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NOTE ON QUARTER-WAVE TIDAL RESONANCE IN THE BRISTOL CHANNEL

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S. W. FONG*   and   N. S. HEAPS

REPORT NO. 63

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NATURAL ENVIRONMENT RESEARCH COUNCIL

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SUMMARY

Quarter-wave tidal resonance in the Bristol Channel - Celtic Sea shelf area is investigated using a simple one-dimensional numerical model. It is found that semi-diurnal resonant modes extending across the shelf are possible. The effects of tidal barrages on resonant period are determined.

INTRODUCTION

The semi-diurnal tides at the head of the Bristol Channel are some of the largest in the world and there has long been speculation that they might be caused by the Channel resonating with the incident North Atlantic tidal wave. On the other hand, a commonly-held opinion is that the high tides are mainly due to the landward convergence and shallowing of the Channel. There appears to be no recorded scientific investigation which resolves this question. The topic has practical significance as well as theoretical interest since cross-channel tidal barrages have been and are being considered for the Bristol Channel and to assess how these might alter the tidal characteristics of the area it is obviously important to understand the tidal dynamics as they presently exist.

Table 1 shows that the main semi-diurnal tidal constituents, of which M_2 is by far the greatest, are all magnified in amplitude in passing across the Celtic Sea shelf, from its oceanic edge, into the Bristol Channel. This suggests that, if there is a resonant response here, then it is a rather broad-band one involving the whole Celtic Sea - Bristol Channel system. Note that the shorter-period constituents are magnified most, consistent with the idea of a wide resonance peak centred on a period of about 11 hours (Flather 1976). The very large flux of energy from ocean to shelf in this region, calculated by Flather in the paper cited, may itself be regarded as an indicator of some form of resonance on the shelf.

Perhaps the most striking evidence for a free mode of oscillation of roughly semi-diurnal period across the Celtic Sea into the Bristol Channel comes from the association of major storm surges at Avonmouth with secondary depressions which move in from the Atlantic and pass over the western sea region of the British Isles at a critical speed of about 40 knots (Lennon 1963). It has been demonstrated (Heaps 1965, figure 9) that an onshore wind stress field (such as might be associated with one of these depressions) sweeping across the area of the Celtic Sea to the south of Ireland, produces a surge pulse of duration between 5 and 6 hours when the speed of movement is equal to the critical speed. The pulse is longer for lower speeds and shorter for higher speeds. It seems reasonable to suppose, therefore, that the critical 5- to 6-hour pulse represents the resonant excitation across the width of the Celtic Sea - Bristol Channel shelf of a free mode approximately half a day in period, resulting in a major surge. Indirectly, we thus have an indication of a natural mode of oscillation, of period between 11 and 12 hours, across the shelf.

The present paper takes a first elementary step in examining natural modes and periods on the west coast of the British Isles by estimating periods of quarter-wave resonance in the Bristol Channel - extended into the Celtic Sea as far as the shelf edge. A simple one-dimensional numerical model is used, in the manner of Duff (1970) for the Bay of Fundy, and it is concluded that tidal resonance across the shelf at semi-diurnal periods is a likely possibility.

THE MODEL

We have constructed a one-dimensional numerical tidal model of the Bristol Channel leading out into the Celtic Sea as far as the edge of the continental shelf (figure 1). Moving seawards from the head of the Channel, vertical cross sections are numbered alternately n = 1(1)60; their distance apart measured along a medial line is 5.11 km. Sections 1(1)19, here, correspond to sections 0(2)36 of a tidal model of the Bristol Channel due to Heaps (1968).

Seawards of section 27, the southern extremities of the various sections in the Celtic Sea lie on a southwesterly-directed line, chosen somewhat arbitrarily to enclose the main Atlantic tidal wave supposedly incident on the Bristol Channel.

The equations of motion, without friction, are (Proudman 1953, p.227) :

$$\frac{\partial}{\partial x}(Au) + b\frac{\partial \zeta}{\partial t} = 0 , \qquad (1)$$

$$\frac{\partial u}{\partial t} = -g\frac{\partial \zeta}{\partial x} , \qquad (2)$$

where x denotes distance measured along the medial line, A cross-sectional area, b cross-sectional breadth, u the current in the x-direction, ζ the elevation of the sea surface above equilibrium, g the acceleration of the Earth's gravity, and t the time. Assuming a simple harmonic solution :

$$\zeta = Z(x)\cos\frac{2\pi t}{T} , \qquad u = U(x)\sin\frac{2\pi t}{T} \qquad (3)$$

representing tidal motion of period T , it follows that

$$\frac{d}{dx}\{A(x)U(x)\} = \frac{2\pi}{T} b(x)Z(x) , \qquad (4)$$

$$\frac{dZ(x)}{dx} = -\frac{2\pi}{gT} U(x) , \qquad (5)$$

which are equations for current amplitude $U(x)$ and elevation amplitude $Z(x)$.

Amplitude U is evaluated at the alternate sections numbered n = 1(1)60 in figure 1, and amplitude Z at the intermediate alternate sections: also numbered n = 1(1)60 in seawards progression. Thus we have a series of interlacing U and Z sections from the head of the Bristol Channel to the edge of the continental shelf. We let

$U = U_n$, $A = A_n$ at U— section n ,
$Z = Z_n$, $b = b_n$ at Z — section n ,

where n = 1(1)60. Then equations (4) and (5) may be written in finite difference form as follows :

$$U_{n+1} = \{ A_n U_n + (2\pi \Delta x\, b_n / T) Z_n \} / A_{n+1} , \qquad (6)$$

$$Z_{n+1} = Z_n - (2\pi \Delta x / gT) U_{n+1} , \qquad (7)$$

where Δx denotes the distance apart of successive U or Z sections, equal to 10.22 km.

Cross-sectional breadth b_n is plotted against n in figure 2 and cross-sectional depth h_n ($= A_n / b_n$) against n in figure 3. Manifestly, h_n increases steeply between n = 54 and n = 60, due mainly to large oceanic depths in the southwestern corner of the model area.

PROCEDURE AND RESULTS

Starting with a no-flow condition at the head section (U_1 = 0) and making an arbitrary choice of Z at the adjacent section (Z_1 = 4m), equations (6) and (7) were used alternately to find U_n , Z_n for n = 1(1)60, stepping outwards from one section to the next in ordered progression. This was done for various values of the period T and, in each case, the position P at which Z first reduced to zero was determined. The distance (L , say) along the medial line between P and the head section corresponds to one quarter of a wavelength of the oscillation of period T . A tidal oscillation of elevation with this period, applied at P , would produce quarter-wave resonance.

The location of P , defined by section number n marked in figure 1, is plotted in figure 4 - see curve (a). Clearly the resonant length L increases as T increases. When P is at the mouth of the Bristol Channel ($n = 19$), approximately $T = 8$ hours. When P lies in the vicinity of the shelf edge ($n = 54$ to $n = 60$) it is apparent that T lies between 12.2 and 12.6 hours. This result suggests that there is the possibility of a quarter-wave tidal resonance, in the semi-diurnal tidal band, between the shelf edge and the head of the Bristol Channel. Clearly there is no possibility of a Bristol Channel semi-diurnal tidal resonance confined to the length of the Channel itself.

The curves (b) to (g) in figure 4 were derived in a similar manner to curve (a) and correspond respectively to the cases of impermeable tidal barrages at sections 3, 5, 7, 10, 15 and 19 distributed along the length of the Bristol Channel. In each case, using (6) and (7) we stepped seawards from the barrage section (where the no-flow condition was assumed) determining the nodal position P for a range of values of T . It is evident from figure 4 that the barrages influence resonant lengths and periods very significantly. For example, with P at the mouth of the Bristol Channel ($n = 19$) the span of resonant periods is from 4.25 to just over 8 hours according to barrage location. However, when P lies in the vicinity of the shelf edge (between sections 54 and 60) the span of possible resonant periods is quite small - between 11.9 and 12.6 hours - for barrages in the upper reaches of the Bristol Channel at sections 1, 3, 5 and 7. This indicates that such barrages will not significantly alter the period of a semi-diurnal tidal resonance which extends across the whole shelf into the Bristol Channel.

CONCLUDING REMARKS

The foregoing work suggests the possibility of a quarter-wave semi-diurnal tidal resonance across the Celtic Sea shelf into the Bristol Channel. Barrages located in the headwaters of the Channel would not appear to significantly influence the period of such a resonance.

Our analysis is a simple and tentative one, omitting the influences of the Irish Sea and the English Channel. However, we think it is valid in pointing to the likelihood of an important tidal resonance on the west coast of the British Isles. A comprehensive and rigorous study of the natural periods and modes of long-wave oscillations in this sea region would clearly be worthwhile.

It should be emphasised that both friction and the effects of the Earth's rotation have been ignored in this paper. These omissions will affect the calculations but the extent of the associated discrepancies are as yet unknown. However, within the context of the broad accuracies expected in the present study, we do not anticipate significantly large errors in the values found.

Table 2, contributed by Dr. R. A. Flather, lists some theoretical estimates of the fundamental seiche period of the Celtic Sea shelf with or without allowance for the Bristol Channel and the adjoining ocean. Our values, included in the list for purposes of comparison, are somewhat higher than the others but it is too early yet to pass judgement on this result. Models 1, 2 and 3 in the table are analytical with rectangular or sloping-bottom topography; uniformity is assumed in the longshore direction; account is taken of friction and the Earth's rotation. Model 4 is a numerical one of the entire continental shelf sea surrounding the British Isles with a grid resolution $1/3^{o}$ longitude by $1/2^{o}$ latitude. The model is two-dimensional; friction and the Earth's rotation are included but there is a radiation condition at the shelf edge which might unduly influence free periods and modes.

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Tidal constituent Period (hours)	K_2 11.97	S_2 12.00	M_2 12.42	N_2 12.66	K_1 23.93
Amplitude (cm) :-					
Offshore Station C_2	10.4	36.2	110.5	24.4	5.5
Offshore Station C_4	10.7	37.3	111.4	24.1	7.7
Offshore Station C_6	10.5	36.6	108.5	23.2	6.7
Roberts Cove (RC)	11.6	42.9	137.2	25.2	1.8
Offshore Station C	16.6	55.9	162.9	30.8	3.8
St. Mary's (SM)	17.5	61.4	177.4	35.2	5.4
Milford Haven (MH)	24.3	81.8	224.9	42.2	5.9
Ilfracombe (IC)	33.5	111.9	308.0	58.1	6.4
Newport (NP)	44.8	148.2	413.3	77.5	7.5
Avonmouth (AM)	42.7	148.2	422.3	73.3	5.9
Magnification from C_2:-					
Offshore Station C_2	1	1	1	1	1
Offshore Station C_4	1.03	1.03	1.01	0.99	1.40
Offshore Station C_6	1.01	1.01	0.98	0.95	1.22
Roberts Cove (RC)	1.12	1.19	1.24	1.03	0.33
Offshore Station C	1.60	1.54	1.47	1.26	0.69
St. Mary's (SM)	1.68	1.70	1.61	1.44	0.98
Milford Haven (MH)	2.34	2.26	2.04	1.73	1.07
Ilfracombe (IC)	3.22	3.09	2.79	2.38	1.16
Newport (NP)	4.31	4.09	3.74	3.18	1.36
Avonmouth (AM)	4.11	4.09	3.82	3.00	1.07

Table 1. Observed tidal amplitudes in the Celtic Sea and the Bristol Channel and their magnification from the shelf edge. The observational positions are marked in figure 1.

Model	Areas included: Ocean	Celtic Sea	Bristol Channel	Fundamental period (hours)
(1) Rectangular shelf (Heaps 1965)	NO	YES	NO	10.21
(2) Shelf with sloping bottom (Flather 1972)	NO	YES	NO	9.82
(3) Shelf and ocean with rectangular step topography (Flather 1972)	YES	YES	NO	11.87
(4) Continental shelf: two-dimensional, numerical (Flather 1976)	NO	YES	YES	11.05
(5) One-dimensional, numerical (Fong and Heaps 1978)	NO	YES	YES	12.2-12.6

Table 2. Estimates of the fundamental seiche period of the Celtic Sea derived from various theoretical models.

LIST OF FIGURES

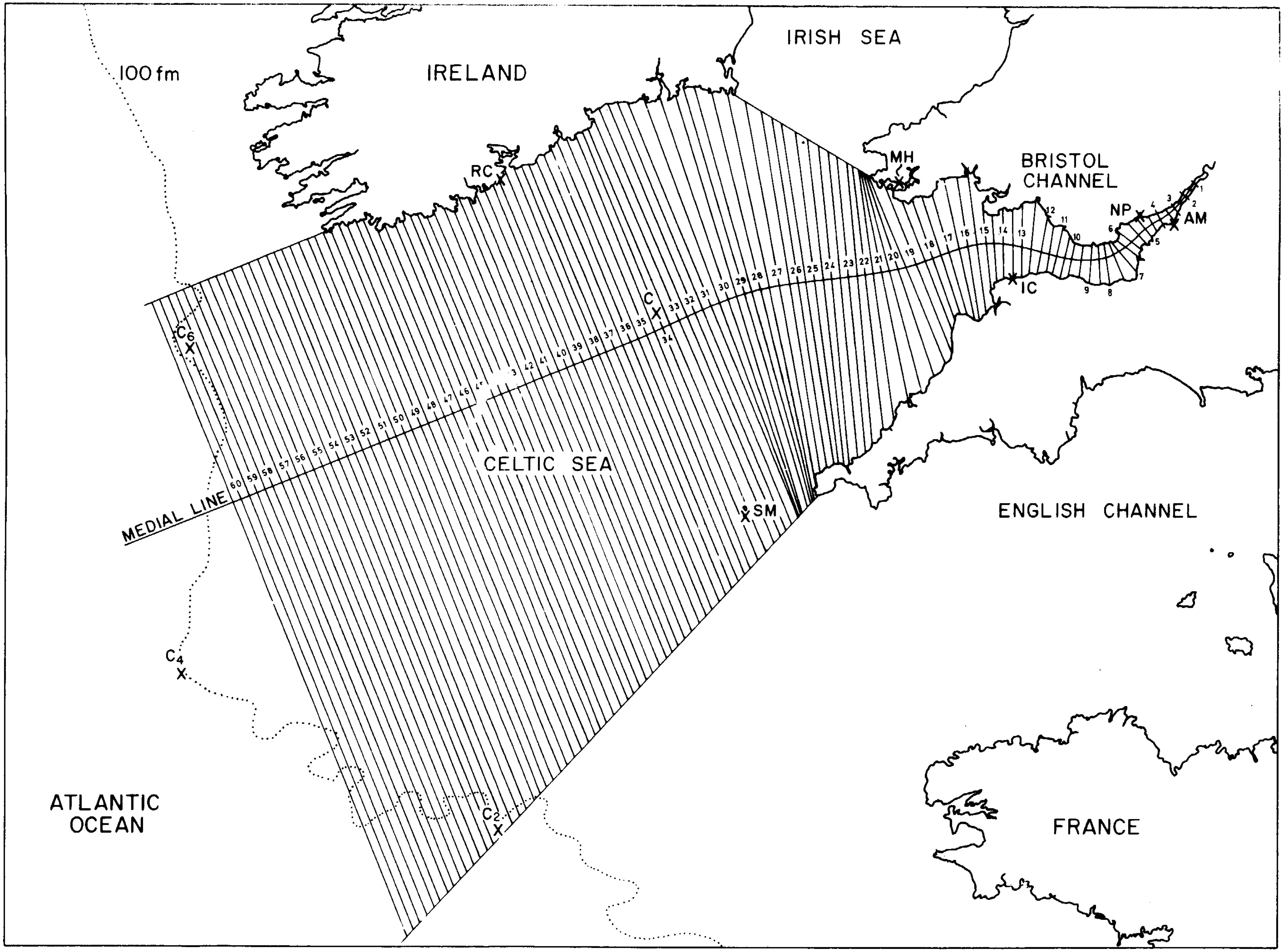

Figure 1. Cross-sections of the one-dimensional model. Alternate sections are numbered n=1(1)60 as shown, moving seawards from the head of the Bristol Channel into the Celtic Sea and thence to the shelf edge. Current is evaluated at these sections and sea surface elevation at the intermediate sections. X = tidal station in Table 1.

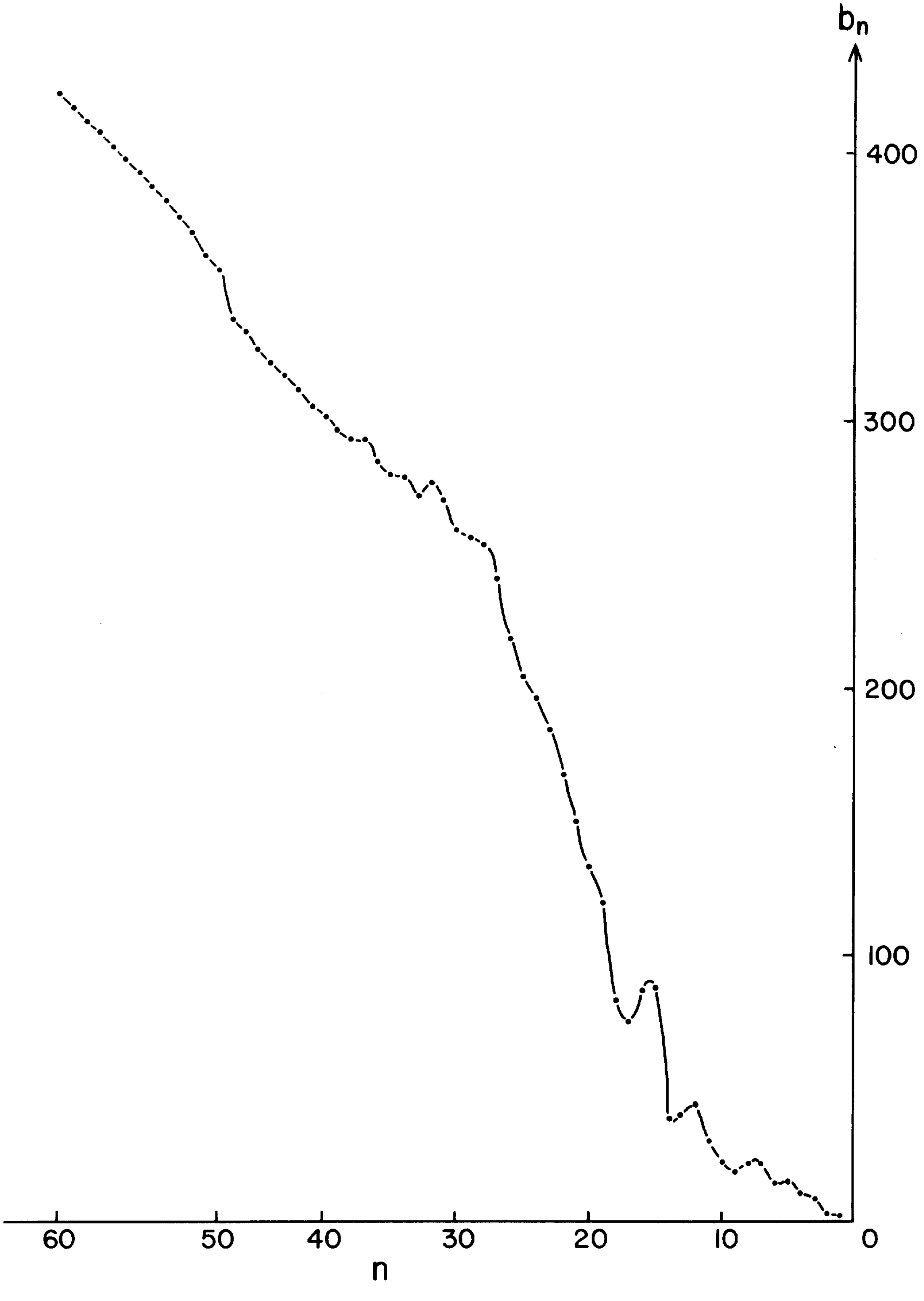

Figure 2. Cross-sectional breadth b_n in kilometres.

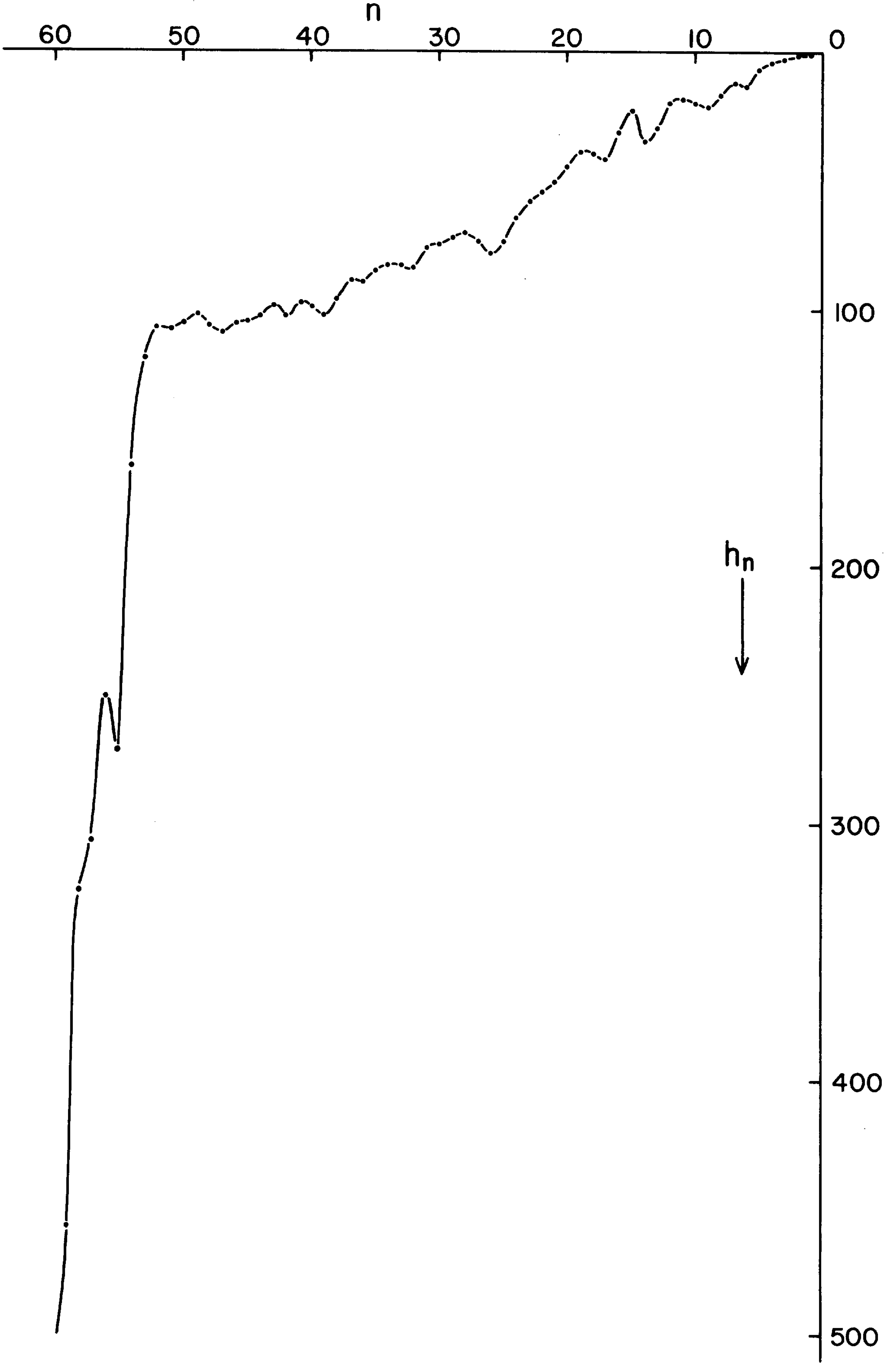

Figure 3. Cross-sectional depth h_n in metres.

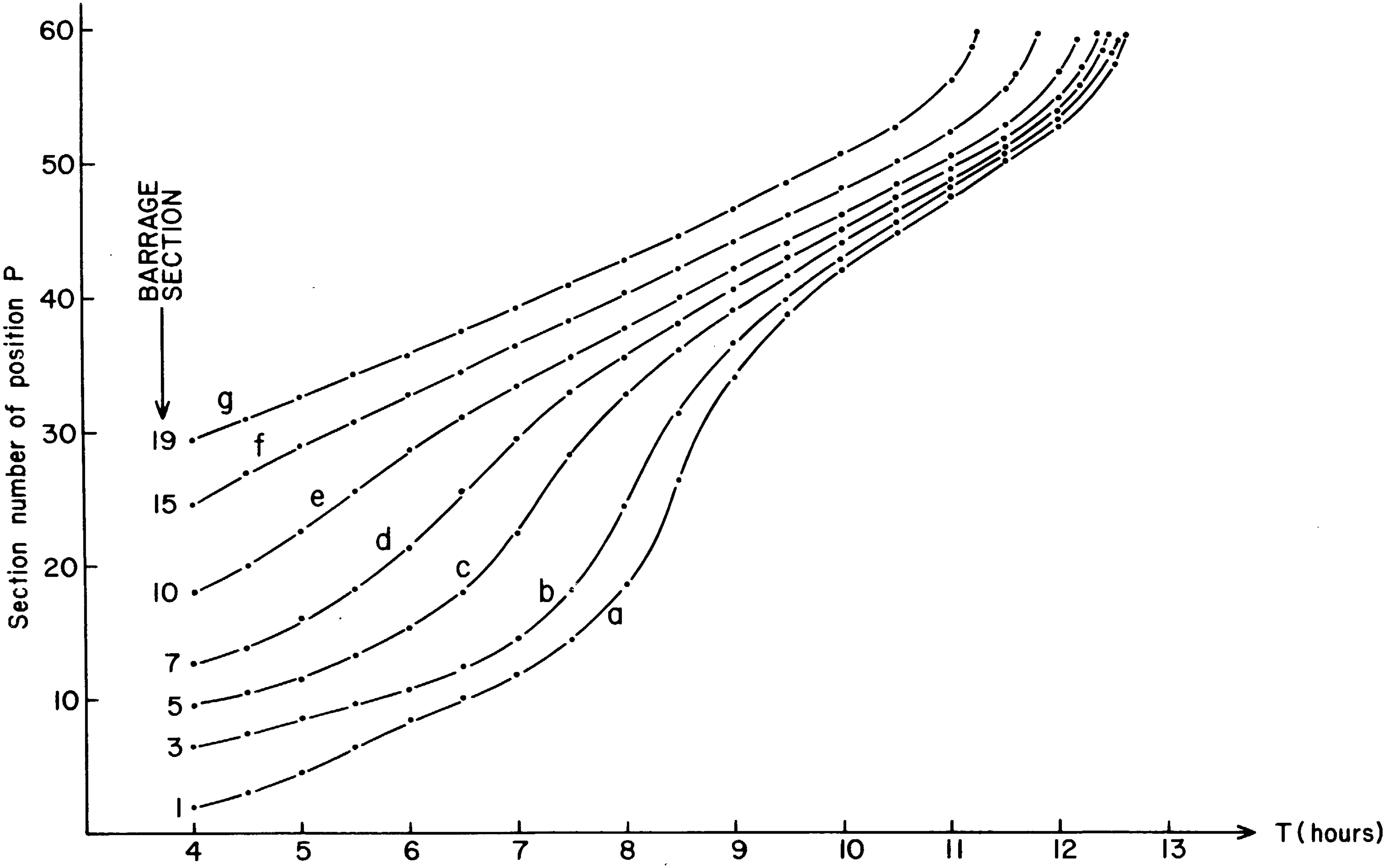

Figure 4. Section number (see figure 1) of nodal position P, plotted against period T for barrages at sections 1, 3, 5, 7, 10, 15 and 19. The distance along the medial line between P and a barrage corresponds to a quarter wavelength of the oscillation of period T, and is the resonant length L.