WAVES RECORDED AT DOWSING LIGHT VESSEL BETWEEN 1970 AND 1979

by

B C H FORTNUM

Data for May 1970 to April 1971
November 1975 to October 1976
and August 1977 to July 1979
at position 53°34.0'N, 000°50'E

Summary Analysis and Interpretation Report

Report No 126

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1. INTRODUCTION

1.1 Site description
The site at which the wave measurements were taken is shown on the map in figure 1.1. It is approximately 48 kilometres east of Spurn Point on the east coast of England, at position 53°34.0'N, 000°50.2'E. It is moored in water of depth about 26 metres; however the sea-bed in its vicinity is very irregular, and several shoals are charted with water depths above them of between 6 and 20 metres. The tidal currents in the area are quite strong, the maximum current being about 2 knots (1 metre/sec), with directions of approximately 150° and 330°. Due to the shoals around the vessel the currents may be even stronger, and may occasionally cause either a real or an apparent increase in the steepness of the waves to an unusually high level. This effect would be most pronounced for short, low-period waves.

1.2 Description of measuring and recording systems
The vessel is fitted with a shipborne wave recorder as described in Tucker (1956), which provides information about the water surface elevation. This information is recorded for a 12-minute period (usually) once every 3 hours by a pen on paper chart rolls. The expression for the correction factor which is applied to the wave height data is given in Appendix IV, together with a table of its values for various values of Tz, using appropriate values for the depth of the pressure transducer on the vessel (1.46 metres for the 1970/71 data, and 0.88 metres for the 1975/79 data). The correction factor is greater than 2 for Tz of 4.1 seconds and less for the 1970/71 data, and for Tz of 3.2 seconds and less for the 1975/79 data. Since the expression for the correction factor is rather inaccurate for these low values of Tz, data with Tz values of 3.5 seconds and less are not considered to be reliable (together with some of the data with Tz values between 3.5 and 4.0 seconds). However all the data have been included in the presentations in this report, with a warning shown on those figures which, if used without regard for this limitation on the data, might yield misleading information.

1.3 Details of calibration and maintenance
During the period covered by this report an N10 4753 (valve) shipborne wave recorder was deployed on the lightvessel.
(i) 20 May 1970 - Instrument was overhauled and re-calibrated whilst the vessel was in dry dock at South Shields.
(ii) October 1971 - Repair and re-calibration of equipment were carried out following report of intermittent fault. The re-calibration showed that there had been a change in sensitivity of -5.6% (i.e. the accelerometers were under-reading the waves by this amount). It was not possible to find out whether the replacement component had contributed to this change, and therefore it was assumed that all data recorded up to the time of the intermittent fault were acceptable.
(iii) September/October 1975 - A replacement wave recorder was installed during the vessel's re-fit.
(iv) May/June 1977 - A re-calibration was carried out during the vessel's re-fit, and the change in sensitivity was found to be -3.2%. Vessel returned to Dowsing station on 23 September 1977.
(v) 12 October 1981 - A calibration was carried out when the vessel came off station, and the change in sensitivity was found to be +9.4%. It has been assumed that the change was linear with time.

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1.4 Wave data coverage and return (figures 1.4(A) - 1.4(L))
The periods covered by the data are 1 May 1970 to 30 April 1971,
1 November 1975 to 31 April 1971, and 1 August 1977 to 31 July 1979.
For this period 158 of the 11688 possible chart records were either
missing or classified as invalid, resulting in a data return of 98.7%;
and 2.06% of the valid records were calms (see Appendix II). No attempt
has been made to correct any bias which may have resulted from
missing/invalid records, because of the uncertain reliability of
available techniques. (Simple gap-filling by linear interpolation, up
to a maximum of 7 consecutive records, has been carried out for the pur-
pose of persistence calculations only: see section 3.6.) The approximate
times when missing/invalid records occurred may be derived from the
plots in figure 1.4 which show Hs as a time series. On these plots each
vertical line represents a valid record, and the height of the line is
proportional to the value of Hs for that record: therefore these plots
also indicate the variation of Hs with time.

2. WIND DATA – COMPARISON WITH THE LONG-TERM AVERAGE
The meteorological station nearest to the wave measurement site is Spurn
Head (53°35’N, 000°07’E) from which wind data have been analysed for the
period January 1957 to July 1979. Winds approaching from directions
which have very limited fetches associated with them have not been con-
sidered, so that only winds in the sector from 320° to 160° have been
considered in this report (including a proportion of calms and
variables). The data used are three-hourly synoptic wind speeds.

2.1 Monthly variation of wind speeds (figure 2.1)
For each month, the mean of the monthly means of wind speed is plotted.
Only two months, January and December, have monthly means which are not
distributed fairly evenly about their 'long-term' monthly means; these
can be seen to be the two stormiest months in the period considered, and
in both cases three out of the four monthly means are higher than the
'long-term' means.

2.2 Yearly variation of wind speeds (figures 2.2(A) - 2.2(C))
The year-to-year variability of wind conditions is illustrated in this
figure. It shows, for each year, the maximum value of wind speed, and
also the means of the next N highest wind speeds, where N = 5, 10, 20,
50, 100 (thus the highest 186 wind speeds are represented). Figure
2.2(A) shows that the wind speeds in 1970/71 lie in the top half of the
range of wind speeds for the 14 year period; from figure 2.2(B) it can
be seen that the very highest wind speeds in 1975/76 are towards the top
of the range, and the mean is in the middle of the range of wind speeds
for the 15 year period; and figure 2.2(C) shows that apart from the very
highest wind speeds, the winds for 1977/79 are generally in the lower
half of the wind speeds for the 15 year period. From these rather
simple analyses, it may be deduced that the 4 years of wave data
presented in this report were recorded during years which were not ex-
ceptionally and consistently stormy, although the very highest storms occurring during the 4-year period may have been amongst the highest in the 'long-term' period.

3. WAVE DATA - DESCRIPTION AND DISCUSSION OF THE PRESENTATIONS
Where figures show seasonal data, the seasons are defined as follows:
spring - March, April, May
summer - June, July, August
autumn - September, October, November
winter - December, January, February

The maximum value of Hs in these four years of data is 6.5 metres; the associated values of Tz and of Hmax(3hr) are also the highest in the data set, being 9.7 seconds and 12.2 metres.

3.1 Statistics of variations of wave heights

3.1.1 Monthly variation of Hs (figure 3.1.1)
For each month, the mean of the monthly means of significant wave height is plotted. February and December have the widest yearly variations in the values of mean Hs. This figure also shows that the highest wave conditions probably occurred during 1978/9 (as experienced in the months of January, February, March and December); and that the least severe conditions were during 1975/6 (only January, September and October are exceptions). This is in agreement with the conclusions drawn from figure 3.1.2 (see section 3.1.2).

3.1.2 Yearly variation of Hs (figure 3.1.2)
The year-to-year variability of wave conditions is illustrated in this figure. It shows, for each year, the maximum value of Hs, and also the means of the next N highest values of Hs, where N = 5, 10, 20, 50, 100 (thus the highest 186 values of Hs are represented). The figure shows that, of the 4 years considered, the highest values of Hs were greatest during 1978/9, and least (marginally) during 1975/6. However, the yearly variation is not great.

3.2 Statistics of wave heights

3.2.1 Occurrence of Hs (figures 3.2.1.1-3.2.1.5)
The percentage occurrence of Hs is shown on histograms. The most frequently occurring values of Hs may be seen, from figure 3.2.1.5, to lie between 0.5 and 1.5 metres, accounting for 57% of the total.

3.2.2 Exceedance of Hs and Hmax(3hr) (figures 3.2.2.1-3.2.2.5)
These graphs may be used to estimate the fraction of the time during which Hs was greater than, or less than, a given height. For instance, from figure 3.2.2.4 it may be seen that during winter the significant wave height exceeded 2.5 metres for approximately 19 per cent of the time.
3.3 Design wave heights
The methods used to calculate the design wave height (the most probable height of the highest wave with a return period of 50 years) are described in Appendix III. The difficulty in fitting one of the extreme-value distributions to this data set may be shown by reference to figure 3.3.2. The upper values of Hs (as shown by the crosses) appear from this figure to be limited in magnitude, possibly due to the shallow water depths in the area. Therefore a Fisher-Tippett III distribution, which assumes an upper limit to Hs, might be expected to fit the data best. A best fit of this distribution to all the data is found for A = 14 metres, resulting in Hs(50) = 7.88 metres. However, since any of the long-term distributions will be greatly influenced by the highest data point, it is useful to examine it more closely. This data point includes the highest value of Hs, which is 6.46 metres; an increase of less than 1% in this value (well within the standard error in Hs of 6% which is inherent in the method of estimation, as described in Appendix II) would move this estimate from the 6.0-6.5 metres class to the 6.5-7.0 metres class. Therefore on figure 3.3.2 an additional point will appear in this top class, and also the probability associated with the next lowest class will be changed (these changes are shown by asterisks on figure 3.3.2). Within the limitations on the accuracy of Hs, this new data set is an equally valid representation of the wave climate at the site as is the original data set. Fitting a Fisher-Tippett I distribution to the whole of this new data set gives Hs(50) = 8.94 metres. Since a small error in the estimate of the maximum Hs leads to a large difference in Hs(50), the distributions described below have been chosen so that, visually, the top point has been given less weight than the others. (Note that the Fisher-Tippett III distribution leads to a much lower value of Hs(50) than do the other two distributions since it is more sensitive to the upper data points.)

3.3.1 Weibull distribution of Hs (figure 3.3.1)
The parameters of the Weibull distribution which most closely fits the data are A = 0.32 metre, B = 1.12 metres and C = 1.25 (figure 3.3.1). Using this distribution, the value of Hs with a return period of 50 years is 8.5 metres. The value of Tz associated with this Hs is approximately 10 seconds, resulting in a value for the design wave height of 16.0 metres.

3.3.2 Fisher-Tippett I distribution of Hs (figure 3.3.2)
The parameters of the Fisher-Tippett I distribution which most closely fits the data are a = 1.45 metre^{-1} and b = 1.28 (figure 3.3.2). Using this distribution, the value of Hs with a return period of 50 years is 9.1 metres. The value of Tz associated with this Hs is approximately 10 seconds, resulting in a value for the design wave height of 17.0 metres.

3.3.3 Fisher-Tippett III distribution of Hs (figure 3.3.3)
The parameters of the Fisher-Tippett III distribution which most closely fits the data are A = 14.0 metres, B = 13.4 metres and C = 14.1 (figure 3.3.3). Using this distribution, the value of Hs with a return period of 50 years is 8.2 metres. The value of Tz associated with this Hs is approximately 9.5 seconds, resulting in a value for the design wave height of 15.5 metres.
3.3.4 Individual wave model (figure 3.3.4)
The value of steepness used in the wave-by-wave method of determining design wave heights (as described in Appendix III) is 1:18, and the inverse mean period is 0.194 Hz. Using these values and the Fisher-Tippett III parameters given in section 3.3.3, the data which appear in figure 3.3.4 are obtained; by interpolation the design wave height is found to be 16.1 metres. A higher value of design wave height is expected from this method than from the methods described above, for the reasons stated in Appendix III.

3.4 Statistics of wave periods
The percentage occurrence of Tz is shown on a histogram.

3.4.1 Occurrence of Tz (figures 3.4.1.1-3.4.1.5)
The most frequently occurring values of Tz in the data set lie between 4.0 and 5.0 seconds (31% of the total), and all values of Tz lie between 2.5 and 10.0 seconds (figure 3.4.1.5).

3.5 Statistics of wave height and period combined
These figures (sometimes called "scatter" plots) show the numbers of wave records having particular combinations of values of Hs and Tz. The numbers of wave records are presented as parts per thousand (the total number of valid observations being shown on each figure), except for those which would be less than one part per thousand; these are shown instead as single occurrences and are distinguished by being underlined.

3.5.1 Occurrences of Hs and Tz combined (figures 3.5.1.1-3.5.1.5)
On these figures points of equal occurrences are joined by contour lines to give an indication of the bivariate probability distribution of Hs and Tz, and to illustrate the correlation between them. A wave "steepness" (as defined in Appendix III) can be calculated for each (Hs,Tz) pair. A line is drawn on figure 3.5.1.5 showing a "steepness" of 1:12, which is the limiting "steepness" for the main body of the data. (Wave "steepnesses" as shown in this figure are less than the maximum of 1:7 for an individual wave, since Hs and Tz are parameters averaged over a number of waves most of which have steepnesses less than this maximum.)

3.6 Statistics of persistence of wave conditions
These figures show the means and standard deviations of the durations of storms and calms against each threshold value of Hs, and also the percentage of the total duration occupied by each event. Gaps in the data series of 7 or less records are filled (for the purpose of persistence calculations only) by linear interpolation; larger gaps are not filled, effectively reducing the series to a number of smaller sub-series, each with a correspondingly smaller total duration. 'Split' seasons (those in which the months are not consecutive) are not used in the persistence calculations. (For storms, the curves showing percentage of time occupied by the events are, for all practical purposes, the same as those showing percentage exceedance of Hs as described in section 3.2.2.)
3.6.1 Persistence of calms of Hs (figures 3.6.1.1-3.6.1.5)
Information about, for example, calms of Hs less than 0.9 metres at the Dowsing station during winter can be derived from figure 3.6.1.4. The mean duration of such calms was about 11.5 hours (with a standard deviation of 18 hours); they occupied about 18% of the total duration of 8652 hours, i.e. about 1560 hours; and therefore there were between 130 and 140 such calm events during this period. Since this represents 4 seasons, there were on average about 35 such events each winter.

3.6.2 Persistence of storms of Hs (figures 3.6.2.1-3.6.2.5)
Similar information can be derived for storms. For Hs of 3.9 metres during spring, figure 3.6.2.1 shows that the mean duration of such storms was approximately 56 hours (the fact that no value of standard deviation is shown for this value of Hs indicates that there was only one event). The total time occupied was approximately 1% of 6573 hours, which confirms that there was only one such storm event in the three springs analysed.

4. ACKNOWLEDGEMENTS
Contributions have been made towards the collection, analysis and presentation of the Dowsing wave data by several members of the Applied Wave Research Group and of the Instrument Engineering Group, both based at the Taunton laboratory of the Institute of Oceanographic Sciences. Thanks are due to Trinity House for permission to install the shipborne wave recorder in the Dowsing Light Vessel; and also to the Meteorological Office for supplying the wind data.

5. REFERENCES


APPENDIX 1

Method of system calibration

1.1 Since there are two types of transducer in the shipborne wave recorder system, it is necessary to divide the calibration procedure into two sections. First the accelerometers are removed from the ship mountings and each is inserted into a rig which allows the transducer to be driven through a vertical circle of diameter 1 metre. The transducer is mounted in gimbals and maintains a vertical attitude during rotation. Two rotation rates are applied: 12 and 18 second periods which are derived from a crystal oscillator. The transducer is connected to the electronics unit in the usual way, and the calibration signal is displayed on the chart recorder. However, because a 1 metre 'heave' is small compared with the wave-heights usually experienced at sea, a precision amplifier (contained in the electronics) is switched into the circuit, converting the 1 metre into an apparent 10 metre signal. The output signal can then be read from the chart record and any corrections to instrument sensitivity made.

The pressure units cannot be easily subjected to a dynamic test since this requires the application of a sinusoidally-varying pressure. Therefore for routine re-calibration a static test is applied. Each pressure unit is fixed to the test rig and a series of discrete pressure levels is applied from a reservoir via a regulator valve. Each pressure level is set manually with the valve by reference to a precise pressure transducer contained within the calibrator unit. The output voltage of the transducer is monitored in the SBWR electronics unit and compared to the original laboratory calibration. Any changes in sensitivity are then compensated for by adjustment of the input amplifier gain.

Full re-calibrations are usually only possible when the ship comes into dock for its 3-yearly re-fit.

1.2 Monthly checks
All light vessel crews are asked to drain water through the valve assemblies to ensure that no blockage prevents the water pressure being transmitted to the pressure sensors, and then to take a test record, on a monthly basis. The test record consists of a short length of pen-trace with all transducers turned off (electrically), followed by a few minutes recording with each transducer on its own. The record thus produced shows two heave records (one from each accelerometer) which should look broadly similar; and also the pressure traces, which may not agree so well, but when compared with other monthly test records should exhibit no systematic error. These tests are not direct checks on calibration accuracy but are often good indicators of a fault condition developing.
APPENDIX II

Definitions of wave parameters and method of analysis of wave chart data

II.1 The technique used to analyse the wave data was that proposed by TUCKER(1961) and DRAPE(1963), and reviewed by TANN(1976). A twelve-minute record is taken every three hours, and from this the following parameters may be derived.

II.1.1 Tz - the mean zero up-crossing period. This is defined as the duration of the record divided by the number of zero up-crossings Nz. (A zero up-crossing is considered to occur when the trace crosses the mean line in an upward direction.)

II.1.2 Hs - the significant wave height. This is defined as 4σ where σ is the standard deviation of the record. (An estimate of σ is obtained from Nz and from the excursions of the two highest maxima, and of the two lowest minima, from the record mean.) For a narrow band random process this parameter approximates closely to the mean height of the highest one-third zero up-cross waves (see LONGUET-HIGGINS(1952)). Comparison between the two definitions is made by GODA(1970,1974). (A zero up-cross wave is defined as the portion of the wave record between two zero up-crossings, and its height is the vertical distance between the highest and lowest points on the wave.)

II.1.3 Hmax(3hr) - the most probable height of the highest zero up-cross wave in the recording interval of three hours. (This is derived from Nz, and the duration of the recording interval.) The parameter Hmax(3hr), which is the mode of the distribution, should not be confused with the expected height of the highest wave in three hours, which is the mean of the distribution. The mean of the distribution is typically 3 per cent higher than the mode (see TANN(1976)).

(A 'calm' record is one for which the sum of the height of the highest crest and of the lowest trough is less than 1 foot or 0.3 metre.)

II.2 References


APPENDIX III

Details of methods used for calculating design wave heights

III.1 By finding the long-term distribution of Hs

III.1.1 Hs is used as a measure of the "sea-state" (i.e. the intensity of wave activity), and it is sampled every 3 hours. It is assumed that a set of Hs data for one year, or an integral number of years, is representative of the wave climate.

For each value of Hs, the probability that this value will not be exceeded is calculated; this probability is then plotted against Hs. The axes are scaled according to a long-term distribution, so that data with a perfect fit would appear as a straight line on the diagram. This procedure is carried out using long-term distributions defined in the following ways:

Weibull

\[ \text{Prob}(Hs < h) = \begin{cases} 
1 - \exp \left[ -\left( \frac{h - A}{B} \right)^C \right], & \text{for } h > A \\
0, & \text{for } h \leq A 
\end{cases} \]

where \( B \) and \( C \) are positive, and \( A \) represents a lower bound on \( h \).

Fisher-Tippett I (first asymptote)

\[ \text{Prob}(Hs < h) = \exp[- \exp(-ah + b)]. \]

Fisher-Tippett III (third asymptote)

\[ \text{Prob}(Hs < h) = \begin{cases} 
\exp \left[ -\left( \frac{A - h}{B} \right)^C \right], & \text{for } h \leq A \\
1, & \text{for } h > A 
\end{cases} \]

where \( B \) and \( C \) are positive, and \( A \) represents an upper bound on \( h \). (See FISHER AND TIPPETT (1928) and GUMBEL (1958) for the derivations of these distributions.)

For each long-term distribution the best-fit straight line is drawn; this line is then extrapolated to the desired probability (see section III.1.2) and the corresponding value of Hs is read off as the "design sea-state".

III.1.2 To calculate the "sea-state" which will be exceeded only once in \( N \) years, a storm duration of \( D \) hours needs to be assumed. The probability that a randomly chosen time will be within this storm is then

\[ \frac{D}{24 \times 365.25 \times N} \]
IOS uses $D = 3$ hours (this choice is discussed in section III.1.5) which gives

$$\text{Probability} = \frac{3.422 \times 10^{-4}}{N} = 6.845 \times 10^{-6} \text{ for } N = 50 \text{ years}.$$  

III.1.3 The value of $T_z$ for the "design sea-state" is required before the highest wave in the storm can be calculated. This is derived from the bivariate distribution of $H_s$ and $T_z$ (figure 3.5.1.5). A line is drawn across this at the "design sea-state" value of $H_s$ and the most likely value of $T_z$ (the modal value) is then estimated using extrapolations of the probability contours.

III.1.4 The most probable value of the highest zero-up-cross wave in the storm is then derived by assuming that the heights of such waves follow a Rayleigh distribution whose probability density function is

$$\text{prob}(h) = \frac{2h}{(H_{rms})^2} \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right]$$

where $H_{rms} \approx \frac{H_s}{\sqrt{2}}$.

Exact theory is not available for zero-up-cross wave heights, but this distribution has been found to be an adequate fit to measured data. If there are $n$ waves in the recording interval (3hr), then the probability that the highest wave, $H$, in three hours is less than $h$ is

$$\text{Prob}(H \leq h) = \left\{1 - \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right]\right\}^n$$

with a corresponding probability density function

$$\frac{2n}{(H_{rms})^2} h \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right] \left\{1 - \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right]\right\}^{n-1}.$$ The most probable value (the mode) of this probability density function is usually used and is given by

$$H_{max}(3\text{hr}) = H_{rms} \sqrt{\Psi}$$

where $\Psi$ is a function of $T_z$ which may be found using either figure 7 or equation 6.1–2 in TANN(1976).

III.1.5 In choosing the value of storm duration $D$, it should be noted that the effect of increasing $D$ is to decrease the value of $H_s$ for a given return period $N$. However, it also increases the ratio of $H_{max}(3\text{hr})$ to $H_s$. It is found that in practice these effects roughly cancel and typically the value of $H_{max}(3\text{hr})$ changes by only 3 per cent for a change of $D$ from 3 to 15 hours. The choice of $D$ is therefore not critical.

Many details of the above procedures may be found in TANN(1976).
III.2 By a wave-by-wave method

III.2.1 BATTJES (1970) shows that the probability that a randomly chosen wave will have a height $H$ greater than $h$ is

$$
\text{Prob}(H > h) = \frac{\int_0^\infty \int_0^\infty R(h, H_s) T_z^{-1} p(T_z, H_s) \, dH_s \, dT_z}{\int_0^\infty \int_0^\infty T_z^{-1} p(T_z, H_s) \, dH_s \, dT_z}
$$

(1)

where $R(h, H_s)$ is the Rayleigh cumulative probability function and $p(T_z, H_s)$ is the joint probability density function of $H_s$ and $T_z$.

III.2.2 TANN makes the following suggestion in an unpublished manuscript. In order that values of $H_s$ higher than those actually measured may be represented in the calculation of this probability, the values of $H_s$ are assumed to have a long-term cumulative probability function $F(H_s)$, and a probability density function $f(H_s) = F'(H_s)$.

For each value of $H_s$ throughout the long-term distribution, an average value of $T_z^{-1}$ is used (denoted by $\overline{T_z^{-1}}(H_s)$). It is defined as

$$
\overline{T_z^{-1}}(H_s) = \int_0^\infty T_z^{-1} \frac{p(T_z, H_s)}{P(H_s)} \, dT_z
$$

where $P(H_s) = f(H_s)$.

Therefore

$$
\int_0^\infty T_z^{-1} p(T_z, H_s) \, dT_z = \overline{T_z^{-1}}(H_s)
$$

which, when substituted into equation (1), allows the probability of exceedance to be written

$$
\text{Prob}(H > h) = \frac{\int_0^\infty R(h, H_s) \overline{T_z^{-1}}(H_s) f(H_s) \, dH_s}{\int_0^\infty \overline{T_z^{-1}}(H_s) f(H_s) \, dH_s}
$$

The value of $\overline{T_z^{-1}}(H_s)$ used with each value of $H_s$ is chosen to satisfy the condition of constant wave "steepness", where the relationship between "steepness" ($l_s : d$), water depth ($d$), $H_s$ and $T_z$ is

$$
T_z = \sqrt{\frac{2 \pi H_s}{g} \coth \left( \frac{2 \pi d}{s H_s} \right)}
$$

The value for the steepness used in this report is given in section 3.3.4.

The long-term distribution used in the computation for this report is the Fisher-Tippett III extreme-value distribution, whose probability density function is

$$
f(H_s) = \frac{C}{A - H_s} \left( \frac{A - H_s}{B} \right)^C \exp \left[ - \left( \frac{A - H_s}{B} \right)^C \right].
$$

The constants $A, B, C$ are determined graphically as described in section III.1.1, and their values as used in this report are given in section 3.3.3.

-VII-
III.2.3 Thus the probability of a wave exceeding each particular wave height may be found, and this probability may be converted into a return period of N years using the formula

\[ N = \frac{1}{365.25 \times 24 \times 3600 \times T_{ave}^{-1} \times \text{Prob}} \]

where \( T_{ave}^{-1} \) = \( \frac{1}{\text{average period}} \).

The value of the average wave period is contained in section 3.3.4. Since \( T_{ave}^{-1} \) is a non-analytic function of \( \text{Prob} \), the simplest way of solving the problem is to calculate \( \text{Prob} \) for various values of \( h \), calculate \( N \) for each of these values of \( \text{Prob} \), and then interpolate to find the height \( h \) corresponding to the required value of \( N \) (in this case 50 years).

Whereas the method described in section III.1 assumes that the highest wave in a 50-year period will come from the most stormy 3-hour period in 50 years, the individual wave method takes into account the probability that storms other than the highest may provide the wave with a 50-year return period. Consequently the height of a 50-year wave as estimated by this method is likely to be greater than that estimated from the method of using a long-term distribution of \( H_s \).

III.3 References


APPENDIX IV

The correction factor applied to wave heights

Two corrections need to be applied to the wave height data:
(i) to compensate for the frequency response of the electronics; and
(ii) to compensate for the hydrodynamic attenuation of the pressure fluctuations.
These are combined into a single correction factor $C$, which is dependent upon $T_z$, and upon the depth of the pressure transducer below the mean water level. The correction factor is

$$C = 0.83 \left(1 + \frac{1}{77.44 \omega^2}\right)^{\frac{3}{2}} \exp \left[\frac{2.5 \omega^2 d}{g}\right]$$

where $d$ is the depth of the pressure transducer,

and $\omega = \frac{2\pi}{T_z}$.

The value of $H_s$ calculated by the method outlined in Appendix II is multiplied by $C$ to obtain the corrected value of $H_s$.

A table is given below showing the values of $C$ for various values of $T_z$ (and for each of the two values of $d$ for the lightvessels on the Dowsing station during the period covered by this report):
- $d = 1.46$ metres for the 1970/71 data,
- and $d = 0.88$ metres for the 1975/79 data).

<table>
<thead>
<tr>
<th>$T_z$(sec)</th>
<th>$C$ $d=0.88m$</th>
<th>$C$ $d=1.46m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>3.455</td>
<td>8.774</td>
</tr>
<tr>
<td>3.0</td>
<td>2.240</td>
<td>4.278</td>
</tr>
<tr>
<td>3.5</td>
<td>1.726</td>
<td>2.777</td>
</tr>
<tr>
<td>4.0</td>
<td>1.458</td>
<td>2.099</td>
</tr>
<tr>
<td>4.5</td>
<td>1.301</td>
<td>1.734</td>
</tr>
<tr>
<td>5.0</td>
<td>1.199</td>
<td>1.514</td>
</tr>
<tr>
<td>6.0</td>
<td>1.081</td>
<td>1.271</td>
</tr>
<tr>
<td>7.0</td>
<td>1.019</td>
<td>1.148</td>
</tr>
<tr>
<td>8.0</td>
<td>0.984</td>
<td>1.078</td>
</tr>
<tr>
<td>10.0</td>
<td>0.952</td>
<td>1.009</td>
</tr>
<tr>
<td>12.0</td>
<td>0.946</td>
<td>0.985</td>
</tr>
<tr>
<td>15.0</td>
<td>0.961</td>
<td>0.986</td>
</tr>
<tr>
<td>20.0</td>
<td>1.021</td>
<td>1.036</td>
</tr>
</tbody>
</table>
LOCATION MAP OF DOWSING WAVE RECORDER

FIG 1.1
FIG 10.4(A)

DOMINOES LV 5/70-4/71, 11/75-10/76, 8/77-7/79

TIME SERIES OF HS

AUG 1970

JUL 1970

JUN 1970

MAY 1970

DATE

DATE

DATE

DATE

Hs (METERS)

Hs (METERS)

Hs (METERS)

Hs (METERS)
TIME SERIES OF HS

DEC 1970

NOV 1970

OCT 1970

SEP 1970

DATE

DATE

DATE

DATE

Hs (METERS)
FIG 2.4(C)
DOMINICAN 5/70-4/71, 11/75-10/76, 8/77-7/79
TIME SERIES OF HS

APR 1977

MAR 1977

FEB 1977

JAN 1977
FIG 10c(D)
DOMSING LV 5/70-4/71, 7/75-10/76, 8/77-7/79

TIME SERIES OF HS

FEB 1976

DATE

JUN 1976

DATE

DEC 1975

DATE

NOV 1975

DATE

Hs (METERS)
FIG 2.4 (F)

DOMSING LV 5/70-4/71, 11/75-10/76, 8/77-3/79

TIME SERIES OF HS

OCT 1976

SEP 1976

AUG 1976

JUL 1976
FIG 10.4(C)

DOMING L.V. 5/70-4/71, 11/75-10/76, 8/77-7/79

TIME SERIES OF HS

NOV 1977

DATE

2.0

1.5

1.0

0.5

0

Hs (METERS)

SEP 1977

DATE

2.0

1.5

1.0

0.5

0

Hs (METERS)

AUG 1977

DATE

2.0

1.5

1.0

0.5

0

Hs (METERS)
Fig. 1.4.(H)

DOWNSING LV
S, 7.70-4.71, 11.765-10.76, 8.77-7.79

Time series of Hs
TIME SERIES OF HS

JUL 1978

DATE

HS (METRES)

JUN 1978

DATE

HS (METRES)

MAY 1978

DATE

HS (METRES)

APR 1978

DATE

HS (METRES)
FIG 1.4(K)

DOMINO L.V 5/70-4/71, 11/75-10/76, 8/77-7/79

TIME SERIES OF Hs

MAR 1979

DATE

FEB 1979

DATE

JAN 1979

DATE

DEC 1978

DATE

Hs (Metres)
FIG 1.4(L)
DOMING LV 5/70-4/71, 11/76-10/76, 8/77-7/79
TIME SERIES OF HS

JUL 1979

DATE

Hs (METERS)

JUN 1979

DATE

Hs (METERS)

MAY 1979

DATE

Hs (METERS)

APR 1979

DATE

Hs (METERS)
FIG 2.1
SPRUNHEAD JAN 1957 - JUL 1979
MIND SPEED
MEAN AND STANDARD DEVIATION OF THE MONTHLY MEAN OF
FIG 2.2(B)

SPRUNGFIELD NOV 1961 - OCT 1976
MEAN OF N LARGEST VALUES OF WIND SPEED

METRES/SEC

MISSING DATA
MAXIMUM

0
10
20
30
40
FIG 2.2(c)
SURLYHEAD AUG 1964 - JUL 1979
MEAN OF N LARGEST VALUES OF WIND SPEED

[Graph showing wind speed data with various conditions and time periods.]
DOMISING LV 5/70-4/73, 11/75-10/76, 8/77-4/79
MONTHLY MEANS OF HS FOR EACH YEAR

FIG 3.10.1

JAN FEB MAR APR MAY JUNE JULY AUG SEP OCT NOV DEC

1978/9 ○
1977/8 ×
1975/6 +
1970/1 *
mean all data •
FIG 3.12

DOMINANT LV 5/70-4/71, 11/75-10/76, 8/77-7/79

MEAN OF N LARGEST VALUES OF H5
DOMINO FALL 5/80-4/81, 11/79-10/76, 8/77-7/79 AUTUMNS

PERCENTAGE OCCURRENCE HISTOGRAM

HS (METRES)

PERCENTAGE OCCURRENCE

20
18
16
14
12
10
8
6
4
2
0

50 OR MORE CASES
500 OR MORE OBSERVATIONS
 Doming LV 5/70-4/71/72-73/74-75/76-77/78-79 WINTER

PERCENTAGE OCCURRENCE HISTOGRAM

HS (METERS)

2359 VALID OBSERVATIONS
PERCENTAGE OCCURRENCE HISTORY

HS (METRES)

PERCENTAGE OCCURRENCE

DOWSING LV 5/70-4/71, 11/75-10/76, 8/77-7/79

1350 DOWSING OBSERVATIONS
221 OF WHICH WERE CALMS

FIG 3.2.1.5
DOWNSING LV 5/70-4/71, 11/75-10/76, 8/77-7/79 - SPRINGS

FIG 3.2.2.1

PERCENTAGE EXCEEDANCE OF $H_s$ AND $H_{max}$ (3HR)

2855 VALID OBSERVATIONS
42 OF WHICH ARE CALMS

$H_s$
Percentage Exceedance of $H_s$ and $H_{MAX(3HR)}$

Wave Height (m)

Percentage Exceedance

143 of which are cold
2261 valid observations
Figure 3.3.2.3

Percentage Exceedance of $H_s$ and $H_{\text{MAX}}(395)$

Wave Height (m)

2885 valid observations, 5% of which are calm

Dowsing LV: 5/70-4/71, 11/75-10/76, 8/77-7/79 - Autumn
PERCENTAGE EXCEEDANCE OF $H_S$ AND $H_{MAX(3HR)}$
Percentage Exceedance of $H_s$ and $H_{max(3hr)}$

Wave Height (m)
Cumulative distribution of wave height, HS

11300 valid observations

Weibull scale

\[ A = 0.32 \]
DOMINING LV 5/70-4/71, 11/75-10/76, 8/77-7/79

Fig 3.3.2

Cumulative distribution of wave height, HS

Wave Height (m)

Return Period (Years)

Probability of Non-exceedance

Fisher-Tippet Scale

See Section 3.3

1750 valid observations
FIG. 3.3.3

DOMINIC LAU 5/70-4/71, 11/75-10/76, 8/77-7/79

CUMULATIVE DISTRIBUTION OF WAVE HEIGHT, HS

WAVE HEIGHT (M)

RETURN PERIOD (YEARS)

FISHER-TIPPETT III SCALE

17520 VALID OBSERVATIONS

A = 14°0
FIG. 3.4

RECORDING L/WAVE HEIGHT - INDIVIDUAL WAVE MODEL

WAVE HEIGHT (M)

RETURN PERIOD (YEARS)

(AVERAGE PERIOD) = 0.194
STEEPLESS = 1.18
Percentage occurrence histogram

TZ (seconds)

This analysis uses data from 11/3/79 to 10/6/79, 8/77 to 7/78.
Percentage Occurrence Histogram

TZ (Seconds)

Percentage Occurrence

This analysis (1431 cases excluded or 50% valid observations)
Figure 3.6.7

Percentage occurrence histogram

TZ (seconds)

Percentage occurrence

The graph shows the percentage occurrence of different time intervals for TZ (seconds), ranging from 20 to 0. Each interval is marked with a bar indicating the frequency of occurrence. The y-axis represents the percentage occurrence, while the x-axis shows the time intervals in seconds.
DOMINGUS LV 5/70-4/71, 11/75-10/76, 8/77-4/79

PERCENTAGE OCCURRENCE HISTOGRAM

TZ (SECONDS)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

PERCENTAGE OCCURRENCE

THIS ANALYSIS
(337 CUMULUS EJECTED FROM
1183 VALID OBSERVATIONS)
FIG 3.5.1.05

DOSSING LUT 5/70-4/71, 11/75-10/76, 8/77-7/79

SCATTER PLOT OF HS AND T2

T2(SECONDS)

11,330 VALID OBSERVATIONS

<table>
<thead>
<tr>
<th>KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>&amp; NO. OF OCCURRENCES (1 PTF)</td>
</tr>
<tr>
<td>11 PARTS PER THOUSAND (PTF)</td>
</tr>
</tbody>
</table>

11,330 VALID OBSERVATIONS

(see section 1.2)
FIG 3.6.1.3

CALMS

DOMINGUE L5/70-4/71 11/71-10/76 8/77-4/79 AUTUMNS

HS THRESHOLD (M)

PERCENTAGE OF TIME OCCUPIED BY EVENT

TOTAL DURATION = 6543 HRS

PERCENTAGE OF TIME

STANDARD DEVIATION

MEAN DURATION

KEY TO PERSISTENCE EVENTS

DURATION OF EVENT (HR)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
0 50 100 150 200 250 300 350 400 450 500
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400 3600 3800 4000 4200 4400 4600 4800
FIG 3.6.2.1

STORMS

DOWSING LU 5/70-4/71 11/71-10/76 8/77-7/79 SPRINGS

HAS THRESHOLD (M)

PERCENTAGE OF TIME OCCUPIED BY EVENT

TOTAL DURATION = 6573 HRS

PERCENTAGE OF TIME

STANDARD DEVIATION AND MEAN DURATION

KEY TO PERSISTENCE EVENTS
STORMS
DOMINOE L5/70-4/71 11/75-10/76 8/77-7/79 SUMMERS
HS THRESHOLD (M)

PERCENTAGE OF TIME OCCUPIED BY EVENT

TOTAL DURATION = 6585 HRS

- = PERCENTAGE OF TIME

- - - - - STANDARD DEVIATION

M = MEAN DURATION

KEY TO PERSISTENCE EVENTS

DURATION OF EVENT (HR)
FIG 3.6.2.3

STORMS

DOMINIC 5/70-4/71 11/71-7/72 8/72-7/73 10/73-10/74 AUTUMN

Hs Threshold (m)

PERCENTAGE OF TIME OCCUPIED BY EVENT

TOTAL DURATION = 6543 HRS

--- PERCENTAGE OF TIME

--- STANDARD DEVIATION

--- MEAN DURATION

KEY TO PERSISTENCE EVENTS

DURATION OF EVENT (HR)
FIG 3.6.2.4

STORMS DOMINATING LATE 1967-68 AND 1969-70 WINTERS

HS THRESHOLD (M)

PERCENTAGE OF TIME OCCUPIED BY EVENT

TOTAL DURATION = 855.2 HRS

PERCENTAGE OF TIME

STANDARD DEVIATION AND MEAN DURATION

KEY TO PERSISTENCE EVENTS

DURATION OF EVENT (HR)
STSORMS
DOMING LV 5/70-4/71 11/71-10/76 8/77-3/79

FIG 3.6_2.5

HS THRESHOLD (M)

PERCENTAGE OF TIME OCCUPIED BY EVENT

DURATION OF EVENT (HR)

TOTAL DURATION = 34830 HRS

PERCENTAGE OF TIME

STANDARD DEVIATION

AND

MEAN DURATION

KEY TO PERSISTENCE EVENTS

000

005

010

015

020

025

030

035

040

045

050

055

060

065

070

075

080

085

090

095

100