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Solving the SUSY flavour and CP problems with non-Abelian family symmetry and supergravity

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ABSTRACT

Can a theory of flavour capable of describing the spectrum of fermion (including neutrino) masses and mixings also contain within it the seeds for a solution of the SUSY flavour and CP problems? We argue that supergravity together with a non-Abelian family symmetry can completely resolve the SUSY flavour and CP problems in a broad class of theories in which family symmetry and CP is spontaneously broken in the flavon sector. We show that a simple superpotential structure can suppress the F-terms of the flavons and GUT scale Higgs fields and that, if this mechanism is implemented, the resulting flavour and CP violation is suppressed and comfortably within the experimental limits. For illustration, we study a specific model based on $SU(3)$ family symmetry, but similar models based on non-Abelian (continuous or discrete) family symmetry will lead to similar results.

1. Introduction

The origin of the flavour structure of the quark and lepton masses and mixing angles is one of the deepest mysteries left unanswered by the Standard Model (SM) and remains one of the main motivations to go beyond it. The introduction of Supersymmetry (SUSY), whilst providing plausible answers to other mysteries left unanswered by the SM (such as the stability and origin of the weak scale, the origin of dark matter, and the question of unification), does not address the origin of flavour. In fact the introduction of TeV scale SUSY gives rise to large flavour changing neutral currents (FCNCs) and electric dipole moments (EDMs), larger than those predicted by the SM, and potentially above the experimental limits [1]. There have been many attempts [2–11] to address these questions in frameworks featuring symmetries stretching over different generations of the SM matter, the so called family symmetries. Apart from providing some insight into the structure of the effective Yukawa sector of the SM, the soft SUSY breaking Lagrangian is typically further constrained, which can in general lead to an alleviation of the SUSY flavour and CP issues.

It has recently been demonstrated that $SU(3)$ family symmetry [12,13] can solve the flavour problem of the SM. In this approach the smallness of neutrino masses is due to the see-saw mechanism [14], and the large lepton mixing is due to the sequential dominance (SD) mechanism [15,16]. Indeed present neutrino oscillation data is consistent with approximate tri-bimaximal lepton mixing [17], and this can be readily achieved with constrained sequential dominance (CSD) [18,19]. In such family symmetry models a potential solution to the SUSY CP problem results if the origin of CP violation is due to the spontaneous breaking of the $SU(3)$ family symmetry via flavon vacuum expectation values (vevs), $\langle \Phi_i \rangle$ [20,21]. In this case CP violation originates in the flavour changing sector (where it is observed to be large) and CP violation in the flavour conserving sector is suppressed by powers of small mixing angles.

In a recent paper three of us [22] analysed this solution of the SUSY flavour and CP problems in a model with gauged $SU(3)$ family symmetry previously introduced to describe quark and lepton masses and mixings, and in particular to generate neutrino tri-bimaximal mixing via CSD. We performed a detailed bottom-up operator analysis of the soft SUSY breaking Lagrangian in terms of a spontaneously broken $SU(3)$ family symmetry, where the operator expansions are $SU(3)$-symmetric. We then made a careful estimate of the mass insertion parameters describing flavour changing and CP violation, keeping track explicitly of all the coefficients, including a careful treatment of canonical normalization effects. The results of this analysis showed that, while all the experimental constraints coming from

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flavour changing and CP violation may be evaded in such a framework, there remained a tension between theory and experiment for $\mu \to e\gamma$ and the EDMs [22]. The common origin of both of these sources of tension resides in the soft SUSY breaking trilinear couplings as was first pointed out for $SU(3)$ family symmetry models without in the framework of supergravity in [20,21]. This study was done in the context of global SUSY and has recently been complemented by another study in the context of supergravity (SUGRA) [23] of models related to those of [21]. Taking the commonly assumed value for the F-terms of the flavons, $F_\Phi \approx m_{3/2}\langle\Phi\rangle$, it was found that the experimental bounds on $\mu \to e\gamma$ and the EDMs exclude significant regions of parameter space and, in the allowed regions, are close to the current experimental limits for SUSY states light enough to be produced at the LHC.

In this Letter we extend these studies to SUGRA models with an underlying $SU(3)$ family symmetry spontaneously broken in a manner that generates tri-bimaximal mixing. With the commonly assumed value for the flavon F-terms we again find a tension with the experimental bounds for $\mu \to e\gamma$ and the EDMs. However we show that this tension can be removed via a simple mechanism for suppressing the flavon F-terms below their commonly assumed value. Though initially formulated in the context of models with $SU(3)$ family symmetry, the mechanism does not rely on the particular choice of the flavour group and can be generalized to other scenarios.

The Letter is organized as follows: In Section 2 we remind the reader about salient features of the class of models with $SU(3)$ family symmetry, first in the globally supersymmetric context and then in SUGRA focusing on the potential tension with respect to the experimental data on lepton flavour violation and EDMs. We recapitulate the simple estimate for the commonly assumed values of the visible sector SUGRA F-terms and argue that supergravity alone does not provide a relief to the strain without further model-building. In Section 3 we discuss a dynamical mechanism yielding a further suppression of the visible sector F-terms which is capable of restoring the full compatibility of the $SU(3)$ model with experimental constraints. In Section 4 we conclude. Some of the technical details are deferred to Appendices A and B.

2. *SU*(3) as an effective family symmetry

We focus on the class of $SU(3)$ flavour models discussed in [19,26] that describes the observed quark and lepton masses and mixings, and in particular generates neutrino tri-bimaximal mixing. These models require three specific types of flavon fields, each of which is an anti-triplet of the $SU(3)$ family symmetry, and each of which has a particular type of vacuum alignment,[1] namely: $\phi_3 \sim (0,0,1)$, $\phi_{23} \sim (0,-1,1)$, $\phi_{123} \sim (1,1,1)$, up to phases. In practice, the desired vacuum alignment must also ensure that $\phi_{23}^\dagger\phi_{123} = 0$, in accordance with the CSD requirements necessary to yield tri-bimaximal neutrino mixing [18,19]. The resulting form of the effective Yukawa superpotential is

$$\tilde{W}_Y = f_a f_b^c \frac{1}{M_f^2}\left\langle y_1^f(\phi_{123})_a(\phi_{23})_b + y_2^f(\phi_{23})_a(\phi_{123})_b + y_3^f(\phi_3)_a(\phi_3)_b + \frac{y_\Sigma^f}{M_f^\Sigma}(\phi_{23})_a(\phi_{23})_b\Sigma\right\rangle H, \tag{1}$$

where we have written the left-handed fermions as f_a and the CP-conjugated right-handed fermions (both family triplets) as f_b^c, where $f = u, d, e, \nu$ and $a, b = 1, 2, 3$ are the $SU(3)$ family symmetry indices. H is the Higgs doublet superfield and the field Σ is a field whose vev generates a relative factor of 3 between the muon and strange quark mass at the unification scale, implementing the Georgi–Jarlskog mechanism [24], and providing a phenomenologically appealing account of charged lepton and down quark masses. In addition the Σ vev suppresses the contribution of the last term to the neutrinos [25], allowing the remaining operators to give rise to tri-bimaximal neutrino mixing via the CSD mechanism. The messenger mass, M_f, and the magnitude of flavon vevs are chosen to generate the desired hierarchical form of the Dirac masses as discussed in detail in [19,26] to which we refer the reader for more details.

At the level of an effective theory (i.e. for energies well below $M_{\rm Pl}$ in Planck-scale mediated SUSY breakdown in SUGRA) one can parametrize the effective soft-SUSY breaking in terms of the coefficients of effective operator expansions. For later convenience in comparing with the experimental bounds we adopt the notation of [22]. Then the leading order contributions have the form:

$$\begin{aligned}
(\hat{m}^2_{f,f^c})_{\bar{a}b} &= m_0^2\left(b_0^{f,f^c}\delta_{\bar{a}b} + b_1^{f,f^c}\frac{\langle\phi_{123}^*\rangle_{\bar{a}}\langle\phi_{123}\rangle_b}{M_f^2} + b_2^{f,f^c}\frac{\langle\phi_{23}^*\rangle_{\bar{a}}\langle\phi_{23}\rangle_b}{M_f^2} + b_3^{f,f^c}\frac{\langle\phi_3^*\rangle_{\bar{a}}\langle\phi_3\rangle_b}{M_f^2} + b_4^{f,f^c}\delta_{\bar{a}b}\frac{\langle\Sigma^*\rangle\langle\Sigma\rangle}{M_\Sigma^2}\right),\\
\hat{A}^f_{ab} &= A_0\frac{1}{M_f^2}\left(a_1^f\langle\phi_{123}\rangle_a\langle\phi_{23}\rangle_b + a_2^f\langle\phi_{23}\rangle_a\langle\phi_{123}\rangle_b + a_3^f\langle\phi_3\rangle_a\langle\phi_3\rangle_b + \frac{a_\Sigma^f}{M_f^\Sigma}\langle\phi_{23}\rangle_a\langle\phi_{23}\rangle_b\langle\Sigma\rangle\right),
\end{aligned} \tag{2}$$

where, for sake of simplicity, we have taken the messenger mass to be the same as that in the expression for the Dirac mass matrices. At this point, the coefficients of these expansions are generic numbers governing the phenomenology analysis. However, once the SUSY breaking mechanism and messenger sector is specified, these parameters become calculable.

2.1. *SU*(3) *family symmetry in supergravity*

In addition to the fermion Dirac mass structure of Eq. (1) it is necessary to specify the general form of the visible sector piece of the Kähler potential. In leading order it is parameterized by the term $\tilde{K}^f_{\bar{a}b} f^{\dagger\bar{a}} f^b + \tilde{K}^{f^c}_{\bar{a}b} f^{c\dagger\bar{a}} f^{cb}$ where

$$\begin{aligned}
\tilde{K}^{f,f^c}_{\bar{a}b} &= \delta_{\bar{a}b}\left(k_0^{f,f^c} + l_0^{f,f^c}\frac{X^\dagger X}{M_{\rm Pl}^2}\right) + \frac{(\phi_{123}^*)_{\bar{a}}(\phi_{123})_b}{M_f^2}\left(k_1^{f,f^c} + l_1^{f,f^c}\frac{X^\dagger X}{M_{\rm Pl}^2}\right)\\
&\quad + \frac{(\phi_{23}^*)_{\bar{a}}(\phi_{23})_b}{M_f^2}\left(k_2^{f,f^c} + l_2^{f,f^c}\frac{X^\dagger X}{M_{\rm Pl}^2}\right) + \frac{(\phi_3^*)_{\bar{a}}(\phi_3)_b}{M_f^2}\left(k_3^{f,f^c} + l_3^{f,f^c}X^\dagger X\right) + \delta_{\bar{a}b}k_4^{f,f^c}\frac{\Sigma^\dagger\Sigma}{M_\Sigma^2}
\end{aligned} \tag{3}$$

[1] The desired vev pattern typically emerges from the minimization of a suitable $SU(3)$-invariant scalar potential. The full discussion of the relevant mechanism is, however, beyond the scope of this study and we defer an interested reader for further details to earlier works on $SU(3)$ flavour models like e.g. [12] and references therein.

and $k_i^{ff^c}$, l_I^{f,f^c} are constants and X denotes a hidden sector field driving the SUSY breakdown. In this we have again assumed[2] that the messenger mass is the same as that in the expression for the Dirac mass matrices.

In terms of Eqs. (3) and (1) the generic formulae for the leading order effective soft SUSY-breaking terms is

$$\hat{m}^2_{\bar{a}b} \approx \left\langle m^2_{3/2}\tilde{K}_{\bar{a}b} - F_{X^\dagger}(\partial_{X^\dagger}\partial_X\tilde{K}_{\bar{a}b})F_X - \sum_{\Phi_I,\Phi_J} F_{\Phi_I^*}\left(\partial_{\Phi_I^*}\partial_{\Phi_J}\tilde{K}^{f,f^c}_{\bar{a}b}\right)F_{\Phi_J} + \cdots\right\rangle \tag{4}$$

and

$$A_{abc} \propto \left\langle F_X\left(\partial_X \frac{K_{\mathrm{hid}}}{M^2_{\mathrm{Pl}}}\right)Y_{abc} + \sum_{\Phi} F_\Phi \partial_\Phi Y_{abc} - \left(F_X\left(\tilde{K}^{-1}\right)_{d\bar{e}}\partial_X\tilde{K}_{\bar{e}a}Y_{dbc} + \sum_{\Phi} F_\Phi\left(\tilde{K}^{-1}\right)_{d\bar{e}}\partial_\Phi\tilde{K}_{\bar{e}a}Y_{dbc} + cyclic(a,b,c)\right)\right\rangle, \tag{5}$$

where Φ stands for all the visible sector fields in the model (in particular the flavons ϕ_3, ϕ_{23}, ϕ_{123} and also Σ).

In the formulae above we have assumed, cf. Eq. (1), that due to the holomorphy of the superpotential the direct couplings of X to the Yukawa sector of the model are absent (which leads to the absence of the $F_X\partial_X Y_{abc}$ terms in Eq. (5)). This is the case for the specific family symmetry model discussed above. In any case, such contribution would necessarily be further (at least $\langle X\rangle/M_{\mathrm{Pl}} \ll 1$) suppressed with respect to the leading order terms emerging from the structure given in formula (1).

2.2. Commonly assumed value for SUGRA F-terms

It is obvious from Eqs. (4) and (5) that the values of the various F-terms provide a crucial ingredient of any detailed analysis of soft terms. In SUGRA the "natural" expectation for F-terms of visible sector superfields, Φ, whose scalar component acquires a vev is given by $F_\Phi \approx m_{3/2}\langle\Phi\rangle$, up to cancellations [20]. This stems from the fact that in Planck units the generic structure of SUGRA F-terms is given by

$$F_I = -e^{G/2}\left(G^{-1}\right)_{I\bar{J}}G_{\bar{J}} = -e^{G/2}\left(K^{-1}\right)_{I\bar{J}}G_{\bar{J}}, \tag{6}$$

where

$$G_{\bar{J}} \equiv \partial_{\bar{J}}(K + \log W^* + \log W) = (W^*)^{-1}(W^*K_{\bar{J}} + W^*_{\bar{J}})$$

and thus

$$F_I = -e^{K/2}\left(K^{-1}\right)_{I\bar{J}}(W^*K_{\bar{J}} + W^*_{\bar{J}}).$$

In the $M_{\mathrm{Pl}} \to \infty$ limit only the global SUSY term $F_I \propto -(K^{-1})_{I\bar{J}}W^*_{\bar{J}}$ survives. Plugging in the gravitino mass $m^2_{3/2} = e^{\langle K\rangle}\langle|W|^2\rangle$ one arrives at

$$F_I = -\left(K^{-1}\right)_{I\bar{J}}\left(m_{3/2}K_{\bar{J}} - e^{K/2}W^*_{\bar{J}}\right). \tag{7}$$

For the 'standard' Kähler potentials like $K \approx \sum_\Phi \Phi^\dagger\Phi$ the first term in (7) provides an "irreducible" contribution to the relevant F-term given by

$$\langle F_\Phi\rangle = m_{3/2}\langle\Phi\rangle. \tag{8}$$

Unless there are cancellations from the second ($W^*_{\bar{J}}$) term in Eq. (7) this provides a lower bound on the F-term. Taking this as a starting point we parameterize the F-terms of the visible sector superfields in the model by

$$F_{(\phi_A)_i} \equiv m_{3/2}x_A\langle\phi_A\rangle_i + \cdots, \qquad F_\Sigma \equiv m_{3/2}x_\Sigma\langle\Sigma\rangle + \cdots \tag{9}$$

and give all our leading order results in terms of the x_A and x_Σ factors (note that the gauge and family symmetries of the model ensure that at the leading order the F-terms are diagonal in the field space).

2.3. The soft SUSY-breaking terms

Using Eq. (9) in Eq. (4) the soft masses are given by

$$\begin{aligned}\left(\hat{m}^2_{f,f^c}\right)_{\bar{a}b} = m^2_{3/2}\Bigg[\delta_{\bar{a}b}\left(k_0^{f,f^c} + l_0^{f,f^c}\frac{\langle X^\dagger X\rangle}{M^2_{\mathrm{Pl}}} - l_0^{f,f^c}\frac{F_{X^\dagger}F_X}{m^2_{3/2}M^2_{\mathrm{Pl}}}\right)\\ + \sum_A \frac{\langle\phi_A^*\rangle_{\bar{a}}\langle\phi_A\rangle_b}{M_f^2}\left(k_A^{f,f^c}(1 - x_Ax_A^*) - l_A^{f,f^c}\frac{F_{X^\dagger}F_X}{m^2_{3/2}M^2_{\mathrm{Pl}}}\right) - k_4^{f,f^c}\delta_{\bar{a}b}\frac{\langle\Sigma^*\rangle\langle\Sigma\rangle}{M^2_\Sigma}x_\Sigma x_\Sigma\Bigg].\end{aligned} \tag{10}$$

The relevant dictionary between the SUGRA setting and the operator coefficients relevant for the effective analysis reads: $m_0 = m_{3/2}$,

$$b_0^{f,f^c} = k_0^{f,f^c} + l_0^{f,f^c}\delta_{\bar{a}b}\left(\frac{\langle X^\dagger X\rangle}{M^2_{\mathrm{Pl}}} - \frac{F_{X^\dagger}F_X}{m^2_{3/2}M^2_{\mathrm{Pl}}}\right), \qquad b_A^{f,f^c} = k_A^{f,f^c}(1 - x_Ax_A^*) - l_A^{f,f^c}\frac{F_{X^\dagger}F_X}{M^2_{\mathrm{Pl}}}. \tag{11}$$

Turning to the trilinear terms, as we now discuss, the dominant term is the second one in Eq. (5). Consider first the flavour violating terms. Since the first term in Eq. (5) is proportional to the relevant Yukawa matrix it does not contribute to flavour violation. Next, due to

[2] Let us point out that the assumption of 'universality' of messenger masses in Eqs. (1), (2) and (3) has been made only for simplicity reasons and (up to numerical details) does not alter the visible sector F-term suppression mechanism which is the merit of this study.

the $\partial_\phi \tilde{K}_{\bar{a}b}$ factor, the terms proportional to F_Φ in the square bracket in Eq. (5) are at least two powers of $\langle\phi\rangle/M^f$ more suppressed than the second term. Concerning the terms coming from the first and last terms of Eq. (3), they are proportional to the Yukawa matrix and hence are also flavour conserving. Finally the terms proportional to F_X coming from the second, third and fourth terms of Eq. (3) are at least two powers of $\langle\phi\rangle/M^f$ more suppressed than the leading terms coming from the second term of Eq. (5). Thus, the $\langle F_\Phi \partial_\Phi Y_{abc}\rangle$ terms govern the SUSY flavour-violation in the trilinear couplings. On the CP side we shall focus on the (flavour conserving) EDMs that provide the most stringent bounds on the CP phases in the trilinear sector. Recall that the relevant quantities are the (1, 1)-entries of the trilinear couplings in the super-CKM basis. As shown in [20] the imaginary parts on the diagonal of the SCKM-basis trilinears from the terms in the square bracket in Eq. (5) are suppressed. Assuming the hidden sector fields X do not couple to the Yukawa sector and the F_Φ-terms have their commonly assumed values, the leading contributions to the SUSY EDMs also come predominantly from the $F_\Phi \partial_\Phi Y_{abc}$ terms in Eq. (5).

From Eqs. (1) and (9) we have

$$\begin{aligned}\langle F_\Phi \partial_\Phi Y^f\rangle_{ab} &\approx \left\langle \left[\sum_{A,c} F_{(\phi_A)_c}\partial_{(\phi_A)_c} + F_\Sigma \partial_\Sigma\right]\hat{Y}^f_{ab}\right\rangle \\ &= \frac{m_{3/2}}{M_f^2}\left\langle \left[y_1^f(\phi_{123})_a(\phi_{23})_b + y_2^f(\phi_{23})_a(\phi_{123})_b\right](x_{23}+x_{123}) + 2y_3^f(\phi_3)_a(\phi_3)_b x_3 + \frac{y_\Sigma^f}{M_f^\Sigma}(\phi_{23})_a(\phi_{23})_b\Sigma(2x_{23}+x_\Sigma)\right\rangle, \end{aligned} \tag{12}$$

which, cf. Eq. (5), gives:

$$a_1^f = y_1^f(x_{123}+x_{23}), \qquad a_2^f = y_2^f(x_{123}+x_{23}), \qquad a_3^f = y_2^f(2x_3), \qquad a_\Sigma^f = y_\Sigma^f(2x_{23}+x_\Sigma), \qquad A_0 = m_{3/2}. \tag{13}$$

2.4. Phenomenology of the SUGRA SU(3) model

2.4.1. Lepton flavour violation ($\mu \to e\gamma$)

It is straightforward now to use the analysis of [22] to determine the phenomenological implications of the $SU(3)$ model. Using the coefficients just determined, the relevant mass insertion parameter, δ, governing the branching ratio of $\mu \to e\gamma$ gives

$$\left|(\delta^\ell_{LR})_{12}\right| \approx 1\times 10^{-4}\frac{A_0}{100\ \text{GeV}}\frac{(200\ \text{GeV})^2}{\langle\tilde{m}_l\rangle^2_{LR}}\frac{10}{\tan\beta}\left(\frac{\bar{\varepsilon}}{0.13}\right)^3 |y_1||x_{123}-x_{23}-x_\Sigma|. \tag{14}$$

2.4.2. Electric dipole moments

Similarly, the mass insertion parameters determining the EDMs are

$$\begin{aligned}\left|\mathrm{Im}\left(\delta^u_{LR}\right)_{11}\right| &\approx 2\times 10^{-7}\frac{A_0}{100\ \text{GeV}}\left(\frac{500\ \text{GeV}}{\langle\tilde{m}_u\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^3\left(\frac{\varepsilon}{0.05}\right)^2\left|y_1^f+y_2^f\right||x_{123}-x_{23}-x_\Sigma|\sin\phi_1, \\ \left|\mathrm{Im}\left(\delta^d_{LR}\right)_{11}\right| &\approx 5\times 10^{-7}\frac{A_0}{100\ \text{GeV}}\left(\frac{500\ \text{GeV}}{\langle\tilde{m}_d\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^5\frac{10}{\tan\beta}\left|y_1^f+y_2^f\right||x_{123}-x_{23}-x_\Sigma|\sin\phi_1, \\ \left|\mathrm{Im}\left(\delta^\ell_{LR}\right)_{11}\right| &\approx 2\times 10^{-7}\frac{A_0}{100\ \text{GeV}}\left(\frac{200\ \text{GeV}}{\langle\tilde{m}_e\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^5\frac{10}{\tan\beta}\left|y_1^f+y_2^f\right||x_{123}-x_{23}-x_\Sigma|\sin\phi_1, \end{aligned} \tag{15}$$

where ϕ_1 is a CP phase associated to the vev of the ϕ_{123} flavon and using Eq. (13). We have chosen to normalize the expansion parameters ε and $\bar{\varepsilon}$ to the values found in a recent fit to the measured masses and mixing angles [27].

Thus, both $\mu \to e\gamma$ and the EDMs are determined by a single combination, Δ, of the x-factors which parametrize the structure of the relevant visible sector F-terms, where

$$\Delta \equiv |x_{123} - x_{23} - x_\Sigma|. \tag{16}$$

The present experimental bound from the non-observation of $\mu \to e\gamma$ is $|(\delta^\ell_{LR})_{12}| \leqslant 10^{-5}$ which is in some tension with this bound requiring, for example, $\tilde{m}_l = 600$ GeV if the remaining factors in Eq. (14) are of $O(1)$. For the EDMs the most stringent bound comes from mercury and corresponds to $|\mathrm{Im}(\delta^d_{LR})_{11}| < 6.7\times 10^{-8}$ and requires $\tilde{m}_d = 1500$ GeV if the other factors are of $O(1)$. This means that SUGRA does not automatically provide a relief from the flavour and CP issues of the $SU(3)$ model under consideration as compared to the effective operator analysis [22], cf. also [28].

3. Suppressing EDMs and $\mu \to e\gamma$

Given this tension it is appropriate to review the possibilities for reducing Δ. The “natural” expectation is that $x_i \simeq 1$, corresponding to the case $\langle\partial W/\partial\phi_i\rangle \simeq 0$. Although Δ does not vanish in this case it is relatively easy to modify the model to arrange for it to do so. This requires that each term in the mass matrix should involve the same number of flavon and Σ fields. A simple illustration of this mechanism is given in Appendix A. Although this mechanism does work it represents an unwanted complication of the model so here we concentrate on a more promising possibility. It turns out that it is relatively easy to modify the model to arrange for a non-zero $\langle\partial W/\partial\phi_i\rangle$ to obtain a cancellation in the relevant F-terms giving $x_i \ll 0$.

3.1. Dynamical suppression of F-terms in SUGRA

We begin by studying a simple class of models provided the following conditions apply:

1. The superpotential of the world can be written as $W = W_{\text{obs}} + W_{\text{hid}}$ where the superpotential of the observable sector may be written as $W_{\text{obs}} = Z(\Psi\bar{\Psi} - M_\Psi^2)$ where Z and Ψ are a visible pair of superfields[3] and the mass scale is below the Planck mass $M_\Psi < M_P$ (notice that this form of W_{obs} is the one which is typically used in the globally-supersymmetric flavour models for sake of arranging the desired patterns of flavon vevs);
2. The relevant Kähler potentials are all of the canonical form, i.e. $\tilde{K}^\Psi = \Psi^\dagger\Psi$, $\tilde{K}^Z = Z^\dagger Z$ at leading order. Since the relevant (flavon and Higgs) vevs are all below the Planck scale this is a reasonable assumption as the canonical form is the leading term allowed in the Kähler potential in a power series expansion of the superfields, cf. formula (3).
3. Negligible D-terms. In fact this proves to be the case for a variety of models. Although our discussion has concentrated on the case of a continuous family symmetry, it applies equally to the case that the structure of the superpotential and Kähler potential is driven by a discrete non-Abelian subgroup of $SU(3)$ [26]. In this case there are no D-terms. Even in the case of a continuous $SU(3)$ the vacuum structure just discussed is not disturbed by D-terms as we demonstrate in Appendix B. Of course one must also check that the D-terms do not introduce unacceptable FCNCs or CP violation; that this is the case is discussed in [29].

Under the above assumptions the scalar potential is given by

$$V = \left(|F_{\text{obs}}|^2 + |F_{\text{hid}}|^2 - 3e^K|W|^2\right), \tag{17}$$

where

$$|F_{\text{obs}}|^2 = |F_\Psi|^2 + |F_{\bar{\Psi}}|^2 + |F_Z|^2 \tag{18}$$

and the individual observable sector F-terms may be written as

$$F_\Psi \approx Z\Psi + m_{3/2}\bar{\Psi}^*, \qquad F_{\bar{\Psi}} \approx Z\bar{\Psi} + m_{3/2}\Psi^*, \qquad F_Z \approx \Psi\bar{\Psi} - M_\Psi^2 + m_{3/2}Z^*, \tag{19}$$

where we have exploited the canonical form of the Kähler potential and used[4]

$$\langle |W_{\text{hid}}| \rangle \approx \langle |W| \rangle \approx m_{3/2}, \tag{20}$$

sticking to the leading contribution from the exponential. We now argue that the potential is minimized for values of the visible sector fields $Z, \Psi, \bar{\Psi}$ such that $F_\Psi \ll m_{3/2}\langle\Psi\rangle$ and $F_{\bar{\Psi}} \ll m_{3/2}\langle\bar{\Psi}\rangle$. It is important to emphasize that we are seeking a minimum of the potential in terms of the observable fields $\Psi, \bar{\Psi}, Z$ and so we may expand the potential as follows:

$$V(\Psi, \bar{\Psi}, Z) \approx |F_{\text{obs}}|^2 - 3|W_{\text{obs}}|^2 - 6\,\text{Re}[W_{\text{hid}}W_{\text{obs}}] + C, \tag{21}$$

where only the leading order contribution of the exponential in Eq. (17) has been retained and C is a constant term driven by the hidden sector dynamics to account for a zero (or negligible) cosmological constant. A necessary condition for a minimum of the potential is that the first derivatives vanish $\partial V/\partial Z = 0$, $\partial V/\partial\Psi = 0$, $\partial V/\partial\bar{\Psi} = 0$. By explicit calculation it can readily be seen that the first derivatives vanish for[5]:

$$\langle Z\rangle = -m_{3/2} + O\left(m_{3/2}^3/M_\Psi^2\right), \tag{22}$$

$$\left|\langle\Psi\rangle\right| = \left|\langle\bar{\Psi}\rangle\right| \quad \text{and} \quad \left|\langle\Psi\rangle\langle\bar{\Psi}\rangle\right| = M_\Psi^2 + O(m_{3/2})^2, \tag{23}$$

with anti-aligned phases on the components of $\langle\Psi\rangle$ and $\langle\bar{\Psi}\rangle$ (up to a possible global phase difference due to a would-be non-zero phase of $\langle Z\rangle$). Inserting these vevs into the F-terms in Eq. (19) it can be seen that

$$|F_\Psi| = |F_{\bar{\Psi}}| = O\left(m_{3/2}^2/M_\Psi^2\right) \times m_{3/2}M_\Psi, \tag{24}$$

which is of the form $F_\Psi = x_\Psi m_{3/2}\langle\Psi\rangle$ with a suppression factor of $x_\Psi = O(m_{3/2}^2/M_\Psi^2)$. Note that $\langle F_Z\rangle$ remains at its commonly assumed value $m_{3/2}\langle Z\rangle$. Moreover, at the minimum corresponding to the field values in Eqs. (22)–(23) we have $W_{\text{obs}} = O(m_{3/2}^3)$ which justifies Eq. (20) a posteriori.

It is straightforward to check that the configuration of Eqs. (22)–(23) corresponds to a (local) minimum by moving away slightly from the minimum. In this case the variation is dominated by the (non-Planck-suppressed) first term in Eq. (17) and clearly increases away from the turning point.

3.2. EDMs and $\mu \to e\gamma$ in SUGRA $SU(3)$ with dynamically suppressed flavon and Σ-field F-terms

The previous section shows how F-terms can be suppressed below their commonly assumed values. In Appendix B we discuss how this can apply to one or more of the flavon fields and to the Σ field. How can this suppression affect $\mu \to e\gamma$ and the EDMs? A particularly simple case is when the Σ field has a suppressed F-term while the flavons have their commonly assumed values. This makes Δ small due to the cancellation between the first two terms in Eq. (16). However, in this case, the cancellation is spoilt by the next-to-leading contributions in Eq. (1) which introduce different powers of the flavon fields in different matrix elements of the Yukawa matrix. As a result our previous estimates are reduced by an extra power of ε in $|\text{Im}(\delta^u_{LR})_{11}|$ and of $\bar{\varepsilon}$ in $|\text{Im}(\delta^d_{LR})_{11}|$ and $|\text{Im}(\delta^l_{LR})_{11}|$ respectively (for further details cf. [22]) to get:

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[3] Here Ψ is typically playing the role of the visible sector superfield whose F-term we wish to suppress while Z is sometimes called the 'driving field' because its F-term prompts the scalar component of Ψ to acquire a non-zero vev.

[4] Note that this result follows from the assumed forms $W = W_{\text{obs}} + W_{\text{hid}}$ where $W_{\text{obs}} = Z(\Psi\bar{\Psi} - M_\Psi^2)$ which implies that at the minimum of the potential $W_{\text{obs}} \ll W_{\text{hid}}$ which is plausible, given the assumed form of W_{obs}, but which can be checked a posteriori.

[5] Here we implicitly used Eq. (20) to give the result in a simple form. An analytic minimization of the potential of Eq. (21) yields: $|\langle\Psi\rangle\langle\bar{\Psi}\rangle| = M_\Psi^2 + m_{3/2}^2 - 3m_{3/2}\langle W_{\text{hid}}\rangle$ and $\langle Z\rangle = -m_{3/2} + 3m_{3/2}^2\langle W_{\text{hid}}\rangle/M_\Psi^2 - 9m_{3/2}\langle W_{\text{hid}}\rangle^2/2M_\Psi^2$ up to higher order terms and an overall phase (due to the phase difference between $\langle\Psi\rangle$ and $\langle\bar{\Psi}\rangle$) in $\langle Z\rangle$.

$$\left|\mathrm{Im}\left(\delta^u_{LR}\right)_{11}\right| \approx 1\times 10^{-8}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{500\ \mathrm{GeV}}{\langle\tilde m_u\rangle_{LR}}\right)^2\left(\frac{\bar\varepsilon}{0.13}\right)^3\left(\frac{\varepsilon}{0.05}\right)^3\sin\phi_1,$$
$$\left|\mathrm{Im}\left(\delta^d_{LR}\right)_{11}\right| \approx 6\times 10^{-8}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{500\ \mathrm{GeV}}{\langle\tilde m_d\rangle_{LR}}\right)^2\left(\frac{\bar\varepsilon}{0.13}\right)^6\frac{10}{\tan\beta}\sin\phi_1,$$
$$\left|\mathrm{Im}\left(\delta^\ell_{LR}\right)_{11}\right| \approx 3\times 10^{-8}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{200\ \mathrm{GeV}}{\langle\tilde m_e\rangle_{LR}}\right)^2\left(\frac{\bar\varepsilon}{0.13}\right)^6\frac{10}{\tan\beta}\sin\phi_1. \tag{25}$$

As before, we are assuming $m_0(M_{\mathrm{GUT}}) \approx 100$ GeV and take into account the RG effects. These numbers are well below the current experimental limits for all the elementary particle EDMs and compatible with those on mercury EDM.

Similarly, for $\mu \to e\gamma$ one gets (multiplying the global SUSY estimate of Eq. (14) by an extra $\bar\varepsilon$):

$$\left|\left(\delta^e_{LR}\right)_{12}\right| \lesssim \left|\left(\delta^\ell_{LR}\right)_{12}\right| \approx 1\times 10^{-5}\frac{A_0}{100\ \mathrm{GeV}}\frac{(200\ \mathrm{GeV})^2}{\langle\tilde m_l\rangle^2_{LR}}\frac{10}{\tan\beta}\left(\frac{\bar\varepsilon}{0.13}\right)^4, \tag{26}$$

which is also compatible with the current bounds, in particular for large $\tan\beta$.

The second possibility is that the flavons also have suppressed F-terms corresponding to a suppression of Δ compared to its commonly assumed value is the factor $O(m^2_{3/2}/M^2_\Psi)$ of Eq. (24). In this case the dominant contribution will be the term proportional to F_X in the last term in brackets in Eq. (5), suppressed by two powers of $\langle\phi\rangle/M^f$, corresponding to a further suppression by the factor ε in $|\mathrm{Im}(\delta^u_{LR})_{11}|$ and of $\bar\varepsilon$ in $|\mathrm{Im}(\delta^d_{LR})_{11}|$ and $|\mathrm{Im}(\delta^l_{LR})_{11}|$ respectively taking the prediction well below the experimental limit even for the EDM of mercury.

4. Conclusions

In this Letter we have analysed the expectation for flavour changing neutral currents and CP violating electric dipole moments in a class of supergravity models with a non-Abelian family symmetry. With the commonly assumed values for the F-terms of the flavon and Georgi–Jarlskog fields there is tension between the experimental limits and the predicted values that requires rather large SUSY particle masses. However we have identified a simple mechanism for suppressing the F-terms and the resulting soft SUSY breaking trilinear couplings. As a result the expectation for lepton flavour violating processes and EDMs in these classes of models may be suppressed to values comfortably within current limits. We emphasize again that the suppression mechanism presented here is applicable to a very large class of models based on non-Abelian (discrete or continuous) family symmetry and SUGRA, which henceforth should be regarded as viable candidates for solving the SUSY flavour and CP problems. On the other hand even with the maximum suppression lepton number violating processes and EDMs are within a factor of 10 of present limits so future measurements capable of improving on the present bounds are extremely important.

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Appendix A. Zero global SUSY *F*-terms and dummy fields

As we mentioned in the body of the text, a relatively simple solution to the slight tension in the SUSY CP sector of the $SU(3)$ model under consideration consists in arranging all the "global" SUSY F-terms in the visible sector (W^*_j in Eq. (7)) to vanish. This can be the case if for instance we put $W_\phi = 0$ by hand and (apart from demanding that Σ develops its GUT-scale breaking vev in a SUSY-flat direction) assume the family symmetry breakdown is triggered by D-terms, see e.g. [26]. In such a case, we get $x_A \approx 1$ for all the flavons and also $x_\Sigma \approx 1$. If, on top of that, we employ a "dummy" Σ_0 field to balance the powers of the first two operators in W_Y so that they are also dimension 7 as the last one with Σ, i.e.:

$$\hat Y^f_{ab} = \frac{1}{M^2_f}\left[\frac{y^f_1}{M^\Sigma_f}(\phi_{123})_a(\phi_{23})_b\Sigma_0 + \frac{y^f_2}{M^\Sigma_f}(\phi_{23})_a(\phi_{123})_b\Sigma_0 + y^f_3(\phi_3)_a(\phi_3)_b + \frac{y^f_\Sigma}{M^\Sigma_f}(\phi_{23})_a(\phi_{23})_b\Sigma\right],$$

which, by the way, leads to the modified form of the trilinear dictionary:

$$a^f_1 = y^f_1(x_{123}+x_{23}+x_{\Sigma_0}), \qquad a^f_\Sigma = y^f_\Sigma(2x_{23}+x_\Sigma),$$
$$a^f_2 = y^f_2(x_{123}+x_{23}+x_{\Sigma_0}), \qquad A_0 = m_{3/2}.$$

The critical bracket in Eq. (16) changes to:

$$|x_{123} - x_{23} + x_{\Sigma^0} - x_\Sigma| \tag{A.1}$$

and $x_{123,23} \approx x_{\Sigma,\Sigma^0} \approx 1$ again provides the desired suppression in $\mu \to e\gamma$ and also in the relevant EDMs.

Appendix B. Dynamical suppression of flavon *F*-terms and *D*-term cancellation

It is straightforward to arrange a dynamical suppression of x_A and/or x_Σ in the $SU(3)$ model under consideration. To suppress all consider the superpotential of the form

$$W_{\mathrm{vis}} \ni \lambda_\Sigma Z_\Sigma\left(\Sigma^2 - M^2_\Sigma\right) + \sum_\Phi \lambda_\Phi Z_\Phi\left(\Phi\bar\Phi - M^2_\Phi\right) + \cdots. \tag{B.1}$$

The discussion given in Section 3.1 applies with the identification $\Psi = \bar{\Psi} \equiv \Sigma$ for the Georgi–Jarlskog field Σ (living in the adjoint of the underlying Pati–Salam group) and similarly to the choice of $\Psi \equiv \Phi$ and $\bar{\Psi} \equiv \bar{\Phi}$ for each of the flavons of the $SU(3)$ model under consideration.

In a specific model like this one can easily inspect the effects of the D-terms that we have just touched upon in the preceding parts. It is clear from Eq. (B.1) that in order for Σ to admit for such a quadratic term in the superpotential it must belong to a real representation of the underlying Grand Unified group and thus its vevs must be real (up to perhaps an irrelevant overall phase). The antisymmetry of the generators in the real and unitary representation together with the canonicity of the relevant Kähler metric then ensure vanishing of the corresponding D-term associated to Σ. The addition of the second term in Eq. (B.1) for each of the flavon species Φ in the model not only does not disturb the F_Σ-suppression mechanism described above but leads to the same suppression mechanism for each of the flavon F-terms.

Let us remark that the symbol $\bar{\Phi}$ in Eq. (B.1) denotes an additional conjugate flavon field, so that $\Phi\bar{\Phi}$ is a singlet under all the symmetries of the model. Note that such extra flavons are usually needed anyway in order to cancel the unwanted D-terms potentially arising at the $SU(3)$ family symmetry breaking scale. At the level of the effective $SU(3)$ SUSY model [22] this is usually ensured by aligning manually the phases of vevs of Φ and $\bar{\Phi}$ against each other. This follows immediately since the F-term has the form given in Eq. (19) for each of the $SU(3)$ components and is minimized for $\langle\Phi\rangle_a = \langle\bar{\Phi}\rangle_a$. This, in turn, ensures that the contribution of the flavon fields to the $SU(3)$ D-terms vanishes (yielding $D_a^\Phi = \Phi^\dagger T_a \Phi$ and $D_a^{\bar{\Phi}} = \bar{\Phi}^\dagger \bar{T}_a \bar{\Phi} = -\bar{\Phi}^\dagger T_a^* \bar{\Phi}$).

To conclude, the dynamics of the system under consideration (cf. Eq. (B.1)) leads to a natural suppression of both Σ and flavon F-terms and thus all the x-factors entering Eq. (16) can be made small, as desired. By restricting the superpotential to have a subset of the terms given in Eq. (B.1) it is straightforward to suppress only a subset of the F-terms.

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