

**ON THE TIDAL SPECTRAL ANALYSIS  
OF 30 DAY RECORDS**

**Y. ANLE**

**(Shandong College of Oceanology, the  
People's Republic of China, on visit  
to I.O.S. Bidston, May - October 1980)**

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## Summary

Some new approaches to the analysis of 30-day tidal records are given, based on smoothed Fourier spectral estimates of both data and potential. The separation of close constituents and the parameterization of non-linear terms are also discussed in relation to results from actual data. The harmonic constants for Southend from the analysis of 30-day records are compared with Amin's results from 18 years of records.

### 1. Introduction

There are a growing number of short term (for instance, 30 days) tidal records which need to be analysed. Especially, more tidal current records are being obtained from auto-recording instruments, because more people have been paying attention to research on tidal currents.

For many years, short term records have been analysed with classical harmonic methods. But those methods only provide one with information about certain harmonic constituents. In fact, there are different shallow water constituents in different areas. These shallow water constituents are also an important characterization of locality.

The following method can be used to obtain not only those harmonics provided by the classical method, but also some of those which are important in particular areas, because the use of many spectral bands allows one to choose which constituents are important.

## 2. 30-day tidal spectra

The spectrum of tidal level  $Y(t)$  can be estimated from a 30-day sample, as follows:-

$$Z_n(p_k) = n^{-1} \Delta t \sum_{j=-n/2}^{n/2} Y(j \Delta t) \exp(2\pi i j \Delta t p_k/n)$$

$$\doteq \sum_{p_n=-n/2}^{n/2} Z(p_n) \frac{\sin \pi (p_k - p_n)}{\pi (p_k - p_n)} \Delta p_n \quad (1)$$

Where  $n = 24 * 30 = 720$ ,  $\Delta t = 1$  hour

Formula (1) is a weighted sum of spectra  $Z(p_k)$  over the whole frequency range. In order to reduce the side-band effect, a function,  $1 + 0.84 \cos(2\pi j \Delta t/n)$ , was applied to (1), we obtain

$$Z_n(p_k) = n^{-1} \Delta t \sum_{j=-n/2}^{n/2} Y(j \Delta t) (1 + 0.84 \cos(2\pi j \Delta t/n)) \exp(2\pi i j \Delta t p_k/n)$$

$$\doteq \sum_{p_n=-n/2}^{n/2} Z(p_n) \frac{\sin \pi (p_k - p_n)}{\pi (p_k - p_n)} \Delta p_n$$

$$+ 0.42 \sum_{p_n=-n/2}^{n/2} Z(p_n) \frac{\sin \pi (p_k - 1 - p_n)}{\pi (p_k - 1 - p_n)} \Delta p_n$$

$$+ 0.42 \sum_{p_n=-n/2}^{n/2} Z(p_n) \frac{\sin \pi (p_k + 1 - p_n)}{\pi (p_k + 1 - p_n)} \Delta p_n \quad (2)$$

In view of Fig. 1, the reduction of the side-band effect is better than if the more usual  $1 + \cos(2\pi j \Delta t/n)$  were used.



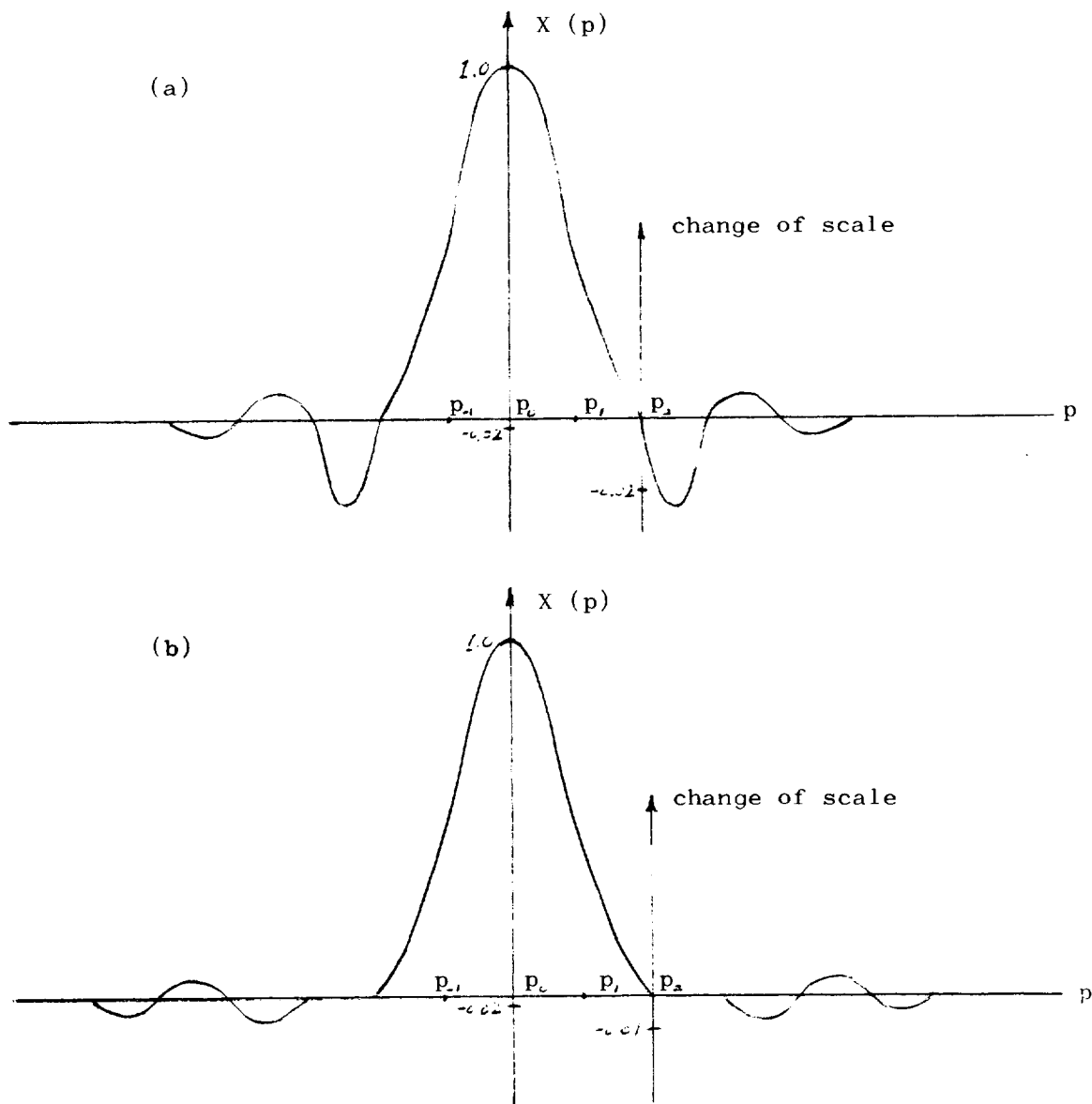


Fig. 1 (a) Spectra of  $(1+\cos(2\pi j \Delta t/n))$   
 (b) Spectra of  $(1+0.84 \cos(2\pi j \Delta t/n))$

In spite of this, formula (2) is still a weighted sum of three spectra

$$Z_n(p_k) = Z(p_k) + 0.42 (Z(p_{k-1}) + Z(p_{k+1})) \quad (3)$$

How to separate them is a matter of concern, because in our case it is possible that the spectra of three principal constituents are overlapped. For instance, when the spectrum of  $M_2$  is expected, what has been obtained is only a weighted sum of the spectra of  $M_2$ ,  $N_2$  and  $L_2$  (not to mention  $\nu_2$  and  $\lambda_2$ ).

For this purpose, two sets of functions are introduced as follows

$$\left. \begin{aligned} ZC(P_{2k-1}) &= Z_n(P_{2k-1}) - 0.42 (Z_n(P_{2k-2}) + Z_n(P_{2k})) \\ ZC(P_{2k}) &= Z_n(P_{2k}) - 0.42 (Z_n(P_{2k-1}) + Z_n(P_{2k+1})) \end{aligned} \right\} \quad (4)$$

$$k = 1, 2, 3, \dots$$

When (3) is inserted into (4), we have two sets of simultaneous equations as follows

$$\left. \begin{aligned} ZC(P_1) &= A * Z(P_1) - B * (Z(P_{-1}) + Z(P_3)) & (a) \\ ZC(P_3) &= A * Z(P_3) - B * (Z(P_1) + Z(P_5)) & (b) \\ ZC(P_5) &= A * Z(P_5) - B * (Z(P_3) + Z(P_7)) & (c) \\ &\vdots \\ &\vdots \\ ZC(P_{2k-3}) &= A * Z(P_{2k-3}) - B * (Z(P_{2k-5}) + Z(P_{2k-1})) & (i) \\ ZC(P_{2k-1}) &= A * Z(P_{2k-1}) - B * (Z(P_{2k-3}) + Z(P_{2k+1})) & (j) \end{aligned} \right\} \quad (5)$$

and

$$\left. \begin{aligned} ZC(P_2) &= A * Z(P_2) - B * (Z(P_0) + Z(P_4)) \\ ZC(P_4) &= A * Z(P_4) - B * (Z(P_2) + Z(P_6)) \\ ZC(P_6) &= A * Z(P_6) - B * (Z(P_4) + Z(P_8)) \\ &\vdots \\ &\vdots \\ ZC(P_{2k-2}) &= A * Z(P_{2k-2}) - B * (Z(P_{2k-4}) + Z(P_{2k})) \\ ZC(P_{2k}) &= A * Z(P_{2k}) - B * (Z(P_{2k-2}) + Z(P_{2k+2})) \end{aligned} \right\} \quad (6)$$

where  $A = 0.6472$ ,  $B = 0.1764$

If the above simultaneous equations are to be solved for  $Z(p_k)$ , terminal conditions must be given, that is to say,  $Z(P_{-1})$  and  $Z(P_{2k+1})$  must be known for equations (5), and  $Z(P_0)$  and  $Z(P_{2k+2})$  for equations (6). How to decide the terminal conditions and how the error caused by the terminal conditions propagate will be discussed later.

For the convenience of discussion on this problem, at first we proceed to establish a set of formulae suitable for computation. Meanwhile, the  $Z(P_{-1})$ ,  $Z(P_{2k+1})$ ,  $Z(P_0)$  and  $Z(P_{2k+2})$  will be put in brackets in order to indicate that they should be considered known.

New arrays  $HC(2k-1)$ ,  $C(2k-1)$  and  $D(2k+1)$  are introduced for simultaneous equations (5).

$$\text{Set: } C(1) = A$$

$$D(3) = B$$

$$\text{and } HC(1) = ZC(P_1) = C(1) * Z(P_1) - D(3) * Z(P_3) - \left[ B * Z(P_{-1}) \right] \quad (a)'$$

$$(a)' * B + (b) * C(1):$$

$$B * HC(1) + C(1) * ZC(P_3) = (A * C(1) - B * D(3)) * Z(P_3) - B * C(1) * Z(P_5) - \left[ B^2 * Z(P_{-1}) \right]$$

$$\text{Set: } C(3) = A * C(1) - B * D(3)$$

$$D(5) = B * C(1)$$

$$\begin{aligned} \text{and } HC(3) &= B * H(1) + C(1) * ZC(P_3) \\ &= C(3) * Z(P_3) - D(5) * Z(P_5) - \left[ B^2 * Z(P_{-1}) \right] \end{aligned} \quad (b)'$$

$$(b)' * B + (c) * C(3)$$

$$\begin{aligned} B * HC(3) + C(3) * ZC(P_5) &= (A * C(3) - B * D(5)) * Z(P_5) \\ &- B * C(3) * Z(P_7) - \left[ B^3 * Z(P_{-1}) \right] \end{aligned}$$

$$\text{Set: } C(5) = A * C(3) - B * D(5)$$

$$D(7) = B * C(3)$$

and

$$\begin{aligned} HC(5) &= B * HC(3) + C(3) * ZC(P_5) \\ &= C(5) * Z(P_5) - D(7) * Z(P_7) - \left[ B^3 * Z(P_{-1}) \right] \end{aligned} \quad (c)'$$

⋮

$$\text{Set: } C(2k-1) = A * C(2k-3) - B * D(2k-1)$$

$$D(2k+1) = B * C(2k-3)$$

and

$$\begin{aligned} HC(2k-1) &= B * HC(2k-3) - C(2k-3) * ZC(P_{2k-1}) \\ &= C(2k-1) * Z(P_{2k-1}) - \left[ D(2k+1) * Z(P_{2k+1}) \right] \\ &\quad - \left[ B^{(2k-1+1)/2} * Z(P_{-1}) \right] \end{aligned} \quad (j)'$$

From (a)', (b)', ..... (j)', we obtain finally

$$\begin{aligned} Z(P_{2k-1}) &= \left( HC(2k-1) + \left[ D(2k+1) * Z(P_{2k+1}) \right] + \left[ B^k * Z(P_{-1}) \right] \right) / C(2k-1) & (j)'' \\ Z(P_{2k-3}) &= \left( HC(2k-3) + D(2k-1) * Z(P_{2k-1}) + \left[ B^{k-1} * Z(P_{-1}) \right] \right) / C(2k-3) & (i)'' \\ Z(P_{2k-5}) &= \left( HC(2k-5) + D(2k-3) * Z(P_{2k-3}) + \left[ B^{k-2} * Z(P_{-1}) \right] \right) / C(2k-5) & (h)'' \\ &\vdots & \\ Z(P_3) &= \left( HC(3) + D(5) * Z(P_5) + \left[ B^2 * Z(P_{-1}) \right] \right) / C(3) & (b)'' \\ Z(P_1) &= \left( HC(1) + D(3) * Z(P_3) + \left[ B * Z(P_{-1}) \right] \right) / C(1) & (a)'' \end{aligned} \quad (7)$$

$$\text{where } C(1) = A$$

$$D(3) = B$$

$$C(2k-1) = A * C(2k-3) - B * D(2k-1) \quad (8)$$

$$D(2k+1) = B * C(2k-3)$$

and

$$HC(1) = ZC(P_1)$$

$$HC(2k-1) = B * HC(2k-3) + C(2k-3) * ZC(P_{2k-1}) \quad (9)$$

A set of formulae similar to (7), (8) and (9) can be derived for simultaneous equations (6).

In view of (7), all spectra  $Z(p_k)$  can be computed explicitly as long as  $\left[ Z(p_{2k+1}) \right]$  and  $\left[ Z(p_{-1}) \right]$  were given. We set  $\left[ \left[ Z(p_{2k+1}) \right] \right] = \left[ \left[ Z(p_{-1}) \right] \right] = 0$ . In fact, usually they are not zero. But a frequency band width can be so selected that terminals of the band are far from dominant constituent frequencies and that the terminal spectra are quite small.

No matter what they are, it is necessary to evaluate the error caused by putting  $\left[ \left[ Z(p_{2k+1}) \right] \right]$  and  $\left[ \left[ Z(p_{-1}) \right] \right]$  equal to zero instead of their real values and to see how the error propagates with computation.

$\left[ Z(p_{2k+1}) \right]$  and  $\left[ Z(p_{-1}) \right]$  will be discussed separately. From formula (7), it can be found that the influence of  $\left[ Z(p_{2k+1}) \right]$  on  $Z(p_{2k-1})$  is

$$\left[ \frac{D(2k+1)}{C(2k-1)} Z(p_{2k+1}) \right]$$

and that of  $\left[ Z(p_{2k+1}) \right]$  on  $Z(p_{2k-3})$  is

$$\left[ \frac{D(2k+1)}{C(2k-1)} \frac{D(2k-1)}{C(2k-3)} Z(p_{2k+1}) \right]$$

⋮

and that of  $\left[ Z(p_{2k+1}) \right]$  on  $Z(p_1)$  is

$$\left[ \frac{D(2k+1)}{C(2k-1)} \frac{D(2k-1)}{C(2k-3)} \frac{D(2k-3)}{C(2k-5)} \dots \frac{D(3)}{C(1)} Z(p_{2k+1}) \right]$$

Now we turn our attention to formula (8).

From (8)

$$\begin{aligned} \frac{C(2k-1)}{D(2k+1)} &= \frac{A * C(2k-3) - B * D(2k-1)}{B * C(2k-3)} \\ &= \frac{A}{B} - \frac{1}{\frac{A}{B} - \frac{1}{\frac{A}{B} - \frac{1}{\frac{A}{B} - \dots - \frac{1}{\frac{A}{B} - \frac{1}{\frac{C(1)}{D(3)}}}}} \end{aligned} \tag{10}$$

$$\therefore \frac{A}{B} = \frac{0.6472}{0.1764} = 3.6689$$

$$\begin{aligned} \therefore \frac{D(3)}{C(1)} &= 0.2726 \\ \frac{D(5)}{C(3)} &= 0.2944 \\ \frac{D(7)}{C(5)} &= 0.2963 \\ \frac{D(9)}{C(7)} &= 0.2965 \\ \frac{D(11)}{C(9)} &= 0.2965 \\ &\vdots \\ \frac{D(2k-1)}{C(2k-3)} &= 0.2965 \\ \frac{D(2k+1)}{C(2k-1)} &= 0.2965 \end{aligned} \quad (11)$$

If in practical computation  $\left| \left[ Z(P_{2k+1}) \right] \right| = 0$  is used instead of its real value of 5 cm say, what will happen? According to (11), we should have

$$\frac{D(2k+1)}{C(2k-1)} \left| \left[ Z(P_{2k+1}) \right] \right| = 0.2965 * 5 = 1.48 \text{ cm}$$

$$\frac{D(2k+1)}{C(2k-1)} \frac{D(2k-1)}{C(2k-3)} \left| \left[ Z(P_{2k+1}) \right] \right| = 0.2965 * 1.48 = 0.44 \text{ cm}$$

$$\frac{D(2k+1)}{C(2k-1)} \frac{D(2k-1)}{C(2k-3)} \frac{D(2k-3)}{C(2k-5)} \left| \left[ Z(P_{2k+1}) \right] \right| = 0.2965 * 0.44 = 0.13 \text{ cm}$$

$$\frac{D(2k+1)}{C(2k-1)} \frac{D(2k-1)}{C(2k-3)} \dots \frac{D(2k-5)}{C(2k-7)} \left| \left[ Z(P_{2k+1}) \right] \right| = 0.2965 * 0.13 = 0.038 \text{ cm}$$

$\vdots$

Therefore, it can be confirmed that if we use  $\left| \left[ Z(P_{2k+1}) \right] \right| = 0$  instead of 5cm, the influence of the error is only limited by computation of (j)" and (i)" , because afterwards the error becomes negligibly small.

If the real value of  $\left| \left[ Z(P_{2k+1}) \right] \right|$  is 2 cm, then the influence is only restricted by computation of (j)" .

As to  $\left| \left[ Z(P_{-1}) \right] \right|$  , it is apparent that its influence is only limited by computation of (a)" , because B (=0.1764) is small.

From the above, all spectra  $Z(P_k)$  can be obtained except the terminal spectra, which probably have larger error.

Spectra of 30-day tide-generating potential  $Z_{pot}(P_k)$  can be computed with the procedure similar to computation of  $Z(P_k)$ .

### 3. Computation of the frequency response function

The output spectra, namely the spectra of a record of sea level or north or east component of current,  $Z(P_k)$ , consist of linear components which are the spectra of real astronomical constituents, and non-linear components which are the spectra of shallow water constituents. For the former, according to the spectra  $Z(P_k)$  and  $Z_{pot}(P_k)$ , the frequency response function can be estimated (with due respect for random errors due to noise in  $Z(P_k)$ ) as

$$F(P_k) = \left| F(P_k) \right| e^{iG(P_k)} = \frac{Z(P_k)}{Z_{pot}(P_k)} \quad (12)$$

This function is a characteristic of the locality of the original record. For the latter which are by definition non-linear, there is no such direct relationship, but from tidal dynamics a kind of indirect relationship between the shallow water constituents and the astronomical constituents concerned exists. For instance, if R and Q are amplitude and phase of an astronomical constituent  $X(t)$  and  $R''$  and  $Q''$  are the corresponding values for  $X^2(t)$ , these are the following relations

$$\begin{aligned}
R''(p_{2k}) &= \frac{1}{2}R^2(p_k) \\
Q''(p_{2k}) &= 2Q(p_k) \\
R''(p_{k+L}) &= 2R(p_k)R(p_L) \\
Q''(p_{k+L}) &= Q(p_k) + Q(p_L)
\end{aligned}
\tag{13}$$

(13) implies for real constituents that

$$\begin{aligned}
\frac{R''(p_{2k})}{R^2(p_k)} &= \text{constant} = C(p_{2k}) \\
\theta''(p_{2k}) - 2\theta(p_k) &= \text{constant} = B(p_{2k}) \\
\frac{R''(p_{k+L})}{R(p_k)R(p_L)} &= \text{constant} = C(p_{k+L}) \\
\theta''(p_{k+L}) - (\theta(p_k) + \theta(p_L)) &= \text{constant} = B(p_{k+L})
\end{aligned}
\tag{14}$$

(14) is equivalent to the following formulae

$$\begin{aligned}
\frac{|Z''(p_{2k})|}{|Z(p_k)|^2} e^{i(G''(p_{2k}) - 2G(p_k))} &= C(p_{2k}) e^{iB(p_{2k})} \\
\frac{|Z''(p_{k+L})|}{|Z(p_k)||Z(p_L)|} e^{i(G''(p_{k+L}) - G(p_k) - G(p_L))} &= C(p_{k+L}) e^{iB(p_{k+L})}
\end{aligned}
\tag{15}$$

According to (12), (15) can be written

$$\begin{aligned}
\frac{|Z''(p_{2k})|}{|Z_{\text{pot}}(p_k)|^2} e^{iG''(p_{2k})} &= C(p_{2k}) |F(p_k)|^2 e^{i(B(p_{2k}) + 2G(p_k))} \\
\frac{|Z''(p_{k+L})|}{|Z_{\text{pot}}(p_k)||Z_{\text{pot}}(p_L)|} e^{iG''(p_{k+L})} &= C(p_{k+L}) |F(p_k)||F(p_L)| e^{i(B(p_{k+L}) + G(p_k) + G(p_L))}
\end{aligned}
\tag{16}$$



There is some resemblance between (12) and (16). Therefore  $|Z_{pot}(p_k)|^2$  and  $|Z_{pot}(p_k)| |Z_{pot}(p_L)|$  could be defined as spectral values of a "tide-generating potential" of shallow water constituents,  $|Z''_{pot}(p_{2k})|$  and  $|Z''_{pot}(p_{k+L})|$  respectively.  $C(p_{2k}) |F(p_k)|^2 e^{i(B(p_{2k}) + 2G(p_k))}$  and  $C(p_{k+L}) |F(p_k)| |F(p_L)| e^{i(B(p_{k+L}) + G(p_k) + G(p_L))}$  could be defined as a "frequency response function" of shallow water constituents,  $|F''(p_{2k})| e^{iG''(p_{2k})}$  and  $|F''(p_{k+L})| e^{iG''(p_{k+L})}$  respectively. These definitions can be expressed as

$$\left. \begin{aligned} |Z''_{pot}(p_{2k})| &= |Z_{pot}(p_k)|^2 \\ |Z''_{pot}(p_{k+L})| &= |Z_{pot}(p_k)| |Z_{pot}(p_L)| \end{aligned} \right\} (17)$$

and

$$\left. \begin{aligned} |F''(p_{2k})| e^{iG''(p_{2k})} &= C(p_{2k}) |F(p_k)|^2 e^{i(B(p_{2k}) + 2G(p_k))} \\ |F''(p_{k+L})| e^{iG''(p_{k+L})} &= C(p_{k+L}) |F(p_k)| |F(p_L)| e^{i(B(p_{k+L}) + G(p_k) + G(p_L))} \end{aligned} \right\} (18)$$

The relative constancy of C and B has been confirmed by calculation of real data. Data from 3 stations, TRINDADE, OLINDA and SOUTHEND, were analysed. For each station, results of several sets of C and B were obtained. They are listed in Table 1 - 7, where it is seen that they are stable.

TABLE 1 VALUES of  $C_{M_4}$  and  $B_{M_4}$  for TRINDADE

	$M_4$		$M_2$		$C_{M_4}$	$B_{M_4}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)		
1	0.96	77.58	41.42	48.54	0.0006	340.46
2	1.10	82.47	41.41	58.74	0.0006	324.99
3	0.94	106.44	40.76	72.72	0.0005	320.90
4	0.96	128.92	40.48	83.27	0.0006	322.38
5	1.13	147.61	39.90	94.41	0.0007	318.79
6	1.05	178.23	40.11	106.28	0.0007	325.67
7	1.22	-145.65	40.14	119.54	0.0007	335.27
8	1.36	-131.01	40.68	129.23	0.0008	330.53
9	1.28	-119.26	40.39	142.09	0.0006	327.56
10	1.03	-82.49	40.31	154.71	0.0006	328.19
11	1.26	-66.94	39.58	164.68	0.0007	323.70

TABLE 2 VALUES of  $C_{M_4}$  and  $B_{M_4}$  for OLINDA

	$M_4$		$M_2$		$C_{M_4}$	$B_{M_4}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)		
1	1.20	-22.70	78.02	8.14	0.0002	321.02
2	1.08	162.96	80.21	-77.85	0.0002	318.66
3	1.03	-9.26	79.28	-163.79	0.0002	318.32
4	1.33	-169.85	79.21	115.48	0.0002	322.19
5	0.92	11.72	77.50	26.46	0.0002	318.78
6	0.95	-161.05	79.98	-62.03	0.0001	323.01

TABLE 3 VALUES of  $C_{M_4}$  and  $B_{M_4}$  for SOUTHEND

	$M_4$		$M_2$		$C_{M_4}$	$B_{M_4}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)		
1	9.32	30.33	196.28	172.73	0.0002	44.56
2	7.72	-142.00	200.66	85.78	0.0002	46.44
3	11.23	81.24	200.06	22.93	0.0003	35.38
4	9.15	-34.39	198.89	-36.23	0.0002	38.07
5	8.07	-114.92	195.39	-74.61	0.0002	34.30
6	8.08	65.66	197.95	-160.99	0.0002	27.64
7	7.82	-153.20	195.71	78.00	0.0002	50.80
8	8.79	61.81	197.87	14.46	0.0002	32.87
9	10.14	-21.43	196.54	-21.10	0.0003	20.77

TABLE 4 VALUES OF  $C_{MS4}$  and  $B_{MS4}$  for SOUTHEND

	$MS_4$		$M_2$		$S_2$		$C_{MS4}$	$B_{MS4}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)	R(cm)	$\theta$ (o)		
1	2.22	-84.50	196.28	172.73	54.02	26.30	0.0002	76.47
2	3.63	170.19	200.66	85.78	44.58	43.51	0.0004	40.90
3	3.27	130.80	200.06	22.93	56.43	65.24	0.0003	42.63
4	5.43	67.73	198.89	-36.23	66.62	63.76	0.0004	40.20
5	5.73	33.88	185.39	-74.61	78.83	50.68	0.0004	57.81
6	6.72	-65.95	197.85	-160.99	77.53	45.12	0.0004	49.92
7	2.47	167.96	195.71	78.00	51.50	27.88	0.0002	62.08
8	3.77	99.90	197.87	14.46	47.16	54.63	0.0004	30.81
9	4.59	81.88	196.54	-21.10	57.48	68.14	0.0004	34.96

TABLE 5 VALUES of  $C_{MO3}$  and  $B_{MO3}$  for SOUTHEND

	$MO_3$		$M_2$		$O_1$		$C_{MO3}$	$B_{MO3}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)	R(cm)	$\theta$ (o)		
1	6.51	-96.01	196.28	172.73	16.22	163.95	0.0020	287.31
2	7.18	106.93	200.66	85.78	18.86	93.25	0.0019	287.90
3	5.43	4.09	200.06	22.93	11.61	75.13	0.0023	266.03
4	5.55	-75.03	198.89	-36.23	13.93	30.46	0.0020	290.74
5	4.73	-132.37	195.39	-74.61	14.45	1.32	0.0017	299.92
6	4.57	110.43	197.95	-160.99	13.42	-41.56	0.0017	312.98
7	6.05	87.57	195.71	78.00	15.73	72.19	0.0019	297.38
8	6.65	-27.11	197.87	14.46	12.96	39.06	0.0023	278.37
9	4.76	-53.99	196.54	-21.10	11.76	39.59	0.0020	287.25

TABLE 6 VALUES of  $C_{2MS6}$  and  $B_{2MS6}$  for SOUTHEND

	$2MS_6$		$M_2$		$S_2$		$C_{2MS6}$	$B_{2MS6}$
	R" (cm)	$\theta$ " (o)	R(cm)	$\theta$ (o)	R(cm)	$\theta$ (o)		
1	4.56	63.86	196.28	172.73	54.02	26.30	$2 * 10^{-6}$	52.38
2	3.85	-88.72	200.66	85.78	44.58	43.51	$2 * 10^{-6}$	45.21
3	4.73	162.81	200.06	22.93	56.43	65.24	$2 * 10^{-6}$	51.71
4	5.89	40.84	198.89	-36.23	66.62	63.76	$2 * 10^{-6}$	49.54
5	6.59	-45.98	195.39	-74.61	78.83	50.68	$2 * 10^{-6}$	52.56
6	6.09	131.64	197.95	-160.99	77.53	45.12	$2 * 10^{-6}$	48.50
7	4.28	-129.22	195.71	78.00	51.50	27.88	$2 * 10^{-6}$	46.90
8	3.62	130.17	197.87	14.46	47.16	54.63	$2 * 10^{-6}$	49.61
9	4.67	78.49	196.54	-21.10	57.48	68.14	$2 * 10^{-6}$	52.55

TABLE 7 VALUES of  $C_{M6}$  and  $B_{M6}$  for SOUTHEND

	$R''$ (cm)	$M_6$ $\theta''$ (o)	$R$ (cm)	$M_2$ $\theta$ (o)	$C_{M6}$	$B_{M6}$
1	5.39	-144.65	196.28	172.73	$7 * 10^{-7}$	57.16
2	5.10	-50.48	200.66	85.78	$6 * 10^{-7}$	52.18
3	4.37	116.56	200.06	22.93	$5 * 10^{-7}$	47.77
4	4.59	-53.63	198.89	-36.23	$6 * 10^{-7}$	55.06
5	4.22	-177.25	195.39	-74.61	$6 * 10^{-7}$	46.06
6	4.84	-64.50	197.85	-160.99	$6 * 10^{-7}$	58.47
7	5.39	-68.30	195.71	78.00	$7 * 10^{-7}$	57.70
8	4.34	95.64	197.87	14.46	$6 * 10^{-7}$	52.60
9	3.80	-6.05	196.54	-21.10	$5 * 10^{-7}$	57.31

4. Resolution of the frequency response function at  $S_2$  and  $K_2$ ,  $K_1$  and  $P_1$  and similar parts of close constituents.

Up to now, the separate frequency response functions at  $S_2$  and  $K_2$  ( $K_1$  and  $P_1$ ) havenot been obtained, because their frequencies are too close to each other for explicit separation. They will be found with an approximate method which is sometimes used in classical tidal analysis.

An approximate relationship between the theoretical constituents and real constituents was derived from their values in the gravitational potential, namely

$$\left. \begin{aligned}
 \frac{R_{K2}}{R_{S2}} &\doteq \frac{0.07993}{0.29400} = \alpha_2 \\
 G_{K2} &\doteq G_{S2} \\
 \frac{R_{P1}}{R_{K1}} &\doteq \frac{0.12205}{0.36876} = \alpha_1 \\
 G_{P1} &\doteq G_{K1}
 \end{aligned} \right\} (19)$$

According to (19), we obtain for  $S_2$  and  $K_2$

$$\left. \begin{aligned}
 R_{S2} &= \frac{R_{S2,K2}}{1 + \alpha_2 \cos(2h + u_{K2})} \\
 G_{S2} &= G_{S2,K2} \\
 R_{K2} &= \alpha_2 R_{S2} \\
 G_{K2} &= G_{S2}
 \end{aligned} \right\} (20)$$

and for  $K_1$  and  $P_1$

$$\begin{aligned}
R_{K1} &= \frac{R_{K1,P1}}{(1 + \alpha_1^2 - 2\alpha_1 \cos(2h + u_{K1}))^{\frac{1}{2}}} \\
G_{K1} &= G_{K1,P1} \\
R_{P1} &= \alpha_1 R_{K1} \\
G_{P1} &= G_{K1}
\end{aligned}
\tag{21}$$

(20) and (21) are equivalent to

$$\begin{aligned}
Z(P_{S2}) &= \frac{Z(P_{S2,K2})}{1 + \alpha_2 \cos(2h + u_{K2})} \\
Z(P_{K2}) &= \alpha_2 Z(P_{S2})
\end{aligned}
\tag{22}$$

and

$$\begin{aligned}
Z(P_{K1}) &= \frac{Z(P_{K1,P1})}{(1 + \alpha_1^2 - 2\alpha_1 \cos(2h + u_{K1}))^{\frac{1}{2}}} \\
Z(P_{P1}) &= \alpha_1 Z(P_{K1})
\end{aligned}
\tag{23}$$

The same consideration for their potential gives

$$\begin{aligned}
Z_{\text{pot}}(P_{S2}) &= \frac{Z_{\text{pot}}(P_{S2,K2})}{1 + \alpha_2 \cos(2h + u_{K2})} \\
Z_{\text{pot}}(P_{K2}) &= \alpha_2 Z_{\text{pot}}(P_{S2})
\end{aligned}
\tag{24}$$

and

$$\begin{aligned}
Z_{\text{pot}}(P_{K1}) &= \frac{Z_{\text{pot}}(P_{K1,P1})}{(1 + \alpha_1^2 - 2\alpha_1 \cos(2h + u_{K1}))^{\frac{1}{2}}} \\
Z_{\text{pot}}(P_{P1}) &= \alpha_1 Z_{\text{pot}}(P_{K1})
\end{aligned}
\tag{25}$$

From (22) - (25), the following relationships are obtained

$$\left. \begin{aligned} F(P_{S2}) &= F(P_{K2}) = F(P_{S2,K2}) \\ F(P_{K1}) &= F(P_{P1}) = F(P_{K1,P1}) \end{aligned} \right\} (26)$$

#### 5. Computation of the harmonic constants

Dr D E Cartwright completed "New Computation of Tide-generating potential" in 1970. Theoretical average amplitudes of all constituents,  $H_{pot}(P_k)$ , have been obtained and listed. It is very convenient for us to compute the harmonic constants using those  $H_{pot}(P_k)$  and frequency response function  $F(P_k)$ . From the definition of  $F(P_k)$ , we obtain the harmonic constants - amplitudes.

$$H(P_k) = |F(P_k)| H_{pot}(P_k) \quad (27)$$

and the harmonic constants - phases

$$G(P_k) = G(P_k) \quad (28)$$

#### 6. Harmonic constants for SOUTHEND and their comparison with results of a 18-year analysis.

We executed 9 computation of tidal spectra for SOUTHEND using the method described above. The nine sets of frequency response functions are listed in TABLE 8.

An average of each constituent response function was used to compute its harmonic constants. The results are listed in TABLE 9. A set of harmonic constants for SOUTHEND, which M Amin obtained in 1976 using 18-years of records, is also listed in the second line of the same TABLE for comparison. The Table shows that the two sets of harmonic constants for SOUTHEND are close.

TABLE 8 Frequency Response Function for Southend

	$M_2$		$S_2$		$K_2$		$N_2$	
	$ F $	$G(o)$	$ F $	$G(o)$	$ F $	$G(o)$	$ F $	$G(o)$
1	3.2310	351.91	2.0071	48.54	2.0071	48.54	2.6941	329.41
2	3.3024	350.69	2.1528	46.93	2.1528	46.93	2.4967	331.06
3	3.2854	349.43	2.1984	46.32	2.1984	46.32	2.3147	335.69
4	3.2656	352.29	1.9168	44.98	1.9168	44.98	2.5245	331.29
5	3.2126	351.43	1.9693	42.44	1.9693	42.44	3.3019	331.15
6	3.2678	351.60	2.0603	49.87	2.0603	49.87	2.9740	327.70
7	3.2517	351.19	2.1103	43.53	2.1103	43.53	2.9840	329.70
8	3.2984	349.67	2.1229	48.84	2.1229	48.84	2.9539	335.25
9	3.2575	351.52	1.9524	48.98	1.9524	48.98	3.2016	331.48
average	3.2636	351.09	2.0545	46.71	2.0545	46.71	2.8272	331.48

	$K_1$		$P_1$		$O_1$		$Q_1$	
	$ F $	$G(o)$	$ F $	$G(o)$	$ F $	$G(o)$	$ F $	$G(o)$
1	0.3281	354.07	0.3281	354.07	0.5538	197.19	0.7468	108.32
2	0.3223	3.96	0.3223	3.96	0.16199	186.21	0.2661	166.88
3	0.3302	11.72	0.3302	11.72	0.3838	203.87	0.5817	156.66
4	0.3508	358.32	0.3508	358.32	0.4595	195.11	1.2005	131.53
5	0.2144	0.52	0.2144	0.52	0.4847	175.85	1.2224	59.87
6	0.2734	342.81	0.2734	342.81	0.4250	191.31	1.0901	103.17
7	0.3229	356.69	0.3229	356.69	0.5115	187.13	0.8801	144.84
8	0.3064	12.78	0.3064	12.78	0.4247	189.70	0.7596	179.75
9	0.3300	358.36	0.3300	358.36	0.3852	200.83	0.5585	113.95
average	0.3087	359.91	0.3087	359.91	0.4720	191.91	0.8118	129.44

	$M_4$		$MS_4$		$MO_3$		$M_6$		$2MS_6$	
	$ F'' $	$G''(o)$	$ F'' $	$G''(o)$	$ F'' $	$G''(o)$	$ F'' $	$G''(o)$	$ F'' $	$G''(o)$
1	0.0021	28.38	0.0013	116.92	0.0036	116.41	0.00002	32.89	0.00004	84.74
2	0.0022	27.82	0.0028	78.52	0.0038	104.80	0.00002	24.25	0.00004	73.52
3	0.0032	14.24	0.0022	78.38	0.0030	99.33	0.00002	16.06	0.00004	76.89
4	0.0021	22.65	0.0031	77.47	0.0030	118.14	0.00002	31.93	0.00004	79.10
5	0.0021	17.17	0.0025	91.72	0.0030	107.20	0.00002	20.35	0.00004	77.86
6	0.0021	10.84	0.0026	91.41	0.0030	135.89	0.00002	33.27	0.00004	81.56
7	0.0021	33.18	0.0014	96.80	0.0031	115.70	0.00002	31.27	0.00004	72.81
8	0.0022	12.23	0.0028	69.32	0.0035	98.74	0.00002	21.61	0.00005	77.97
9	0.0031	3.81	0.0025	75.44	0.0025	119.60	0.00002	31.87	0.00004	84.57
average	0.0023	18.92	0.0024	86.22	0.0031	112.87	0.00002	27.06	0.00004	78.78



TABLE 9 Comparison of two sets of harmonic constants for SOUTHEND

	$M_2$		$S_2$		$K_2$		$N_2$		$K_1$		$P_1$		$O_1$		$Q_1$	
	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)
(1)	206.2	351.1	60.4	46.7	16.4	46.7	34.2	331.5	11.4	359.9	3.8	359.9	12.4	191.9	4.1	129.4
(2)	204.4	353.5	59.0	49.4	17.2	45.9	34.9	329.3	11.1	11.2	4.1	353.8	13.1	187.7	4.3	133.1

	$M_4$		$MS_4$		$MO_3$		$M_6$		$2MS_6$	
	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)	H (cm)	G (o)
(1)	8.4	18.9	4.8	86.2	5.0	112.9	5.0	27.1	4.6	78.8
(2)	9.7	8.7	3.5	72.4	5.1	117.1	4.6	35.9	4.8	85.9

(1) Results of this paper. (2) From Amin (1976)

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