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INSTITUTE OF OCEANOGRAPHIC SCIENCES

RETURN WAVE HEIGHTS AT SEVEN STONES AND FAMITA ESTIMATED FROM MONTHLY MAXIMA

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The work described in this report was supported financially by the Departments of Industry and Energy

Report No. 66
1978

NATURAL ENVIRONMENT RESEARCH COUNCIL

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ABSTRACT

Monthly maximum values of significant wave height have been extracted from Shipborne Wave Recorder records obtained by the Seven Stones Light Vessel during seven years between 1968 and 1977, and by the Norwegian Rescue Ship Famita during six winters between 1969 and 1976. Fifty-year return wave heights have been estimated for each month and for the year from these maxima, assuming they come from a Fisher-Tippett Type I extreme distribution.

The yearly values were found to be lower than some monthly values. Statistical analysis and a study of month-to-month variations in the values of the two parameters determining the Fisher-Tippett distribution indicate that the distribution of maximum significant wave height varies from month to month. Estimating the fifty-year return wave height from annual maxima gives too low a value.

Fifty-year return values of significant wave height have been calculated by combining the monthly probability distributions; they are approximately 15m at Seven Stones and 18m at Famita. Two-year return values have also been calculated, for each month and for the year.

Confidence limits for the monthly return values have been derived. The broad limits for the fifty-year return period illustrate the inadequacy of only six or seven years of data for determining these values.

CONTENTS

INTRODUCTION

Analogue records of wave heights have been obtained at three-hourly intervals with few breaks for about seven years from the Seven Stones light vessel, and for six years during the winter months (October - March) from the rescue ship Famita. Detailed studies of wave heights and periods from many of these records have been reported by Fortnum and Tann (1977) and by Fortnum (1978), including an analysis of seasonal variations and estimates of the fifty-year return height obtained by fitting a whole year's data to various distributions.

This report examines possible month-to-month variations in the fifty-year return wave height, based upon analysis of the maximum significant wave height recorded each month.

DATA

The Seven Stones light vessel is moored in about 60 metres of water approximately 17 kilometres north-east of the Scilly Isles in position 50°03.8'N, 06°04.4'W.

The Norwegian m.v. Famita occupies a meteorological station and serves as a rescue vessel during the winter months in the northern North Sea. Its station position is 57°30'N, 03°00'E, where the water depth is about 66 metres.

Both vessels are fitted with Shipborne Wave Recorders, described by Tucker (1956). These instruments provide a 12 to 15 minute analogue record of surface wave height, usually obtained at three-hourly intervals. The largest and second largest crest heights and trough depths from the record were used to estimate wave conditions; the method of analysis is given by Tann (1976). Parameters estimated include:-

a) Significant wave height (H_s), which is defined as $4\sqrt{m_o}$ where m_o is the mean square water elevation.

b) Most likely height of the highest zero-up-crossing wave during a three-hour interval ($H_{max(3hr)}$), derived assuming that sea state remains unchanged during the interval and that zero-up-crossing wave heights have a Rayleigh distribution with standard deviation $H_s/\sqrt{2}$.

The months of data from the two vessels which are considered in this report are shown in Tables 1a-1c, which give the number of records missing each month, from a maximum possible of eight per day. The tables show that very few records from Seven Stones are missing, but rather more from Famita. A study of the Famita records shows that there are gaps during some storms, possibly because she was off-station in her rescue role, so the highest sea state may have been missed. The percentage of data missing from each month during the five or six years of records varies between 21% and 33% with a mean of about 26%.

The largest gap in the Seven Stones data is in October 1976, with no records from the 18th to the 22nd, but during this five-day period the wind speeds were comparatively low, with a maximum mean hourly wind speed at Scillies of 22 knots. Earlier in the month, a maximum significant wave height of 8.4m was recorded with a wind speed at Scillies of 47 knots. So it is most unlikely that a wave higher than 8.4m occurred during October 1976.

The gap at Seven Stones from 2100 on 20th January to 1500 on 23rd January 1977 inclusive was during a period of stronger winds. The significant wave height at 1800 on the 20th January had risen to 7.1m when the ten-minute mean wind at Scilles was 180°/34 knots. The wind speed decreased slowly, to 190°/33 knots at 1600, 190°/27 knots at 1800, and 200°/25 knots at 2100 then remained below 25 knots throughout the rest of the wave data gap. So the wave height possibly rose slightly above 7.1m but seems unlikely to have exceeded the 7.8m recorded later in the month, with a wind speed of 58 knots.

Inspection of other gaps in the Seven Stones records indicate that they were generally most unlikely to have included the monthly maximum wave heights, with the exception of two small gaps, one in August 1969, the other in December 1973. The gap in December 1973 consists of one missing record immediately following the recording of that month's highest wave of 7.04m (the second lowest maximum for the seven Decembers).

The maximum value of H_s obtained from each month's records are given in Table 2, together with the maximum for each twelve-month period (not necessarily January through to December) at Seven Stones, and for each winter (October to March) at Famita.

No corrections have been made to the data for any possible variations from the long-term average in storminess during the period 1968 to March 1977. Fortnum (1978) finds that mean wind speeds at Famita during 1969-1975 were higher than the mean during 1962-1975. But Jenkinson (1977) concludes that the decade 1966-1976 was within 1% of the average of extreme winds over the U.K. and the North Sea for the period 1881-1976.

Nor has any correction been made to allow for depth of water. At Seven Stones and Famita the water depth is 60m and 66m respectively, so short-period waves would not be affected; but the long-period wave components (10-20 seconds) with wave lengths in deep water of 150-600m would 'feel' the bottom, and the height of these components would be reduced by a few percent from their deep water value (1%-7% assuming no friction loss and no refraction). The Seven Stones Light Vessel is slightly sheltered by the Scilly Isles to the W.S.W., but any reduction in wave height would be countered by refraction around the ridge on which the Isles and the Light Vessel lie.

METHOD OF ESTIMATING THE FIFTY-YEAR RETURN SIGNIFICANT WAVE HEIGHT FROM MONTHLY AND ANNUAL MAXIMA

The five to seven maximum values for each month in Table 2, each of which is the maximum of some 240 values, are assumed to be from a Type I distribtuion given by Fisher and Tippett (1928); likewise the annual maxima. This distribution, also known as 'Gumbel' or 'double exponential' is given by the cumulative distribution function:-

$$P(X \leq x) = \exp\left[-\exp -\left(\frac{x-A}{B}\right)\right] \qquad (B>0) \qquad (1)$$

i.e.

$$x = A - B\,\ln(-\ln P) \qquad (2)$$

Estimates of the parameters A and B have been obtained using the method of maximum likelihood, and the fifty-year return value computed by equation (2), with $P = 1 - \frac{1}{50} = 0.98$ for Seven Stones, assuming maxima are from a complete data set, and with $P = 1 - \frac{0.74}{50} = 0.9852$ for Famita with about 26% of the data missing.

Another method of estimating A and B, used for example by Ewing et al (in press), is to obtain a least-squares fit to a plot of x: -ln(-lnP). This method has also been applied in this report, to obtain comparisons with the maximum likelihood estimates, but it has several drawbacks:-

a) There is the difficulty of deciding what probability value to assign to the ordered maxima. The mean value of the m^{th} largest value of the variable

$$y = \frac{x-A}{B}$$

can be obtained from the generating function given by Gumbel (1956) (para 6.1.4 equation 2) and is given by:

$$\bar{y}_m = \sum_{j=0}^{m-1} {}^nC_{m-1} \cdot {}^{m-1}C_j \cdot (-1)^j \frac{n-m+1}{n-m+j+1} \cdot \left\{ \gamma + \ln(n-m+j+1) \right\} \quad (3)$$

where m = 1 is the largest, m = 2 the second largest etc of n values and γ is Euler's constant, 0.5772 ... The corresponding cumulative frequency is given by

$$\hat{P}_m = \exp\left[-\exp(-\bar{y}_m)\right] \quad (4)$$

Gringorten (1963) proposes a plotting position for a Fisher-Tippett I distribution, if n$\geqslant$ 20, of

$$\hat{P}_i = (i - 0.44)/(n + 0.12) \quad (5)$$

where i = n is the largest etc., i.e. i = n + 1 - m; P_i being a close approximation to equation (4) for large n. This plotting position is advocated in Flood Studies Report (1975) and was used by Ewing et al (in press); but in this report, with n between 5 and 7, values have been taken from equations (3) and (4). These values are given in Table 3 together with Gringorten's approximations. (The table shows that equation (5) is quite a close approximation even with n = 6 and 7. Fifty-year wave heights for several months were re-calculated using equation (5) and the results differed by less than 0.01m).

b) The distribution of y_m is not normal, and varies with the value of m, so the method of simple linear regression is not strictly valid; in particular it gives equal weight to the highest value, which has relatively large variance.

The method of maximum likelihood does not require a 'plotting position', nor does it assume a normal distribution, but it does give a biased estimate for A and B, and for the fifty-year return value (under-estimating its value by approximately 4%). Annex A explains the method used to determine the bias, and gives the derivation of confidence limits for the maximum likelihood fifty-year return values.

These maximum likelihood confidence limits do not indicate how well the given n data values fit the derived Fisher-Tippett distribution. They describe the range of an estimate of the fifty year return value determined from n data values chosen randomly from that particular distribution. An indication of how well the observed maxima fits the distribution is given by the standard error of the height of the linear regression line at the probability of the fifty-year return value - determined from the estimated variances of the line's slope and centroid.

RESULTS OF MAXIMA ANALYSES

Tables 4 and 5 give the maximum likelihood estimates of A and B in equation 1, together with the resulting value of the fifty-year return significant wave height, this value corrected for bias, and its 90% confidence limits, for each month of data; also for the yearly maxima at Seven Stones and for the winter maxima at Famita.

The tables also include estimates of the fifty-year wave height obtained by linear regression and the standard errors of those estimates.

There are two sets of values for Seven Stones January maxima. The variance of the highest crest and of the lowest trough are about one-third greater than the variance of the second highest and lowest so an estimate of significant wave height from these second greatest values alone might be expected to give more consistent results. This method was applied to the Seven Stones records, and the resulting estimate of the fifty-year wave heights for each month were little changed, with a mean difference from the 12 maximum likelihood values given in Table 4 of 0.15m. The greatest difference was the result for January with a maximum likelihood estimate of 10.67m and a regression estimate of 11.28 (cf. with 9.63m and 10.64m respectively); the correlation coefficient was improved from 0.849 to 0.936. (These results have been included in Table 4). The situation was reversed for July, the second highest estimates for the fifty-year height were lower (4.89m and 5.19m compared with 5.71m and 5.47m) and the correlation coefficient was reduced from 0.96 to 0.82.

There are also two sets of results for Seven Stones December analysis. The one with six values omits December 1973 when the maximum height was possibly missed because of a gap in the records; again the results are close, with a difference of 0.3m in the maximum likelihood estimate of fifty-year return wave height.

In the analysis of annual maxima at Seven Stones and in figure 4 the original data sets were used. Replacing the highest value in table 1b for July '73 - June '74 by the January maximum calculated from the second highest and second deepest trough in the record (10.64m instead of 10.43m) would increase the estimated annual fifty-year return value from 12.4m to 12.5m.

Two sets of results are given for Famita winter maxima: one for five winters - omitting 1971-72 which contains no records for October or for February - and one for six winters, with no allowance for the missing months; the results are in close agreement.

The maximum likelihood and the regression lines fitted to the data, plotted with probabilities given by equation 4, are shown in figures 1-4.

The bias - a multiplication factor - for A, B, and the fifty-year return wave height were taken to be for Seven Stones: 0.99, 1.13, and 1.04 respectively, and for Famita: 0.99, 1.18 or 1.14 (for 5 or 6 data points) and 1.05 except that for Famita October: 0.96, 1.18 and 1.09.

MONTH TO MONTH VARIATIONS

The monthly analyses of Seven Stones data clearly indicate that maximum wave heights during the six summer months come from a different population from those during the winter months. More surprising is the high estimate of the fifty-year wave height in March compared with other winter months. Likewise, at Famita one winter month appears to have an outstandingly high fifty-year wave height, but here it is the month of November. Figures 1 and 2 show that during March at Seven Stones, the monthly maximum wave heights were generally lower than during

December and January, but on occasions very high waves occurred; the consequent "steepening" of the regression lines resulted in these high fifty-year wave height estimates. The maximum significant wave heights at Famita during November were generally higher than for any other month.

The very broad confidence limits suggest that these anomalously high fifty-year wave heights in March and November might result from sampling. Even if this is not the case, and the maxima come from different populations during these months, each with its Fisher-Tippett Type I distribution, the maxima from the annual distribution will also have a Fisher-Tippett Type I distribution. However, if this annual distribution is used to estimate the fifty-year return value, then a constraint (that there is a fixed probability of the annual maximum coming from each month) has been lost, so the result will not be the true fifty-year return value. It is shown in Annex B that this result will be less than the true value.

On the other hand, if the maxima for the winter months can be considered as coming from one population then, assuming month-to-month independence, all winter monthly maxima can be combined to give a larger data set from which to estimate the fifty-year return wave height, with a consequent considerable narrowing of the confidence band.

Saetre (1974) analysed Famita data from three Winters (1969-70, 1971-72, 1972-73) and noted a monthly variability, with November as the worst month.

An analysis of variance was carried out - using the logarithm of maximum wave heights so that distributions were approximately normal - to test the assumption that different months were from the same population. This assumption when applied to twelve months of data from Seven Stones was found to be highly unlikely - with a probability of less than 5%. Thus the analysis of variance provides statistical evidence of differences between months. However, this analysis ignores the ordering of the months; variations from month-to-month are indicated in figures 5 and 6 which show plots of the monthly maximum likelihood estimates of A and B, with 90% confidence limits. (The derivation of the confidence limits are explained in Annex A). The monthly mean values of significant wave height are also shown.

Figure 5 shows that the estimates of A from the Seven Stones data vary in a regular fashion with a clear annual cycle, similar to the curve of monthly mean significant wave height. Estimates of B have, for some months, a large S.E., and it is harder to see a pattern to the month-by-month variations, but there does appear to be a cycle with two peaks during the year, in March and September. Figure 6, with only six months of records from Famita, is harder to interpret; the estimates of A are scattered but there is the suggestion of a six-month cycle in B with minimum around December or January (compared with January at Seven Stones). Thus fig. 5 clearly supports an assumption that maxima populations vary at Seven Stones from month to month, but fig. 6 is inconclusive concerning Famita.

Further support for monthly variations comes from studies of wind speeds. Analysis by the Meteorological Office of wind speeds recorded in the U.K. for more than thirty years, shows significant differences in fifty-year return values from month-to-month (private communication, D.J. Painting).

Information on possible monthly variation in the occurrence of severe storms in the North Sea is provided by the list of sea floods which have caused losses of land and/or cost many lives, given by Lamb (1977) in his table 13.3. The earliest flood noted with a precise date was in Fresland on 26th December 838 (new style calendar); the earliest recorded in November was in Holland in 1170. The greatest loss of life seems to have been during the period 31 October to 2 November 1570 when possibly 400 000 were drowned in the Netherlands. The number of floods recorded in each month is given in Table 8. These storm surges depend upon the state of the tide as well as a storm in the northern North Sea or in the Norwegian Sea. Moreover, as Lamb points out, there have been climatic variations during the 1100 years spanning these events, and there has been local land sinking in the southern part of the North Sea. Nevertheless, it is interesting to note that November - the month with the highest fifty-year wave estimate at Famita - has experienced the greatest number of these disastrous floods.

FIFTY-YEAR RETURN WAVE HEIGHT

Assuming that the highest wave from successive months are independent, and that the probability of the highest wave being less than x in month m is given by equation 1.

Then the probability that the highest wave throughout the year is less than x is given by

$$P(X < x) = \prod_{m=1}^{12} \exp\left[-\exp-\left(\frac{x - A_m}{B_m}\right)\right] \qquad (6)$$

Solving for x with values of A_m, B_m, from table 4 and with P = 0.98 gives a fifty-year return wave at Seven Stones of 14.25m. The contribution of the summer months to this value is very small; if the equation is evaluated only for months October - March, then the fifty-year is estimated as 14.1m.

Using unbiased values of A_m and B_m does not give an unbiased estimate of the fifty-year return wave (but using the biased values - particularly the biased value of B - probably leads to an under-estimate of the fifty-year wave height) correcting this by the monthly fifty-year return wave height bias of 1.04, gives a value of 14.8m. (Using the unbiased values for A_m and B_m gives 15.2m).

At Famita, assuming that the summer months are unimportant, the fifty-year wave height can be calculated using A_m and B_m from table 5 to solve equation 6 with P = 0.9852. The result is 16.74m; correcting for bias of 1.05 gives 17.6m (using the unbiased values for A_m and B_m gives 18.1m).

TWO-YEAR RETURN VALUES

Estimates of two-year return significant wave heights for each month are given by equation (2) with P = 0.5 for Seven Stones and with P = 0.63 for Famita.

The value of A dominates the result, so for Seven Stones a consistent month-to-month variation is obtained. (Strictly the argument should be reversed : A is the 1.6-year return wave height which can be determined within narrow limits given seven years of data, so any monthly variations should be readily discernable). Table 6 gives the results, determined using the maximum likelihood estimates of A and B in Tables 4 (bias can be shown to be negligible with values differing by about 0.1m). The 90% confidence limits have been derived from Annex A fig 3.

Solving equation 6 for P = 0.5 and P = 0.63 gives estimates of the two-year return wave height at Seven Stones and during October-March at Famita. Results are 9.4m and 10.6m respectively.

DISCUSSION

a. Data

The Seven Stones wave data constitute an almost complete set of monthly maximum heights covering seven years. However the method of deducing the significant wave height from the Shipborne Wave Recorder traces does introduce some uncertainty; it is estimated from the two highest crests and two lowest troughs in the twelve minute record, and the method makes no allowance for deviations of these four values from their expected values. The estimates for Seven Stones in January obtained using only the second biggest crest and trough appears to be more consistent with the December and February results, and give a higher estimate for the fifty-year return wave height. So these values have been plotted in the figure and used in this report.

The Famita data covers only five or six years with about $\frac{1}{4}$ of the records missing. The method used to allow for the missing records when estimating the fifty-year return value is only approximate. It assumes that this value is given by a cumulative probably of 0.9852 and makes no allowance for the variations in the numbers missing from month to month; nor has any attempt been made to determine - from the size of the gap and the wind speed at the time - what proportion of those missing records might have contained a monthly maximum value. The average percentage of data missing from each month varies from 21% (February) to 33% (November), which give corresponding cumulative probabilities for the fifty-year return value of 0.9842 and 0.9866, resulting in differences in the fifty-year return value from those in Table 5 of less than 0.2m. However, the number of missing records varies considerably between individual months;

for example, more than 50% missing in October 1975 - and no allowance has been made for this.

b. Method of analysis.

The method used, of analysing only the highest value from each month, appears to disregard much of the available information, but other methods, whilst making fuller use of the data, introduce further problems.

A common method is to attempt to fit all the wave data to some distribution. For example, L. Draper (1976) uses a log-normal distribution and a two-parameter Weibull distribution. Fortnum and Tann (1977) use these distributions plus a three-parameter Weibull, Fisher-Tippett Type I and Fisher-Tippett Type III. A drawback to this method is the lack of physical or theoretical justification for any of these distributions, so while they may appear to give a reasonable fit over the few years of data (particularly those distibutions with three assignable parameters) extrapolation to the fifty-year wave is highly questionable. (This method was devised to handle only one year of data which is frequently all that are available). Moreover, this method takes the fifty-year return value as that with a probability of $1 - \frac{1}{50n}$ where n is the number of observations in one year (2920 for three-hourly data), and assumes consecutive data to be independent, which tends to over-estimate the return value.

The fifty-year return value H*, thus defined by the population distribution is not the same as that defined from annual (or monthly) maxima, H_{50}. The former considers all data values; the latter only the largest value in each year. H* is exceeded on average once every fifty years. Occasionally H* will be exceeded more than once during a single year, but these 'multiple exceedances' are not taken into account when determining H_{50}, the value of which will consequently be lower. The probability of H* being exceeded once or more than once in a year is 0.0198 while the probability of it being exceeded more than once is 0.0002 (assuming a Poisson distribution). Therefore the number of years containing one or more exceedances is 1% less than the actual number of exceedances, so H* corresponds to the 50.5-year return value from annual maxima analysis. The 50-year and the

50.5-year return value of significant wave height differ by about 1cm, so in practice the difference resulting from the two definitions of fifty-year return value is negligible.

The method which Ewing et al (in press) use is to take the maximum wave height from each storm and to fit a Fisher-Tippett Type I; but this method raises questions concerning the significance of how a storm is defined.

The method in this report assumes that three-hourly significant wave heights may be regarded as independant and stationary over the period of analysis and that the maximum wave heights have a Fisher-Tippett Type I extreme value distribution. It may be unnecessary to assume independant of wave heights - see Watson (1954)). There must be an absolute maximum wave height, determined by depth of water and other physical constraints, so a Fisher-Tippett Type III distribution might appear more appropriate. However, assuming this upper bound is considerably higher than the 10-15m fifty year wave heights derived in this report (and figures 1-4 indicate nothing to the contrary), then the error in using a Fisher Tippett I is small (but on the safe side). Anyway, determining the upper bound from so few data points is impracticable.

The differences between the fifty-year return wave heights estimated by linear regression and by maximum likelihood are small compared with size of the 90% confidence bands for the latter - but a drawback to the linear regression method is that confidence bands cannot be computed.

c. Comparison with other fifty-year return wave height estimates.

Fortnum and Tann (1977) derive a fifty-year return value for $H_{max(3hr)}$ at Seven Stones, by fitting a Fisher-Tippett Type III distribution to all the data from 1968 to 1974, The result is 23.4m.

Assuming a Rayleigh distribution with standard deviation $H_s/\sqrt{2}$, then a good approximation for the most likely maximum wave in 3 hours is given - from Longuet-Higgins (1952) equation (68) - by

$$H_{max(3hr)} = H_s \sqrt{\frac{\ln N_z}{2}} \qquad (6)$$

where N_z is the number of zero-up-crossing waves in 3 hours. Assuming zero-up-crossing wave period (T_z) is 15 seconds gives

$$H_{max(3hr)} = 1.81H_s \qquad (7)$$

(Clearly, from equation 6, the exact value of N_z is not crucial).

So Fortnum and Tann's estimate corresponds to a fifty-year significant wave height of 12.9m; which - considering the variation they found when analysing the Seven Stones data year by year - is in good agreement with the value from Table 4 of 12.4m (maximum likelihood estimate corrected for bias), and well inside the 90% confidence limits of 10.9 to 15.4m. However, assuming a month-to-month variation, these analysis of annual data are incorrect, the fifty-year return wave height is estimated in this report as 14.8m.

Draper and Driver (1971) estimate the fifty-year return value of $H_{max(3hr)}$ at Famita (from the 1969/70 data only) as 27.4m; which corresponds to a significant wave height of 15.1m.

Saetre fits three winters' data from Famita to a Fisher-Tippett I and to a three-parameter Weibull distribution, he also uses a "storm model" and obtains fifty-year return values for H_s of 16.3m, 15.2m, and 14.6m respectively.

Ewing et al (1978), using hindcast wave data from severe storms during the period 1966 to early 1976, obtain a fifty-year return value for H_s at Famita of 16.2m. Fortnum (1978) gives the fifty-year return value of $H_{max(3hr)}$ at Famita - derived using a Fisher-Tippett Type III distribution with the 1969 to 1974 data, but ignoring the data gaps - as 27.5m, corresponding to H_s of 15.2m. Using a Fisher-Tippett Type I distribution he obtains 28.1m (H_s = 15.5m). This report suggests a fifty-year return value of significant wave height of 17.6m.

These comparisons are summarised in Table 7 which shows that the estimates in this report of fifty-year return values at Seven Stones and Famita are higher than previous estimates - but not significantly so, considering the broad confidence limits.

CONCLUSIONS

Shipborne Wave Recorder records from Seven Stones Light Vessel between January 1968 and March 1977 provide an almost complete set of maximum significant wave heights for each month for seven years. Records from Famita during the winter months, October to March, from 1969 to 1976 provide estimates of maximum significant wave heights for each month for five or six winters, but on average some 26% of the records are missing from each month; only approximate allowance has been made for these data gaps in this report, and results from the analysis of the Famita records should be treated with caution.

Estimates of fifty-year return values of significant wave height have been derived assuming monthly maxima and annual (or winter) maxima are from Fisher-Tippett Type I distributions Estimates for the two parameters in this distribution have been obtained using the method of maximum likelihood, corrected for bias.

Estimates of fifty-year return wave heights from Seven Stones' seven annual maxima and Famita's six winter maxima are generally within 1m of estimates derived by others (See Table 6). However, the assumption made to obtain all these results that wave height distributions are independent of month is probably wrong. For example, larger values are estimated from some monthly maxima (March at Seven Stones and November at Famita).

Analysis of variance shows significant differences between months, both at Seven Stones and at Famita. Moreover plots of the two Fisher-Tippett I distribution parameters against month from the Seven Stones analysis (fig. 5) indicate that there is a variation in these parameters from month-to-month. The steady changes in the location parameter (equivalent to the intercept of the regression line, equation 2) is particularly convincing - and appears similar to the graph of mean monthly significant wave height. The scale parameter (or regression line slope) is not well-defined by only seven data points, but it appears to have two maxima during the year, one in March and the other in September; perhaps associated with equinoxial gales. Similar plots of the parameters from the Famita analysis are not so convincing, but it seems unlikely that there could be a month-to-month variation in maximum wave height distribution at Seven Stones and not at Famita.

Therefore it is likely that maximum wave height populations vary from month-to-month, so maxima from different months should not be analysed together. One result of so-doing is shown in Table 7 : the fifty-year return significant wave height is under-estimated. The best estimate for these from the analysis in this report are : Seven Stones : 14.8m
Famita : 17.6m

However, any implication that these estimates are accurate to within 0.1m is unjustified, and it would seem more sensible to use 15m and 18m respectively. The very broad confidence limits for the monthly estimates given in Table 5, from which these two values are derived, follow from their being determined from only six or seven data points. The rather smooth fluctuation throughout the twelve months of the year of the two parameters in the Fisher-Tippett I distributions for Seven Stones - particularly the 'position' parameter - indicate that these estimates of the fifty-year return values might be more accurate, but no confidence limits have been determined.

Estimates of the two-year return wave heights for each month and for the year at Seven Stones and for the winters at Famita are given in Table 6.

Figures A1 - A4 in the annex indicate the need to obtain further years of wave records from Seven Stones if significant improvement is to be made in estimating fifty-year return values. More data from Famita might also be useful, but gaps in the records - especially for periods of high sea states - introduce possible errors which cannot be quantified using the method in this report of analysing monthly maxima.

Annex A.

METHODS OF ESTIMATION

The major method of estimation used in this report is the method of maximum likelihood. Briefly this technique involves maximising a function of the data called the likelihood, which is related to the probability of the sample occuring given a certain distribution, in our case the Fisher-Tippett I distribution. The likelihood function $L(\underline{\theta} ; x_1 \ldots x_n)$ is given by

$$L(\underline{\theta} ; x_1 \ldots x_n) = \prod_{i=1}^{n} f(x_i ; \underline{\theta})$$

where $x_1 \ldots x_n$ is our sample.

$\underline{\theta}$ is a vector of unknown parameters to be estimated, (in this case $\underline{\theta} = (A, B)$.

$f(x ; \underline{\theta})$ is the probability density function from which the sample is drawn, in this case

$$f(x ; A, B) = \frac{1}{B} \exp\left(-\left(\frac{x-A}{B}\right) - \exp\left(-\left(\frac{x-A}{B}\right)\right)\right)$$

The likelihood is maximised with respect to $\underline{\theta}$ to give the maximum likelihood estimator $\hat{\underline{\theta}}$. (For a more complete discussion of maximum likelihood; see for example Hoel (1965) or Kendall and Stuart (1967)). For the Fisher-Tippett I distribution we obtain the following equations for $\hat{A}$ and $\hat{B}$:

$$\sum_{j=1}^{n} \exp[-(x_j - \hat{A})/\hat{B}] = n$$

$$\sum_{j=1}^{n} (x_j - \hat{A})[1 - \exp(-(x_j - \hat{A})/\hat{B})] = n\hat{B}$$

These have to be solved numerically to obtain estimates A and B (for details see Johnson and Kotz (1970)).

In general maximum likelihood estimators possess certain desirable properties and asymptotically they are optimal. However because of the small sample sizes considered in this report, most of these asymptotic results do not hold. One of the major problems is that for small sample sizes the estimators of A and B are biased. This means that the expectation (the long term average value) of our estimators is not the true population value of the parameter. This is not so disastrous as it might first appear. Estimators with small bias and moderate variance for instance may be preferable to those with no bias and large variance. For this reason when comparing estimators the criterion used is normally mean square error (variance + bias squared) rather than variance or bias alone. However it is usual to replace biased estimators by related unbiased estimators (for example when estimating the variance of a normal distribution). This is done by finding the expectation of the biased estimator and then finding the multiplicative factor that will make this estimator unbiased. It should be noted here that this need not produce a better estimate in any particular situation, it can quite easily produce a worse one, it is only the expected (or average) estimate that is improved.

Unfortunately it is not possible to find the expectations of the maximum likelihood estimators for a Fisher-Tippett I distribution theoretically and therefore to find unbiasing factors Monte Carlo methods have to be used. Since the bias alters with n (the sample size) and probably with A and B as well, simulations must be produced for varying values of A, B and n. It was decided to use values for n of 5, 6 and 7 since these were the sample sizes we are interested in and to vary A and B between 5 and 9 and 0.5 and 1.75 respectively. 250 samples were generated for each combination of A, B and n and maximum likelihood estimates of A, B and H50 (the fifty year return period) produced for each sample. These were then averaged over the 250 samples and estimates of the expectations formed. Estimate of the variance and mean square error were also produced as were equivalent estimates for the regression method to allow some comparison to be made. The results briefly are as follows. As would be expected (maximum likelihood estimators are asymptotically unbiased), the bias in $\hat{A}$, $\hat{B}$ and $\widehat{H50}$ decreases as n increases. The bias in $\hat{A}$ increases with B but falls with increasing A However it is reasonably constant over the region we are interested in. There are theoretical reasons for the bias in $\hat{B}$ to be independent of both A and B and it appears to be so, however it does show some apparently random variation, which implies that 250 is probably too small a number of samples to take. In spite of this variation (both random and deterministic) it is possible to extract unbiasing factors that are reasonably constant over the values of A and B we are interested in.

Although it is not possible to derive theoretically the bias in A, B or H50 it is possible using an approximation due to Lawless (1974) to produce approximate confidence intervals for A, B and any return period desired.

Fig. A1 - A4 enable confidence intervals for the maximum likelihood estimators of A, B, H2 and H50 respectively to be constructed. The 90% intervals for A and B are exact from Thoman et al (1969). If z_1 and z_2 are the values of z taken from the figure, our intervals are:

$$\hat{A} - z_1\hat{B} \leq A \leq \hat{A} - z_2\hat{B}$$
$$z_1\hat{B} \leq B \leq z_2\hat{B}$$
$$\hat{A} - z_1\hat{B} \leq H2 \leq \hat{A} - z_2\hat{B}$$
$$\hat{A} - z_1\hat{B} \leq H50 \leq \hat{A} - z_2\hat{B}$$

As stated in the report these intervals do not show how well the data fits the Fisher-Tippett 1 distribution since they are produced under the assumption that this is known to be the distribution of the population. What they do tell us is how confident we can be about our inferences, for example if we have a 90% interval about a parameter then it means that nine times out of ten this interval will cover the true value, i.e. that there is a 10% chance that this interval does not cover the true value of the parameter.

These confidence intervals are rather wide at the sample sizes we are considering, hardly surprising as we have very little data. However they narrow quite rapidly after only a moderate increase in sample size. After this the increase in accuracy is only marginal and further collection of data may be unprofitable from the point of view of narrowing confidence intervals.

Finally a comparison of the maximum likelihood and regression methods: As stated earlier in the simulation experiment both maximum likelihood and regression were used to estimate A, B and H50 so that a comparison could be made. These results showed that although the regression method was unbiased it did have a large variance and hence a considerably larger mean square error than maximum likelihood. Also it is only with the maximum likelihood estimators that we can produce realistic confidence intervals, the considerably narrower intervals produced around regression estimates are based upon incorrect implicit distributional assumptions. So apart from the bias, which is easily removed, maximum likelihood is to be preferred to the regression method.

Annex B

A COMPARISON OF RETURN PERIOD FROM MONTHLY AND ANNUAL MAXIMA

Consider a random variable X that is measured on n independent populations, in each of which X has a different distribution $F_i(x)$, $i = 1, n$. These distributions may have a completely different form or differ only in the values of some parameters.

If we sample each of these populations separately, taking a sample size m_j from the j^{th} population, then the distribution of the largest of these values is given by

$$G(x) = P(X_{max} < x) = \prod_{j=1}^{n} [F_j(x)]^{m_j}$$

On the other hand we could sample from the separate populations after aggregating them, i.e. consider them merely as one population. The distribution of a single member of this sample is

$$F(x) = \sum_{j=1}^{n} p_j F_j(x)$$

where p_j is the probability that any member of the large population taken at random belongs to the j^{th} population. The distribution of the largest of a sample size n^* is seen to be

$$G^*(x) = \left[\sum_{j=1}^{n} p_j F_j(x)\right]^{n^*}$$

We are interested in the return periods of events and in particular whether the return periods from one method are larger or smaller than those from the other. Return period is linked to the distribution function by the following relationship. If x has an n year return period

$$F(x) = 1 - \frac{1}{n}$$

(strictly speaking year should be replaced by event, but it is simpler to consider annual data). If we have two distributions F(x) and G(x) then the following two statements are equivalent since F and G are monotonically increasing.

The n-year return period from F is greater than that from G

$$F(x) < G(x)$$

In our case we want to show that the return period obtained from the individual populations is greater than that obtained from the aggregated one. Hence using the same notation as above we have to prove

$$G(x) < G^*(x) \qquad \text{for all } x$$

$$\text{i.e.} \quad \prod_{j=1}^{n} [F_j(x)]^{m_j} < [\sum_{j=1}^{n} p_j F_j(x)]^{n^*}$$

where

In general this is not true. However if we consider the practical situation of proportional sampling, $m_j = n^* p_j$ we have

$$\prod_{j=1}^{n} [F_j(x)]^{m_j} < [\sum_{j=1}^{n} \frac{m_j}{n^*} F_j(x)]^{n^*}$$

$$\left[\prod_{j=1}^{n} [F_j(x)]^{m_j} \right]^{\frac{1}{n^*}} < \frac{1}{n^*} \sum_{j=1}^{n} m_j F_j(x)$$

Now the L.H.S. of this equation is the geometric mean of the n^* $F_j(x)$ and the R.H.S. is the arithmetic mean. Since $F_j(x)>0$ for all x,j this proves the required result (For a proof of this see Kendall and Stuart (1958)). If all the F_j's are the same we have equality, as would be expected.

The above result shows that if we have a situation where we have several independent populations to obtain the correct estimate of return period we must

(1) sample from each population separately
(2) use proportional sampling.

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Table 1a SEVEN STONES : Number of records missing during each month analysed in this report.

Year	J	F	M	A	M	J	J	A	S	O	N	D
1968	0	0	1	0	2	0	1	0	0	0	0	1
1969	0	0	0	2	2	26	13	5	2	3	13	0
1971							1	1	2	0	0	0
1972	1	2	6	1	0	0	12	0	0	0	0	2
1973	0	0	0	24	0	0	0	1	0	1	0	3
1974	1	0	1	0	1	1						
1975				1	0	0	0	2	0	4	1	0
1976	1	0	0	0	1	0	0	1	0	42	0	2
1977	24	0	1									

Table 1b SEVEN STONES : Twelve months used for annual analysis and number of records missing.

Year				Records missing	% missing
1	January	-	December 68	5	0.2
2	January	-	December 69	66	2.3
3	July 71	-	June 72	14	0.5
4	July 72	-	June 73	38	1.3
5	July 73	-	June 74	9	0.3
6	April 75	-	March 76	9	0.3
7	April 76	-	March 77	71	2.4

Table 1c FAMITA : Number of records missing during each month analysed in this report.

Winter	O	N	D	J	F	M	Total
1969-70	43	75	57	62	45	67	349
1971-72	240	89	47	61	232	46	715
1972-73	47	80	41	85	29	71	353
1973-74	61	107	68	94	81	97	508
1974-75	68	46	78	126	43	31	392
1975-76	126	71	43	39	44	34	357
Total	345*	468	334	467	242*	346	2677
% missing	28*	33	22	31	21*	23	

* in 5 months

Table 2a SEVEN STONES : Maximum significant wave height (m*) each month.

Year	J	F	M	A	M	J	J	A	S	O	N	D
1968	8.02	5.06	7.26	5.46	5.06	3.72	3.60	5.85	7.20	6.10	5.98	8.81
1969	8.26	6.68	4.36	6.13	3.66	5.30	4.73	3.66	3.84	5.27	8.69	7.74
1971							2.96	4.36	4.94	6.37	7.29	8.11
1972	8.26	8.45	9.88	8.29	7.59	4.60	4.15	4.82	3.26	6.07	7.59	7.65
1973	7.93	8.38	6.13	5.79	5.85	4.27	3.93	5.03	5.88	4.76	5.85	7.04
1974	10.43	9.33	6.13	4.12	6.07	4.66						
1975				6.59	3.78	3.26	4.27	4.02	7.53	5.43	6.95	6.86
1976	8.02	6.62	11.04	5.18	4.36	3.81	3.78	2.56	6.43	8.38	6.77	9.36
1977	7.84	7.53	7.96									

Table 2b SEVEN STONES : Maximum significant wave height (m*) each year.

Year		Wave height	Month
1	January - December 68	8.81	December
2	January - December 69	8.69	November
3	July 71 - June 72	9.88	March
4	July 72 - June 73	8.38	February
5	July 73 - June 74	10.43	January
6	April 75 - March 76	11.04	March
7	April 76 - March 77	9.36	December

Table 2c FAMITA : Maximum significant wave height (m*) each month. (Maximum each winter under-lined).

Winter	O	N	D	J	F	M
1969-70	8.32	12.32	8.20	7.13	7.38	9.18
1971-72		10.06	6.62	8.72		11.16
1972-73	8.14	8.63	8.05	6.98	7.47	5.30
1973-74	4.09	10.43	8.26	6.13	5.67	5.40
1974-75	6.77	8.29	8.87	8.20	5.52	5.46
1975-76	2.50	6.22	7.32	9.24	4.66	5.52

* original data in feet, to one decimal place

Table 3. Plotting Probabilities for Fisher-Tippett I Distribution

Number of Values	ordered position	probability	Gringorten's approximation
5	1*	0.8938	0.8906
	2	0.7099	0.6953
	3	0.5203	0.5
	4	0.3286	0.3047
	5	0.1361	0.1094
6	1*	0.9107	0.9085
	2	0.7562	0.7451
	3	0.5972	0.5817
	4	0.4368	0.4183
	5	0.2753	0.2549
	6	0.1135	0.0915
7	1*	0.9229	0.9213
	2	0.7898	0.7809
	3	0.6529	0.6404
	4	0.5149	0.5
	5	0.3761	0.3596
	6	0.2367	0.2191
	7	0.0973	0.0787

* largest value

Table 4 Results of analyses of Seven Stones data

	Nb of data	Max. likelihood estimates						Linear regression ests.	
		A (see equation 1)	B	50-yr ht (m)	50-yr ht corrected for bias	90% confidence limits		50-yr ht (m)	S.E. of 50-yr ht
Jan	7	8.0817	0.3965	9.63	10.0	9.1	11.5	10.64	0.59
Jan*	7	8.1452	0.6464	10.67	11.1	9.8	13.8	11.28	0.45
Feb	7	6.7479	1.3276	11.93	12.4	10.1	18.3	11.40	0.56
March	7	6.4968	1.8417	13.68	14.2	11.1	22.5	14.14	0.53
April	7	5.3654	1.0247	9.36	9.7	7.9	14.3	9.65	0.27
May	7	4.5749	1.0439	8.65	9.0	7.2	13.7	9.25	0.25
June	7	3.9175	0.5681	6.13	6.4	5.4	8.9	6.20	0.13
July	7	3.6479	0.5280	5.71	5.9	5.0	8.2	5.47	0.15
August	7	3.8251	0.9807	7.65	8.0	6.3	12.4	7.30	0.30
Sept	7	4.8120	1.4216	10.36	10.8	8.4	17.2	10.09	0.61
Oct	7	5.5757	0.7765	8.61	9.0	7.5	12.3	9.33	0.37
Nov	7	6.5832	0.7542	9.53	9.9	8.5	13.2	9.81	0.18
Dec	7	7.5372	0.6882	10.22	10.6	9.3	13.5	10.53	0.16
Dec‡	6	7.6884	0.7259	10.52	10.9	9.5	14.7	10.66	0.22
Year	7	9.0773	0.7278	11.92	12.4	10.9	15.4	12.32	0.18

* Estimates from second highest crest and second lowest trough of wave record.

‡ December 1973 omitted

Table 5 Results of analyses of Famita data

Month	Nb of data	Maximum likelihood estimates						linear regression estimates	
		A (see equation 1)	B	50-yr ht (m)	50-yr ht corrected for bias	50-yr ht 90% confidence limits		50-yr ht (m)	S.E. of 50-yr ht
Oct	5	4.7753	2.2038	14.04	15.3	10.0	29.3	13.63	1.83
Nov	6	8.3612	1.8081	15.97	16.8	12.8	25.8	15.87	0.72
Dec	6	7.5039	0.7426	10.63	11.2	9.3	14.7	10.26	0.40
Jan	6	7.1945	0.9730	11.29	11.9	9.6	16.6	11.40	0.41
Feb	5	5.5975	0.9425	9.56	10.0	7.8	16.1	9.90	0.69
March	6	5.9805	1.4842	12.22	12.8	9.6	20.3	14.43	1.55
Winter	6	9.4765	1.0345	13.83	14.5	12.0	19.4	14.70	0.38
Winter*	5	9.2876	0.9287	13.19	13.9	11.5	19.6	14.72	0.62

* Winter 1971-72 omitted.

Table 6

Two-year return values of significant wave height.

Month	2-yr return value (m)	90% confidence limits	
SEVEN STONES			
Jan	8.4	7.9	9.1
Feb	7.2	6.2	8.7
March	7.2	5.7	9.2
April	5.7	4.9	6.9
May	5.0	4.1	6.1
June	4.1	3.7	4.7
July	3.8	3.4	4.4
Aug	4.2	3.4	5.2
Sept	5.3	4.2	6.9
Oct	5.9	5.3	6.7
Nov	6.9	6.3	7.7
Dec	7.8	7.2	8.5
Dec*	8.0	7.3	8.9
Year ‡	9.4		
FAMITA			
Oct	6.5	3.3	8.9
Nov	9.8	7.4	11.3
Dec	8.1	7.1	8.7
Jan	8.0	6.7	8.8
Feb	6.3	5.0	7.1
March ‡	7.1	5.2	8.4
Winter ‡	10.6		

* December 1973 omitted.

‡ Cumulative probability from monthly values.

Table 7 Various estimates of fifty-year return value of significant wave height.

Source	Height (m)	Comment
SEVEN STONES		
This report	14.2	Value for March (90% prob 11.1-22.5m)
" "	12.4	from annual maxima (90% prob 10.9-15.4m)
" "	14.8	Product twelve monthly maxima probabilities
Fortnum & Tann (1977)	12.9	Fisher-Tippett III to all $H_{max(3hrs)}$ values 1968-74.
FAMITA		
This report	16.8	Value for November (90% prob 12.8-25.8m)
" "	14.5	from 6 winters max. (90% prob 12.0-19.4m)
" "	17.6	Product of six monthly maxima probabilities
Draper & Driver (1971)	15.1	Winter 1969-70 data fitted to log-normal distribution
Ewing et al (1978)	16.2	hindcast wave data from 42 severe storms 1966-76. Fisher-Tippett I distrib.
Fortnum (1978)	15.5, 15.2	1969-76 data fitted to Fisher-Tippett I & III respectively.
Saetre (1974)	16.3	1969-70, 71-72, 72-73, fitted to Fisher-Tippett I.
	15.2	1969 fitted to Weibull distribution
	14.6	1969 storm model with Fisher Tippett I

Table 8 Numbers of sea floods each month in the North Sea 838 - 1973 (from Table 13.3 of Lamb (1977))

Month	Number	
August	1	19-21 August 1573
September	3	
October	13	Including 31 Oct - 2 Nov 1570
November	20	Including 30 Nov - 1 Dec 1936
December	14	
January	16	
February	9	
March	7	
April	1	10 April 1446

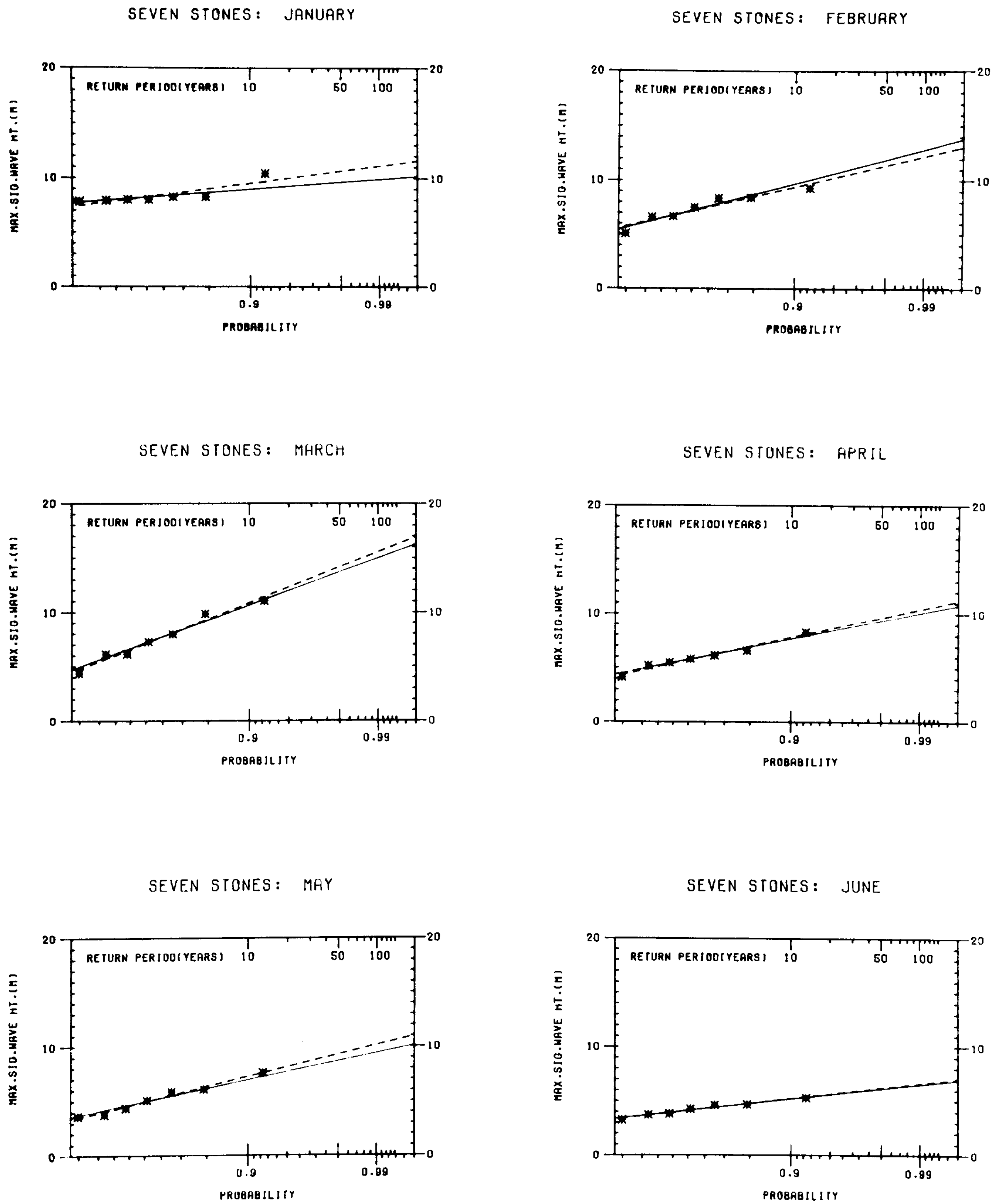


Fig.1 Fisher-Tippett Type I distribution fitted to monthly maximum values of significant wave height.

———— Maximum likelihood

– – – Linear regression

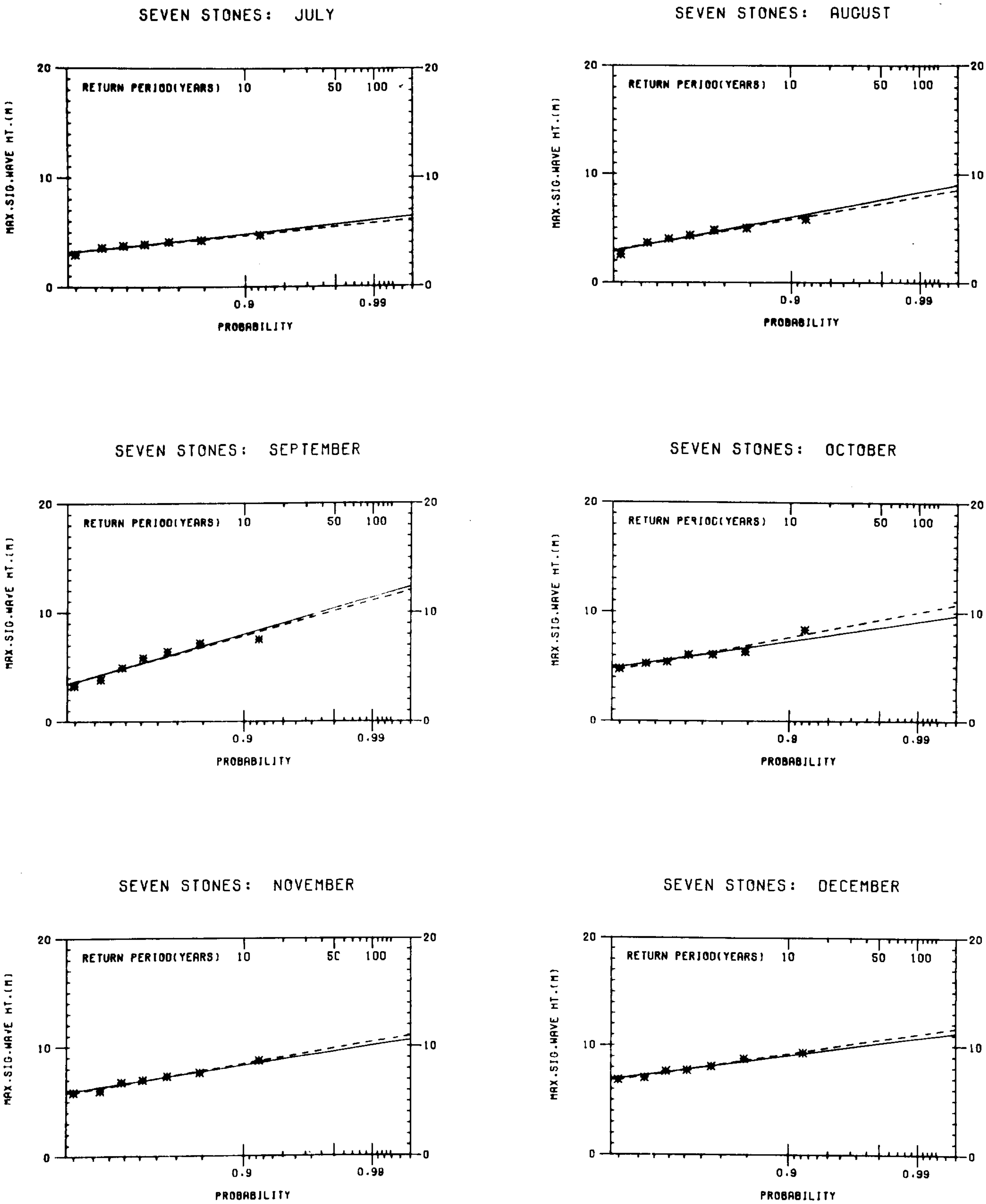


Fig. 2 **Fisher-Tippett Type I distribution fitted to monthly maximum values of significant wave height.**

——— Maximum likelihood
– – – Linear regression

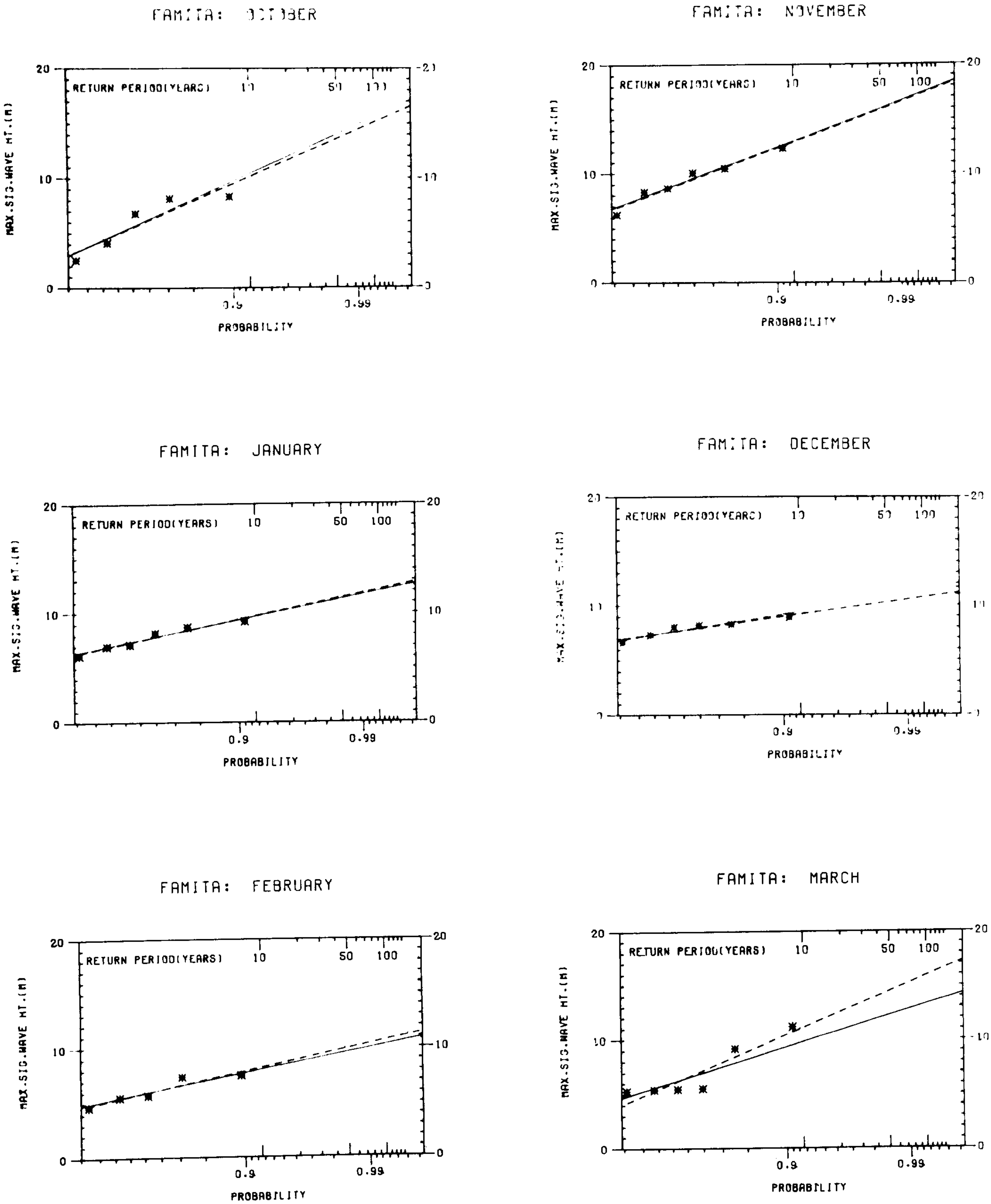
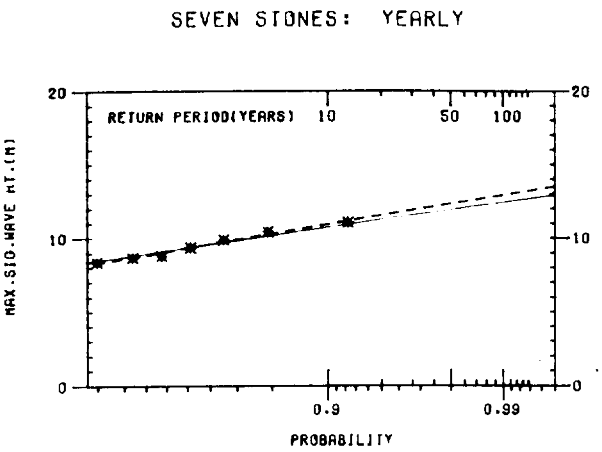


Fig. 3 Fisher-Tippet Type I distribution fitted to monthly maximum values of significant wave height.

——— Maximum likelihood

– – – – Linear regression



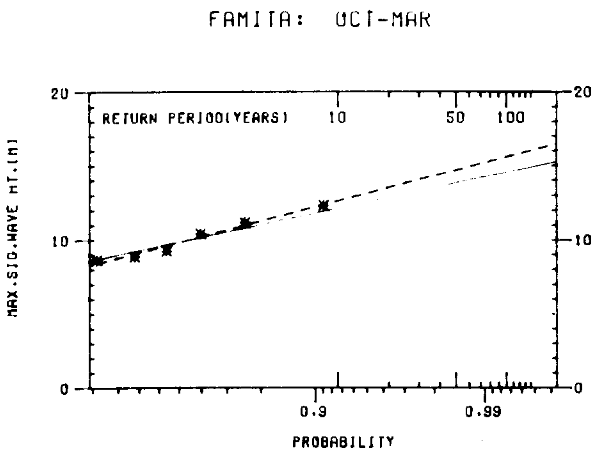


Fig. 4 Fisher-Tippett Type I distribution fitted to monthly maximum values of significant wave height.

——— Max likelihood
- - - - Least squares

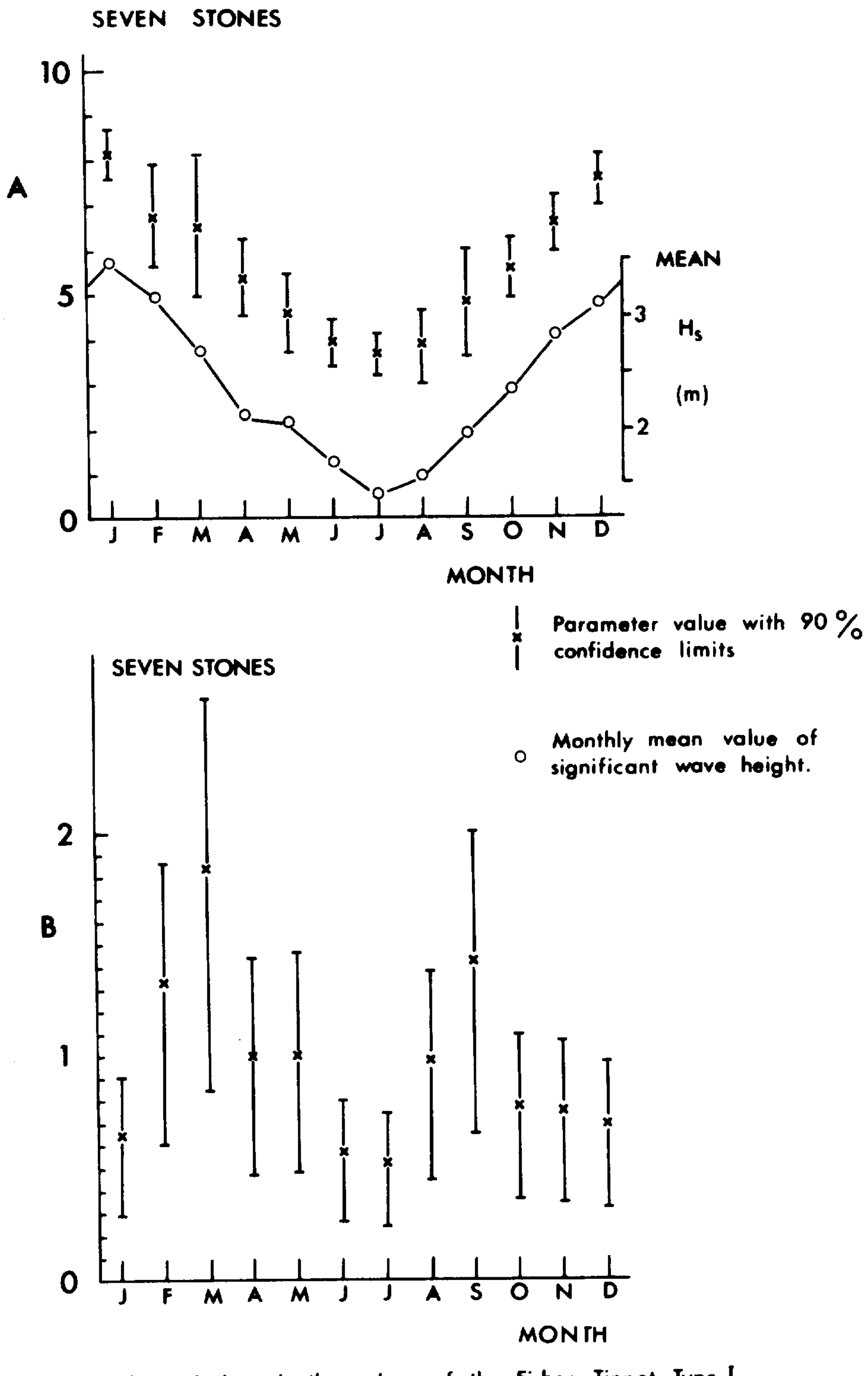


FIG. 5 Month-to-month variations in the values of the Fisher-Tippet Type I distribution parameters A & B defined by equation 1.

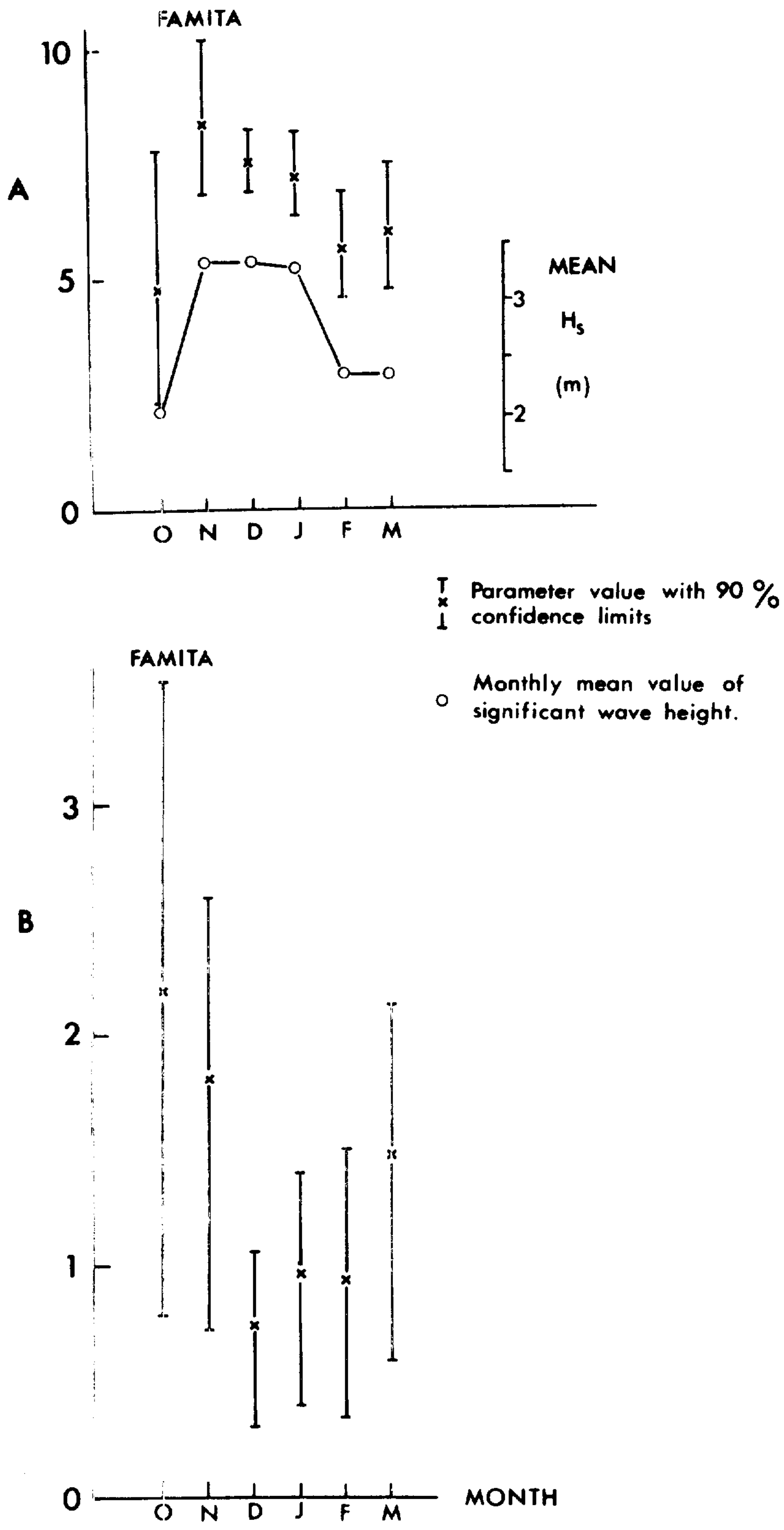
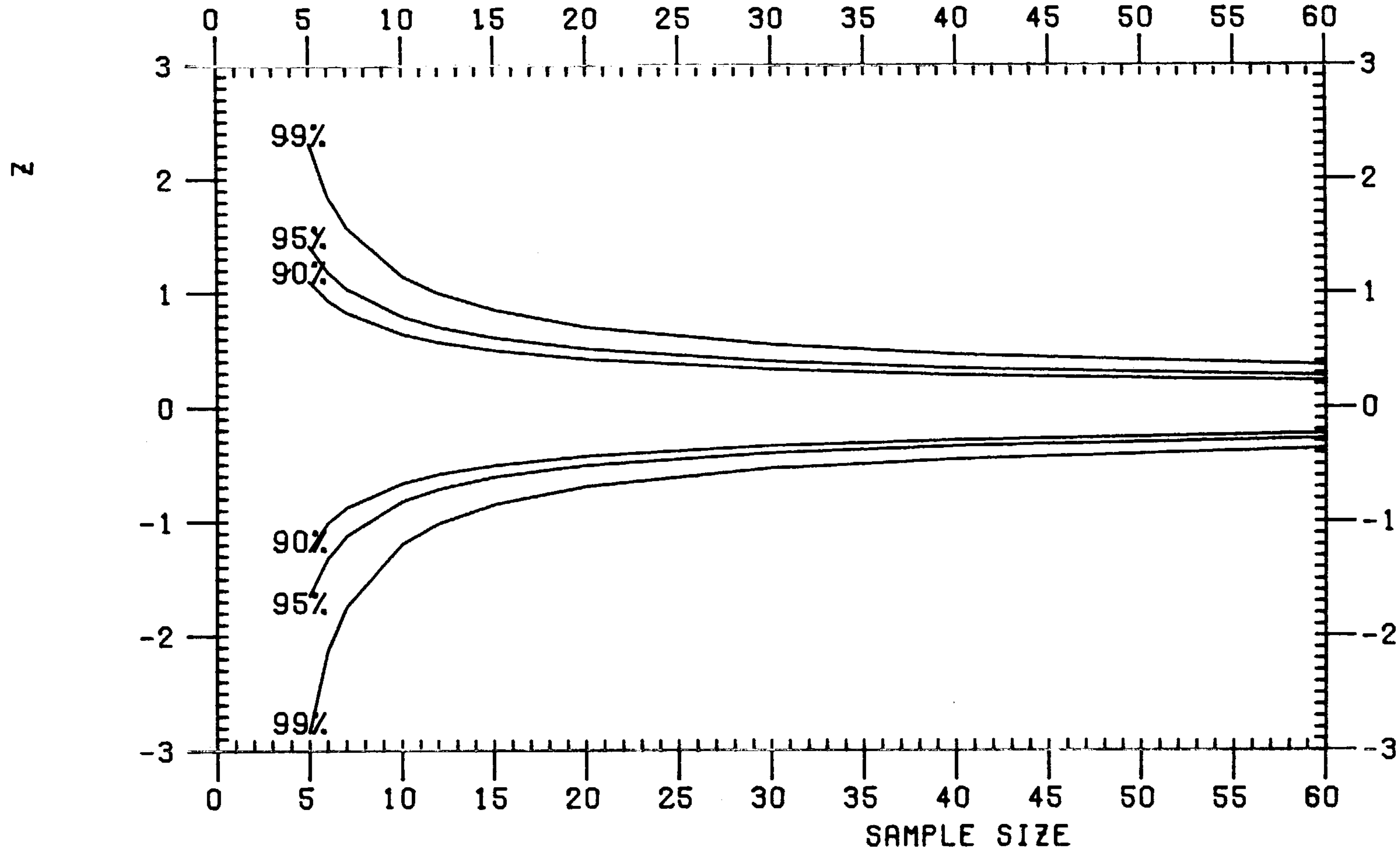
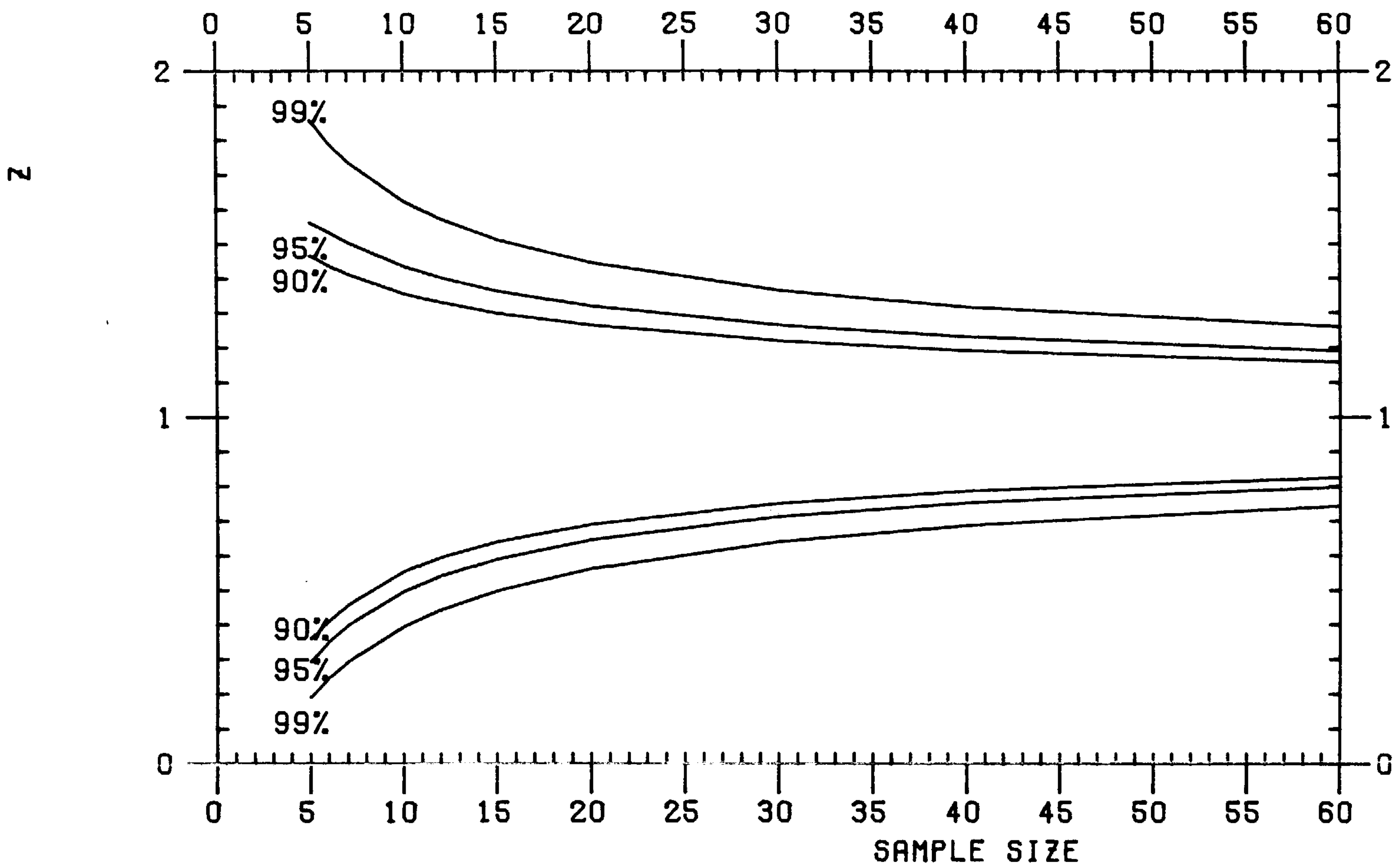


FIG. 6 Month-to-month variations in the values of the Fisher–Tippett Type I distribution parameters A & B defined by equation 1.



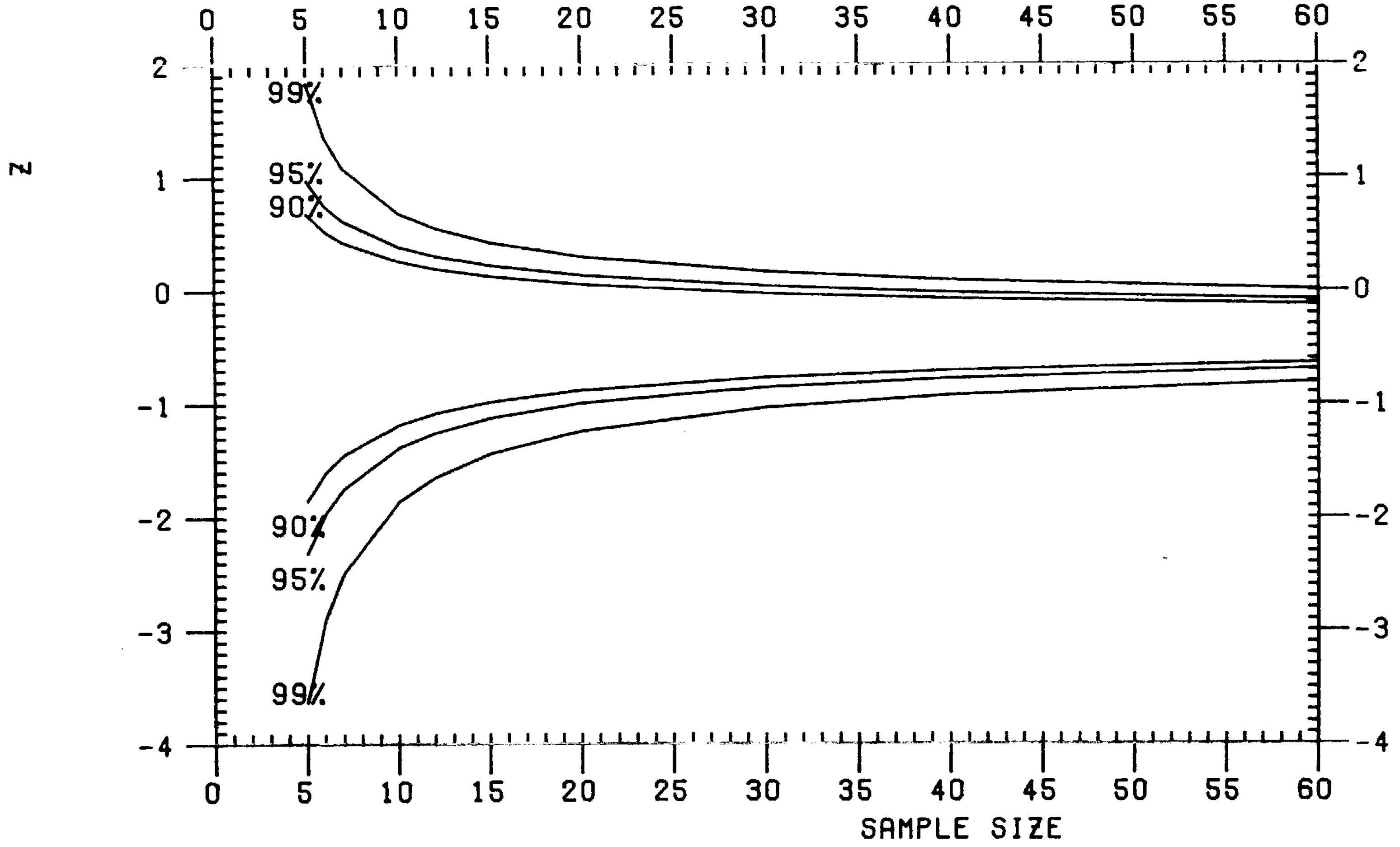
CONFIDENCE INTERVALS
FOR A F-T 1

FIG. A.1



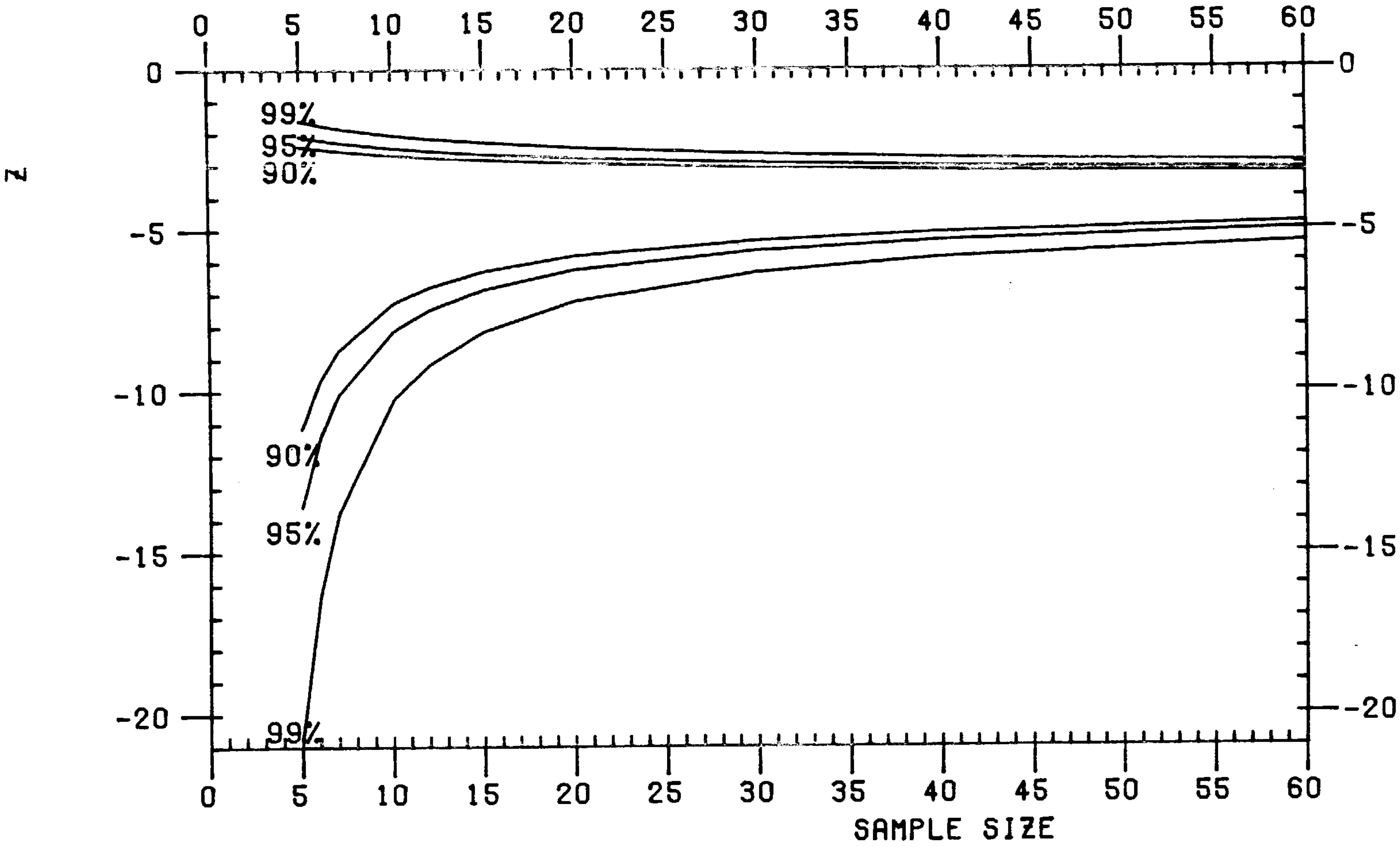
CONFIDENCE INTERVALS
FOR B F-T 1

FIG. A.2



CONFIDENCE INTERVALS
FOR 2 YR RETURN PERIOD F-T 1

FIG. A.3



CONFIDENCE INTERVALS
FOR 50 YR RETURN PERIOD F-T 1

FIG. A.4