WAVES RECORDED BY M. V. FAMITA
IN THE NORTHERN NORTH SEA

B. C. H. FORTNUM

Data for winters of 1969 to 1976 at position
57°30'N 03°00'E

Summary Analysis and Interpretation Report

The preparation of this report and the collection of some
of the data contained in it have been financed by the
Departments of Energy and of Industry.

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INSTITUTE OF OCEANOGRAPHIC SCIENCES

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INTRODUCTION

The Institute of Oceanographic Sciences and its forerunner, the National Institute of Oceanography, have measured waves at the location of m.v. Pamita since 1969, and have produced two previous reports containing analyses of these data. The first report uses data from the winter of 1969/70 (Draper and Driver, 1971) and the second contains an analysis (including spectra) of the high wave conditions from the winters of 1969/70, 1971/72 and 1972/73 (Saetre, 1974). The present report is similar to the first but contains an analysis of all the wave measurements from October 1969 to March 1976 taken together.

The m.v. Pamita is a Norwegian vessel which serves both as a meteorological station in the northern North Sea, and as a rescue vessel when the need arises. When on station its location is 57°30'N 3°00'E (see figure at the beginning of this report), where the depth of water is approximately 66 metres. Normally it operates only from October to March.

The vessel is fitted with a shipborne wave recorder (Tucker, 1956) maintained by the Institute of Oceanographic Sciences. Twelve-minute samples of the variation of surface elevation are taken every three hours and recorded on paper chart rolls. Wave records are considered usable only when the vessel was on station during the recording period and not when it was under way for rescue purposes. For this reason a large number of wave records has been missed, and it is probable that these lost records would have contained many high waves associated with the storm conditions which necessitated the rescue operations.

Calibrations were carried out during the period covered by this report, and these confirmed that the instrument was behaving satisfactorily. An appendix to this report contains the results of these calibrations.

Wave data were recorded during the six months from October to March from 1969 to 1976. During the six-month period in 1970/71, so few data were successfully recorded that this period has not been included in the analysis. Consequently the data which are summarized in this report are as follows:

<table>
<thead>
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<th>% valid data</th>
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<tbody>
<tr>
<td>1969/70</td>
</tr>
<tr>
<td>1971/72</td>
</tr>
<tr>
<td>1972/73</td>
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<td>1973/74</td>
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<td>1974/75</td>
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<tr>
<td>1975/76</td>
</tr>
<tr>
<td>Average</td>
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The particularly low data return for 1971/72 is due to data from two entire months (October and February) being unusable or non-existent.
METHODS OF ANALYSIS

The technique used to analyse the wave data was that proposed by Tucker (1961) and Draper (1963) and reviewed by Tann (1976). For each three-hour interval the corresponding twelve-minute record gives values of the following parameters.

1. The number of zero up-crossings, $N_z$. A zero up-crossing is considered to occur when the trace crosses the mean line in an upward direction.

2. The number of crests, $N_c$.

3. The mean zero up-crossing period, $T_z$. This is defined as the duration of the record divided by $N_z$.

4. The standard deviation, $\sigma$.

5. The significant wave height, $H_s$. This is defined as $4 \sigma$. For a narrow band random process this parameter approximates closely to the mean height of the highest one-third zero up-cross waves (Longuet-Higgins (1952)). Comparison between the two definitions is made by Goda (1970, 1974).

6. The most probable height of the highest zero up-cross wave in the three-hour interval, $H_{\text{max}} (3 \text{ hr})$.

7. The bandwidth parameter, $\epsilon$, which is a measure of the range of frequencies present. $\epsilon = \sqrt{1 - \left(\frac{N_z}{N_c}\right)^2}$

The parameter $H_{\text{max}} (3 \text{ hr})$, which is the mode of the distribution, should not be confused with the expected height of the highest wave in three hours, which is the mean of the distribution. The mean of the distribution is typically $3\%$ higher than the mode (Tann (1976)).

This method of analysis introduces statistical scatter leading to a standard error of $6\%$ in the abstracted wave height parameters (Tann (1976)). It is of little importance in the majority of applications, but when extrapolating the long-term cumulative distribution of wave heights (Figures 23 to 27) it is important that
the large values are estimated as accurately as possible. To ensure this, the records giving the forty-three largest values of significant wave height were digitized and new values of \( H_s \) and \( H_{\text{max}} \) (3 hr) were obtained from the computed root-mean-square surface elevation.

Mathematical formulae for the four cumulative distributions used for \( H_{\text{max}} \) (3hr) in this report are:

1. **Weibull**

   \[
   \text{Prob}(X \leq x) = \begin{cases} 
   1 - \exp \left[ - \left( \frac{x - A}{B} \right)^C \right], & \text{for } x > A \\
   0, & \text{for } x \leq A
   \end{cases}
   \]

   where \( B \) and \( C \) are positive, and \( A \) represents a lower bound on \( H_{\text{max}} \) (3hr).

2. **Log-Normal**

   \[
   \text{Prob}(X \leq x) = H \left( \frac{\ln \frac{x}{B}}{\delta} \right)
   \]

   where \( H \) is the normal distribution function

   \[
   H(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\theta} \exp \left( -\frac{t^2}{2} \right) dt.
   \]

3. **Gumbel's Third Asymptote**

   \[
   \text{Prob}(X \leq x) = \begin{cases} 
   \exp \left[ - \left( \frac{A-x}{B} \right)^C \right], & \text{for } x \leq A \\
   1, & \text{for } x > A
   \end{cases}
   \]

   where \( B \) and \( C \) are positive. This is the extreme value distribution first considered by Fisher and Tippett (1928) for a variable bounded above by \( A \) (see Gumbel (1958)). Letting this bound tend to infinity gives the first asymptotic distribution, as below.

4. **Gumbel's First Asymptote**

   \[
   \text{Prob}(X \leq x) = \exp \left[ -\exp(-ax+b) \right].
   \]
DISCUSSION OF RESULTS

The wave data have been treated in this report as a single six-month 'season' extending from October to March. Division into two three-month 'seasons', each containing successive months, was not considered justifiable due to the comparatively small variability of wave conditions between the six calendar months.

The low data return leads to two limitations in this data presentation: the omission of persistence diagrams, and of yearly cumulative distribution plots. Persistence diagrams are not included because the data contain too many discontinuities. Yearly cumulative distribution plots containing six months of data are excluded because the wave data may be seriously deficient in high waves (as noted in the introduction), and may therefore give misleading information about the lower probability parts of the distributions.

FIGURE 1 Percentage exceedance of Hs and Hmax (3 hr)

This graph may be used to estimate the fraction of the time during which Hs or Hmax (3 hr) exceeds a given height. For instance, the significant wave height exceeded four metres 19% of the time, and Hmax (3 hr) exceeded seven metres 24.5% of the time.

FIGURE 2 Frequency histogram for Tz

From this figure it may be seen that almost all zero-crossing periods were between 3.0 and 12.5 seconds, with a maximum number of zero-crossing periods between 6.5 and 7.5 seconds.

FIGURE 3 Scatter diagram

The numbers of occurrences of particular pairs of values of significant wave height and zero-crossing period, expressed in parts per thousand, are shown in this diagram. Joining up points with equal numbers of occurrences as shown ('contouring') gives a representation of the bivariate probability distribution and illustrates the correlation between Hs and Tz. Theoretically the limiting value of wave steepness for a progressive wave is 1 : 7, where wave steepness is defined as the ratio of wave height to wave length. However, both significant wave height and zero-crossing period are average quantities, and on the scatter diagram for these two quantities there is a fairly well defined limit of about 1 : 10. (Steepness is defined for this purpose as \( \frac{2\pi H_s}{g T_z^2} \)). The diagram is based on 6050 valid records.
FIGURE 4  Frequency histogram for $\varepsilon$.

Almost all values of $\varepsilon$ were between 0.25 and 0.95, with a peak at 0.65.

FIGURES 5 to 8  Cumulative distributions of $H_{\text{max}}$ (3 hr)

The cumulative distribution of $H_{\text{max}}$ (3 hr) for the whole period covered by this report is plotted on four scales as shown and straight lines fitted where appropriate. This is equivalent to fitting the corresponding distribution function to the data. Having found the distribution which gives a satisfactory fit, the line is extrapolated to find heights which have a very low probability of being exceeded.

For the Weibull distribution shown in Figure 5, a value 1.0m for the lower bound, $A$, was found to produce the best alignment of data.

For Gumbel's Third Asymptotic distribution shown in Figure 8, both the position of the line and the value of 193m for the upper bound $A$, were chosen to give a least-squares best fit.

Due to the relatively high value of the upper bound in Gumbel's Third Asymptotic distribution, this distribution appears to be very similar to Gumbel's First Asymptotic distribution. Consequently the heights of the wave with a 50-year return period taken from these two distributions are in good agreement at 27.5m and 28.1m respectively. The Weibull distribution indicates a height of 28.3m for the 50-year wave, and the log-normal a height of 30.5m.

Confidence limits of 70% have been drawn about the extrapolated part of the regression line in Figure 8 (Gumbel's Third Asymptotic distribution). The confidence limits represent an equal probability of the extreme wave height lying either side of the regression line, although on the scales shown this requires the confidence intervals above and below the regression line to be of unequal widths. At the probability corresponding to the 50-year return period, the upper and lower confidence limits are 31.0m and 26.2m respectively.

A cumulative probability of 0.9999863 has been used for these extreme wave height estimations, which corresponds to 50 six-month periods. However the data set is incomplete, and moreover the very highest waves may not be fully represented for the reasons given in the introduction; these restrictions should be borne in mind when interpreting the cumulative distributions.

It should be realised that the 50-year extrapolated value of $H_{\text{max}}$ (3 hr) is not quite the same as the most probable height of the highest wave in fifty years. Correlation between successive values of $H_{\text{max}}$ (3 hr) has been ignored, as has the
fact that the largest wave might occur when $H_{\text{max}}$ (3 hr) is not at its maximum. The errors contributed by these two effects have been roughly estimated under certain assumptions to be $-2.3\%$ and $+6\%$ respectively (Tann (1976)). Thus assuming the 50-year storm has duration twelve hours, the most likely height of the largest wave in fifty years will typically be $25\%$ higher than the 27.5m quoted above for Gumbel's Third Asymptotic distribution.

FIGURE 9 Month-to-month variability of significant wave height

The average value of significant wave height for each separate month of data is calculated, and for each of the months from October to March the mean and standard deviation of the six averages are plotted (except for October and February, where only five months of data were available for each).

November, December and January had very similar characteristics, each with a mean significant wave height of about 3.3m, although December had a standard deviation about half that of the other two months. The mean significant wave heights for October, February and March were similar too, varying between 2.0m and 2.4m.

WIND DATA

The wave data at the Famita location cover six six-month periods, whereas good coverage of wind data is available for thirteen six-month periods. These wind data may indicate how representative wind conditions were between 1969 and 1975 at the Famita location (no wind data for 1976 were available at the time of preparation of this report). This in turn may be taken as an indication of how representative the wave data described in this report are for the Famita location.

The wind data used were recorded on board the vessel; they extend from 1962 to 1975, and were made at three-hour intervals coincident with the wave records. These wind data include measurements made when the vessel was standing-by for rescue purposes, but this is unlikely to affect significantly the wind statistics calculated for this report.

FIGURE 10 Month-to-month variability of wind speeds

This figure shows the mean and the standard deviation of the average wind speeds for each month; data are shown both for the years covered by this report (except 1975/76) and for the full thirteen six-month periods. Only for the months of February and March were the mean wind speeds for the years 1969 to 1975 less than the mean wind speeds for the years 1962 to 1975; and both these differences are 0.5m/sec which is equal to the limiting accuracy of all the wind data.
FIGURE 11 Year-to-year variability of wind speeds

For each six-month period shown, the mean wind speed is plotted. The figure shows that the mean wind speeds recorded from 1969 to 1975 lie mostly in the upper half of the range of mean wind speeds recorded during the thirteen six-month periods from 1962 to 1975. Consequently the wind conditions, and therefore the wave conditions also, observed during the years 1969 to 1975 appear to be more severe than average.

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REFERENCES


APPENDIX

When a calibration is carried out on the shipborne wave recorder, the figure by which the calibration of wave height differs from the previous calibration is noted, and a change of not more than 5% is considered acceptable. These figures are shown below:

September 1969  1st calibration (before installation)

August 1971  2nd calibration - 4.9% low
(An intermittent fault was detected during September and October 1971; the electronics unit was changed on 25 October 1971, and a new calibration was carried out).

August 1972  2nd calibration - 3.0% high
(new electronics)

August 1973  3rd calibration - 2.0% high

August 1974  4th calibration - No change

February 1975  5th calibration - 2.0% low

April 1976  6th calibration - 2.0% low

In addition, monthly operational checks were made by the crew throughout the period of the installation and these revealed no significant malfunction of the equipment.
Map to show location of Famita Wave Recorder.
Fig. 1

Percentage exceedance of $H_s$ and $H_{max}(3\text{hr})$. 
Frequency histogram for zero-crossing period.

Fig. 2
Scatter diagram in parts per thousand.

Fig. 3
Frequency histogram for Bandwidth parameter.
Fig. 4
Cumulative distribution of $H_{\text{max}}$ (3 hr) Log normal scale.

Fig. 6
Cumulative distribution of $H_{\text{max}}$ (3hr) Gumbel III scale ($A=193\text{m}$).

Fig. 8
The mean and standard deviation of the average value of significant wave height for each month.

Fig. 9
The mean and standard deviation of the average value of wind speed for each month.

Fig. 10