), but the authors find this model unconvincing since the definition of the unsteady separation criterion is similar to that for steady flow. COLLINS (1963) has adopted a different approach. Knowing the residual velocity in a laminar wave boundary layer (LONGUET-HIGGINS (1953)), Collins has defined transition to turbulence as occurring when the observed velocity departs from this. Unfortunately, Collins' criterion may be theoretically insupportable, since

LONGUET-HIGGINS (1958) has shown that the residual velocity is the same at the edge of the boundary layer for both laminar and turbulent flows.

The values obtained by various workers for Reynolds numbers at transition to turbulence are shown in Table 2.1. In the first place, it can be seen that there is not even complete agreement as to which is the relevant Reynolds number to adopt. The contribution of EINSTEIN (1972) is a summary of the results obtained by LI (1954), MANOHAR (1955) and KALKANIS (1957), and the formula of SLEATH (1974c) is based on the results of the dye injection experiments of LI, MANOHAR, L'HERMITTE (1961) and VINCENT (1957). The previous dimensional arguments, relating to Figs 2.2 and 2.3, indicated a relationship at transition from laminar to smooth turbulent flow of the form:

$$\frac{\bar{U}_\infty\big|_{max}\, k}{\nu} = \{Constant\}\left(\frac{k}{A}\right)$$

Sleath's presentation of results would appear to show, therefore, that the turbulent flow after transition is neither hydraulically rough nor smooth in the conventional sense.

Under field conditions, the wave velocity amplitude in the free stream may be anything up to 40 cms/sec and the wave period may lie between 5 and 15 secs. Typical granular beds may be composed of roughness elements of up to 5 cms in height (although the distinction between ripples, which are presumed two-dimensional, and sand or cobbles, three-dimensional, is unclear in the present context). Thus it is possible to obtain values of Reynolds numbers up to:

$$Re_1 = 7 \times 10^{-5} \quad ; \quad Re_2 = 2 \times 10^{4} \quad ; \quad Re_3 = 8 \times 10^{2} .$$

According to the criteria in Table 2.1, it is therefore possible for the wave boundary layer in the sea to become turbulent. However it is not clear at present when this will happen.

2.4 Turbulent boundary layer representations

(i) Formulation of the problem

In a laminar boundary layer, Eq (6) is taken as the equation of motion. Following the discussion in the previous section, the linearized equation of motion for a turbulent boundary layer becomes

$$\frac{\partial \bar{U}}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial x} \qquad (10)$$

where the stress τ is given by

$$\tau = (\mu + \varepsilon_m) \frac{\partial \bar{U}}{\partial z} \approx \varepsilon_m \frac{\partial \bar{U}}{\partial z} \tag{11}$$

since $\varepsilon_m \gg \mu$ for fully developed turbulent flow. Since $\tau \to 0$ as $z \to \delta$, the linearized equation of motion for the flow outside the boundary layer is

$$\frac{\partial \bar{U}_\infty}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial x} \tag{12}$$

which, when subtracted from Eq (8) gives

$$\frac{\partial \bar{U}}{\partial t} = + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\varepsilon_m \frac{\partial \bar{U}}{\partial z} \right) + \frac{\partial \bar{U}_\infty}{\partial t} \tag{13}$$

The adoption of this form, which as in the laminar case omits the non-linear terms, can be justified if the total thickness of the bottom frictional layer is very much smaller than the wavelength of the potential surface wave. Since $\bar{U}_\infty$ is prescribed (apart from a certain ambiguity in the boundary layer thickness), solutions for the instantaneous velocity profile $U(z,t)$ can be obtained if ε_m is known. Hence, from the velocity profile, the instantaneous value of the shear stress can be calculated. A slightly different formulation has been adopted by JOHNS (1975) who, working explicitly with a free surface, has studied the turbulent boundary layer characteristics under a long wave.

(ii) The empirical approach

Rather than solving Eq (13) above, subject to some assumption about ε_m, KALKANIS (1957) has assumed that the form of the laminar solution (7) is preserved in the case of turbulent wave boundary layers such that

$$\bar{U} = \bar{U}_\infty \Big|_{max} \left\{ \sin \omega t - f_1(z) \sin (\omega t - f_2(z)) \right\} \tag{14}$$

for a uniform oscillation. The functions $f_1(z)$ and $f_2(z)$ have been the subject of experimental determination above an oscillating bed. There is no indication that Kalkanis has attempted to verify experimentally the laminar flow solution (7). However, for smooth beds, and both two and three-dimensionally rough beds, the functions f_1 and f_2 have been obtained by curve fitting. For all beds, the function f_1 has been assumed to take the form $f_1 = (\text{constant})_1 \exp\{-(\text{constant})_2 \, z\}$; while $f_2 \propto z^{1/3}$ for the smooth bed, and $f_2 \propto z^{2/3}$ for both varieties of rough bed (the phase shift being identical, in fact, in these cases). From the results of the curve fitting, it is apparent that two-dimensional roughness elements permit the velocity amplitude $f_1(z)$ to increase much faster away from the bed than do the three-dimensional roughness elements. Solutions having the form (14) may be expected to hold only in the range of values of z over which the curve fitting was carried out.

KAJIURA (1968) has criticized these empirical results on the grounds that the flows in question may not have been in the range of fully developed turbulence. His criticism concerns the fact that the viscosity ν appears in the empirical representations of f_1 and f_2, and he argues that this should not be the case for fully developed turbulent flow over a rough bottom. However Kalkanis does not claim the forms f_1 and f_2 to be any more than a convenient representation; indeed, he suggests that the true distributions may follow some other law.

A second purely empirical assumption to account for departures from the laminar solution (7) over sand beds has been suggested by SLEATH (1970b) (see section 2.2). He shows, at least as far as the variation in amplitude of velocity with height above the bed is concerned, that the replacement of β by (β/χ) in Eq (7), where χ is a constant in any given experiment, accounts adequately for the differences in the momentum transfer in the flow over a sand bed, compared with the case of laminar flow over a smooth bed. Sleath has proposed an empirical representation for χ and has found its value to vary in the range $1.0 < \chi < 1.8$.

(iii) Eddy viscosity representations

The empirical approach outlined above was developed because it was found that the velocity distribution could not be described using methods of vorticity and momentum exchange known to be successful in uni-directional flows. However this conclusion is not at all well borne out by the detailed arguments presented by KALKANIS (1957), who manages to show only that certain specific analytical assumptions made on the basis of laboratory observations are incompatible with an approach based on the definition of an eddy viscosity of some sort. Indeed, several authors have achieved a measure of success employing the eddy viscosity approach, eg KAJIURA (1968) and JOHNS (1970 and 1975). Furthermore, the very representation of Eq (14) adopted by Kalkanis and others can be shown to be capable of interpretation in terms of eddy viscosity. In particular, Eq (14) can be shown to be compatible with (13), under assumption (11), only if certain relationships involving the eddy viscosity are satisfied.

The main point at issue here seems to be whether the eddy viscosity representation adopted should be a function only of height z above the (flat) bed, or a function of both z and time t . Kajiura has adopted the former course of action, defining an effective viscosity in three regions of the boundary layer (inner layer, overlap layer and outer layer) by analogy with the steady flow case. Kajiura's assumption has been criticised by HORIKAWA and WATANABE (1968) on the basis of their experimental work. This shows quite clearly that the viscosity at a given depth is a function of t (although, in fairness to Kajiura, it is never his main intention to produce information about instantaneous velocity profiles or bottom stresses, rather he has aimed to obtain information about time averaged quantities in the analysis). Horikawa and Watanabe's experimental results also show that the mean value of the eddy viscosity in a wave cycle is a continuous function of the depth, and this supports Kajiura's approach in dealing with certain aspects of the boundary layer problem. JOHNS (1970) adopted a time independent eddy viscosity, but more recently (1975) has taken a time varying representation for rough turbulent flow, which is based on a mixing length assumption.

The experimental results of both JONSSON (1963) and HORIKAWA and WATANABE (1968) suggest a periodic eddy viscosity having twice the wave frequency[1]. Also it is interesting to note that Jonsson's calculated values of the mean eddy viscosity are an order of magnitude greater than those which can be deduced from Kalkanis' results; this emphasises some of the inconsistencies noted earlier concerning the definitions of transition to turbulence.

A peculiar feature in the results of both Jonsson and Horikawa and Watanabe is that, at certain instants in the wave cycle, their calculations suggest that the eddy viscosity takes a negative value. This is clearly a physical nonsense which is brought about by calculating instantaneous values of ε_m , at height z and at various phase intervals, from (13) according to the rule

$$\varepsilon_m = \int_z^{\delta} \frac{\partial}{\partial t}\left(\bar{U} - \bar{U}_\infty\right) dx \Big/ \left(\frac{1}{\rho}\frac{\partial \bar{U}}{\partial z}\right) \qquad (15)$$

Horikawa and Watanabe explain the negative values by the possible failure of Eq (13) over a rippled bed where vertical velocities, which are not included in the simple formulation, are significant. These vertical velocities are said by the authors to be associated with vortex formation in the lee of the ripples. However, since negative eddy viscosity values are also produced for the flat bed

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1 Note. The latter authors in the text of their paper incorrectly suggest that the eddy viscosity has the same frequency as the surface wave motion; an inspection of their results indicates that twice the frequency is what they intend.

case, this explanation is suspect. Jonsson explains the negative values in his calculations in terms of 'the inertia of the shear stress' in a turbulent oscillatory boundary layer. Another curious and unexplained feature in Horikawa and Watanabe's approach is the occurrence in the outer layer of a viscosity value which is smaller than the molecular viscosity. Whether or not this is supposed to indicate a tendency towards potential flow at a distance above the bed is not made clear by the authors.

(iv) The velocity profile

Although it is clear that a periodic eddy viscosity is appropriate where a solution for the instantaneous velocity profile is required, the exact way in which the stress τ ($\approx \varepsilon_m \frac{\partial \bar{U}}{\partial z}$ from (11)) should be parameterised remains a fundamental problem. We have noted above, in Eq (14), the empirical assumption concerning the form of the velocity profile made by Kalkanis. We turn now to velcoity profiles arrived at on the basis of specific assumptions about the stress, and to those measured experimentally. Representations of the velocity amplitude and phase angle as functions of height above the bed are of particular concern, as in the laminar flow case.

JONSSON and CARLSEN (1976) have claimed from their experimental results that the instantaneous velocity profile near the bottom is logarithmic. A similar picture emerges from JOHNS' (1975) numerical solution in which, under an eddy viscosity assumption made for a rough turbulent flow ($30z_0 \gg \delta$ where z_0 is the roughness length), a logarithmic velocity profile approximately describes the velocity structure in a thin layer near the bed. This layer has a mean thickness of the order of 3 mm, the thickness being a function of the phase, amplitude and frequency of the wave. In Johns' model, the logarithmic velocity profile close to the bottom gives way fairly rapidly to the free stream velocity above it. The boundary layer produced by Johns is an order of magnitude thinner than that measured by Jonsson in the laboratory; however, the peculiar nature of the bed adopted by Jonsson must raise a question as to whether the results in the two cases are at all comparable.

KAJIURA (1968), in his three layer model, has produced theoretical solutions containing exponential and Bessel function terms, and his final solution for the velocity profile has involved matching the solutions obtained in adjacent layers. The result is a logarithmic velocity profile in the overlap layer for a smooth bed. Comparing his theoretical profiles with Jonsson's experimental results for rough turbulent flow, Kajiura claims reasonable agreement in both velocity amplitude and phase angle. Discrepancies arise, however, when the theory is compared with Kalkanis' results for a smooth bed, but these discrepancies are explained by Kajiura in terms of Kalkanis' experimental conditions not being fully rough turbulent, as they had been supposed at the time. On the other hand, for a three-dimensionally rough bed with $Re_2 > 1500$, Kajiura has found his theory and Kalkanis' results in reasonable agreement, at least in velocity amplitude. Kajiura further suggests that if $Re_2 < 1000$, the turbulence in the outer layer is not fully developed and the velocity profile can be better described by (14) than by his own model.

Kajiura's theory has been tested experimentally by HORIKAWA and WATANABE (1968). For a smooth bed the theory was found to predict the velocity amplitude well, but not the phase angle. (Horikawa and Watanabe have commented that Kajiura's solution differs from (7) by only about 1% in both velocity amplitude and phase angle, for flow in the transition range over a smooth bed.) A similar pattern was found when the theory was compared with results obtained over a rough rippled bottom. In general, Horikawa and Watanabe found Kajiura's theory to be in better agreement with Jonsson's results than with their own. This may indicate that the latter were not obtained in a fully rough turbulent flow.

In studies of the sediment transport problem, it is instantaneous velocity profiles and bed shear stresses which are of paramount importance, mainly because of the fact that sediment transport is a threshold phenomenon. Therefore, whether or not a given theory correctly predicts the velocity amplitude or energy dissipation averaged over a wave cycle is somewhat irrelevant, and it follows that, for our purposes, the approach of Johns is likely to be of far greater value than that of Kajiura. Finally, we note here that the bottom permeability has been found experimentally to have little effect on the velocity distribution (SLEATH (1970b)); this is discussed in Chapter 4.

(v) The bottom stress

A knowledge of the velocity field, together with the use of Eqs (10) and (12), implies a knowledge of the shear stress distribution in the flow. Thus, as well as showing that the velocity profile near the bottom is logarithmic, JONSSON's (1963) experiments have indicated that a phase difference exists between the bottom shear stress and the reference velocity outside the boundary layer. The results show that the fluid stress is periodic with the wave period, and that its magnitude decreases away from a maximum value close to the bed to zero in the potential flow region at the edge of the boundary layer.

Jonsson has calculated the instantaneous value of the shear stress from velocity measurements in two ways. Firstly, he has integrated Eq (10), using (12), numerically to obtain the stress at depth z and time t from

$$\tau = \rho \int_{z}^{\delta} \frac{\partial}{\partial t} \left(\bar{U} - \bar{U}_{\infty} \right) dz \qquad (16)$$

taking as the upper limit of integration a height (17 cms) above the bed at which $\tau = 0$. The stress calculated in this way is obviously very sensitive to errors in $\bar{U}_{\infty}$. The first approach has indicated that the phase angle of the shear stress at the bed leads the shear stress in the flow above. Secondly, for comparison, Jonsson has fitted an assumed logarithmic velocity profile in the boundary layer, having the usual form for steady turbulent flow over a rough bottom, namely

$$\frac{\bar{U}}{U_*} = 2.5 \ln \left(\frac{30 z}{k_s} \right) \qquad (17)$$

in which z is the height above the 'theoretical bed level' and k_s is the Nikuradse roughness parameter. The stress at the bed, τ_o, has then been calculated from $\tau_o = \rho U_*^2$. Equation (17) has been used on the assumption that the acceleration terms are small near the bed, and from the experimental results it appears to hold between heights of 0.3 and 2 cms above the bed. The two methods have produced comparable maximum values of the stress, but there is a clear tendency for the logarithmic profile method to yield too small a bottom stress at other instants in the wave cycle. When the pressure gradient is zero, Jonsson has observed a linear decrease in stress away from the bed in the bottom 6 cms.

There are two objections to Jonsson's second indirect method of determining the stress. In the first place, Eq (17) is derived using the assumption of a constant stress layer close to the bed and there is considerable uncertainty as to whether this is valid in oscillatory flow. (Indeed, Jonsson himself notes a linear decrease in the stress away from the bed.) Secondly, it may be impermissible to assume that the fluid acceleration and the pressure gradient are negligible. As seen earlier, JOHNS (1975) has produced a logarithmic velocity profile in the layer adjacent to the bed. But he points out that this cannot be identified with the steady flow profile (17), and concludes that any assumption of the type (17) is bound to result in an underestimation of the bottom stress, which is consistent with Jonsson's findings. In particular, Johns predicts an underestimation of 9% when the pressure gradient is a minimum and 23% when it is a maximum. (Thus the lower percentage errors occur when the thickness of the boundary layer is greatest, and vice versa.) This point is reiterated by KOMAR and MILLER (1973).

KAJIURA's (1968) model, for the case of a smooth bed, produces a small variation in the shear stress in the lower part of the overlap layer, thus supporting the idea of a constant stress layer. In this connection, HORIKAWA and WATANABE (1968) have shown experimentally that, for a smooth bed, Kajiura's theory adequately predicts the amplitude of the shear stress and the phase difference between the shear stress and U_∞, although there is little difference between Kajiura's solution and Eq (7) in this case. Also we note that realistic instantaneous stress profiles are unobtainable from Kajiura's formulation.

The general picture that emerges for rough turbulent flows, is that there is both experimental and theoretical evidence of a logarithmic velocity profile in a thin layer adjacent to the bed. However this is not associated with a constant stress layer and is not directly comparable with the usual logarithmic profile for steady flow.

A knowledge of the stress and velocity distributions in the flow provides sufficient information for the calculation of the energy dissipation in the boundary layer. In particular, energy loss due to bottom friction over an impermeable surface is calculated from a dissipation function, $\tau \frac{\partial \bar{U}}{\partial z}$, such that

$$\text{Energy loss/}_{\text{the bed}}^{\text{area of}} = \int_0^{\delta} \tau \frac{\partial \bar{U}}{\partial z}\, dz \qquad \text{per unit time} \tag{18}$$

Both JONSSON (1976) and KAJIURA (1968) have employed this formulation. At the bed itself the stress is often expressed in terms of the 'wave friction factor', f_w, which is normally defined by

$$\tau_0\Big|_{max} \equiv f_w \cdot \tfrac{1}{2} \rho \left\{ \bar{U}_\infty\Big|_{max} \right\}^2 \tag{19}$$

(eg JONSSON (1966)). However, KAJIURA (1968) has defined the friction coefficient, C, in his analysis by

$$\tau_0 = C \cdot \rho\, \bar{U}_\infty\, \bar{U}_\infty\Big|_{max} \tag{20}$$

JONSSON (1966) has shown that f_w is a function of the Reynolds number and, for turbulent flows, also a function of the relative roughness of the boundary. Only for laminar flow can f_w be evaluated analytically, and the classical solution yields

$$f_w = \frac{2\sqrt{\omega\nu}}{U_\infty} = 2\left[\frac{U_\infty A}{\nu}\right]^{-1/2} \qquad (21)$$

Clearly Eq (19) defining f_w contains no reference to phase in the wave cycle at which the maximum bed shear stress is achieved. The use of the friction factor is helpful, therefore, only in studies which are not concerned with instantaneous values of fluid shear stress and velocity during the wave cycle. It is, of course, precisely this information which is needed in the study of the sediment transport problem.

The energy loss at the bottom boundary may be enhanced by the porosity of the seabed. This topic is the subject of Chapter 4 and we confine ourselves here to some general comments about the effects of porosity on the progressive waves at the water surface. REID and KAJIURA (1957) have presented a model of an inviscid oscillatory flow over a porous bed, and have solved a potential equation in both upper and lower layers of an interacting two layer model. The potential equation applied within the bed is produced from Darcy's Law. Percolation is shown to have a negligible effect on the wave length, phase velocity and group velocity of the surface waves. It does, however, alter slightly the relative phase of the horizontal velocity and fluid pressure, and also the relative phases of these quantities and the free surface elevation. Energy loss is thus confined to wave amplitude, and the rate of decay of amplitude is shown to depend upon the wave frequency. So a wave group, containing many different frequencies, travelling over a permeable bed will suffer a selective attenutation, such that the energy spectrum may be diminished for a certain range of intermediate frequencies and remain essentially unchanged elsewhere in the spectrum.

The rate of damping of waves also depends rather critically upon the nature of the boundary layer (see COLLINS (1963)). For a turbulent boundary layer, long waves are damped at a faster rate than short waves. On the other hand, for a laminar boundary layer, the reverse is true. Finally, we note that ripples on the seabed increase the energy dissipation in the flow by the generation of vortices in the lee of the crests; this topic is examined in Section 3.4.

CHAPTER 3 SEDIMENT TRANSPORT ASPECTS

We have argued above that energy loss from a wave train may arise by frictional dissipation in the layer of the flow adjacent to the bed and, to a lesser extent, through bottom percolation. For sufficiently active oscillatory flows, we can add to these two categories a third, namely energy dissipation by sediment transport.

We start this section by noting the usual separation of sediment transport into studies of the 'bed load' and the 'suspended load'. Both of these aspects have been studied extensively for steady flows, on a semi-theoretical basis as well as in the laboratory, and YALIN (1972) has provided a comprehensive review of the existing sediment transport theories for this case. However consideration of the steady flow problem is helpful only to a limited extent in treating the case of sediment transport in an oscillatory flow. As we have seen, the passage of surface waves is associated with non-uniformities in the fluid pressure both above and within the bed, in the direction of wave travel. These non-uniformities, which do not arise in steady flows, cause additional forces to act on the grains of the bed. This leads, for instance, to different threshold conditions for sediment motion between the steady and unsteady flow cases, in terms of the principal flow parameters. In spite of these differences, the semi-theoretical methods which have been developed to tackle the steady flow problem may still be utilised in the case of oscillatory flow, even if the detailed results for steady flow may not. We look at some of the existing semi-theoretical models in this review.

Another point which we stress initially is the important role played by ripples in the sediment transport problem. In general an oscillatory flow, which is sufficient to cause sediment motion, is likely to deform the bed. This affects both the threshold conditions for sediment motion and the overall sediment mass transport. For instance, sediment motion may be caused on a rippled bed by a flow which produces no motion on a flat bed. Furthermore the development or the destruction of a ripple pattern on the bed as a response to surface waves takes a considerable time. In the laboratory, experimental beds can be allowed sufficient time to establish an equilibrium configuration under the action of a monochromatic wave train. In natural condition however, this is not the case, as the wave conditions are always changing, albeit slowly, and an equilibrium configuration of the bed may seldom exist. Thus observations of sediment motion on a natural rippled bed are very likely to be of a non-equilibrium condition.

In what follows we review the problem of sediment transport in oscillatory flows. We are concerned throughout with sediment transport by the first order oscillatory motion, and not with residual transport associated with any non-zero steady second order velocities present, wave-induced or otherwise.

3.1 A descriptive model

As wave activity over a flat sand bed increases, the first sign of sediment motion is likely to be a sporadic movement of grains. This occurs very roughly at the parts of the wave cycle when the speed of the fluid outside the boundary layer above the grains achieves its maximum value; the direction of the motion will be that of the fluid velocity. This condition, which is described usually as 'incipient motion', may occur when the boundary layer is laminar or turbulent. We discussed earlier the form of the laminar boundary layer caused by a progressive wave over a smooth horizontal bed, and showed that at any point the bed shear stress, $\tau_{o\ell}$, leads the velocity of the flow outside the boundary layer by a phase angle of $\pi/4$ (or one eighth of the wave period). We might, therefore, expect sediment movement to occur well in advance of the maximum fluid speed outside the boundary layer. However a rough sand bed differs from a smooth horizontal bed in the matter of phase, as has been shown in the laboratory by SLEATH (1970b), and a smaller phase difference between the stress and the velocity of the potential flow is found in practice (see Section 2.2). This is so for both laminar and turbulent boundary layers. We therefore start with the picture of incipient motion on a flat bed occurring at the time of the wave cycle shortly before the maximum velocity is achieved outside the boundary layer.

We are concerned in discussing 'incipient' and later 'general' motion of grains, with a layer at the surface of the bed of thickness no more than a few grain diameters. In other words, we are concerned with bed load motion. As the level of wave activity increases beyond the point at which 'incipient motion' occurs (and at constant frequency, for the sake of argument), a more general 'to' and 'fro' motion of grains will occur, the whole surface of the bed being taken into motion for a greater portion of the wave cycle than is the case for incipient motion. This condition of 'general motion' of the surface of the bed may be expected to be associated with ripple formation. The height and length of the ripples formed will depend upon both the material of which the bed is composed and the principal features of the oscillatory fluid motion. We will return to this matter later, but at this stage can make the crude statement that, when general motion of the bed has produced an equilibrium ripple pattern, the vast majority of potentially mobile grains will remain a part of just one ripple thereafter.

The final sediment transport condition which arises for still more active waves than those causing general motion, is the 'suspension' of grains. Here the grains are carried up into the flow as a result of the vertical hydrodynamical forces acting on them being greater than the downward force of gravity. Grains transported in this way can be found at heights which are orders of magnitude greater than those attained by grains of the bed load. Now if the development of a suspension requires hydrodynamical forces on grains which are directed vertically upwards, there must be a component of the fluid velocity also directed upwards. In laminar flows under progressive waves, vertical velocities are associated only with the wave action in both the potential flow region and the boundary layer. Furthermore since, as we saw earlier, the boundary condition $W = 0$ is satisfied at the bed $z = 0$, the vertical velocities near the bed are necessarily small in most situations. (we could complicate the discussion here by suggesting a relaxation of the bottom boundary condition due to percolation into the bed, but we show in Chapter 4 that the effect of percolation is unlikely to be significant in the problem as a whole.) Therefore it is generally unlikely that significant amounts of sediment will be suspended in predominatly laminar flows, at least when the bed is comparatively flat. The oscillatory flow caused by waves over a rippled bed (see Section 3.4) may be associated with vortex formation in the lee of ripples, and the flow separation thus involved may provide a means by which grains can be projected into the flow to heights which are significant locally. Generally however, we expect 'suspension' to be an important sediment transport condition only when the flow is turbulent, and to be caused by the combination of wave induced and turbulence induced vertical velocities.

Let us now look briefly at the probable mechanisms of sediment transport, with particular reference to the incipient motion condition. When a grain moves from its resting position on the bed, we can say that the resultant force on the grain is greater than the forces resisting its motion. The latter forces comprise the immersed weight of the grain and reaction forces (normal forces, roughly speaking) resulting from its contacts with surrounding grains. The resultant hydrodynamical force can be calculated in principle, by integrating the force per unit area over the surface of the grain. This surface 'pressure' will be composed of three parts. The first is a component from the overall pressure field driving the flow; this is therefore a component associated with the acceleration of the fluid locally (Eq (10) . The second is an additional component of pressure on the surface of the grain, associated with the deformation of the streamlines around it. (The sum of these two components constitutes the 'form drag'.) The third is associated with the formation of a boundary layer on the surface of the grain (ie the 'skin friction drag').

Although the rough separation of the 'pressure' on the surface of a grain into these three component parts is a helpful starting point in considering the incipient motion problem, it is difficult to establish their magnitudes. The flow field close to the bed will be greatly affected by the local topography, and grains on the bed surface are likely to be influenced by the wakes from grains upstream. Also, measurements of the instantaneous pressure around the surface of a sand grain are most difficult to make. The nearest attempt of this kind in the laboratory has been that of EINSTEIN and EL SAMNI (1949) who measured the pressure at points on the surface of a bed composed of an arrangement of hemispheres. This study revealed the fluctuating nature of the pressure on grains in a turbulent flow and enabled the authors to define a turbulent lift coefficient, but it cast no light on the different roles of the two pressure components described above. In particular, the experiments demonstrated the Gaussian distribution of the instantaneous value of the lift force on the bed about its mean value. This result lies at the heart of one of the principal semi-theoretical approaches to the problem of bed load sediment transport, which can be attributed to H A Einstein (see Section 3.2). Measurements of the mean pressure distribution, also over a bed composed of hemispherical roughness elements, have been made by CHEPIL (1958).

A theoretical contribution on this topic has been made by SLEATH (1976b) who has presented results from numerical and analytical solutions for the drag and lift forces exerted in laminar flow on a two-dimensionally rough bed by wave action. His arguments are based on the underlying contention that the flow near the seabed remains laminar at much higher Reynolds numbers than is often supposed, and that therefore it is possible to investigate forces on the bed analytically or by numerical techniques. Thus Sleath has computed the lift and drag forces on a roughness element and has compared their magnitudes for various roughness sizes. As expected, when this size is small compared with the thickness of the viscous boundary layer, the lift force is found to be negligible compared with the drag (except at very high values of Re_3). Also, following the above comments about the forces acting on the surface of a grain, Sleath finds the component of drag due to shear stress to be much larger than the resolved component of pressure on the bed. For particles of a size which is large compared with the thickness of the viscous boundary layer, neither of these conclusions is true. Even at small Re_3

the resolved component of pressure is an important part of the total drag, and at moderate to high values of Re_3 the lift may completely dominate the initial motion of the sediment. We emphasise that all these conclusions are for essentially laminar flow, unlike the flows studied experimentally by Einstein and El-Samni.

We conclude the present discussion with the remark that the relative importance both of the component parts of the pressure on the surface of a sand grain and of the drag and lift forces, have not yet been clearly established. It is of fundamental importance to the study of sediment transport in the sea that this matter be resolved for reasons which are dealt with in more detail later. As we will argue, the reason why little information on the subject has been forthcoming from laboratory studies is that, in a uniform wave train in a flume, the parameters of importance in incipient sediment transport can be handled in dimensionless groups, and thereby detailed consideration of the processes involved in sediment transport by unsteady flows can be overlooked. To fully understand sediment transport in the sea, where waves of a single frequency at a particular location persisting for a reasonable duration are the exception rather than the rule, the processes involved in the phenomenon must be assessed and then modelled in detail.

3.2 Semi-theoretical models of bed load transport

(i) The approach of H A Einstein

We have referred above to the probabilistic approach which was first introduced by EINSTEIN (1950) and which has been used in the study of oscillatory flow problems by ABOU-SEIDA (1965), KALKANIS (1964) and LIANG and WANG (1973). Under various assumptions which are outlined briefly below, the semi-theoretical development has been taken to the point where a sediment transport rate formula has been proposed, initially for steady flows for which it is assumed that a logarithmic velocity profile extends right down to the level of the bed. The adaptation of the theory to the case of oscillatory flows involves no new physical assumptions, merely the use of a velocity profile having the form of Eq (14).

Einstein's approach is based on the fundamental assumption that a grain will be set in motion if the upward lift force on it due to hydrodynamical effects, exceeds its immersed weight. The lift forces at the bed are associated entirely with the turbulent velocity fluctuations in the fluid; in fact, EINSTEIN (1950) argues that what is important in the present context is 'turbulence generated at the bed'. Thus it is assumed that separation occurs in the flow around individual grains, causing low pressure wakes on their downstream sides. It follows that the resultant of the normal pressure on a grain has a significant component in the direction of flow and it is argued that this becomes so large that all viscous shear can be neglected. The upward force on a grain is associated with the fluctuating form drag by a coefficient of lift. Once a grain is disturbed from rest, it is assumed somewhat arbitrarily that it makes a jump of 100D (where D is the diameter of an equivalent spherical grain) to its next resting place, where it is again subject to further movement depending upon the instantaneous value of the upward lift force at that point. The model is therefore one of the grain being subjected to an impulsive force arising from the turbulence in the flow, this force always having the same effect in terms of the grain's motion, namely the jump of 100D.

A second characteristic of a grain of known diameter is assumed to be its 'exchange time', which is defined as the time which is needed to replace a displaced particle of the bed by a similar one. This quantity is employed in the formulation of the probability of lift-off p which is defined both as the fraction of the total time during which at any one point on the bed the local flow conditions cause a sufficiently large lift on a particle to remove it, and alternatively as the fraction of the bed on which at any time the lift on a particle of given diameter is sufficient to cause motion. (Einstein invokes the statistical equivalence of all points of the bed in presenting these alternatives.) However it is our opinion that the entire argument concerning the exchange time and the grain jump of 100D is unnecessary. Essentially, both of these quantities are introduced in order that a dimensional probability p_s, the probability of a grain being moved in a second, can be replaced by the non-dimensional probability p. The conversion to this required probability p can be produced by

$$p_s \,.\, [\text{time}] = p$$

where $[\text{time}]$ may be taken as any suitable length of time. YALIN (1972) refers to this as the 'time of the grain', which Einstein takes as proportional to $\dfrac{\{\text{grain diameter}\}}{\{\text{settling velocity of the grain}\}}$. Neither the grain jump length nor the 'time of the grain' seem to be quantities having any direct physical relevance, and they are introduced simply because they have a convenience value in the context of Einstein's fictive bed.

These detailed considerations do not, of course, throw doubt on what appears to be basically a sound approach to the problem. We do, however, draw attention to an error in Einstein's original (1950) analysis, which seems to have been overlooked by the workers mentioned at the start of this section. This error, which has been pointed out by YALIN (1972), is that in the analysis as it stands it is the magnitude of the lift force on a grain, rather than simply the lift itself, which must exceed the immersed weight of the grain for motion to occur. Thus large negative (downward) lift forces are capable of causing motion in Einstein's original formulation, and clearly this is unacceptable.

Further refinements in the model include an allowance for the differences in grain size in the surface layer of the bed (ie a sheltering factor depending upon the grain sizes present), and an allowance for differences in the coefficient of lift which arise in mixtures of grains. Also it is assumed that the time history of the lift force at a point on the bed is such that the lift is normally distributed about its mean value, and therefore a probability of motion can be assigned to the grains of the surface layer. Once grains are in motion, their probability of being redeposited is assumed to be equal at all points of the bed where the local flow would not immediately remove them again.

The probabilistic argument has been investigated also by GRASS (1970) who has compared the temporal probability of the shear stress (as opposed to the lift) attaining a particular value at a point on the bed and the spatial probability of the bed being capable of erosion at a particular instant in time. The approach of Grass complements that of Einstein, but it rests (as does Einstein's) on the assumption of the equivalence of the temporal and spatial probability distributions. The conditions attached to this equivalence need to be defined carefully, although for the case of a uniform homogeneous bed surface the equivalence is likely to provide a very good first approximation.

Einstein's approach to the problem is, as we have seen, limited to turbulent flows, since no mechanism for grain motion is proposed for laminar flow states. While this limitation is probably not too serious at high sediment transport rates when the flow, in general, will be turbulent, there is evidence suggesting that incipient motion may arise in laminar flows (see Section 3.3) and the theoretical framework proposed by Einstein is incapable of handling this. Nevertheless the probabilistic formulation of the sediment transport problem is a realistic approach and one which may offer the best chance of success.

(ii) The approach of A T Ippen and P S Eagleson

A rather different approach to the sediment transport problem, and one which takes into account the threshold of sediment motion in a rather more formal way than does that of Einstein, has been developed by Ippen, Eagleson and others (eg IPPEN and VERMA (1953); IPPEN and EAGLESON (1955); EAGLESON, PERALTA and DEAN (1958)). Initially much of the work was carried out with a view to tackling the problem of the development of the beach profile as a result of wave action; this has been reviewed by JOHNSON and EAGLESON (1966). Variable depth is not an aspect of the problem which has been considered in the present review although, of course, it does represent the natural condition of most sand beds in the sea. We will refer here only to the use of Eagleson's approach, as it has been exploited by CARSTENS ET AL (1969), and others, in flat and rippled bed problems. On this scale, the slope of Eagleson's beach becomes the slope of a face of a ripple.

It is assumed that the bed is composed of spherical particles, for which the angle of repose is known. The hydrodynamical forces on the grains are identified and are interpreted in terms of the 'effective' bed slope they produce. Locally this may be greater than the angle of repose and, if the bed is flat and horizontal, sediment movement will result. In general, however, grain motion will only occur if the combination of actual slope and effective slope exceeds the angle of repose. In a periodic flow the picture is more complicated than for steady flow, in that the instantaneous values of the effective bed slope will be periodic also.

Incipient sediment motion occurs when the instantaneous hydrodynamical forces cause a static instability of the bed sediment particles. The condition of instability is determined from consideration of the resultant force acting on a potentially mobile grain, and this is evaluated from its weight, the normal reaction forces it experiences due to the presence of the adjacent grains and the hydrodynamical forces acting on it. The viscous resistance and apparent mass forces are assumed to act through the upper edge of the particle. All other forces are assumed to act through its centre. Thus the condition of instability is determined by taking moments about the point of contact with the appropriate neighbouring grain. The laminar shear wave solution is adopted and it is further assumed (JOHNSON and EAGLESON (1966)) that incipient motion always occurs under the wave crest, which is to say when the velocity achieves its maximum value. On the basis of the earlier discussion, it is by no means certain that this particular assumption is valid. Also the assumptions concerning the lines of action of the hydrodynamical forces, not to mention the structure of the drag coefficients introduced, leave the theory rather loose in many ways.

The consideration of 'established' or 'general' sediment motion is conducted on comparable lines, but now it is assumed that a sediment particle remains in motion throughout the entire wave cycle. Again a force balance procedure is adopted in which the bottom now merely contributes a resisting force proportional to the sediment particle velocity, and the role of phase is introduced by means of an apparent inertia force. In other words, the acceleration of the fluid and the effect which this has on the grains, as well as the drag forces arising from the local fluid velocity, are now taken into account. Again the instantaneous fluid particle velocities at the edge of and within the bottom boundary layer are given by the laminar shear wave relation, and, further, it is assumed that the elevation at which the effective instantaneous fluid velocity and acceleration are taken from this solution is time independent.

CARSTENS, NEILSON and ALTINBILEK (1969) have performed experiments on sediment transport and ripple development in an oscillating water tunnel, and have analysed the phenomena in terms of the above theory. Incipient motion on a flat bed was arbitrarily defined as the motion(rolling back and forth) of about 10% of the particles on the surface (this criterion was introduced previously by CARSTENS and NEILSON (1967)). Interestingly, the authors confirmed experimentally the physical hypothesis for incipient motion described earlier, that grains move only when the fluid velocity is near its maxima, that is when the acceleration in the flow is negligible. Consequently they have not included in their analysis forces arising from the inertial reaction of the particles and the fluid around the particles (ie virtual mass and added mass forces). Having assumed in their formulation of the hydrodynamical forces on the grains that the flow is turbulent, the authors have conducted a force balance and thus have derived a 'universal incipient motion criterion'. They suggest that this criterion is satisfactory for both rippled and flat beds, although it is necessarily deficient if the potentially mobile grains are submerged in a laminar sublayer. Further a Shields type curve for the threshold of sediment motion is deduced on semi-theoretical arguments, but, unfortunately, it is not compared with the original Shields curve in any systematic way.

The approach adopted by Eagleson and others has been used extensively. However it has to be concluded that the assumptions that the grains are spherical, that lines of action of forces are reasonably well known and that the structure of the hydrodynamical forces on the grains are known save for empirically determined coefficients, leave the theory incomplete. In addition, the use of the laminar shear wave solution together with turbulent drag forces on grains is dangerous to say the least.

(iii) The approach of R A Bagnold

A further semi-theoretical approach is that of Bagnold. Essentially BAGNOLD (1963) has adapted his well known energetic arguments, presented initially for steady flows, to the oscillatory flow case. Thus a certain proportion of the 'stream power' is said to be available for sediment transport, this proportion being determined by the 'efficiency' of the stream. In particular, the stream power is taken to be the decrement of the transmitted power per unit distance travelled by progressive waves in shallow water, on the assumption that this is due entirely to energy losses by fluid drag at the bed surface. Bagnold suggests that the mean rate of to- and fro- transport of sediment is likely to be proportional to this quantity. In his arguments dealing with oscillatory flows, Bagnold is concerned principally with the residual transport by secondary currents of the grains which are set in motion by the first order oscillation. Since no fundamentally new arguments arise in the theoretical formulation in the oscillatory flow case compared with the steady flow case, we will not embark here upon a detailed examination of Bagnold's approach.

Bagnold's overall view of the sediment transport problem is clearly an attractive one. If, indeed, sediment transport rates can be quantified on the basis of the efficiency of the stream to cause grain motion and if, further, Bagnold's assumptions about the sediment transport rate in oscillatory flow are valid, then the problem reduces simply to that of the empirical determination of the efficiency factor. However, we feel at present that the underlying physical processes in the problem must be examined in rather more detail than would be required for an investigation of this kind, although we would not wish to preclude the possibility of adopting Bagnold's formulation ultimately.

3.3 Studies of incipient sediment motion in the laboratory

(i) General discussion

A considerable amount of work on the problem of sediment transport by oscillatory flows has been carried out in the laboratory, and this has produced much information on the magnitudes of the parameters of importance in the problem. Most of the studies have dealt with the definition of the conditions required for incipient sediment motion (eg KOMAR and MILLER (1974), MANOHAR (1955), BAGNOLD (1946)), while others have proceeded to the definition of other sets of critical conditions, such as those required for suspension or ripple formation (eg CHAN, BAIRD and ROUND (1972)). A number of results from the studies of incipient motion are presented and discussed in Part 2 of this report (DAVIES, FREDERIKSEN and WILKINSON (1977)), in the light of the experimental results presented there. In this section, the parameters of importance in the problem are discussed and the experimental techniques which have been used in the laboratory are examined.

Generally speaking, in the laboratory an oscillation of a single frequency is produced in the fluid above a flat loose granular bed, and the amplitude of the oscillation (ie the excursion of the fluid particles) is increased until grain motion is observed. (Alternatively the bed may be oscillated below a stationary fluid depending upon the apparatus in use; however, as we argue later, this method may give rise to misleading results.) The visual detection of grain motion clearly implies that a rather subjective judgment is involved, a point which has been stressed by CHAN, BAIRD and ROUND (1972) who attempted to draw a distinction between an 'incipient motion regime' and 'general surface motion'. They defined the former condition by the motion of virtually any material of the bed, however slight this motion might be, and they concluded that it was very difficult to define this condition at all adequately in terms of the flow parameters. The latter condition, which was far more easily defined, consisted of a general motion of the uppermost particles on the bed surface. Thus, we see initially that the definition of incipient motion presents difficulties. A comprehensive list of incipient motion formulae found from laboratory studies has been given by SILVESTER and MOGRIDGE (1970). However care must be exercised in the application of any of these formulae since there is always doubt as to whether incipient motion consisted of the sporadic movement of a few grains or the general movement of the surface of the bed.

We have suggested earlier that incipient motion may occur when the flow is laminar or turbulent. For instance, BAGNOLD (1946) has proposed his incipient motion formula for flow which he judged to be laminar on the basis of dye injection tests, and MANOHAR (1955) has presented criteria for incipient motion in both laminar and turbulent flows. A curious and somewhat controversial aspect of Manohar's results for the turbulent flow case, however, is that his incipient motion criterion contains the kinematic viscosity ν , a quantity only expected to be of importance in laminar flows. KOMAR and MILLER (1974) have combined what they regard as the most satisfactory of the available formulae for incipient motion and have presented general incipient motion conditions (see Fig 3.1). For $D < .05$ cm the threshold is reached when the flow is still laminar, while for $D > .05$ cm the flow is turbulent. In the figure, the distinction between these regimes is more evident at shorter oscillation periods, where the two suggested curves are non-intersecting. Komar and Miller have also noted various inconsistencies between Bagnold's and Manohar's experimental results, notably that Bagnold claims that the flow stays laminar at much higher Reynolds numbers than is suggested by Manohar. Discrepancies of this kind can probably be attributed to the problem of the subjectivity involved in the definition of transition to turbulence in the laboratory by visual means.

As far as the processes in the problem of incipient motion are concerned, KOMAR and MILLER (1973) pointed in their initial consideration of the phenomenon to fundamental differences between entrainment forces in unsteady oscillatory flows and those in steady uni-directional currents. In particular, they argued that the reason for the apparent dependence of threshold conditions on the wave period was probably the importance not only of the orbital velocity, but also the value of the end-of-stroke acceleration which, for the same orbital velocity, increased with decreasing wave period. This was challenged by MADSEN and GRANT (1975) who expressed the view that the differences between steady and oscillatory flows are not as pronounced as Komar and Miller had suggested, and the latter authors (1975a) subsequently accepted this criticism. By re-plotting the data on which Komar and Miller drew their initial conclusions, Madsen and Grant concluded in turn that the usual Shields' function, with all its shortcomings, may serve as a relatively reliable and quite general criterion for the threshold of sediment movement under water waves. This would seem to exclude the possibility of the pressure gradients in the flow producing significant forces on the grains on the bed. Such a conclusion would appear at present to be unwarranted, as we argue in a full discussion of the topic in Part 2, and this view is supported by CHAN, BAIRD and ROUND (1972) who have compared the threshold of particle motion in steady and unsteady flows. Their results indicate that, in a wave cycle, the peak oscillatory shear stress needed to bring about incipient motion is about twice the corresponding shear stress for steady flow. This is consistent with the oscillatory velocity threshold measured above the boundary layer being significantly lower than the steady flow threshold, at low amplitudes and high frequencies. They argue that the difference can be explained qualitatively in terms of a 'response time' which must elapse before a particle can be dragged out of its site on the bed surface. This response time is unimportant in the case of a steady shear stress, but with an oscillatory stress it results in a higher value at the threshold of motion. Their explanation is curiously devoid of any mention of the possibility of the pressure gradient having a role in the entrainment process. Nevertheless their experimental results lend weight to one of the main conclusions in this review, that the fluid acceleration may play an important part in bringing about incipient motion.

The analysis of laboratory results has usually involved the association of incipient motion with a 'critical wave'. Thus information about instantaneous velocities, pressure gradients and phase angles, at the instant when grain motion commences, is not available from such studies. Rather, grain motion is associated with the 'critical wave' in terms of the following sediment, fluid and flow parameters:

ρ_s, the density of the sediment;
D, the representative grain diameter;
ν, the kinematic viscosity of the fluid;
ρ_f, the density of the fluid;
g, the acceleration of gravity;
$\bar{U}_\infty\big|_{max}$, the amplitude of the fluid velocity near the bed;
ω, the wave frequency.

The argument proceeds, in principle, to the formation of four dimensionless groups based on these seven parameters. However, in practice, a rigorous dimensional analysis is not usually presented. Instead incipient motion results tend to be expressed in terms of a single dimensionless grouping containing most, if not all of the above variables, with the possible addition of a rider stating the range of applicability of the given formula in terms of a second group (or even dimensional quantity in some cases). If the bottom stress τ_0 is introduced into the argument, on the grounds that it is the quantity of fundamental importance in incipient motion studies, this is usually done by way of Eq (19) involving the definition of the friction factor f_w. Clearly the relation (19) contains no reference to phase angle in the wave cycle; indeed $\tau_0\big|_{max}$ and $\bar{U}_\infty\big|_{max}$ will occur typically at different phases in the cycle.

The approach outlined above, while having considerable merit as a first step towards an understanding of incipient motion conditions, does have severe limitations from the points of view, firstly of extending it to a form useful in studies in natural conditions, and secondly of forming a basis for comparison with sediment motion data such as that obtained in the present study (see Part 2). In particular, since for a monochromatic wave train the amplitude of the velocity U_0 is proportional to ($a\omega$) (see Eq (3)), it follows that a rather more extensive set of dimensionless groups is required if waves of more than a single frequency and amplitude are present($a \rightarrow a_1, a_2 \ldots ; \omega \rightarrow \omega_1, \omega_2 \ldots$). In short, the approach based on dimensionless groups soon becomes impracticable, and needs to be replaced by an approach based on instantaneous bed shear stresses and instantaneous fluid velocities at initiation of motion (ie including phase angle information). In this way the processes involved in incipient sediment transport would not be masked in a proliferation of dimensionless groups.

SILVESTER and MOGRIDGE (1970) have provided a general discussion in which they suggest a number of possible reasons why incipient motion in the sea is likely to differ from the predictions made in the laboratory. They note that the presence of ripples on the ocean floor will produce incipient motion at smaller velcoity amplitudes than for a flat bed, on account of the fact that the velocities of fluid particles near the bed may be higher than is suggested by small amplitude wave theory. This point, which is also discussed by KOMAR and MILLER (1975b), is closely connected with the discussion in Section 2.1. To make allowance for a rippled bottom, Silvester and Mogridge propose curves which purport to define rather more realistic incipient motion conditions for application in the ocean generally. Further, they suggest that the interaction of wave trains of slightly differing period may generate greater instantaneous velocities than implied by linear theories, and, finally, they suggest that shell debris on the sea floor can initiate the movement of sand particles sooner than for either smooth or rippled beds. In this context they refer to CARSTENS and NEILSON's (1967) tests which showed that, if small protuberances exist on the bed, motion is initiated at rather smaller velocity amplitudes than for a flat sand bed.

We conclude this section by suggesting that the correct way to tackle the incipient sediment motion problem in the sea is not by studying the gross features of the 'critical wave' as has been the case in most laboratory investigations, or by developing analytical models in which only time averaged values of the basic flow parameters are handled, but by measuring and modelling the instantaneous flow parameters through the wave cycle. This point of view is expressed also by KOMAR and MILLER (1975b).

(ii) Comments on laboratory techniques

We have commented earlier that certain experimental results may have to be treated with some caution on account of the method used in obtaining them. Basically, experimental studies in the laboratory have been of three types:

1. oscillatory flows in channels having a free sufrace (wave flumes);
2. oscillatory flows in closed conduits (oscillatory water tunnels);
3. flows induced in an otherwise stationary fluid by an 'oscillating bed'.

In the first category, waves are generated at the free surface and these produce an oscillatory motion close to the bed. The surface waves can be of small or finite amplitude. However the disadvantage of this method is that waves of the period, length and amplitude, found in nature cannot be reproduced; typically experiments are restricted to waves of 2 seconds period, when what is required is a 10 second period.

This problem can be overcome to a great extent if the free surface is replaced by the upper surface of a closed conduit (category 2 above). In this apparatus a reversing flow is driven by the generation of an oscillatory pressure gradient. The flow is uniform and thus, from Eq (12), $\frac{\partial U_\infty}{\partial t} \propto \frac{\partial p}{\partial x}$. The oscillating water tunnel is capable of correctly modelling waves of the period found in nature although, of course, non uniformities in the fluid velocity in the x -direction and variations in the vertical velocity, both of which arise under a progressive wave in the sea, are not present. If the wave length of the 'associated surface wave' is long compared with the orbital excursion of a fluid particle just above the boundary layer, it is probably reasonable to assume that the problem of longitudinal non-uniformity is not important. In any case, at a point on the bed, the oscillatory boundary layer is adequately and correctly modelled, except possibly in the matter of transition to turbulence (see section 2.3).

While results obtained in an oscillating water tunnel can be accepted for our purposes, this cannot so readily be said to be the case when the 'oscillating bed' method (category 3) has been used, at least as far as sediment transport studies are concerned. The oscillating bed method involves the rocking of a bed in its own plane below a stationary fluid. Again, as for category 2, longitudinal non-uniformities and vertical velocities are both absent. Now, for problems which do not involve the movement of sediment, the approach is exactly equivalent hydrodynamically to the oscillating water tunnel; the only difference between the two cases is that the driving force ($\propto \partial p/\partial x$) in Eq (12), is replaced by the oscillation of the bed itself. However, if sediment motion problems are studied using this method there is a danger that the phenomenon may be modelled incorrectly. This is because the inertia of the sediment grains being oscillated to and fro may mask the forces which are acting on them as a result of hydordynamical action. Thus, for instance, incipient motion may be observed on an oscillating bed for fluid flow conditions which cause no motion in an oscillating water tunnel. In an extreme situation, the inertia forces may by themselves cause grains to move in relation to the bed. To illustrate this problem, let us consider two simple models of the situation.

Case (i)

Firstly, we will make use of the laminar oscillatory boundary layer solution for a flat bed (see Section 2.2), adapted in such a way that the boundary conditions become

$$U_{bed} = U_0 \cos \omega t \quad \text{at} \quad x = 0$$
$$U \to 0 \quad \text{as} \quad x \to \infty$$

The governing equation (cf Eq (6)) is now

$$\frac{\partial U}{\partial t} = \nu \frac{\partial^2 U}{\partial x^2}$$

and its solution, see LAMB (1932), is such that

$$\tau_0 = \mu \left. \frac{\partial U}{\partial z} \right|_{z=0} = \rho_f \, U_0 \sqrt{\omega \nu} \, \cos (\omega t + \pi/4)$$

where $\mu = \rho_f \nu$. The acceleration of a grain on the oscillating bed is clearly $\{- U_0 \omega \sin \omega t\}$ and, therefore, the inertia force of a grain of immersed mass m is $\{- m \, U_0 \omega \, \sin \omega t\}$. If we now assume that the "effective" area of the bed occupied by a grain of diameter D is $\approx D^2$, we can argue that the forces on the grain due to the fluid stress at the bed and due to the inertia of the accelerated grain, may be expected to lie approximately in the ratio

$$- U_0 \, \rho_f \, D^2 \sqrt{\omega \nu} \, \cos \left(\omega t + \frac{\pi}{4}\right) \; : \; - \frac{\pi}{6} \gamma \, \rho_f \, D^3 \omega \, U_0 \, \sin \omega t$$

where $\gamma = (\rho_s - \rho_f)/\rho_f = 1.65$. We can see at once that the forces are not in phase, and that at certain instants of the wave cycle the inertia force will have a magnitude greater than that due to the bottom stress. This being the case, we can suggest that the swinging bed method is satisfactory only if the following condition is satisfied:

$$\left\{\begin{matrix}\text{Amplitude of force arising}\\ \text{from bottom stress}\end{matrix}\right\} \gg \left\{\begin{matrix}\text{Amplitude of the}\\ \text{inertia force}\end{matrix}\right\}$$

or

$$1 \gg \frac{\pi}{6}\,\gamma D \sqrt{\frac{\omega}{\nu}}$$

If D = 0.1 cms and $\omega = 2\pi/10$ rads/sec, the right hand side of this inequality is approximately equal to 0.7. Furthermore, the higher the frequency of oscillation, ω, the larger this value becomes, and it is at frequencies higher than $(2\pi/10)$ rads/sec that most laboratory experiments have been performed.

Case (ii)

Suppose that we now adopt an alternative argument in which we represent the exposed portion of a grain as a semi-circular obstruction on the bed, lying in the plane orthogonal to the direction of fluid flow in the boundary layer. We can then obtain an estimate of the force on a grain by calculating the dynamic pressure on its upstream face, assuming the static pressure on its downstream face. In addition, following TAYLOR (1946), we will assume here for simplicity that the velocity profile in the portion of the wave boundary layer close to the bed is linear ($U = \alpha z$, α = constant). The force on the grain is then given by

$$\int_A \tfrac{1}{2}\,\rho_f\,U^2\,dA$$

where dA is an element of area of the semi-circle. Under the assumed linear velocity profile

$$\int_A \tfrac{1}{2}\,\rho_f\,U^2\,dA = \pi \rho_f\,\alpha^2 D^4$$

in which

$$\alpha = \left[\frac{\partial U}{\partial z}\right]_{z=0} = -\,U_0\,\omega^{1/2}\,\nu^{-1/2}\cos\left(\omega t + \pi/4\right)$$

We then obtain for the force on the grain the expression

$$\pm\;\frac{\pi \rho_f D^4 U_0^2\,\omega}{\nu}\left\{\frac{1-\sin 2\omega t}{2}\right\}$$

in which the sign depends upon the direction of flow. If we now make the same comparison as before, the ratio of the hydrodynamical and inertia forces is approximately

$$\pm\;\frac{\pi \rho_f D^4 U_0^2\,\omega}{\nu}\left\{\frac{1-\sin 2\omega t}{2}\right\} \;:\; -\;\frac{\gamma\,\rho_f\,\pi D^3\,\omega\,U_0}{6}\sin\omega t$$

We can observe again that there is a phase difference between these forces and suggest that, under the assumptions made in this second case, the swinging bed method is satisfactory if

$$\left\{ \begin{array}{l} \text{Amplitude of force arising from} \\ \text{dynamic pressure effect} \end{array} \right\} \gg \left\{ \begin{array}{l} \text{Amplitude of the inertia} \\ \text{force} \end{array} \right\}$$

or

$$1 \gg \frac{\gamma \nu}{6 \mathcal{D} U_0}$$

With $\mathcal{D}$ = 0.1 cms and U_0 = 10 cms/sec, the right hand side of this inequality is equal to 2.75×10^{-3}. If the frequency of oscillation increases while the 'stroke' remains constant, this value decreases.

It is clear that in this second case, the forces due to the inertia of grains on a swinging bed are two (possibly three) orders of magnitude smaller than the hydrodynamical forces. However, in practice, grains on the bed shelter one another in a way which is not accounted for in the simple model above, such that the calculated dynamic pressure is likely to be an overestimate.

The picture emerging from Cases (i) and (ii) is that the inertia force of a grain on a swinging bed is less than the hydrodynamical force. The relative importance of these two forces, however, depends upon how fluid momentum in the boundary layer is transmitted to the grains on the bed. We have argued earlier that this is not well understood. If 'skin friction' is the dominating process in incipient motion, then results obtained with an oscillating bed must be treated with caution. Whereas, if velocity induced drag forces are dominant, there may be no dangers associated with use of the oscillating bed technique.

KENNEDY and FALCON (1965) have also been critical of the use of oscillating beds in the study of wave generated ripples, for the same reasons. On the other hand, SLEATH (1976a) justifies his use of an oscillating bed in moving grain studies by pointing to the good agreement (at high Reynolds numbers) between results of ripple length for wave generated ripples and those produced on oscillating beds.

A variation of the flat bed oscillating in its own plane is the curved bed swinging in an arc, as used by BAGNOLD (1946). A similar piece of equipment, a swinging flume, has been used by DAS (1961). Bagnold's apparatus introduces a complicating factor into the discussion above, in that the resolved component of the force of gravity on the grain, acting tangentially to the arc , must now be included in the calculations. From representative values of frequency and amplitude of motion in Bagnold's experiments, it is clear that the maximum value of this resolved component of weight may be of the same order of magnitude as the inertia force. Furthermore, the maxima of the inertia and resolved gravity forces will occur simultaneously, twice per cycle at the instants when the cradle is stationary (at its positions of greatest excursion). However, the directions of the forces will oppose one another throughout the cycle of motion, thus making Bagnold's cradle a rather better piece of equipment for sediment transport studies than a flat oscillating bed. Despite this, we can conclude generally that the presence of 'rogue' forces in moving bed experiments, possibly having a similar magnitude to the forces which are supposedly being examined, can only harm the credibility of the experimental results obtained. We have only concerned ourselves in this short discussion with the amplitudes of the forces which act on a grain of the bed; a fuller consideration accounting for the phases of action of these forces would modify the results of the calculations in a minor way, but would not alter the basic conclusion.

3.4 The formation of ripples in oscillatory flow

We have argued in the previous sections that ripples play a critical part in the problem of sediment transport in oscillatory flows. Initially we pointed out that the potential flow problem for oscillatory flow over a rippled bed has not yet been solved. It follows that if the boundary layer is taken as the region adjacent to the bed in which departures from the inviscid solution occur, the boundary layer thickness over a rippled bed cannot yet be defined even for laminar flows. This means, furthermore, that velocity profiles and bottom stress distributions over a ripple are also subject to a considerable amount of uncertainty, and this point is emphasised further when, in Part 2 of this report, we discuss results for incipient motion obtained from a field experiment.

In the range of Reynolds numbers of practical importance, the problem is complicated by the possibility of vortices being formed in the lee of each ripple, with all that this implies in terms of the velocity distribution, bottom stress and boundary layer thickness. In general, the energy dissipation increases as a rippled bed develops, and particularly so when vortices are formed. A number of workers, for example HOMMA, HORIKAWA and KAJIMA (1965) have observed that the eddies generated behind sand ripples develop and separate upwards from the ripple every half period of the wave. This increases the intensity of 'turbulence' and the diffusion rate for fluid momentum, and also complicates the overall picture of energy dissipation in the oscillatory flow. In addition, vortex shedding has important consequences in terms of instantaneous values of sediment in suspension during a wave cycle (see Section 3.5 and also KENNEDY and LOCHER (1972)).

The phenomenon of ripple development has been the subject of numerous experimental and semi-theoretical studies. Most of these have been concerned with the problem of ripple formation in steady open-channel flows, but a significant minority have treated the problem in oscillatory flows. The former case has been reviewed by KENNEDY (1969) and also, in considerable detail, by YALIN (1972). Kennedy has noted that four different analytical approaches have been adopted in the studies of the problem over the years. The first of these is one which has been developed by Kennedy himself, although it apparently originated many years earlier. In this approach the characteristics of various bed forms are predicted by introducing a continuity equation for sediment movement, an assumed sediment transport rate formula and a suitable criterion defining the length of the ripples most likely to develop on the bed. This first approach has also been used in the oscillatory flow problem by KENNEDY and FALCON (1965) and will be discussed in rather more detail shortly. The second treats the bed as a fluid of large viscosity and seeks the form of instabilities of Helmholtz type at the interface between the two 'fluids'. This approach is criticized by Kennedy as being unrealistic, and by Yalin on account of the lack of agreement with experimental results. The third, which overlooks the details of the mechanisms involved in ripple formation, attempts to produce a working understanding of the occurrence and size of bed forms on the basis of dimensional arguments. Finally, the fourth involves the description of ripples and dunes in terms of their statistical properties and attempts by means of spectral analysis to arrive at conclusions about the processes involved in their formation. A recent contribution in this fourth category has been that of JAIN and KENNEDY (1974). The statistical approach has been developed because of the fact that, although in the first place ripples arise having a regular formation, some time after their formation when they have approached or achieved their equilibrium dimensions their shapes become somewhat random and short-crested and their arrangement becomes disordered. Mature ripples and dunes of this kind have characteristic frequency distributions of lengths and amplitudes, and are said to be best described by their spectral density functions.

The analytical approaches used in the steady flow problem are likely to be useful also in the case of oscillatory flow, and for the sake of brevity we concentrate in the remainder of this review only on the latter topic.

In terms of the geometry of the bed forms, the main difference between the two cases is that in the former the ripples are strongly asymetrical with much steeper downstream facing (lee) slopes than upstream facing (stoss) slopes, while in the latter case the ripples are symmetrical about their crests in the direction of the fluid oscillation. Also wave generated ripples are generally long-crested and better ordered in their geometry and behaviour than ripples generated by uni-directional water flows. KENNEDY and FALCON (1965) have given an interesting historical account of the developments in the understanding of the oscillatory flow problem during the last 100 years, and we refer the reader to this as far as the early work is concerned.

In considering the problem of ripple formation from a general standpoint, we can identify a basic sequence which takes place on an initially flat sandbed as the intensity of wave activity, characterized by the single wave frequency ω and amplitude of oscillation $A/2$, increases. For low values of $\bar{U}_\infty|_{max}$ $\left(= \frac{A\omega}{2}\right)$, no sediment motion will occur, but at some critical value of $\bar{U}_\infty|_{max}$ incipient sediment motion will take place. This is followed shortly by ripple formation. There is then a range of values of $\bar{U}_\infty|_{max}$ in which ripple development takes place, the upper limit being that value at which the bed becomes flat again. Thereafter the sediment moves as a sheet across the surface.

Of the many descriptions of ripple formation which have appeared in the literature, a notable one for an oscillatory flow is that of CARSTENS and NEILSON (1967). They carried out experiments in an oscillating water tunnel, at a fixed frequency but variable amplitude of oscillation, over an initially flat sand bed. They were able to distinguish two types of ripples; in the first place 'rolling grain ripples' which they referred to simply as 'ripples', and secondly 'vortex ripples' which they referred to as 'dunes'. The former were regarded by Carstens and Neilson simply as the initial transient bed forms in the formation of the latter, but as we will indicate later their occurrence may be rather more general than this. There is a certain amount of disagreement in the literature concerning the classification of ripples, but we regard Bagnold's scheme as the best to follow. Bagnold was the first to introduce the terms 'rolling grain' and 'vortex ripples', together with a third three-dimensional class, namely 'brick pattern ripples'. Following the description of CARSTENS and NEILSON (1967), a 'rolling grain ripple' is taken as one in which the flow causes parallel transverse bands of grains to roll back and forth, separated by bands in which the grains are stationary. The development of rolling grain ripples from a flat bed sees the immediate establishment of the final ripple length, just as in the case of steady flows (see YALIN (1972)). Carstens and Neilson concluded that this stage is transient only, however, and that if the oscillation continues the ripple height increases until separation occurs and vortices are formed. They detected the formation of visible vortices in the lee of each crest twice in each wave cycle, and noted that these were ejected from the trough up into the main flow slightly before flow reversal occurred.

One point not mentioned so far, which has an important bearing on the problem, is that ripples can be induced to form at flow conditions less active than those required for incipient sediment motion on a flat bed. Carstens and Neilson were able to show this in the laboratory by placing an obstruction on the bed, from which a ripple system developed. In view of the fact that any natural bed is almost certain to be covered with obstructions of various kinds, a disturbed-flow deformed-bed incipient motion criterion, such as that proposed by CARSTENS, NEILSON and ALTINBILEK (1969) must be taken as the lower limit for ripple development in practice, rather than an undisturbed-flow flat-bed criterion. Carstens et al argue that such a criterion should be independent of the period of the oscillatory motion, and they claim to have demonstrated this even though their laboratory experiments were carried out at a single frequency only.

Ripples are seldom, if ever, stationary features on the bed. Rather they migrate to and fro about their mean position. Their wavelength is governed by the frequency ω and orbital diameter A, and may be a few inches to one foot or more. Their height (trough to crest) to wavelength ratio is typically in the range 0.1 to 0.2, and the ratio of ripple wavelength to the orbital diameter varies from 0.5 to 1.4 (KENNEDY and LOCHER (1972)). This latter conclusion is in disagreement with Kennedy and Falcon who earlier stated, on the basis of their experiments, that the ripple length is always smaller than the orbital diameter. Homma, Horikawa and Kajima have discussed experimental results on ripple development by a monochromatic wave train, on the basis of various dimensionless parameters characterizing the flow and the ripples. For ripples of height 'H' and length 'L' they have suggested the relationship

$$\frac{H}{L} = 0.231 \left\{\frac{H}{A}\right\}^{0.16}$$

which is independent of the particle size. A further relationship between L/A and the Reynolds number $\frac{\overline{U_\infty}|_{max} A}{\nu}$ has been found to be dependent on the grain size and by implication on the frequency also. From the value of L thus obtained, the height H can be calculated from the equation above. In discussing the height to which ripples grow in unidirectional flow, Yalin proposes a criterion which effectively states that ripples will grow until the kinetic energy of the main flow per unit area of the bed (averaged in the direction of flow) is minimized. This implies a fixed volumetric flow rate in a channel flow of given mean depth. The use of a criterion such as this in developing an understanding of the limiting ripple height seems to make very good sense, although it is perhaps more likely to be the vorticity than the kinetic energy which should be minimized. This is an area into which future research effort should be directed.

We have commented already that there is a range of maximum bed velocities $\overline{U}_\infty|_{max}$ for which ripples can form, and that above a critical value of $\overline{U}_\infty|_{max}$ the ripples give way to flat bed. KOMAR and MILLER (1975b) have discussed this aspect of the problem and have observed that in the régime of 'sheet flow', sediment motion occurs as a sheet of sand within a few centimetres of the bottom, moving to and fro with the intense orbital motions. The disappearance of ripples and the appearance of sheet flow is said to take place gradually. Nevertheless, using the data of Manohar (1955) and others, Komar and Miller have attempted to define zones corresponding to 'no motion', 'ripples' and 'sheet flow' (see Fig 3.2). No mention is made as to whether the ripples are of rolling grain or vortex type in their presentation.

We turn now to consider briefly some of the theoretical models of ripple formation in oscillatory flow which have been proposed. These illustrate the first of the four methods of approach to the problem listed above. One of the most promising models to have been developed is that of Kennedy and Falcon. This is a kinematical potential flow model based on an assumed representation for the deformation of an initially flat bed. The analysis, which is restricted to small amplitude ripples in order that a linearized boundary condition can be applied at the bed, involves neither flow separation nor vortex formation. The ripple wavelength is assumed to be very much smaller than the length of the surface water waves, and the bed is deformed on the basis of a sediment continuity equation which relates local changes in the bed elevation to local changes in the sediment transport rate. An empirical law for sediment transport is assumed in which the sediment transport rate is expressed as a power law in the difference between the potential velocity at the level of the bed and the threshold velocity for sediment movement. The expression for the continuity of sediment motion then yields an equation for ripple amplitude as a function of time. Stable, unstable or neutrally stable conditions are defined corresponding to the decay, growth or neutral stability of the bed forms of a particular length. The model is a linearized one capable only of predicting the trends in the early stages of ripple development under these crude, albeit valuable, headings. A physical argument is then introduced in determining the dominant ripple length of all the possible ripples that might be formed; the dominant ripple wavelength is likely to be that for which the growth rate of small amplitude disturbances is a maximum. An analytical expression for the ripple length is thus obtained. The growth of a ripple will not continue indefinitely in practice, as we noted above, and factors not explicitly accounted for in the model must intervene as the ripples achieve a finite amplitude to fix the equilibrium height of the bed forms. The analysis deals purely with the kinematics of the fluid and sediment motion, and avoids the far more difficult problem of treating the dynamics of particle motion.

An important factor in the analysis of Kennedy and Falcon and one which is a feature of Kennedy's work is the phase difference δ which is introduced between the sediment movement and the water motion. Kennedy describes δ as the 'factor of ignorance' in the problem, but suggests possible reasons why its inclusion is needed. Essentially its role can be interpreted as accounting for the local sediment transport rate not being able to adjust itself instantaneously to the local fluid velocity. On making different assumptions about the form of δ, different expressions for the ripple wavelength are forthcoming. The factor δ is thought to be a function of the ripple height H, which means that it is likely to play an important part in determining the limiting equilibrium ripple height. Also Kennedy makes the rather unexpected observation that δ is temperature dependent.

Throughout their analysis, Kennedy and Falcon have considered small amplitude ripples. In their experimental work, ripples were generated on an initially flat sand bed by progressive waves of length much greater than the ripple length. The orbital velocities involved were in the lower part of the range capable of ripple formation, and the outcome of the experiments appears to have been much the same as that of Carstens and Neilson who used an oscillating water tunnel. The equilibrium ripple wavelength was observed to form at an early stage of growth for all the sediment materials used, and the ripple wavelength never exceeded the orbital diameter of the fluid motion at the bed in any of the experiments. Also the ripple wavelength was found to increase with both maximum bed velocity and orbital diameter. The work of Kennedy and Falcon clearly falls into the category of 'rolling grain ripples', although this is not stated explicitly. One further point worth mentioning is that various shear flow stability analyses have been attempted, as opposed to the potential flow analysis pursued by Kennedy and Falcon. It is arguable whether or not this is the next logical development from the comparatively simple potential flow model. Certainly Kennedy suggests that while it is a refinement which does make the overall picture more realistic, this is at the expense of a far more complicated analysis and little improvement in the quantitative accuracy of the theoretical predictions.

A rather different approach to the problem of the formation of rolling grain ripples is that of SLEATH (1976a). Assuming that the height H to length L ratio of the bed features is small, Sleath has developed analytical relations for the ripple wavelength on the basis of solutions of the two-dimensional equation of vorticity. Essentially his method has been to expand the stream function as a power series in (H/L) which, as we have said, is small. The velocity is taken as zero at the (impermeable) bed and ($U_{\infty m} \cos \omega t$) at 'infinity'. Thus it is assumed that the length of the water waves is very much larger than that of the ripples. Analytical solutions have been sought for the two cases $\frac{U_{\infty m}}{\omega L}$ small and $\frac{U_{\infty m}}{\omega L}$ large, even though the first of these is thought to have little if any practical application. The former case involves a further power series expansion, this time in $\frac{U_{\infty m}}{\omega L}$, while the latter (under the additional assumption that $\beta L \gg 1$ where $\frac{1}{\beta} = \sqrt{\frac{2\nu}{\omega}}$) leads to the Rayleigh equation for which solutions are known. For small $U_{\infty m}/\omega L$ Sleath has determined a limiting ripple wavelength arguing, purely in terms of mass transport velocities, that close to the bed the fluid particles tend to drift towards the ripple crests and that the sediment particles close to the bed will have the same behaviour. By maximizing this residual drift of fluid, but without introducing a threshold of sediment motion or any other device to model the sediment transport, Sleath has proposed as a realistic range of ripple lengths: $4.5 < \beta L < 17.7$. For large $\frac{U_{\infty m}}{\omega L}$ the residual streamlines again show a tendency for fluid motion towards the crests at a small distance above the bed, as in the previous case. Sleath has proceeded as before by assuming that the ripples which form are those for which the net displacement of fluid particles towards the crest is a maximum. This is shown to occur for the maximum value of the orbital excursion $\frac{2\,U_{\infty m}}{\omega}$ for which particles remain predominantly over a single ripple. Again no account is taken of the details of sediment transport. It is concluded that $\left.\frac{U_{\infty m}}{\omega L}\right|_{crit}$ is independent of βL , but displays a functional relationship with βD (see Fig 3.3). A simple physical argument is used to relate the length of ripples which will form on a real sand bed to those which can be deduced from this graph. Although the initiation of rolling grain ripples on an initially flat bed is not discussed as such, Sleath argues that the roughness of individual grains of sediment is sufficient to initiate the process leading to ripple formation.

To examine his theoretical predictions, Sleath has investigated ripple formation in the laboratory, using three different sediment sizes on a bed oscillating in its own plane. (This experimental method has been discussed in Section 3.3(ii)). Sleath found that in only certain of his tests were rolling grain ripples a transitory stage in the formation of vortex ripples, whereas Carstens and Neilson took this sequence to be the general behaviour. Sleath also observed that while the rolling grain ripples persisted, their wavelengths appeared to remain constant as their heights grew. In addition, ripples would not form below some limiting amplitude of oscillation of the bed, a result which Sleath has shown to be consistent with predictions of mass transport velocity from his analytical model. In particular, he concludes that if ripples are to grow then βD must be less than some limiting value in the range $\beta D = 1.6$ to 3.4. This argument is based purely on a consideration of residual velocities, and is not in any way connected with the fact that on a real sand bed the larger grains will be less easily shifted in any case. Generally, Sleath's experimental findings appear to be consistent with the theoretical arguments presented for the case $\frac{U_{\infty m}}{\omega L}$ large.

One of the most interesting points to emerge from Sleath's work is that, as well as having an importance in the initial stages of ripple formation, rolling grain ripples are also found as a transitional regime between vortex ripples and the 'sheet bed' state. This is borne out by results from other experiments, rather than from Sleath's own findings. In other words, as the flow intensity increases, the sequence of bed forms may be as follows: from no motion and flat bed, to rolling grain ripples, to vortex ripples, to rolling grain ripples again and, finally, to 'sheet bed'. It would seem possible, therefore, that rolling grain ripples are of greater practical importance than has been usually supposed.

SLEATH (1975a) has also tackled the problem of vortex ripples, in this case pursuing a numerical solution of the two-dimensional equation of vorticity. As in the analytical solutions for rolling-grain ripples of small amplitude described above, it is found that the fluid close to the bed drifts towards the crests and it is suggested again that this provides a mechanism for the formation of vortex ripples by water waves. Much hangs on Sleath's underlying suggestion that the flow may remain reasonably laminar under conditions of practical importance, to the extent that the transfer of fluid momentum between different fluid layers remains dominated by essentially laminar effects such as vortex formation and decay. The numerical approach differs from the purely analytical formulation for the rolling grain ripple case, in that the condition of small (H/L) is now relaxed (the relaxation, in fact, leads to the breakdown of the purely analytical approaches described above, since the series expansions in (H/L) are no longer adequately convergent). This leads to a rather more vigorous residual flow pattern than in the case of small (H/L), and it is this which is taken by Sleath as indicating vortical activity in the flow. However, whether this pattern in the results from the numerical model can truly be identified with vortices as observed in the laboratory, is somewhat debatable. The residual circulation cells above both the lee and stoss faces of the ripples appear simply as steady flow features and are not linked directly to the flow and its spearation as it passes over a ripple crest. In other words, there is no evidence either in the form of instantaneous streamlines or velocity vectors of vortices developing in the lee of ripples. However, Sleath goes on to draw a distinction between two types of vortex ripples: (a) weak vortex in the lee and (b) strong vortex in the lee. In the former case, which he claims to be modelling, the grains are assumed to remain in close contact with the bed and the lee vortex is assumed to be insufficiently strong to lift the grains up into the flow away from the bed when they are carried over the crest from the stoss face. In this situation very little exchange of sediment occurs between adjacent ripples, and ripple growth occurs according to criteria similar to those described above, based only on the residual fluid motion. In the latter case of a strong vortex in the lee, the sediment carried over the crest is lifted by the lee vortex away from the bed and is deposited on the next ripple or beyond. This complicated situation, referred to by Sleath as 'ripples of the second type', is not dealt with in the paper.

As far as the ripples with a weak vortex in the lee are concerned (and, as we have suggested above, the 'vortex' appears to be so weak that it may be imperceptible in the instantaneous streamlines), Sleath produces from the numerical model the equation

$$\frac{\beta}{k} = 0.229 \left\{ \frac{U_{\infty m}}{\sqrt{\omega \nu}} \right\}^{0.936} \qquad \text{where} \quad k = 2\pi/L$$

for the calculation of the length of vortex ripples most likely to grow. The equation is shown to be little affected by the ripple steepness provided that it is vortex action which is the cause of the ripples. Experimental results are used to show that the equation is valid up to the point at which Sleath suggests 'ripples of the second type' are found. This would seem, therefore, to extend the sequence of occurrence of bed forms (at a particular grain size) listed above, by the subdivision of 'vortex ripples' into firstly 'ripples with a weak vortex in the lee' and secondly, at higher flow stages, 'vortex ripples of the second type'.

Sleath's theory of ripple formation, which is essentially the same for both rolling grain and vortex ripples, is based on the assumption that the grains involved are never transported in the flow at a distance away from the bed, but only in a layer close to it. It is interesting, therefore, to note that Yalin has suggested that, in steady flows, ripples seem to form best when the flow is hydraulically smooth, in which case the granular roughness of the initial plane bed is completely submerged within the laminar sublayer. As we have seen, Sleath argues that in oscillatory flows ripples are formed at even lower flow stages than this when momentum transfer in the flow can be treated as an essentially laminar feature.

In conclusion, we consider that good attempts have been made only in the study of rolling grain ripples. Reasonable criteria have been developed for the determination of the length of the ripples most likely to grow in a given situation, although the equilibrium height of these ripples still poses a difficult problem. As far as vortex ripples are concerned, we are aware of no theoretical treatment of the topic which has yet been carried through successfully. In relation to experimental work on ripple formation, it is important to note that there are likely to be differences between the results of field and laboratory studies. The reason for this is that, in the field, an equilibrium bed will be found only rarely, whereas in the laboratory sufficient time can be allowed for equilibrium to become established. All of the results discussed in this section were obtained in the laboratory.

3.5 Sediment in suspension

We have seen already that oscillatory flows are generally more effective than steady flows in bringing about particle motion. However, as has been pointed out by CHAN, BAIRD and ROUND (1972), oscillatory flow appears to be somewhat less effective in bringing about suspension. This is consistent with the tendency for purely oscillatory flows to remain laminar at velocities which, in steady flow, would correspond to turbulent conditions. Essentially, the presence of material in suspension requires flow conditions in which the forces on grains due to upward vertical velocities are sufficient to support their immersed weight. Upward vertical velocities arise firstly from the wave action in the potential flow region, secondly on account of the presence of ripples on the bed, thirdly as a result of the possible presence of vortices in the lee of the ripples, which may be shed into the main flow in certain circumstances, and fourthly as a result of turbulence in the flow. The relative importance of these four factors appears to be poorly understood at present. However it is generally accepted that away from the bed the wave action is the predominant mechanism in establishing and maintaining a suspension, while close to the bed the turbulence has the greatest relative importance. We note also that the rate at which sediment is entrained is governed largely by the ripple spacing, the strength of the lee vortices and the resulting intensities of the velocity and shear stress just upstream from the ripple crests.

Experimental and semi-theoretical studies of the suspension problem have been reviewed by KENNEDY and LOCHER (1972). As far as the theoretical work is concerned, almost without exception the basic aim has been to produce expressions for the average sediment concentration $\bar{c}(z)$ over a wave cycle. This time averaged quantity can then be used in conjunction with any secondary residual flow present to evaluate the overall movement of suspended sediment. The first order problem of the prediction of concentraton, $c(z,t)$, at height, z, and time, t, has not been attempted. A number of authors have tackled the time averaged problem making use of the usual Rouse formulation often used for steady flow:

$$\frac{d}{dz}\left(K_z \frac{d\bar{c}}{dz}\right) + w_s \frac{d\bar{c}}{dz} = 0 \qquad (22)$$

where K_z is the sediment diffusion coefficient and w_s is the settling velocity of the grains in still water. The choice of K_z which has been made often in steady flow problems is $K_z = \varepsilon$ where ε is the fluid momentum diffusion coefficient or eddy viscosity. This assumption is less than adequate even in steady flows however, a proportionality between the two quantities being a somewhat better assumption. In the case of oscillatory flows, the complicated nature of the vertical velocity field, and also the lack of knowledge about the eddy viscosity, make any assumption about K_z difficult to justify. HATTORI (1969) has suggested that the assumption K_z = constant is reasonable for $z/h > 0.2$, where h is the water depth. On the other hand, DAS (1971), from experiments in a swinging flume, has concluded that the diffusion coefficient varies linearly with distance above the bed. A slightly more general form than equation (22) has been proposed by KENNEDY and LOCHER (1972). In their equation the wave effects are separated from the turbulence, and the authors have examined the two limiting cases of the dominance of the wave term and the dominance of the turbulence term. A solution for $\bar{c}(z)$ from Eq (22), or any similar equation, only gives the relative concentration of sediment at different heights above the bed. Therefore, a reference concentration is needed, at some known height in order to attach an absolute scale to the theoretically derived profile.

HORIKAWA and WATANABE (1970) have carried out laboratory experiments on sediment in suspension in an attempt to separate basic wave effects (as calculated from the water surface elevation) from the wave-induced turbulence. They were able to show that the vertical intensity of turbulence (defined as the rms value of the turbulent fluctuations) was almost depth independent above both trough and crest of a ripple, unlike the horizontal intensity which decreases rapidly with distance from the bed. Thus, although the total turbulence intensity decreased away from the bed, the quantity of most relevance to the sediment diffusion coefficient did not. Concentrations of suspended sediment were measured optically, and it was found that the profiles were similar over both the trough and the crest, predictably showing a decreasing concentration away from the bed. The experimental results have been used to deduce the diffusivity K_z on the basis of an assumed settling velocity. The authors have compared this value of K_z with deduced values of ε, and, although a similarity in the two quantities is noted, they have concluded that there is the need for the development of a generalised relationship between them. From their experiments they suggest that the functional representation for the diffusivity K_z requires a maximum value at about 5 cm from the bed, this maximum being far more pronounced in K_z than in ε.

HOMMA, HORIKAWA and KAJIMA (1965) have adopted a sediment diffusion coefficient derived on the basis of von Karman's mixing length assumption. Curiously, in their formulation of the diffusion coefficient, they have employed the inviscid small amplitude wave theory solution for the horizontal velocity. Since an essentially dissipative phenomenon is being described this appears to be questionable, although, of course, periodic vertical velocities capable of transporting material do exist in the inviscid region under progressive waves. The diffusion coefficient proposed is a function of the ripple shape as well as of the parameters defining the surface waves.

WANG and LIANG (1975) have also adopted the Rouse formulation for calculation of the mean suspended sediment concentration in a wave cycle, taking a diffusivity proportional to the amplitude of the vertical velocity component in the potential flow region under progressive waves. They have found a reasonable agreement between field observation and theory for waves of a single frequency, and argue that if the flow is characterized by a spectrum, then the contributions to the sediment concentration at discrete wave frequencies can be linearly superimposed. Further, by superimposing mass transport velocities in the same way and assuming that the sediment transport velocity is the same as the fluid mass transport velocity, the suspended sediment transport rate can be calculated. This use of spectra is an interesting idea which has possible future application. The field results made use of by Wang and Liang can be contrasted with those described by Homma, Horikawa and Kajima, which were obtained in a far less regular wave field. These latter authors have compared their laboratory results with some field results, and have noted discrepancies between the two cases in what appear to be comparable wave conditions. They explain these discrepancies in terms of the difference between the regular waves in the laboratory and the irregular waves in the sea, which were characterized by mean values of significant wave height and frequency over the observation period.

So far we have mentioned only mean concentrations of suspended sediment over a wave cycle. When consideration is given to the instantaneous concentration the situation can apparently be rather complicated. For instance, over a rippled bed, HOMMA and HORIKAWA (1963) have observed four maxima in the concentration in one wave cycle, at a height of 1 cm above a ripple crest (ripple length 5.8 cms). The same pattern was observed over the trough, but there it was less distinct. The double peak in each half cycle was explained as the initial predictable peak caused by entrainment from a crest as a result of the high velocity over it, being followed by a second peak due to transport of grains from the crest of next ripple upstream. The time interval between the peaks was said to support this contention, and clearly three or more peaks could be envisaged as occurring in a half cycle in different circumstances. The overall concentration was found to decay quite sharply between 1 cm and 4 cm above the bed, and also there was a phase difference in the peak of concentration at the point above the trough and the point above the crest, in relation to the phase of the surface elevation at both positions. Since most of the experimental results discussed in Section 3.4 suggested that the ripple wavelength never exceeds the orbital diameter of the fluid motion at the bed, the presence of two peaks in concentration in a wave half-cycle possibly indicates that the ripple patterns produced by Homma and Horikawa were not in equilibrium with the flow.

HOMMA, HORIKAWA and KAJIMA (1965) noted essentially the same features in their laboratory study, and concluded that, while close to the bed the picture was a most complicated one, away from the bed it was less so. In particular, they identified a distinct boundary between a clear and a turbid layer. The latter layer had a depth of the same order as the ripple length and moved up and down with the period of the wave motion. The concentration in the upper part of the turbid layer was found to change little during a wave period while, close to the bed, phase differences of the kind described above between trough and crest were noted. These were explained by pointing out that the maximum sediment concentration above a ripple crest appears when a shed vortex passes the crest, while above the ripple trough it occurs while the vortex is in the trough. This explanation of concentration maxima differs somewhat from the one given by Homma and Horikawa. In general, the effect of vortex activity was found to be important in approximately the bottom 3 cms of the flow. Above this level the concentration appeared to behave roughly in phase with the speed of the horizontal motion of the potential flow.

There is practically no analytical treatment of the transport of suspended sediment in a wave field, which goes beyond a crude attempt to calculate the mean concentration in a wave cycle. This mode of transport is likely to be one of great significance in rough conditions in the sea and it calls for a good deal of further study.

CHAPTER 4 THE EFFECTS DUE TO A PERMEABLE BED

Progressive waves passing over the water surface cause the pressure on the seabed to vary in a way which can be calculated approximately from small amplitude wave theory (see Section 2.1). Thus, at any instant in time and at any point on the bed, there are pressure gradients present in the direction of wave travel, and the question posed here is 'What effects are associated with these spatial and temporal variations in the pressure when the bed is permeable?'. The problem can be split into various aspects: (i) what are the seepage flows in the bed?; (ii) what influence do these flows have on individual particles?; (iii) do the seepage flows at the edge of the bed interact with the main flow and change the nature of the boundary layer?; (iv) do the pressure variations have any overall effect on the stability of the bed? The permeability of a granular material behaves approximately as the square of the grain size, and so any such effects will be greatest for coarser materials. However, we argue in this chapter that, even though permeability has some influence on sediment transport by waves, this is of only secondary importance.

There have been many studies of the flows induced in permeable beds by the passage of surface waves (eg REID and KAJIURA (1957), SLEATH (1970a), LIU (1973) and MOSHAGEN and TORUM (1975)), the majority having been carried out from the point of view of energy dissipation leading to attenuation in the surface wave height (see Section 2.4). The problem is usually formulated in terms of a permeable layer of finite thickness, sometimes assumed to be isotropic, sometimes not. The upper boundary condition is taken as the pressure distribution under a small amplitude wave, while the lower condition is taken as one of no flow across the bottom boundary. Pressure changes are assumed to take place slowly enough for the simple steady Darcy's Law to apply to the flow in the layer, and a further assumption made is of the incompressibility both of the soil skeleton and of the pore water. The result of this assumption is that the pressure distribution is independent of absolute permeability, only the ratio of the permeabilities in the vertical and horizontal directions being important in this respect (eg SLEATH (1970a)). Furthermore it follows in the above formulation, that there is no variation in the phase angle of the pore pressure fluctuation with depth in the permeable layer. This fact, together with the predicted amplitude of the pressure fluctuation, has been checked in the laboratory by SLEATH (using in situ measured permeabilities) and found to agree approximately with the theory.

MOSHAGEN and TORUM (1975) have allowed the pore water to be compressible, but have neglected the buoyancy terms, thus retaining a linear set of governing equations. As a result, both the phase and amplitude of the pore pressure fluctuations are depth dependent, and become more so with decreasing permeability (ie with smaller grain size). In a discussion of this paper PRÉVOST et al (1975) have pointed out that the compressibility of the pore water is an order of magnitude less than that of the soil skeleton (SCOTT (1963)), and have concluded that the solution of Moshagen and Torum is unrealistic. However, even though their results have not been compared with any laboratory or field measurements, these do seem sensible in that the depth of influence of the pressure fluctuations diminishes with decreasing permeability; and, in the limit of an impervious bed, the fluctuations do not penetrate at all. Since the compressibility of the pore water is very small compared with the skeleton, perhaps the correct trends have been found by Moshagen and Torum, but on the wrong assumptions. The work needs to be continued further for the case of a deformable soil skeleton only, and any numerical results obtained need to be checked against laboratory or field data. The differences between the incompressible solutions (eg SLEATH (1970)) and the compressible are greatest for fine materials ($D_{10} < 0.2$ mm according to Moshagen and Torum), and therefore for medium to coarse sands it can be assumed that the incompressible solution is satisfactory.

It is well known that seepage flows through permeable materials have a major influence on the stability of the soil mass. The 'seepage force', as it is known, has a magnitude of $(-\nabla p_b)$ per unit volume, where p_b is the pore pressure in the bed; this force on the soil skeleton is in the direction of flow. The classic example of the force in soil mechanics arises when a vertical seepage flow causes a seepage force equal, and in the opposite direction, to the submerged weight of the soil (per unit volume). Under these critical conditions the effective stress in the soil (ie that between the granules) becomes zero, the shear strength disappears, and 'boiling' or a 'piping' failure occurs. Similar arguments applied to flows vertically into or out of a porous bed beneath a fluid flow give rise to questions about the possible effects of seepage forces on sediment motion. However, in this connection, it must be realized that the term 'shear strength of a soil' and the concept of a seepage force are normally associated with regions away from the boundaries of a permeable material, where, even though the structure is not homogeneous, the soil is treated as a continuum. Near a boundary, or on the boundary itself for the present purpose of the consideration of the erosion and deposition of sediment grains, inhomogeneities in the structure become significant and the continuum model becomes inaccurate. Nevertheless its simplicity is attractive and it is generally accepted.

Most investigations of the seepage force have been carried out with reference to unidirectional flows, although MARTIN (1970) does make mention of wave motion. The effects of seepage on sediment transport in a fluid flow can be split into two aspects: (i) the influence of inflow and outflow (suction and blowing) on the flow in the boundary layer and, hence, on the boundary shear stress; (ii) the change in the 'shear strength' of the surface due to seepage flows. MARTIN and ARAL (1971) have isolated the effect of seepage on the stability of surface particles, finding in their experiments the angle of repose of a sand bed under various seepage conditions, with no surface shear stress applied. The seepage force has been calculated on the assumption of a homogeneous continuum, and the edge effects have been determined experimentally in terms of a coefficient C, equal to $\{$ the surface seepage force $\}$ $\div$ $\{$the interior seepage force$\}$. The results in this connection showed a great deal of scatter, the value of C varying between 0.3 and 0.5. The investigation of MARTIN (1970), on the influence of seepage on incipient sediment motion due to steady flow, indicated that motion can be enhanced or hindered depending upon the relative importance of effects (i) and (ii) above. For example, seepage into the bed causes a downward force on the surface particles making them harder to move, but the accompanying suction of the boundary layer causes an increase in the boundary shear stress, which has the opposite effect. On the other hand, seepage out of the bed has a less pronounced effect than seepage into it, because, as soon as a sediment particle 'lifts off' the bed, the upward seepage force on it is greatly reduced; the boundary shear stress is decreased in this case. Now, if the vertical seepage velocity at the interface is calculated for an isotropic coarse sand ($\mathcal{D} \sim$ 1mm) it can be easily shown from SLEATH (1970b) that

$$\frac{W|_{z=0}}{U_\infty} \lessapprox 7 \times 10^{-4}$$

the two velocities in the quotient being in phase. This figure is orders of magn magnitude less than the seepage velocities used in Martin's experiments, and it can be concluded (in agreement with Martin) that 'a porous bed under oscillatory waves would not have much effect on incipient motion or on sediment transport'. SLEATH (1968) has shown that the magnitude of the perturbation of the horizontal velocity ($O(1)$) from the shear wave solution, introduced by a percolation velocity into and out of the bed of amplitude V_0, is of the order of ($V_0^2/4\omega\nu$).

In a typical situation in the field (see Part 2 of this report - DAVIES, FREDERIKSEN and WILKINSON (1977)) this quantity takes a value less than 7.5×10^{-5} and so the effect of permeability on the flow in the boundary layer is also likely to be small.

Up to this point we have concentrated on the response of the surface particles to wave induced seepage forces, in terms of their susceptibility to erosion. However the stability of the sea bed, permeable or otherwise, must also be considered as a whole. HENKEL (1970) has considered the effect of a sinusoidal surface pressure distribution, induced by the passage of progressive water waves, on the stability of soft underconsolidated deltaic sediments. In particular, evidence has been found of underwater landslides in areas such as the Mississippi Delta where the average bottom slope is only about $\frac{1}{2}^{\circ}$, flat enough to be stable despite the very weak underconsolidated state of the bottom sediments . Henkel has used a simple 'slip circle' analysis (see TERZAGHI and PECK (1967)) with a quasi-static sinusoidal surface loading to show that the bottom pressure effects due to waves are more than adequate to cause shear failure in such sediments. An alternative type of failure, the 'infinite slope' type, has been investigated by WRIGHT and DUNHAM (1972) using a quasi-static finite element analysis; this too has shown that surface waves may produce relatively high stresses and large displacements in the sea floor sediments to considerable depths. In addition, studies of the topic have been undertaken by MITCHELL, TSUI and SANGREY (1972) and MITCHELL and HULL (1974), using laboratory models. In both cases an order of magnitude agreement has been found between a rather oversimplified theoretical model and the experimental results. The quasi steady nature of these analyses means that the predicted 'failure'conditions are only momentary and would not necessarily, therefore, result in soil movement; however it might also be argued that, once the bottom has become unstable, the effect of gravity may be to initiate and sustain movement of the bed. The work of Mitchell et al has also suggested that the continually fluctuating pressure load due to the waves could possibly remould the bottom sediments significantly, reducing their strength and leading to failure. All the above examples have been of possible wave induced instabilities to cohesive sediments with no mention of seepage at all, although MOSHAGEN and TORUM (1975) have indicated that the large values of the vertical pressure gradient induced in very fine grain sediments will have an influence on erosion.

The action of waves on non-cohesive materials (eg sand and gravel) is considered by MADSEN (1974a and 1974b) in two ways. In a simplified analysis, he has assumed that a linear horizontal pressure gradient is transmitted through the top layer of the sea bed, resulting in a horizontal seepage force and with 'failure' occurring when this exceeds the shear strength available. In a more sophisticated analysis, a circular failure surface is assumed and the method of slices is used (see TERZAGHI and PECK (1967)) to solve for the angle which the failure surface subtends in terms of the soil parameters and the linear horizontal pressure gradient. Both solutions give a critical pressure gradient:

$$\left.\frac{\partial p}{\partial x}\right|_{crit} = (\rho_s - \rho_f)\, g \tan\phi$$

where ϕ is the angle of repose of the sediment. If the bottom pressure is taken as approximately equal to the hydrostatic (dynamic effects being ignored), this gives a critical water surface slope of $(\rho_s/\rho_f - 1)\tan\phi$. For a loosely packed rounded sand, this quantity takes a value of approximately 0.5. This is far greater than could arise in the context of small amplitude wave theory, but is not by any means large in the context of breaking waves. Indeed, such values have been reported by Madsen on the basis of his tests in the labroatory. However, it must be remembered that the proposed failure mechanism is a momentary one, and its full relevance to sediment transport is at present unclear.

All the effects discussed in this Section have been of an instantaneous kind. They are often small compared with other effects simultaneously present, and in such cases can be ignored. However, second order effects may have an important influence on results in the long term. For instance, LOFQUIST (1975) has considered the influence of permeability on residual sediment transport rates, studying the difference between the average residual stress on both permeable and impermeable rippled beds. In his formulation the non-linear terms have been retained in Eq (10), which is taken as the equation of motion for the incompressible flow with and without seepage present. This equation is then integrated with respect to z between the bed and a level at which the shear stress is negligibly small compared with τ_0. The spatial and temporal average of the difference (ie the added residual stress due to seepage) is shown to be always in the direction of wave travel and of magnitude $\frac{1}{2}\rho_f U_b W_b$, where ($U_b$, W_b) are the amplitudes of the seepage velocity components at the surface of the bed. Although it is a small residual, the fact that this added stress is inexorably in the wave direction could be important on geological time scales. However, laboratory experiments with a permeable bed in an oscillating water tunnel (with the seepage modelled) failed to demonstrate the phenomenon, the small residuals being undetectable in the presence of the first order sediment motion.

CONCLUSIONS

Despite a considerable amount of theoretical and experimental work on the problem of sediment transport by wave action, many important questions remain unresolved. In this review the problem has been broken down into several aspects, and we have attempted to assess the present state of knowledge of each of these.

Previous research relevant to the present study can be divided into two kinds. The first is the study of the wave field itself and the character of the flow in the oscillatory boundary layer. The second includes studies in the laboratory of sediment transport by wave action together with various theoretical attempts to develop quantitative relationships between the driving flow and the resulting sediment movement. Let us take these topics now in turn.

Wave theory has been highly developed over many years and, in many situations of practical importance, small amplitude progressive wave theory appears to offer a convincing theoretical framework. In particular, this theory appears to be adequate for the case of waves having both small steepness and small wave height to mean water depth ratio. Swell waves travelling across the continental shelf towards the shore line meet these requirements. Close to the shore in shallowing water, the waves inevitably steepen and break and here the theory is no longer applicable.

Although small amplitude wave theory is expected to be valid offshore in the main body of the flow, it is not expected to hold in the layer adjacent to the bed in which a boundary layer will be formed and the influence of sand ripples will be felt. Ripples will be found on the bed in most natural situations. The theory has only been developed for a flat horizontal bed and a flat bed of uniform slope, and theoretical predictions concerning the amplitude and phase relationships between the pressure and velocity components still need to be confirmed experimentally in the field.

The boundary layer developed close to the bed will be laminar or turbulent depending upon characteristics both of the flow field and of the bed roughness. The form of the relevant Reynolds number in this context is not yet clear: in particular, whether it is the sand grains or the ripples on the bed which provide the relevant length scale in the transition from laminar to turbulent flow is open to doubt at present. What is needed is a quantitative means of assessing whether or not transition has occurred, and we have suggested that a suitable parameter in this context is the effective viscosity. This will take the value of the molecular viscosity if the flow is laminar, and a greater value if the flow is turbulent. There are two factors which complicate the picture however. In the first place, laminar and turbulent flow states may arise in different parts of a single wave cycle. Secondly, "turbulence" in the oscillatory flow problem may comprise a far more ordered and repeatable behaviour in the flow than is usually implied by use of the term. This turbulence may be partly the result of vortices being shed from the lee of ripples upon flow reversal.

The presence of ripples on the bed serves to complicate the picture greatly, and close to the bottom the simple potential solution for flow over a flat bed becomes invalid. It is important for an appropriate potential solution to be developed in the case of a rippled bed, for then the zone of frictional influence adjacent to the bed could be identified by departures from theoretical predictions. Thus the velocity profile and stress distribution within the boundary layer could be modelled, and the boundary layer thickness could be defined. This would enable a theoretical understanding of the effect of sand ripples to be obtained, at least in the case of a non-separating flow. The flow separation involved in vortex formation raises further problems and has not yet been modelled successfully.

The ultimate aim of such a boundary layer study would be to obtain predictions of the stress at the bed with a view to developing an understanding of the conditions needed to bring about threshold sediment motion. The main difficulties here are that the threshold of sand movement appears to be in the transition region between laminar and turbulent flow, and that the processes involved in sediment transport in unsteady flows are poorly understood at present. Although the velocity induced bed shear stress is likely to be the most important factor in moving sediment, the pressure gradients driving the flow will also act directly on the exposed grains on the surface of the bed and may have a secondary influence on incipient motion conditions. No laboratory studies have explored the phase relationship between these two quantities explicitly. Just as most studies of the wave field have been concerned with the case of a monochromatic wave train, so also have most laboratory studies of sediment transport in oscillatory flows. The usual laboratory technique has been to identify the "critical wave" capable of moving sediment, in terms both of the wave parameters and sediment characteristics. The presentation of results has invariably been in the form of dimensionless groups, and from these it is not possible to recover phase information which would indicate the phase angle in the wave cycle at which sediment movement was detected. Consequently when analysing results obtained in a comparatively irregular flow field, such as that associated with swell waves in the sea, it is not possible to state with certainty what conditions are likely to produce sediment movement on account of the lack of a good basis of comparison with laboratory results. Such a basis must be sought, however, even though it is likely to be very approximate.

When threshold conditions have been exceeded, sediment movement will occur either as 'bed load' or 'suspended load'. A certain amount of progress has been made in treating the bed load problem in oscillatory flows. Here various models of the transport phenomenon, originally developed for steady flows, have been adapted for use in the unsteady case. From a quantitative point of view, it is hard to assess how successful these models are in oscillatory flows, since the problems involved in measuring transport rates are formidable, particularly in the field. Further progress in this direction seems to be almost entirely dependent upon the development of new instrumentation to quantify bed load transport rates. In connection with the suspended load, laboratory studies have revealed that the instantaneous sediment concentration at positions in the flow close to the bed varies with time in a very complicated manner, especially when the bed is rippled and separation is occurring. It is perhaps not surprising, therefore, that no attempts have been made to model the instantaneous sediment concentration profile, and that all that has been done is to model the average concentration over a wave cycle as a function of the depth.

One aspect of the problem which is often discussed in connection with nearly all of the topics raised so far, is the effect of the permeability of the bed. For typical sand sizes such effects are likely to be of only marginal importance. Possibly, over long periods, the total residual sediment transport may be enhanced or diminished somewhat on account of permeability, but it seems clear that the first order problem of the instantaneous sediment transport within a wave cycle can be studied without modelling the permeability if the surface waves have a small steepness.

Surprisingly few field experiments have been carried out up to the present. Some attempts have been made to monitor the flow field under progressive waves, but no studies of the threshold of motion in the sea are known to have been carried out. In Part 2 of this report (DAVIES, FREDERIKSEN and WILKINSON (1977)), a field experiment is described which is the first in a series of such experiments aimed at elucidating some of the aspects of the problem described above. The study concentrates mainly on instantaneous aspects of the phenomenon of sediment transport by waves, with a view to examining threshold conditions for sediment motion in the sea.

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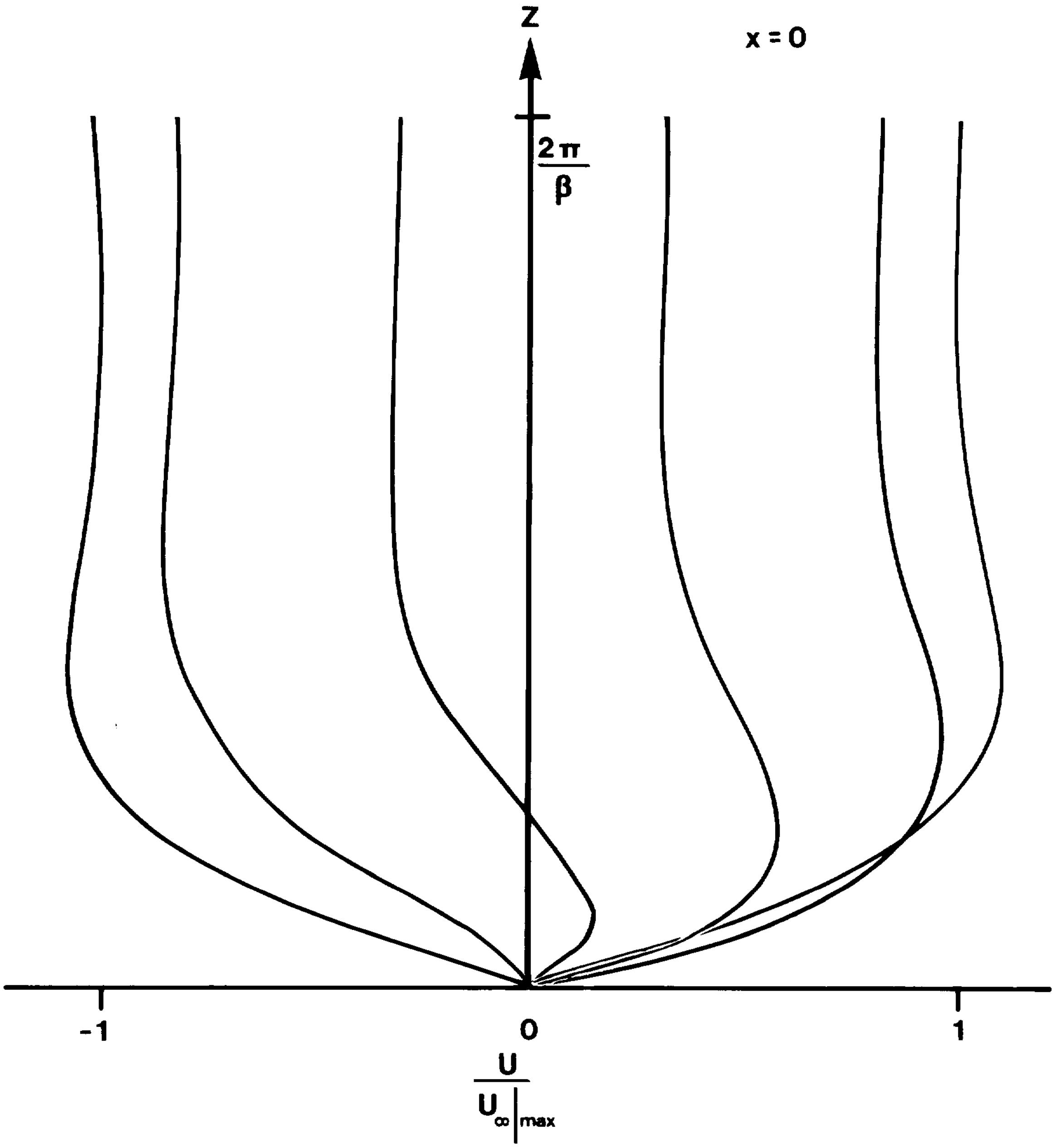
Author	$Re_1 = \frac{\hat{U}_0|_{max} A}{\nu}$	$Re_2 = \frac{\hat{U}_0|_{max} k}{\nu}$	$Re_3 = \frac{\hat{U}_0|_{max}}{\beta \nu}$	Comments	Apparatus and Method
Vincent (1957)		18 33 30		$\frac{\delta_v}{k} < \begin{cases} 19 \\ 8.5 \\ 6.5 \end{cases}$ where δ_v is an undefined boundary layer thickness	Dye observations in wave flume
Collins (1963)			160	Transition hypothesis unconvincing	Observations of residual velocity
Kajiura (1968)		2000	$35 < Re_3 < 920$ $141 < Re_3 < 1414$	(Upper bound?) Smooth bed Rough bed	? ? ?
Carstens et al (1969)	3.75×10^4	50.6	193.6	Only one test performed $A/k = 370$	Dye observations in oscillating water tunnel
Einstein (1972)	3.4×10^5			Smooth plate	Dye observations in oscillating bed apparatus
		640 (2-D) 104 (3-D)		Rough plate – for hydraulically rough flow, $A/k < 532$ (2-D) and $A/k < 3260$ (3-D).	
Sleath (1974c)		$8440 \left(\frac{k}{A}\right)^{0.55}$		Not original results – fitted to work of Li (1954), Manohar (1955) Vincent (1957) and L'Hermitte (1961). Described as 'fully developed mixing' $3.55 \times 10^{-2} < Re_2 \times \left(\frac{k}{A}\right) < 987$	

Note: If the laminar shear wave solution is valid, $Re_1 = (Re_3)^2$

TABLE 2.1 Transition to turbulence in wave boundary layers



Veiocity profiles in the laminar shear wave layer.

(from LAMB (1932))

Fig. 2·1

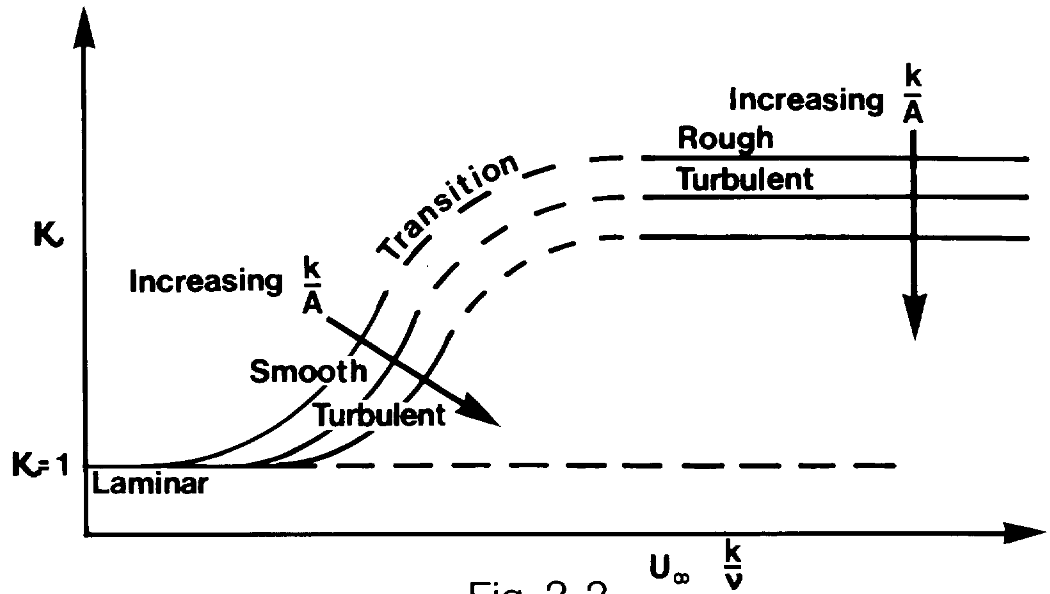


Fig. 2·2

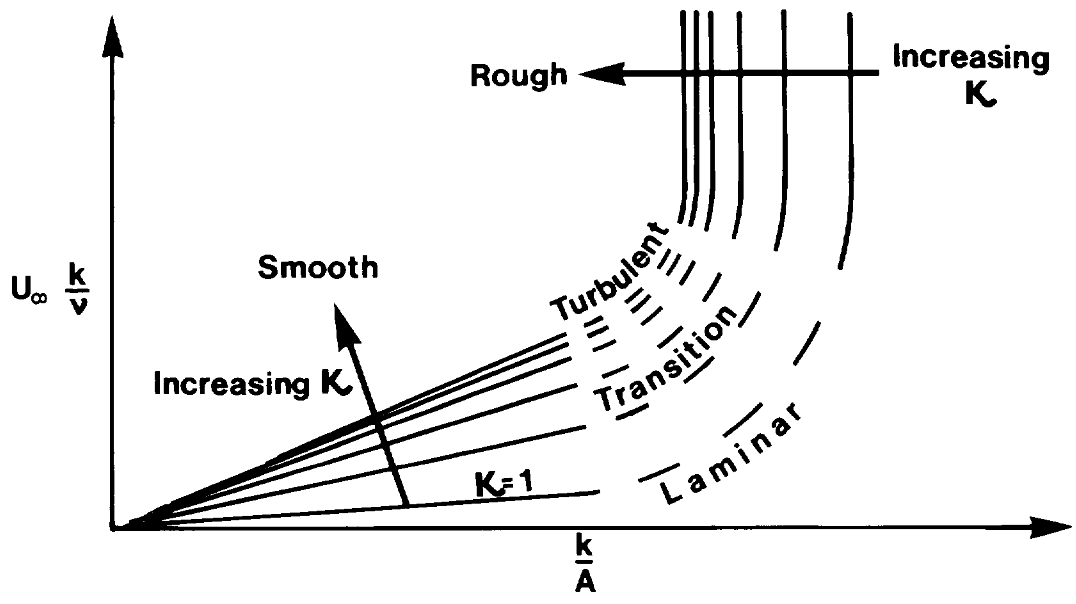
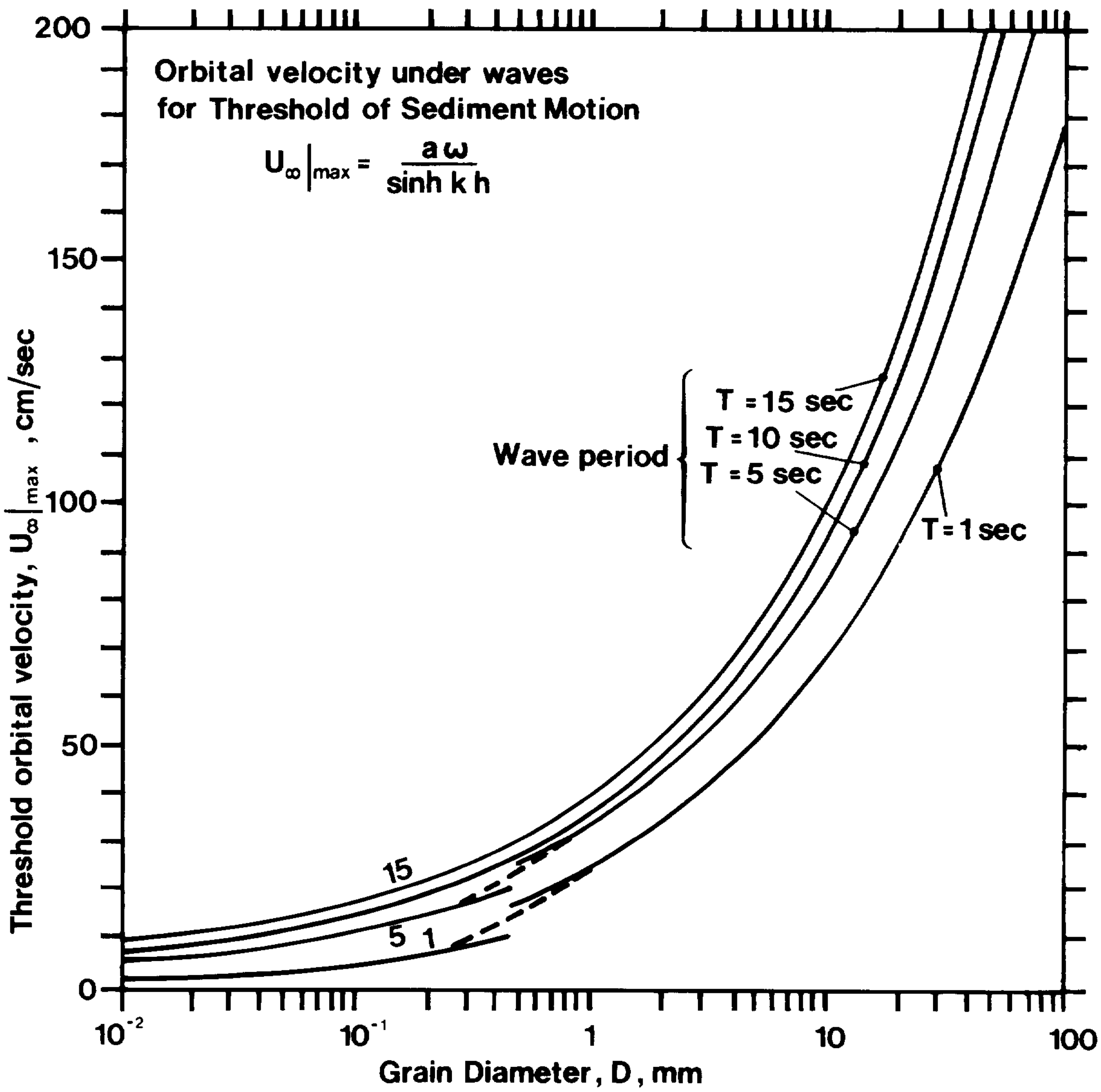


Fig. 2·3

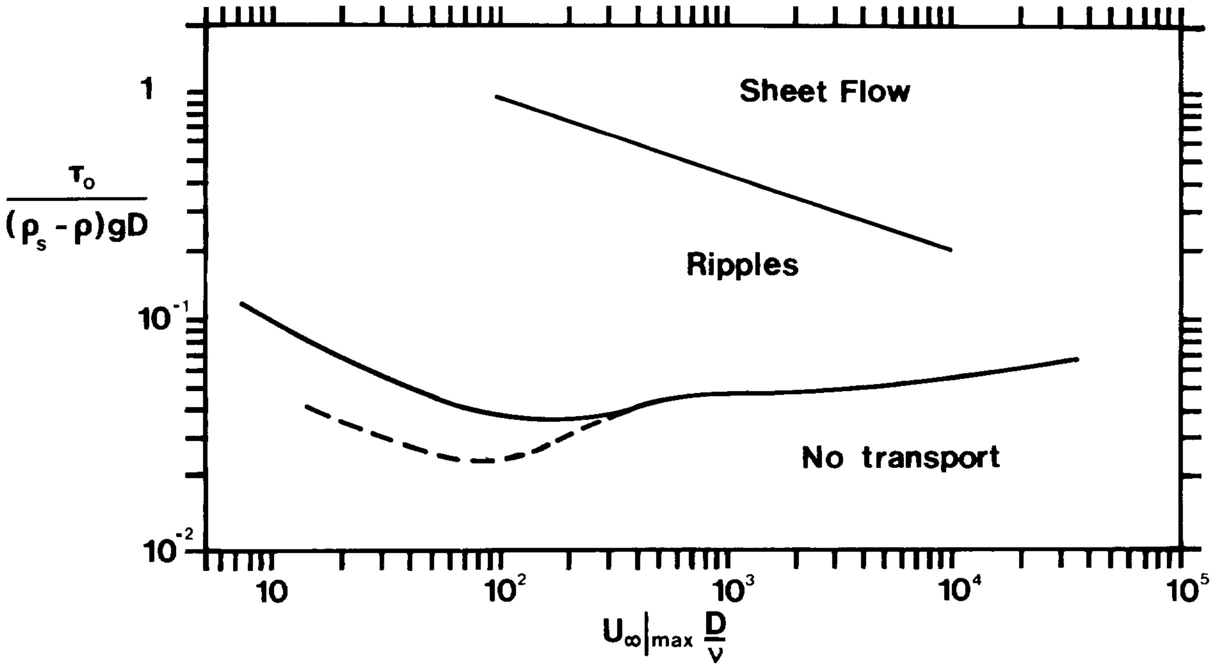
Transition from Laminar to Turbulent flow. *(Caption for Figs 2·2 & 2·3)*



The near-bottom orbital velocity $U_\infty|_{max}$ for sediment threshold under waves.

(from KOMAR & MILLER (1974))

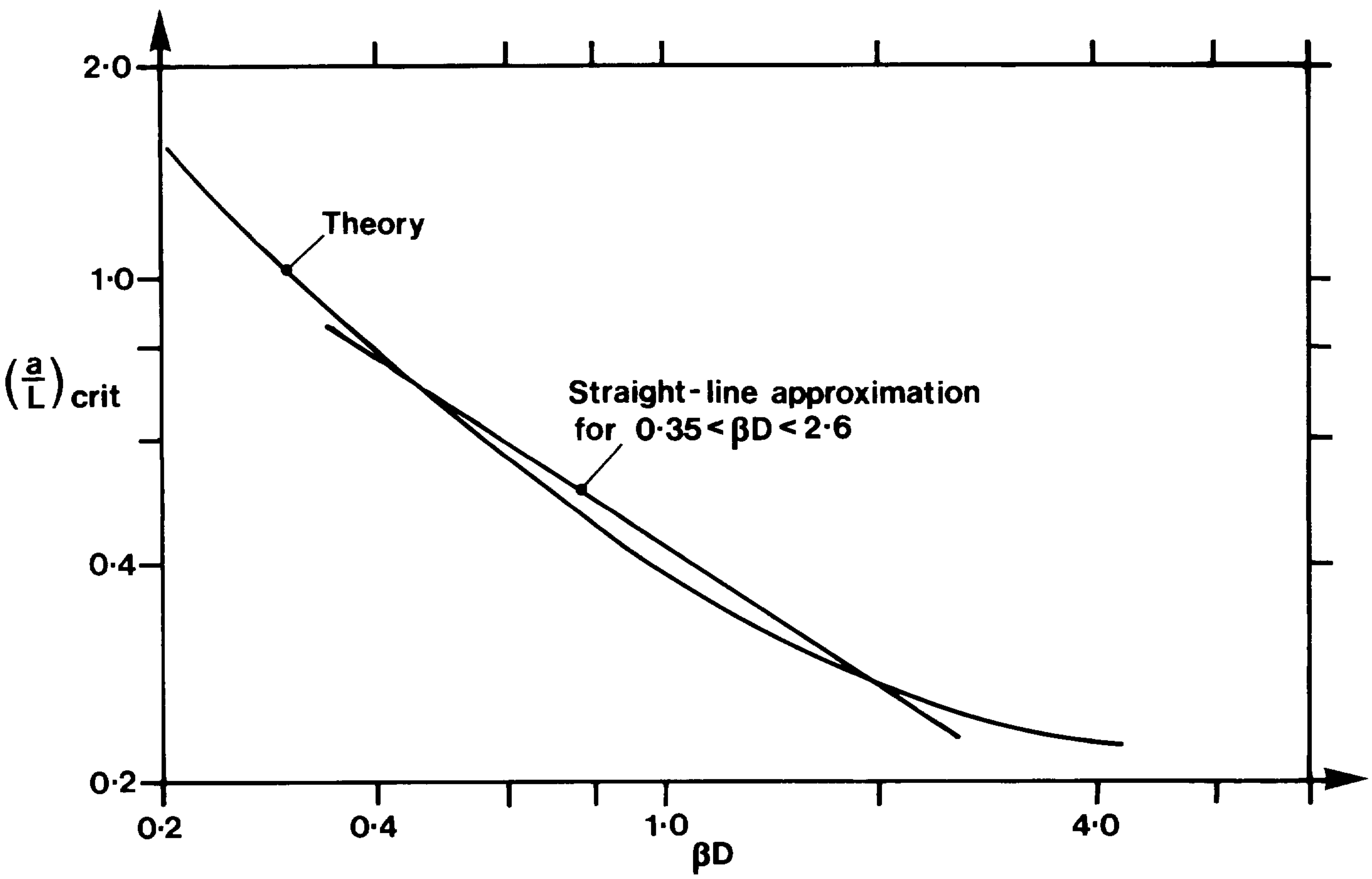
Fig. 3·1



Occurrence of bed forms.

(from KOMAR & MILLER (1975b))

Fig. 3·2



Variation of $\left(\frac{a}{L}\right)_{crit}$ with βD for $\beta L = 60$ $\quad (a\omega = U_\infty|_{max})$

(from SLEATH (1976 a))

Fig. 3·3