I.O.S.

REGRESSION TECHNIQUE FOR FORECASTING SURGES AT LIVERPOOL

M. AMIN

REPORT NO. 80

1979

NATURAL ENVIRONMENT RESEARCH COUNCIL
INSTITUTE OF OCEANOGRAPHIC SCIENCES

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Contents

Abstract

The application of a multiple regression technique to forecast surges at Liverpool is investigated. Both a linear and non linear scheme has been examined. In this study only extreme positive surges are considered. The solution of the regression equation is obtained with reference to sea level data for the standard port of Liverpool and meteorological variables (barometric pressure and wind speed and direction) as observed at the closeby Bidston Observatory. Winds from the south-west rather than from any other direction have the most pronounced effect, and are the most salient meteorological parameter, in raising the sea levels to generate extreme surges. This conforms with Lennon's (1963) findings concerning the synoptics of meteorological conditions considered with the occurrence of extreme surges in the Liverpool area. Extension of the model to include surges from Fishguard, a Welsh port some 80 miles south of Liverpool, did not provide any significant improvement. A simple scheme of functional approximation is applied to enhance the model and to estimate the non-linear components of the surge, which are shown to be significant. The results are applied within a real time operational scheme suggested for the forecasting of flood surges.

Introduction

Variation in the barometric pressure and wind stress on the sea surface are responsible for generation and propagation of surges. These surges may cause a hazard for navigation or endanger coastal areas by the potential of flooding. The study here is confined to those positive surges which are responsible for flooding in the vicinity of Liverpool. The observed sea level ζ_t, measured as deviation of the sea level above the mean sea level, at any time t can be expressed as

$$\zeta_t = \zeta_t^{(T)} + S_t + Z_t \qquad (1)$$

where
- t is the time
- $\zeta^{(T)}$ is the tidal component
- S is the weather induced perturbation 'surge'
- Z is the residual variation which is not accountable in terms of readily measured forces

The tidal component $\zeta_t^{(T)}$ can be predicted accurately by existing tidal prediction methods e.g. Munk & Cartwright (1966) and Rossiter & Lennon (1968).

Subtracting the tide from observed sea level, equation (1) becomes

$$R_t = \zeta_t - \zeta_t^{(T)} = S_t + Z_t \qquad (2)$$

where the residual R is the comination of surge (S) and the uncorrelated term or 'noise' (Z).

Any comprehensive technique to predict surges requires the use of numerical models (Heaps 1969, Heaps & Jones, 1975), where proper consideration can be given to the dynamics of the ocean system. The hydrodynamical equations governing the flow and changes in the sea level, in linearised form (Proudman, 1954) are

$$\frac{\partial \bar{u}}{\partial t} - \gamma \bar{v} = -g \frac{\partial}{\partial x}(\zeta - \zeta') + \frac{1}{\rho h}(F_x^{(w)} - F_x^{(b)}) \qquad (3)$$

$$\frac{\partial \bar{v}}{\partial t} + \gamma \bar{u} = -g \frac{\partial}{\partial y}(\zeta - \zeta') + \frac{1}{\rho h}(F_y^{(w)} - F_y^{(b)}) \qquad (4)$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = -\frac{1}{h}\frac{\partial \zeta}{\partial t} \qquad (5)$$

where
- x, y, z are cartesian co-ordinates, in which x, y lie in the horizontal plane of the mean sea level surface, and z is positive below that surface,
- h is the depth below mean sea level,
- ρ is the density of water,

ζ is the elevation of the sea surface about the mean sea level plane x, y,

$\bar{u}, \bar{v}$ are depth mean values of u and v (current components in the increasing x, y direction)

$$\bar{v} = \frac{1}{h}\int_o^h vdz \quad , \quad \bar{u} = \frac{1}{h}\int_0^h udz$$

$F^{(w)}, F^{(b)}$ are wind stress component on the sea surface and component of bottom frictional stress on water at bottom,

γ is the geostrophic co-efficient, regarded as constant,

$\bar{\zeta}$ elevation of the sea surface due to atmospheric pressure and the equilibrium tide.

These equations are usually solved by explicit finite difference schemes with inputs of meteorological data at selected grid points and observed surges at selected boundary points.

Doodson (1924), Rossiter (1959), Cartwright (1968) and more recently Amin (1977) used the regression technique to examine and estimate surges. In the regression technique a surge is expressed as

$$S_t = \alpha_1 (P_{t-\tau_1} - \bar{P}) + \alpha_2 U_{t-\tau_2} \, |U_{t-\tau_2}| + \alpha_3 V_{t-\tau_3} \, |V_{t-\tau_3}| \qquad (6)$$

where P is the barometric pressure with annual mean $\bar{P}$,

U is the east-west component of the wind,

V is the north-south component of the wind,

τ are averaged time lags of maximum correlation of the surge (S) with meteorological parameters P, U, V.

Equation (6) which is a simple linear form does not adequately balance the hydrodynamical equations (3), (4), (5), but under specified condition, e.g. when a location is fixed and winds coming from certain directions only are considered, many variables of equations (3), (4), (5) will behave as constants and we can then represent the system by equation (6).

Linear Surge

Six extreme flood generating surges for Liverpool were selected and three of them

(Jan. 1965, Jan. 1976, and Nov. 1977) were used to develop the linear regression model, the remaining three being used as test cases to check the validity of a surge prediction scheme. Meteorological data from Bidston Observatory were used, being available at hourly intervals, the same time interval as the sea levels being considered in the model. The solutions of equations (3), (4), and (5) or (6) will give only linear components of the surge, since non linear terms are not included in this system. The cross-correlation functions of Liverpool surges with meteorological variable P, U and V (from Bidston), Figure 1, show that there is some pronounced relationship amongst them. It suggests that the sea responds almost immediately to westerly winds and with a slight delay to southerly winds. The westerly winds are of a considerably greater magnitude than any associated southerly winds and consequently are the predominant contribution to the surge generation. For the time sub-set $t = [1,N]$, equation (1) gives a redundant system which when solved by the least squares method, such that $\langle (R-S)^2 \rangle$ is minimum, will give estimate of a solution

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} C_{PP} & C_{PU} & C_{PV} \\ C_{UP} & C_{UU} & C_{UV} \\ C_{VP} & C_{VU} & C_{VV} \end{bmatrix} \begin{bmatrix} C_{PR} \\ C_{UR} \\ C_{VR} \end{bmatrix} \tag{7}$$

where C's are autocorrelations and cross correlations of respective variables (R, $P - \bar{P}$, $U\,|U|$ and $V\,|V|$) with predetermined time lags from individual correlations of meteorological variables with surge. The solution of equation (7) for the three principal surges, Jan. 1965, Jan. 1976 and Nov. 1977, gave on averaging;

$$\hat{S}_t = 0.0093\ (P_{t-1} - \bar{P}) + 0.0028\ U_{t-1}\,|U_{t-1}| + 0.0040\ V_{t-8}\,|V_{t-8}| \tag{8}$$

The forecast of the independent surges of Jan. 1974, Feb. 1970 and Nov. 1973, as computed by equation (8), are shown in the Figure (2). It is important to recognise that in equation (8) the forecasting time lag on the principal parameters P and U has been established to be 1 hour. Therefore it would be necessary to utilise meteorological forecasts (of P, U) if this equation is to be extended to predicting events in advance of one hour.

Some discrepancies are visible between the observed and forecast surges. The improved performance of numerical models, Heaps and Jones (1975), suggests that

---

* $\hat{S}, \hat{\alpha}$,..........denote the estimates of S, α ,..........

Fishguard surges may be introduced as a space-variable in equation (5). To investigate the effectiveness of this on the regression model, and additional parameter included in equation (1) gave,

$$\hat{S}_t = \alpha_1(P_{t-1}-\bar{P}) + \alpha_2 U_{t-1} \mid U_{t-1} \mid + \alpha_3 V_{t-8} \mid V_{t-8} \mid + \alpha_4 S^{(F)}_{t-4} \qquad (9)$$

where $S^{(F)}$ is the Fishguard surge with associated time lag.

Examination of the cross correlative function between Fishguard and Liverpool surges, Figure 1, shows that a relationship exists of about 4 hours time lag, this relationship disappears or becomes negligible in the simultaneous solution of equation (9). It was therefore considered inappropriate to include Fishguard surges in the model.

Non-linear surges

The surges computed from linear regression equation (8) do not reproduce the diurnal and higher frequency variations shown in the observed surges. These components principally result from surge-tide interactions. The technique used by Cartwright (1968) may prove useful to predict these non-linearities. Another useful proposition may be to change equation (6) to a non-linear regression equation of the form

$$\hat{S}_t = \alpha_1(P_{t-1}-\bar{P}) + \alpha_2 U_{t-1} \mid U_{t-1} \mid + \alpha_3 V_{t-8} \mid V_{t-8} \mid + \alpha_4 f(S, \zeta^{(T)}) \qquad (10)$$

where function $f(S, \zeta^{(T)})$ may represent the surge-tide interaction. A simple test was carried out by choosing a function representing the surge tide interaction as $f(S, \zeta^{T}) = S. \zeta^{(T)}$. This generated a component with tidal characteristics superimposed on the surge. The contributions from the function $f(S, \zeta^{(T)})$ were of an oscillatory tidal form and found to be similar for each of the different surges considered. The observed fluctuations (about the surge profile) although showing some oscillatory tidal form were not adequately reproduced by the function $f(S, \zeta^{(T)})$. The function $f(S, \zeta^{(T)})$ as considered here was in its most simplest form and it would be useful to examine further in this context such effects as the surge/tide gradients.

The regression technique, as described in equation (8) is a linear process and cannot reproduce the non-linear components of the surge. To determine somehow the non-linear components, further schemes were considered.

(i) An autoregression scheme was tested on the residual surge $S^{(R)}(=R-\hat{S})$ as described by Amin (1977).

(ii) A 25 hour polynomial or harmonic functional representation of the residual surge, and then extrapolation of the computed function was examined.

Scheme (i) also did not produce any significant improvements in the results. Some large oscillations in the non-linear component as observed are aperiodic, therefore it is difficult to deduce generalised autoregressive equation which could be applied to all the surges.

In view of these difficulties, scheme (ii) was principally investigated. In this scheme a functional approximation of the residual surge was estimated over a 25 hour period as

$$S_t^{(R)} = a_o + \sum_r (a_r \cos \sigma_r t + b_r \sin \sigma_r t) \tag{11}$$

where a, b are constants,

σ represents the speed of central lunar terms M_1, M_2 and M_4 for r = 1, 2, 3 respectively

In the time subset t = $[-12, 12]$, equation (11) gives a redundant system of equations

$$\begin{aligned} a_o + \sum_r (a_r \cos \sigma_r(-12) + b_r \sin \sigma_r(-12)) &= S_{-12}^{(R)} \\ a_o + \sum_r (a_r \cos \sigma_r(-11) + b_r \sin \sigma_r(-11)) &= S_{-11}^{(R)} \\ \text{-------------------------------} & \\ a_o + \sum_r (a_r \cos \sigma_r(12) + b_r \sin \sigma_r(12)) &= S_{12}^{(R)} \end{aligned} \tag{12}$$

In matrix notation, the system of equation (12) can be written as

$$\underline{M}\ \underline{A} = \underline{S} \tag{13}$$

where

$$\underline{M} = \begin{vmatrix} 1 \cos \sigma_1(-12) & \sin \sigma_1(-12) & \text{-----} & \cos \sigma_3(-12) & \sin \sigma_3(-12) \\ \text{-----} & & & & \\ \text{-----} & & & & \\ \text{-----} & & & & \\ 1 \cos \sigma_1(12) & \sin \sigma_1(12) & \text{-----} & \cos \sigma_3(12) & \sin \sigma_3(12) \end{vmatrix} \tag{14}$$

$$\underline{A} = (a_o\ a_1\ b_1\ a_2\ b_2\ a_3\ b_3) \tag{15}$$

$$\underline{S} = (S_{-12}^{(R)}\quad S_{-11}^{(R)}\quad - \quad - \quad -S_{12}^{(R)}) \tag{16}$$

The estimate $\hat{\underline{A}}$ of $\underline{A}$ is obtained by least squares solution from system of equation

---

* $\underline{M}$, $\underline{A}$, $\underline{S}$ represent matrices or vectors and their transposes are represented by $\underline{M}'$, $\underline{A}'$, $\underline{S}'$.

(13) such that $\langle (S^{(R)} - \hat{S}^{(R)})^2 \rangle$ is minimum, as

$$\underline{\hat{A}} = (\underline{M}'\underline{M})^{-1} \underline{M}'\underline{S} \qquad (17)$$

The matrix $\underline{M}$ is unique for any given time span and constituents, therefore $(\underline{M}'\underline{M})^{-1}\underline{M}'$ can be computed once for subsequent regular use. This matrix is given in Table 1.

The nominal components M_1, M_2 and M_4 are selected to represent the tidal spectral bands of high concentration of energy. The quarter diurnal term M_4 was resolved but subsequently it was not used in extrapolation as it proved to be variable over extrapolation. This scheme can easily be operated on a small programmable calculator and permits one to extrapolate both the linear and non-linear surge components up to 2 hours in advance. It is interesting to note that the second order term, as in equation (10), which was also harmonic in character did not help to improve the surge prediction. There is a marked difference in the two schemes. Equation (10) is generalising the system for all surges but the harmonic scheme, described, uses the up-to-date information in a stepwise fashion and predicts only 2 hours ahead. When extrapolation of residual surge was attempted more than 2 hours in advance, this scheme also started diverging.

Discussion of Results

The most successful technique of forecasting surges presented here is similar to that used in the Torres Strait, Amin (1977), but it differs in its functional approximation of non-linear components. It has been established that winds coming from the west and south west direction are responsible for generation of high positive surges in Liverpool area. The coefficient associated with north south component of wind, in equation (8) suggests that strong winds from south can also develop high positive surges. This could not be confirmed in this study because such an event is not observed. Investigation has shown that surges are composed of two distinct components; (a) locally generated surge by east-west component of wind and (b) external surge propagated into Irish Sea by winds coming from the south. All the observed extreme positive surges are associated with predominantly westerly winds. Lennon (1963) showed that high positive surges at Liverpool are associated with deep secondary depressions moving in a north eastward direction from Atlantic towards Scotland in the manner as shown in Figure 3. As shown by Graff (1978), the surge associated with the extreme sea levels observed at Liverpool on 11th November 1977 was directly associated with this synoptic weather pattern. Westerly winds in the Liverpool area are in fact generated by such patterns of depressions. The response of the sea level to meteorological forces induced by barometric pressure and westerly winds is almost immediate with a one hour time lag on average. The effect of north-south component of wind is delayed suggesting that it may be responsible for some mechanism in the transportation of water mass from the Celtic sea region.

Three surges of Jan. 1965, Jan. 1976 and Nov. 1977 were analysed to compute regression coefficients. The forecasts of these surges and independent surges (not included in the analysis) of Jan. 1974, Feb. 1970 and Nov. 1973 are shown in Figure 2. The plots of independent surges indicate that accuracy of their prediction is similar to those of 1977 and 1976. A large difference between the peaks of forecast and observed surges of 1976 is due to the fact that the wind speeds were not recorded at Bidston on that occasion (breakdown), and instead Speke observations are used. Westerly winds are weaker at Speke than at Bidston because Speke is situated inland.

The results obtained here are comparable with results from numerical model, Figure 4. However, validity of this statement should not be generalised as the regression model is too simplified. The tests were performed on the limited number of surges generated by winds coming from limited directional ranges. The product term of tide and surge elevations, as in equation (10) did not prove successful but this will not be a valid reason to conclude that interaction does not exist. It may be that this simple term could not simulate the complex situation, therefore more investigations are required. Finally, the improvement obtained by functional extrapolation is limited both in magnitude and on time scale, and its application may prove difficult under ordinary circumstances.

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Algorithm for an operational Surge Forecast scheme based on multiple regression technique.

Procedure suggested when a surge forecast computation is required.

(1) Obtain the barometric pressure and wind observations from Bidston Observatory at hourly intervals, over 36 hour period of immediate past.

Up-to-date meteorological observations can forecast surges only one hour in advance, therefore, these observations must be augmented by weather forecast to predict surges sufficiently in advance.

(2) Resolve wind speed into east-west (u-component) and north-south (v-component), and subtract annual mean $\bar{p}$ ($\bar{p}$ = 1013.5 mb for Bidston) from barometric pressure.

(3) Compute Linear surge $\hat{S}$ using equation (8).
If estimate of non-linear component of surge is not required then go to step 8.

(4) Calculate tidal residuals R by subtracting predicted tide from tidal observations.

(5) Compute residual surge $S^{(R)} = R - \hat{S}$.

(6) Estimate functional approximation from residual surge of previous one day as:

(i) Multiply 25 elements of each column of transpose of matrix $(\underline{M}'\underline{M})^{-1}\ \underline{M}'$, given in Table 1, by respective values of residual surge then add up. This operation will give the 7 elements of vector A.

(ii) Substitution of these values in equation (11) with t = 14 will give $\hat{S}^{(R)}$ for 2 hours ahead.

(7) Improved surge $S^{(I)} = \hat{S} + \hat{S}^{(R)}$.

(8) Substitute new meteorological observations for weather forecast values and extend present data with new weather forecast, if available, and repeat from step 1 for as long as necessary

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* In this scheme it will be necessary to interpolate any meteorological forecast

data into hourly intervals.

** Units used are metres for surges, millibars for barometric pressure and metres/second for wind components.

Figure Captions

Figure 1. Cross-correlation functions of observed surge at Liverpool with:

_______________, barometric pressure;

- - - - - - - - -, U | U| component of wind;

____ _ ______ _, V | V| component of wind;

..............., Fishguard surge.

Figure 2. Elevations of observed surges and surges predicted by the regression technique:

(a) _______________, observed surge;

- - - - - - - - -, surge ($\hat{S}$) predicted by linear regression;

..............., improved surge estimate ($\hat{S}^{(I)}$) obtained by using functional extrapolation.

(b) _______________, non-linear surge component ($R-\hat{S}$);

- - - - - - - - -, extrapolated non-linear surge.

Figure 3. Depression tracks for large surges at Liverpool.

Figure 4. Jan. 1965 surge at Liverpool: ______________________, observed;

- - - - - - - - -, predicted by numerical model (Heaps & Jones, 1975).

Table 1.

Transpose of matrix $(\underline{M}\,\underline{M})^{-1}\,\underline{M}$.

0.0386	-0.0767	-0.0086	0.0753	0.0170	0.0698	0.0333
0.0391	-0.0731	-0.0283	0.0583	0.0531	0.0083	0.0799
0.0398	-0.0650	-0.0463	0.0265	0.0758	-0.0628	0.0515
0.0403	-0.0522	-0.0613	-0.0128	0.0795	-0.0764	-0.0252
0.0402	-0.0351	-0.0724	-0.0498	0.0633	-0.0192	-0.0782
0.0400	-0.0155	-0.0789	-0.0747	0.0313	0.0556	-0.0577
0.0400	0.0047	-0.0804	-0.0805	-0.0085	0.0782	0.0170
0.0404	0.0240	-0.0768	-0.0656	-0.0462	0.0276	0.0758
0.0408	0.0416	-0.0683	-0.0341	-0.0742	-0.0486	0.0634
0.0408	0.0572	-0.0554	0.0055	-0.0804	-0.0788	-0.0086
0.0404	0.0701	-0.0390	0.0430	-0.0683	-0.0349	-0.0725
0.0398	0.0789	-0.0201	0.0695	-0.0391	0.0417	-0.0684
0.0395	0.0820	0.0000	0.0790	0.0000	0.0789	0.0000
0.0398	0.0789	0.0201	0.0695	0.0391	0.0417	0.0684
0.0404	0.0701	0.0390	0.0430	0.0683	-0.0349	0.0725
0.0408	0.0572	0.0554	0.0055	0.0804	-0.0788	0.0086
0.0408	0.0416	0.0683	-0.0341	0.0724	-0.0486	-0.0634
0.0404	0.0240	0.0768	-0.0656	0.0462	0.0276	-0.0758
0.0400	0.0047	0.0804	-0.0805	0.0085	0.0782	-0.0170
0.0400	-0.0155	0.0789	-0.0747	-0.0313	0.0556	0.0577
0.0402	-0.0351	0.0724	-0.0498	-0.0633	-0.0192	0.0782
0.0403	-0.0522	0.0613	-0.0128	-0.0795	-0.0764	0.0252
0.0398	-0.0650	0.0463	0.0265	-0.0758	-0.0628	-0.0515
0.0391	-0.0731	0.0283	0.0583	-0.0531	0.0083	-0.0799
0.0386	-0.0767	0.0086	0.0753	-0.0170	0.0698	-0.0333

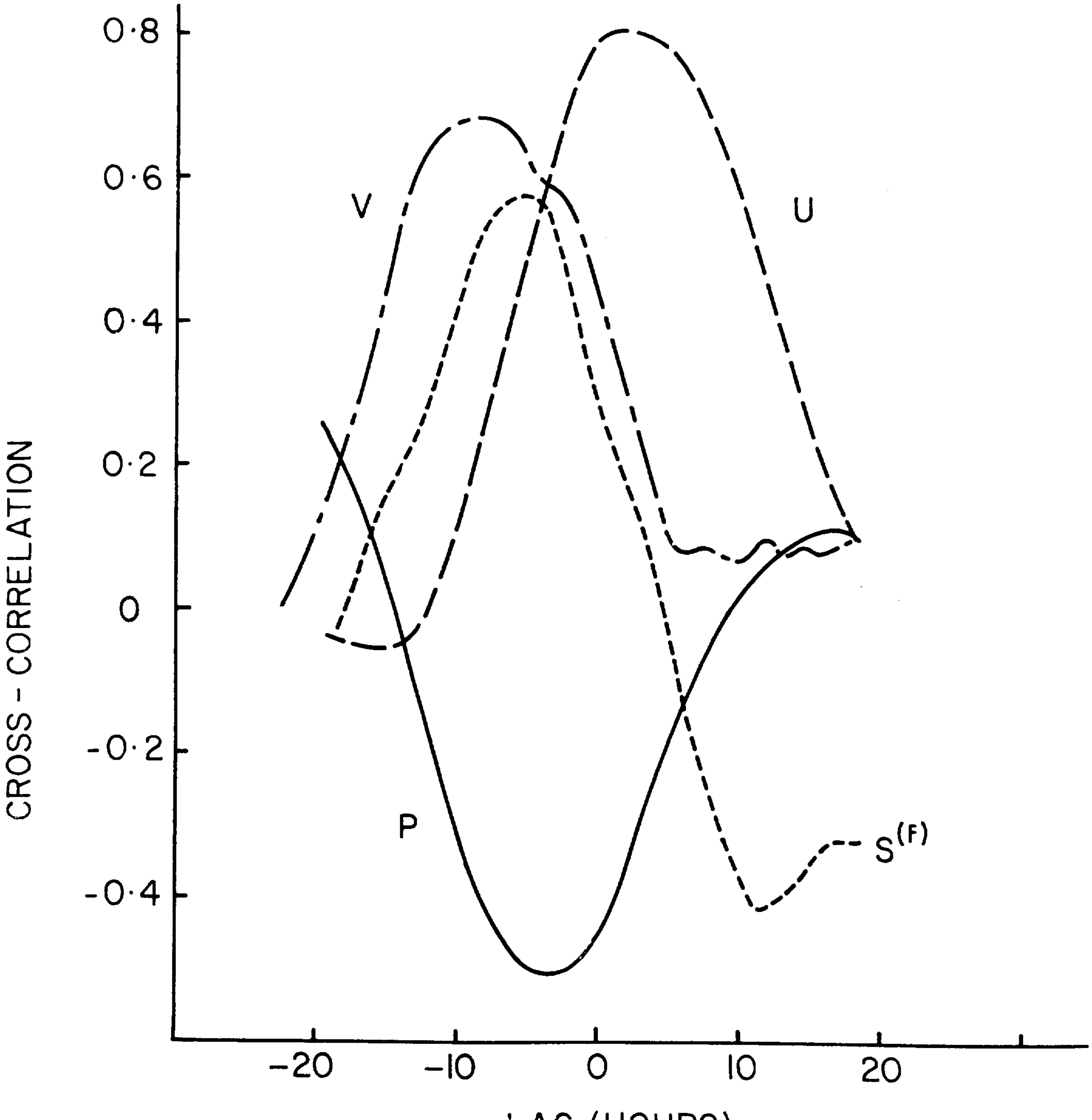
CROSS - CORRELATION
0·8
0·6
0·4
0·2
0
-0·2
-0·4
V
U
P
S(F)
-20
-10
0
10
20
LAG (HOURS)

FIGURE 1

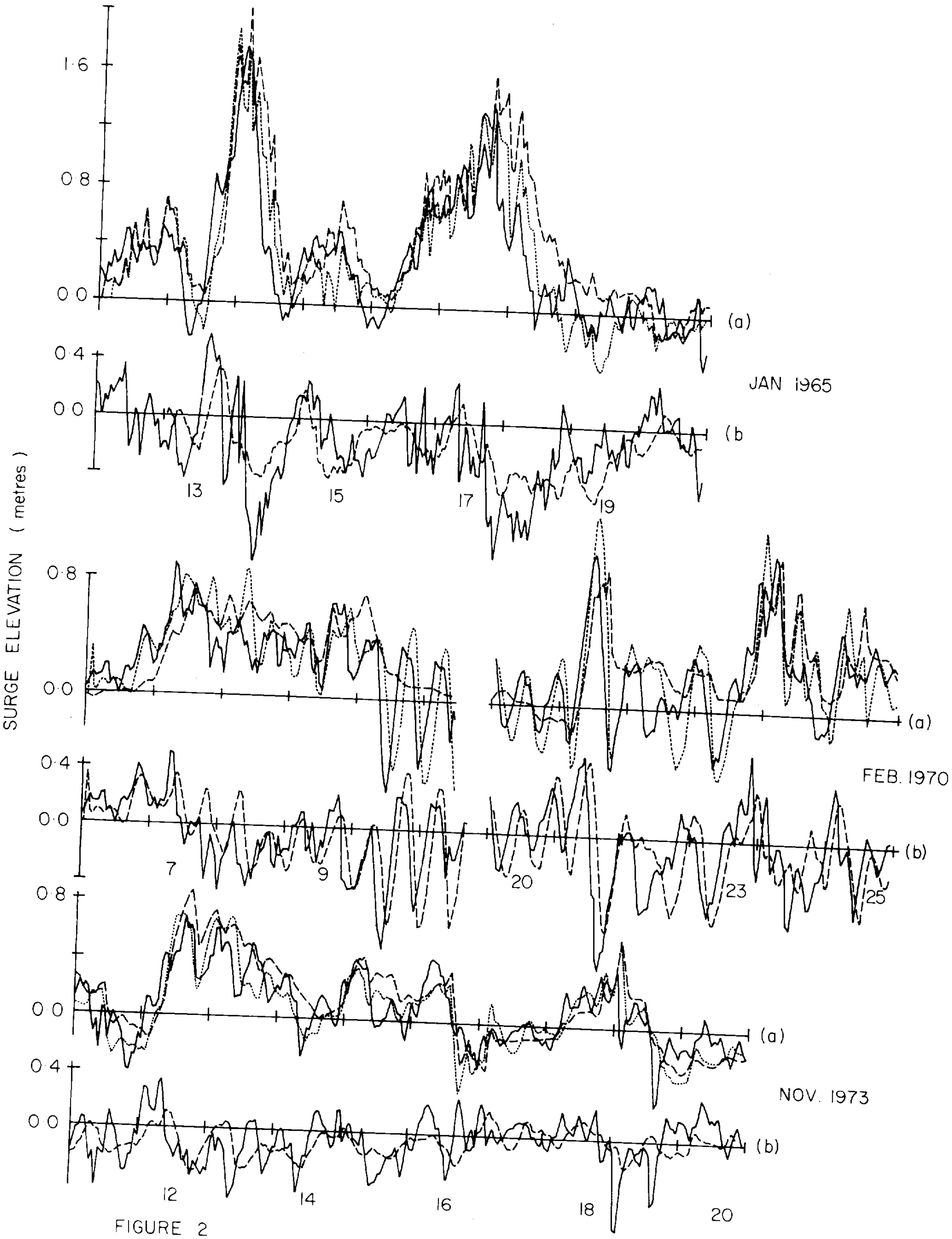
SURGE ELEVATION (metres)
1·6
0·8
0·0
(a)
0·4
0·0
(b)
JAN 1965
13
15
17
19
0·8
0·0
(a)
0·4
0·0
(b)
FEB. 1970
7
9
20
23
25
0·8
0·0
(a)
0·4
0·0
(b)
NOV. 1973
12
14
16
18
20

FIGURE 2

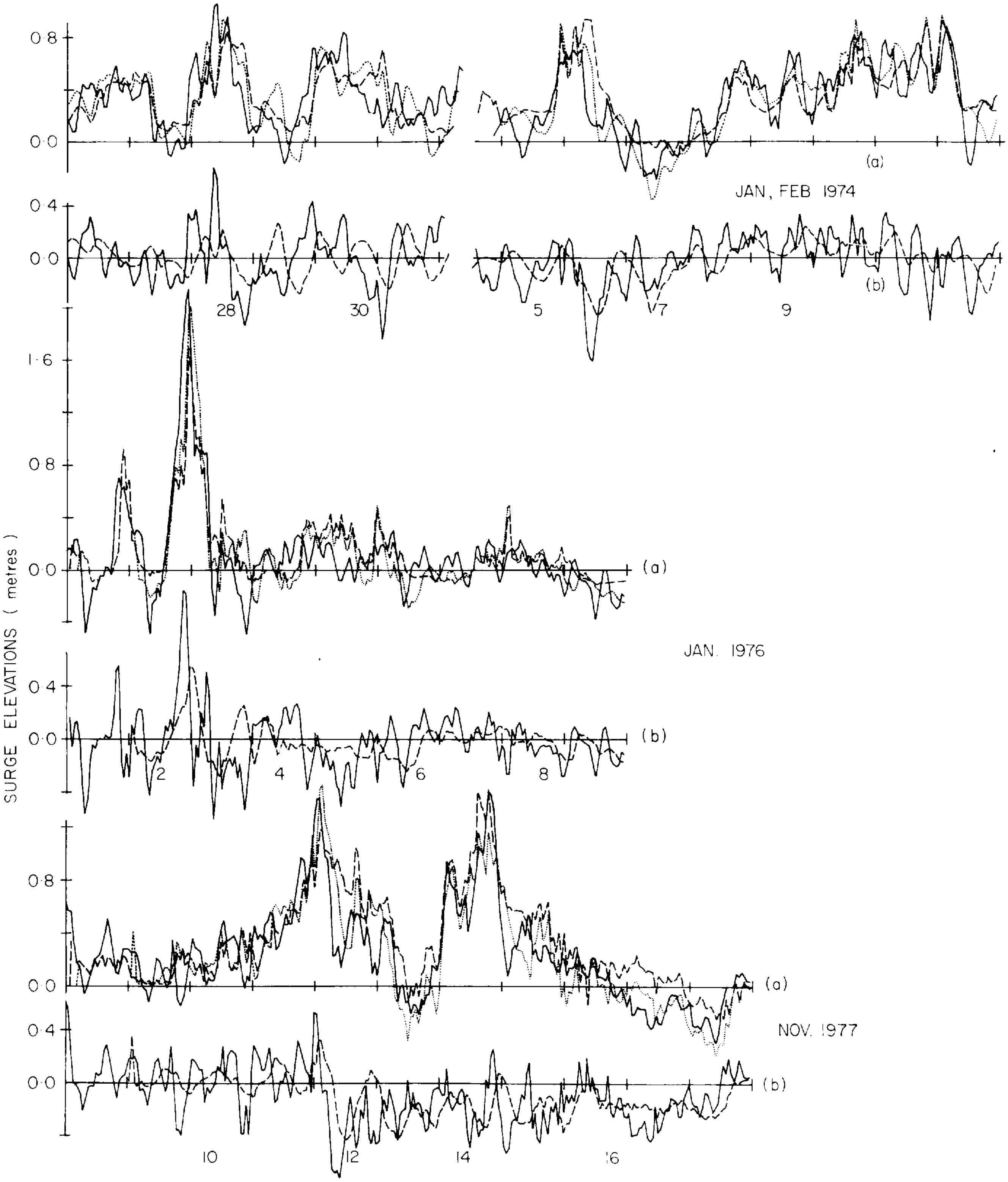

FIGURE 2 (cont.)

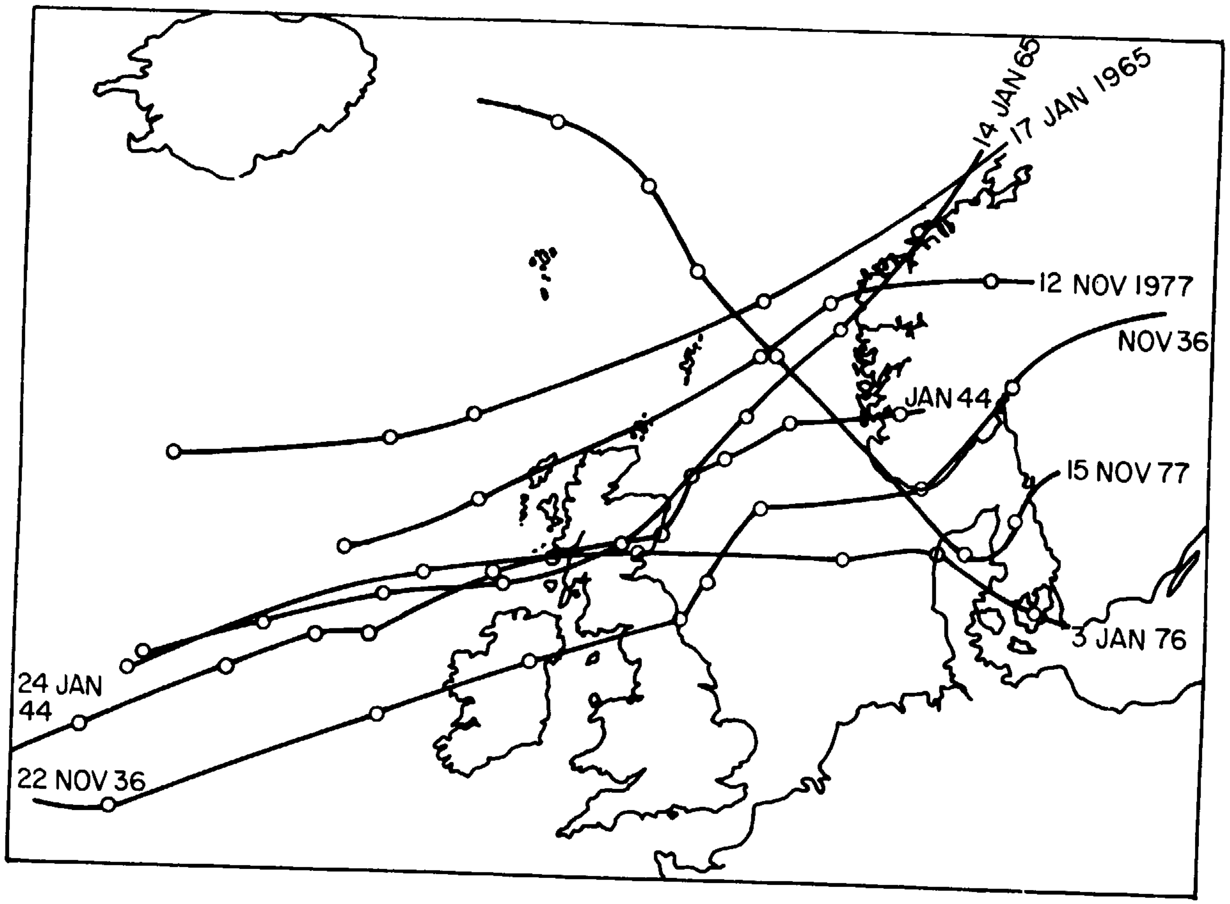
14 JAN 65
17 JAN 1965
12 NOV 1977
NOV 36
JAN 44
15 NOV 77
3 JAN 76
24 JAN
44
22 NOV 36

FIGURE 3

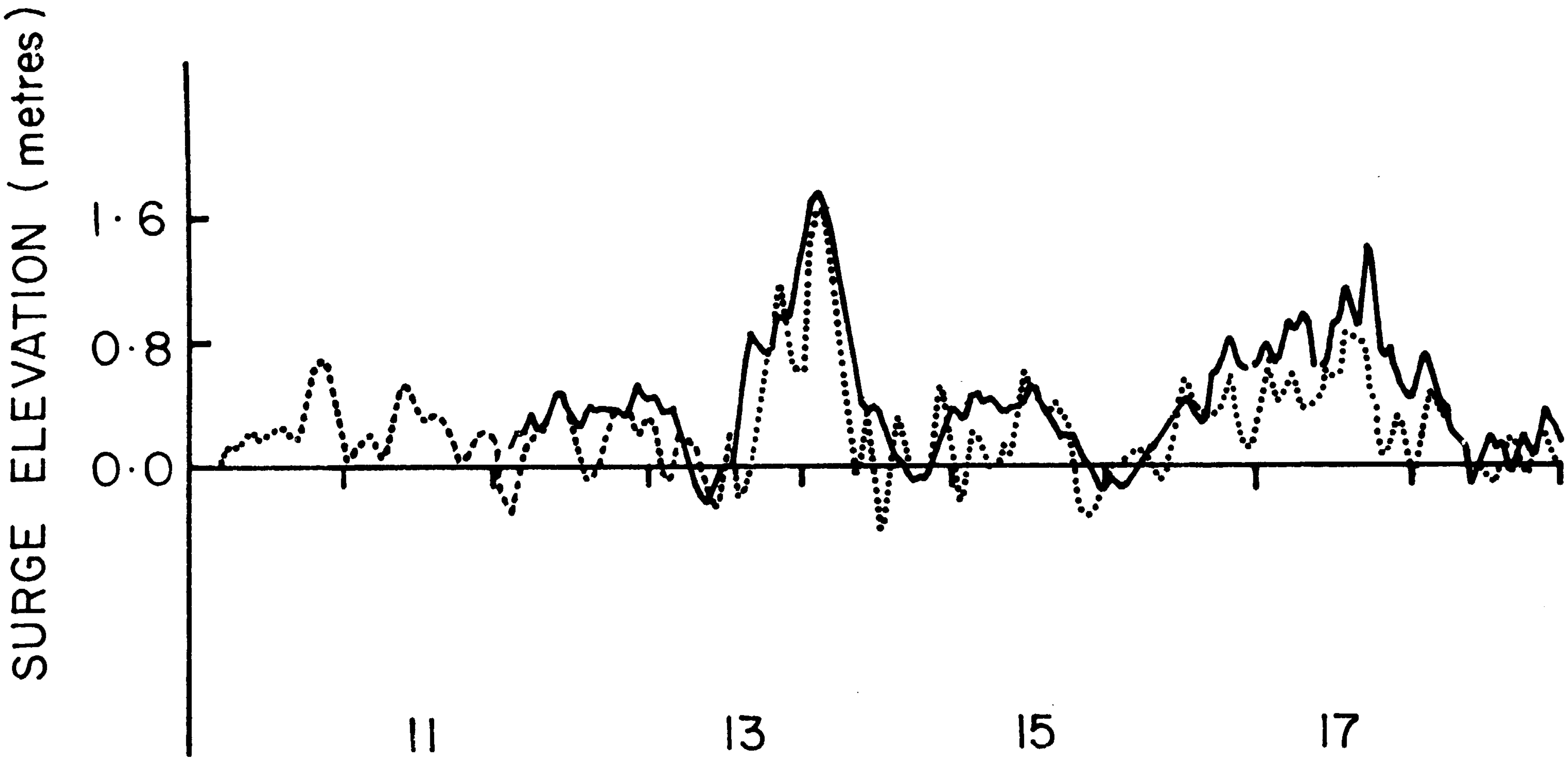

FIGURE 4