THE RESPONSE OF ELECTRO-OPTICAL TURBIDITY METERS TO COHESIVE SEDIMENTS

T J SMITH

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1. INTRODUCTION

Electro-optical turbidity meters (EOTMs) have been used for many years as a method of determining the concentrations of suspended particulate matter in the marine environment. Various single or multi-channel EOTMs have been described using constant intensity or pulsed light sources and single or dual beams, for example Jones and Wills, 1956; Thorn, 1975; Basano et al, 1976; Brenninkmeyer, 1976; Nakato et al, 1976; Shepherd, 1978; Ohm, 1979; Smith et al, 1980. In a constant intensity EOTM the turbidity is a function of the magnitude of the light intensity at the sensor. However, in a pulsed light EOTM the turbidity is a function of the amplitude of the light intensity fluctuations about an arbitrary mean. Thus, constant intensity EOTMs respond to any ambient light reaching the sensor whereas pulsed light EOTMs are independent of ambient light as this only influences the mean intensity and not the high frequency (>10 Hz) fluctuations about the mean.

An EOTM responds to the solids concentration, size, size distribution, shape and density of the suspended particles and, consequently, only where these last four parameters are constant does the instrument respond solely to changes in solids concentration. In many applications, the suspended solids have been non-cohesive particles whose size, size distribution, shape and density could be easily characterised and which were invariant throughout the suspension. However, EOTMs have been increasingly used to monitor the concentrations of cohesive sediments in rivers and estuaries (Thorn, 1975; Kirby and Parker, 1975). These sediments flocculate and hence, while the constituent particles of the flocs can be characterised by their size, size distribution, shape and density (eg Bryant and Williams, in press), the characterisation of the flocs themselves, ie the particles the EOTM observes, is dependent on the constituent particle characteristics and also on a number of external parameters, including the solids concentration, salinity, pH and local hydrodynamic conditions. Thus, in cohesive sediment suspensions the floc characteristics vary throughout the suspension and hence the influence of typical variations in floc size, size distribution, shape and density on the response of an EOTM must be investigated.

This report presents a theoretical analysis of the response of an EOTM to cohesive sediments. This is then shown to predict the measured response. The dependence of an EOTMs response to changes in parameters other than solids concentration when used with cohesive sediments, has been recognised for some time. Consequently, when EOTMs are used in the field they are calibrated in-situ. However, as the number and variability of the parameters affecting the EOTMs response is so large, it is often not possible to conduct a complete calibration.
Consequently, the theoretical analysis is used to investigate the implications of the use of EOTMs in field studies of fine sediment transport.

Finally, an analysis is presented of two potential sources of error in EOTMs: the intrusion of ambient light and lens contamination. This analysis is then used to derive a procedure for determining and correcting for these errors in the results of a single beam, constant intensity EOTM; a type in common use at IOS and other marine and hydraulic laboratories.

2. THE RESPONSE TO NON-COHESSIVE SEDIMENTS

2.1 Uniform Spherical Particles

The theoretical response of an EOTM to a homogeneous suspension of opaque, uniform, spherical particles is derived. The derivation is presented in full for later reference, although it is not original, having been first presented, albeit neglecting diffraction effects, by Rose (1950).

Consider a volume, $V$, containing $n^1$ identical, opaque, spherical particles of diameter, $d_p$, and density, $\rho_p$. The concentration, $C$, expressed as the mass of solids per unit volume of suspension is given by

$$C = \frac{n^1 V \rho_p}{V}$$

where $V$ is the volume of one particle. Thus

$$C = \frac{1}{6} \rho \pi d_p^3$$

where $n = \frac{n^1}{V}$ is the particle number density, i.e., the number of particles per unit volume of suspension. Note also that the solids volumetric concentration, $\phi$, defined as the total solids volume per unit volume of suspension is given by

$$\phi = \frac{1}{6} \pi d_p^3$$

and hence

$$\phi = C \rho_p^{-1}$$

Consider now an EOTM as sketched in Figure 1. Let the diameter of the source and sensor be $D_s$ and their separation be $L$. Assume that the sensor emits a
a parallel beam of light having a total intensity $I_0$. Consider a thin section perpendicular to the longitudinal axis of the light beam and let the length of this section parallel to the longitudinal axis of the light beam be $\delta x$. Let the particle concentration in the beam be $\phi$ and assume that the particle distribution, though random, is statistically homogeneous throughout the beam. Also assume that $\delta x$ is sufficiently thin and $\phi$ sufficiently small that no one particle obscures all or part of any other particle in the section when viewed along the axis of the system.

Thus, the number of particles in a section of the beam of length $\delta x$ and volume $V^1$ is $N$ where

$$N = n V^1 = \frac{3}{2} \phi \frac{D_s^2}{d} \delta x$$

from equation (3). The total cross-sectional area of the $N$ particles is $A$ where

$$A = \frac{3}{8} \pi D_s^2 \frac{\phi}{d} \delta x$$

and, hence, the particle blockage in the section, $B^1$, is given by

$$B^1 = \frac{\text{cross-sectional area of particles}}{\text{cross-sectional area of beam}}$$

ie

$$B^1 = \frac{3}{2} \frac{\phi}{d} \delta x$$

For small particles ($d_p \lesssim 1 \mu m$) light energy is diffracted around the particle and the light beam is reformed within a short distance behind the particle. Thus, the optically apparent blockage, $B$, is given by

$$B = K (d_p) B^1$$

where $K (d_p)$ is the extinction coefficient, a function of particle diameter.

For larger particles ($d_p \geq 4 \mu m$) a shadow zone is created behind each particle and the reduction in light intensity is proportional to the projected area of the particle such that the proportionality constant $K_0 \approx 1$. Thus, the optical blockage is given by

$$B = \frac{3}{2} K \frac{\phi}{d} \delta x$$
\[
\begin{align*}
K \left( \frac{d_p}{d_p} \right) &= \frac{d_p}{\lambda} \\
\text{where } K &= \\
\begin{cases}
K_0 & \text{if } d_p > 4 \mu m \\
0 & \text{if } d_p \leq 4 \mu m
\end{cases}
\end{align*}
\]

The functional form of $K$ is given by the Mie theory of light scattering, (see Allen 1975 for details).

If the light intensity entering the section from the source is $I^1$ and the light intensity emerging from the section is $I^1 + \delta I^1$ then

\[
\delta I^1 = -\delta \cdot I^1
\]

ie

\[
\delta I^1 = -\frac{3}{2} K_0 \frac{\Phi}{d_p} I^1 \delta x
\]

or in the limit $\delta x \to 0$

\[
\int_I \frac{d I^1}{I} = \begin{cases} L & -\frac{3}{2} K_0 \frac{\Phi}{d_p} \int_0 \delta x 
\end{cases}
\]

which leads to

\[
\frac{I_L}{I_0} = \exp \left\{ -\frac{3}{2} KL \frac{\Phi}{d_p} \right\}
\]

where $I_L$ is the light intensity at the sensor. Equation 11 describes the attenuation of a light beam of length $L$ by opaque, uniform spherical particles. In addition to the concentration, size and density of the particles the attenuation, and hence the response of the EOTM, is also dependent on the source-sensor separation. However, this is usually a constant for a particular EOTM.

2.2 Non-uniform, Non-spherical Particles

Homma and Horikawa (1963) extended Rose's (1950) analysis to include the effect of non-spherical particles having a non-uniform particle size described in terms of a discrete distribution. The analysis is presented in full for later reference.

Let $n_i$ denote the number of particles of effective spherical diameter, $d_p^{(i)}$, per unit length of the light path, let $N_i = n_i L$ denote the total number of particles of diameter $d_p^{(i)}$ in the light path and let $N$ denote the total number of particles in the light path.
Thus, for \( m \) particle size class intervals

\[
N = \sum_{i=1}^{m} N_i \quad \quad \quad n = \sum_{i=1}^{m} n_i
\]  

(12)

Let the mean effective particle diameter be \( \bar{d}_p \) where

\[
\bar{d}_p = \frac{1}{N} \sum_{i=1}^{m} N_i d_i
\]  

(13)

and define

\[
d_p^{(i)} = \bar{d}_p + \langle d_p^{(i)} \rangle
\]  

(14)

Then the volumetric concentration of particles in the light path, \( \phi \), is given by

\[
\phi = \frac{2}{3} \sum_{i=1}^{m} \frac{N_i (d_p^{(i)})^3}{D_s^2 L}
\]  

(15)

The expectation, \( E \) of any variable, \( \theta \) is defined as

\[
E(\theta) = \frac{1}{N} \sum_{i=1}^{m} N_i \theta_i
\]  

(16)

Thus, by definition

\[
\sum_{i=1}^{m} N_i (d_p^{(i)})^3 = N \ E (d_p^3)
\]

and hence

\[
\phi = \frac{2}{3} \frac{N \ E (d_p^3)}{D_s^2 L}
\]

from which

\[
N = \frac{3}{2} \frac{L D_s^2}{E (d_p^3)} \phi
\]  

(17)

The number of particles in a section of the beam perpendicular to its longitudinal axis having a length \( \delta x \) is \( N \delta x / L \) and the total projected area of these particles, \( A \) is given by
\[ A^1 = \frac{\pi}{4} \sum_{i=1}^{m} N_i \left( \frac{d_{p}^{(i)}}{d_p} \right)^2 \frac{\delta x}{L} \]  

Assume that the smallest particle diameter class interval \( d_{p}^{(i)} \) is greater than 4 \( \mu m \) such that the extinction coefficient is a constant, \( K_o \), for all particle size class intervals. Thus, the optically apparent projected area of particles in the section, \( A \), is given by

\[ A = \frac{\pi}{4} K_o \sum_{i=1}^{m} N_i \left( \frac{d_{p}^{(i)}}{d_p} \right)^2 \frac{\delta x}{L} \]  

Define the standard deviation of the particle size distribution, \( \sigma \), as

\[ \sigma^2 = \frac{1}{N} \sum_{i=1}^{m} N_i \left( \frac{d_{p}^{(i)}}{d_p} - \bar{d_p} \right)^2 \]  

and hence

\[ \sum_{i=1}^{m} N_i \left( \frac{d_{p}^{(i)}}{d_p} \right)^2 = N (\bar{d_p}^2 + \sigma^2) \]  

Thus, from equation (19)

\[ A = \frac{\pi}{4} K_o N \bar{d_p}^2 \frac{\delta x}{L} \left\{ 1 + \frac{\sigma^2}{\bar{d_p}^2} \right\} \]  

The optically apparent blockage, \( B \), then becomes

\[ B = \frac{K_o N \bar{d_p}^2}{D_s L} \left\{ 1 + \frac{\sigma^2}{\bar{d_p}^2} \right\} \delta x \]  

which on using equation (17) gives

\[ B = \frac{3}{2} \frac{K_o \phi}{d_p} \left\{ 1 + \frac{\sigma^2}{d_p^2} \right\} \delta x \left\{ E \frac{(d_p^3)}{d_p^2} \right\} \]  

However, as the particles are not spherical they may take up a preferred orientation, such that the blockage is less than that obtained from the effective spherical diameter. Thus, the actual apparent blockage, \( B_A \), is given by
\[ B_A = \alpha_p B \]  

(25)

where \( \alpha_p \) is the geometric shape factor such that, if \( a, b \) and \( c \) are the linear dimensions of the particle along three mutually perpendicular axes such that \( a \) is the smallest possible dimension, then the shape factor is defined as

\[ \alpha_p = a (b/c)^{-\frac{1}{3}} \]  

(26)

Proceeding as for the case of uniform, spherical particles gives the attenuation of the light intensity at the sensor as

\[ \frac{I_L}{I_o} = \exp\{-k_o \frac{\phi}{d_p}\} \]  

(27)

where

\[ k_o = \frac{3}{2} \alpha_p K_o L \frac{(1 + \sigma^2/d_p^2)}{E(d_p^3) d_p^3} \]  

(28)

Thus, for non-uniform sediments the response of an EOTM is also sensitive to the standard deviation of the distribution and to the expectation, \( E(d_p^3) \).

2.3 Effect of Sensor Characteristics

Equation (28) describes the attenuation of a light beam by opaque, suspended particles. However, an EOTM output is also dependent on the response characteristics of the sensor, two basic types of which are available:

(i) active
(ii) passive

Active sensors, for example photo-cells, photo-diodes and photo-transistors generate an emf related to the intensity of the incident light falling on the sensor whereas passive sensors, usually photo-resistors, change their electrical resistance. Both types of sensor have an operating range of light intensity in which their response is linear i.e.

(i) active sensors

\[ V^1 = F_A I_L \]  

(29)

for \( I_{A_1} \leq I_L \leq I_{A_2} \), where \( V^1 \) is the generated emf and \( F_A \) is a constant. Hence, if the electronic amplification is by a factor \( \beta \), the output voltage, \( V = \beta V^1 \) of an active sensor equipped EOTM is given by
\[
\frac{V}{V_0} = \exp \left\{ - k_o \frac{\phi}{d_p} \right\}
\]

where \( V_0 \) is the EOTM output voltage when \( \phi = 0 \)

(ii) passive sensors

\[
R = R_o - F_p (I_L - I_p)
\]

for \( I_{p1} \leq I_L \leq I_{p2} \) where \( R \) is the sensor resistance, \( R_o \) is the resistance at \( I_L = I_p \) and \( F_p \) is a constant. The sensor usually forms one arm of a simple bridge circuit and it can be shown that the output voltage of a passive sensor equipped EOTM is given by

\[
\frac{V}{V_1} = \frac{1 - \exp \left\{ - k_o \phi/d_p \right\}}{1 + G \exp \left\{ - k_o \phi/d_p \right\}}
\]

where \( V \to V_1 \) as \( I_L \to 0 \) (ie \( \phi \to 1 \)) and \( G \) is a constant which depends on the basic sensor resistance and those in the bridge circuit. For values of \( G \ll 1 \) which corresponds to low sensitivity, high operating range conditions, equation (32) reduces to

\[
\frac{V}{V_1} = 1 - \exp \left\{ - k_o \frac{\phi}{d_p} \right\}
\]

Equation (30) has been shown to predict the response of an EOTM for well sorted artificial and natural sediments with effective spherical diameters in the range 100 \( \mu \text{m} \leq d_p \leq 350 \ \mu\text{m} \) and for poorly sorted sands with diameters in the range 60 \( \mu \text{m} \leq d_p \leq 210 \ \mu\text{m} \) (Smith et al, 1980).

3. THE RESPONSE TO FLOCCULATED SEDIMENTS

3.1 Non-uniform, Non-spherical Floccs

The response of an EOTM to flocculated sediments is similar to its response to non-cohesive sediments (equations 30 and 33) provided that the volume fraction, the mean particle diameter and the particle shape and size distribution are representative of the flocs and not the flocs' constituent particles. Thus, define the apparent volume fraction, \( \Psi \), as
\[ \psi = \frac{\text{total volume of all flocs}}{\text{volume of suspension}} \]  \hspace{1cm} (34)

and the volume fraction of an individual floc, \( \phi_f \), as

\[ \phi_f = \frac{\text{volume of solids in a floc}}{\text{volume of floc}} \]  \hspace{1cm} (35)

Note that, by definition

\[ \phi = \phi_f \psi \]  \hspace{1cm} (36)

and that \( \psi \) is a function of the floc size distribution while \( \phi_f \) is a function of the floc diameter.

Using the definitions given by equations (34) to (36) and proceeding as in Section 2, the attenuation of a light beam of length \( L \) by cohesive sediment is given by

\[ \frac{I_L}{I_0} = \exp \left\{ -k_{fo} \frac{\psi}{d_f} \right\} \]  \hspace{1cm} (37)

where \( \bar{d}_f \) is the mean floc diameter,

\[ k_{fo} = \frac{3}{2} \alpha_f K_o L \frac{(1 + \sigma^2_f/\bar{d}_f^2)}{E(\bar{d}_f^3)/\bar{d}_f^3} \]  \hspace{1cm} (38)

\( \alpha_f \) is the floc shape factor and \( \sigma_f \) is the standard deviation of \( d_f \).

Thus, the response of an EOTM to cohesive sediments is given by (cf Section 2.3)

(i) active sensors

\[ \frac{V}{V_0} = \exp \left\{ -k_{fo} \frac{\psi}{d_f} \right\} \]  \hspace{1cm} (39)

(ii) passive sensors

\[ \frac{V}{V} = \frac{1 - \exp \left\{ -k_{fo} \frac{\psi}{\bar{d}_f} \right\}}{1 + G \exp \left\{ -k_{fo} \frac{\psi}{\bar{d}_f} \right\}} \]  \hspace{1cm} (40)

Equations (39) and (40) in themselves are of little use as \( \psi \) and \( \bar{d}_f \) are functions, as yet unknown, of the solids volume fraction \( \phi \) as well as a number of other additional parameters, for example the local hydrodynamic and chemical conditions of the suspending medium and the mineralogy and heterogeneity of the
flocs constitute particles. Consequently, in order to understand the response of an EOTM to cohesive sediments it is necessary to be able to parameterise the effects of these variables on the degree of flocculation in a particular suspension.

3.2 A Simple Model for Sediment Flocs in a Turbulent Shear Flow

A simple semi-empirical model for a floc in a turbulent shear flow has been developed to predict the mean properties $(\bar{d}_f, \psi)$ of a flocculated suspension given the solids concentration and the local hydrodynamic and chemical conditions. This model was derived with the aims of:

(i) identifying those parameters to which the response of an EOTM is sensitive under typical laboratory and field conditions;
(ii) explaining the difference between laboratory calibrations and in-situ field calibrations.

For a flocculated suspension in which the primary particles are of uniform mineralogy and size, $d_p$, at a solids volume concentration, $\phi$, the mean floc diameter, $\bar{d}_f$, and the floc volume concentration, $\psi$, are functions of $\phi$, the representative shear rate across the flocs, $\gamma$, the kinematic viscosity of the suspending medium, $\nu$, and the chemical state (pH, ionic strength etc) of the suspending medium. In marine systems the variability in the chemistry of the suspending medium is sufficiently small that its effect is small in comparison with the effects of the variability in the other parameters and, consequently it is neglected for the purpose of this simple analysis.

For constant chemical conditions in the suspending medium, assume that

$$\psi = f_1(\phi) \ h_1(\Gamma)$$

$$\frac{\bar{d}_f}{d_p} = f_2(\phi) \ h_2(\Gamma)$$

where $\Gamma = \gamma \ d_p^2 / \nu$ is a non-dimensional shear rate and $f_i, h_i$ ($i = 1, 2$) are, as yet, unknown functions.

The simplest approximations to these functions are

$$f_i = a_i \phi^{m_i} \quad h_i = b_i \phi^{n_i} \quad (i = 1, 2)$$

where $a_i, b_i, m_i$ and $n_i$ are constants. However, equations (43) are unrealistic as they do not satisfy the limiting conditions on $f_i, h_i$ as $\phi \to 0, 1, \Gamma \to 0, \infty$.

The limiting conditions to be satisfied by equations (41) and 42) under steady state conditions are:
\[(i) \quad \phi \to 0 \quad \psi \to 0 \quad \bar{d}_f + d_p\]

\[(ii) \quad \phi \to 0 \ (I) \quad \psi \to 0 \ (I) \quad \bar{d}_f + d_1\]

\[(iii) \quad \Gamma \to 0 \quad \psi \to 0 \ (I) \quad \bar{d}_f + d_2\]

\[(iv) \quad \Gamma \to \infty \quad \psi \to 0 \quad \bar{d}_f + d_p\]

where \(d_1\) and \(d_2\) are floc diameters defined by the above limiting conditions.

The simplest general functional forms of \(f_1\) which satisfy the above limiting conditions are:

\[f_1 = a_1 \phi \left\{ I + \frac{(1 - a_1)}{a_1} \phi^{m_1} \right\}\]  \hspace{1cm} (44)

\[f_2 = 1 + \frac{(d_1 - d_p)}{d_p} \phi^{m_2}\]  \hspace{1cm} (45)

where \(m_1, m_2 > 0, a_1 > 1\) to be physically realistic.

Similarly, Williams and James (1978) showed that the simplest functional form of \(h_1\) satisfying the above conditions can be written as

\[h_1 = \phi + \frac{(1 - \phi)}{1 + b_1 \Gamma n_1}\]  \hspace{1cm} (46)

\[h_2 = 1 + \frac{(d_2 - d_p)}{d_p (1 + b_2 \Gamma n_2)}\]  \hspace{1cm} (47)

where \(n_1, n_2, b_1\) and \(b_2 > 0\) to be physically realistic.

Consider now the role of the individual terms in equations (44) to (47).

Term I represents the increase in floc volume fraction over the solids volume fraction \((a_1 > 1)\) due to the incorporation of pore water within the flocs. This yields a greater effective particle volume for the flocculated system. Term II represents the perturbation to the linear increase in \(\psi\) with \(\phi\) to account for the volume of the suspension occupied by the flocs.
This term is only important at high solids volume concentrations. Thus, for low concentration ($\phi \ll 1$) cohesive sediment suspensions typical of those in the natural environment, equation (44) can be approximated by

$$f_1 \approx a_1 \phi$$  \hspace{1cm} (48)

Term III represents the minimum floc diameter permissible, i.e., the diameter of the constituent particles. Term IV then represents the increase in effective particle diameter due to flocculation. Except at very low concentrations ($\phi < 10^{-4}$) or very high shear rates ($\Gamma_v \bar{d}_p^2 > 100$ s$^{-1}$) marine cohesive sediment suspensions are strongly flocculated so that, in general $(d_1 - d_p) \phi^{m_2} \gg 1$ and hence equation (45) can be approximated by

$$f_2 \approx \frac{(d_1 - d_p) \phi^{m_2}}{d_p}$$  \hspace{1cm} (49)

Term V represents the minimum floc solids volume concentration after the complete breakdown of flocs into their constituent particles under the action of hydrodynamic forces. This condition is only achieved at very high shearing rates, generally much greater than those encountered in the environment. Hence, term V is very much smaller than term VI which represents the volume fraction of the flocculated suspension under environmental shear rates. Equation (46) can then be approximated by

$$h_1 \approx \frac{(1 - \phi)}{1 + b_1 \Gamma^{n_1}}$$  \hspace{1cm} (50)

Finally, term VII represents the minimum floc diameter after complete breakdown of the flocs by hydrodynamic forces and is much less than the mean floc diameter represented by term VIII. Therefore, equation (47) can be written as

$$h_2 \approx \frac{(d_2 - d_p)}{d_p (1 + b_2 \Gamma^{n_2})}$$  \hspace{1cm} (51)

The simple model for flocculated sediments can now be written using equations (48) to (51) as

$$\psi \approx \frac{a \phi (1 - \phi)}{1 + b_1 \Gamma^{n_1}}$$  \hspace{1cm} (52)

$$\frac{\bar{d}_f}{d_p} \approx \frac{(d_1 - d_p) (d_2 - d_p)}{d_p^2 (1 + b_2 \Gamma^{n_2})} \phi^{m_2}$$  \hspace{1cm} (53)

Under environmental and laboratory conditions $d_1 \approx d_2 > d_p$ and $\phi \ll 1$  Hence equations (52) and (53) can be taken to be
\[ \psi \simeq \frac{a_1 \phi}{1 + b_1 \phi^n} \]  

(54)

\[ \frac{\bar{d}_f}{d_p} \simeq \left( \frac{d_1}{d_p} \right)^2 \frac{\phi^{m_2}}{1 + b_2 \phi^{n_2}} \]  

(55)

Equations (54) and 55) represent the empirical model adopted to study the response of an EOTM to cohesive sediment. However, at this stage it is interesting to note that, using equation (36), equation (54) predicts that \( \bar{d}_f \) is independent of \( \phi \) which implies that, as \( \bar{d}_f \) increases with increasing \( \phi \) (\( m_2 > 0 \) by definition), the floc density is independent of the floc size.

Values of the empirical constants:

Stevenson (1972) considered the results of Camp (1968) and Lagvankar and Gemmel (1968) and concluded that

\[ n_1 = n_2 = \frac{2}{3} \]  

(56)

Previous experimental calibrations of EOTMs with cohesive sediments have shown that \( \psi^d \bar{d}_f \) increases as \( \phi \) increases (Thorn, 1975) and hence \( (1 - m_2) > 0 \) and as, by definition, \( m_2 > 0 \), we require \( 0 < m_2 < 1 \).

Data presented by Ward and Chickwanha (1980) for three different fine sediment samples having particle sizes in the ranges \(<50 \, \mu m, 6-8.5 \, \mu m and 12-18 \, \mu m\) lead to the conclusion

\[ m_2 \simeq 0.07 \]  

(57)

This implies that the floc diameter is relatively insensitive to solids concentration, a result recently confirmed over a wide concentration range by the in-situ settling velocity data from the Severn Estuary quoted by Odd (1981).

The value of the constant \( a_1 \) has been estimated for kaolinite suspensions (James and Williams, 1978) and for natural cohesive sediments (Bryant and Williams, in press) and can be taken to be

\[ 5 \leq a_1 \leq 10 \]  

(58)

Finally, Williams and James (1978) demonstrated that for a flocculated kaolinite suspension at 2% by volume solids the constants \( b_1 \) and \( b_2 \) were given by

\[ b_1 \simeq 0.065 \, \sqrt{2/3} \, d_p^{-4/3} \]  

\[ b_2 \simeq 19.2 \, \sqrt{2/3} \, d_p^{-4/3} \]  

(59)

where the numerical values have dimensions of \( s^{2/3} \). The floc diameter for the same suspension in the limit \( \Gamma > 0 \) was taken to be \( d_2 \simeq 10^4 \, \mu m \) and as it is assumed that \( d_1 \simeq d_2 \) we have \( d_1 \simeq 10^4 \, \mu m \). Note, however, that this applies to a suspension at
very high solids volume concentration (\(\phi \sim 2\%\)). At lower concentrations more
typical of the natural environment this is probably reduced by an order of magnitude
and hence for the purpose of this report the value of \(d_1\) will be taken to be
\[
d \approx 10^3 \mu m
\]

(60)

3.3 Effect of Flocculation on an EOTM Response

Combining the theoretical expression for the attenuation of a light beam by
flocculated sediments, equation (37), with the simple floc model given by
equations (54) and (55) leads to
\[
\frac{I_L}{I_0} = \exp \left\{ - k_{fo} a_1 \frac{d_p}{d_1} \phi^{1-m_2} \frac{1 + b_2 \Gamma^{2/3}}{1 + b_1 \Gamma^{2/3}} \right\}
\]

(61)

At typical environmental and laboratory shear rates it can be shown that
\(b_1 \Gamma^{2/3} < 1\) and \(b_2 \Gamma^{2/3} >> 1\). Hence, to a first approximation equation (61) becomes
\[
\frac{I_L}{I_0} = \exp \left\{ - k_{fo} a_1 \frac{d_p}{d_1} b_2 \phi^{1-m_2} \frac{2}{3} \right\}
\]

(62)

which, when written in the form
\[
\frac{I_L}{I_0} = \exp \left\{ - \kappa_o \frac{\phi^{1-m_2}}{d_1} \right\}
\]

(63)

where \(\kappa_o = k_{fo} a_1 b_2 \left( \frac{d_p}{d_1} \right) \Gamma^{2/3} \)

(64)

is very similar to the form for non-cohesive sediments, equation (27), albeit with
two important differences. Firstly, the calibration "constant" \(\kappa_o\) is a function of
the hydrodynamic shear rate, \(\Gamma\), and secondly the volume fraction is raised to a
power slightly less than unity. This result is important as it shows that, with
the exception of a region very close to the bed where \(\Gamma\) varies significantly, an
EOTM responds predominantly to changes in mass concentration of suspended fine sediment

Finally, Table 1 shows an example of the comparison between the predicted
response of an EOTM to flocculated sediments given by equation (40) with

\[
k_{fo} \frac{\psi}{d_F} = \kappa_o \frac{\phi^{0.97}}{d_1}
\]

(65)
as given by equations (63) and (64) with constant shear rate experimental calibrations at low and high instrument sensitivities. It can be seen that the theoretical response accurately reproduces the observed response.

4. EFFECT OF AMBIENT LIGHT AND LENS CONTAMINATION

4.1 Introduction

In field use the lenses of EOTMs can quickly become contaminated by dirt and normal marine growth enhanced by the light and heat output of the instrument. This changes its calibration characteristics. Frequent cleaning of the lenses minimises this problem but a constant check on the calibration is required as lens contamination is very rapid in the early stages. Similarly, when used in near surface waters or in deeper water at low sediment concentrations, the sensor can detect ambient light. Hence, variations in ambient light can be interpreted as variations in sediment concentration. Ambient light effects can be eliminated by using a pulsed light source and laboratory and field EOTMs based on this principle have been developed (Basano et al, 1976; Shepherd, 1978; Smith et al, 1980). However, the EOTMs used by the Institute of Oceanographic Sciences have a constant intensity source with a single beam and hence are sensitive to ambient light.

Even in environments with low concentrations of suspended fine sediment $100 \leq \rho \leq 1000 \text{ gm}^{-3}$ ambient light penetrates only a short distance into the water column. However, it is still necessary to be able to determine the depth of penetration and to estimate its effect and that of lens contamination on the calibration characteristics of an EOTM so that any errors can be estimated and, if necessary, corrected.

4.2 An Analysis of the Combined Effects of Ambient Light and Lens Contamination

Consider the optical arrangement of constant intensity source and sensor shown in Figure 2. The light intensity emitted by the source, $I_E$, is attenuated by the contamination on lens 1 according to Lambert's Law for filters (Longhurst, 1957) such that the intensity is reduced to a proportion $e_1$ of the intensity emitted by the source. Similarly, the light intensity from the source falling on the lens in front of the sensor, $I_L$, is attenuated by the contamination of lens 2 such that a proportion $e_2$ of the light intensity arriving at the lens is transmitted through to the sensor. In addition, the ambient light intensity, $I_A$, at lens 2 is similarly attenuated by the factor $e_2$. Therefore, the total light intensity arriving at the sensor, $I_T$, is given by
\[ I_T = e_2 (I_L + I_A) \]  \hspace{1cm} (66)

where \( I_L \) is dependent on the attenuation of the light beam due to the suspended sediment in the light path. Equations (27) and (63) can be written in the general form,

\[ \frac{I_L}{I_o} = \exp \{ -k_o^{(1)} L \theta \} \]  \hspace{1cm} (67)

where, for cohesionless sediments, \( k_o^{(1)} = k_o / d_p \), \( \theta = 1 \) and for cohesive sediments \( k_o^{(1)} = k_o / (d_L) \), \( \theta \approx 0.93 \). Thus with \( I_o = e_1 I_E \) we have from equations (66) and (67)

\[ \frac{I_T}{I_E} = e_1 e_2 (\frac{I_A}{I_E} + \exp \{ -k_o^{(1)} L \theta \}) \]  \hspace{1cm} (68)

If ambient light levels are negligible (\( I_A / I_L \ll 1 \)) then the effects of lens contamination can be eliminated by using two light beams of different pathlength taken from the same source (Thorn, 1975). However, with a single beam EOTM a procedure for determining the factors \( e_1 \) and \( e_2 \) is required as well as an estimate of the ambient light intensity at the measurement depth.

4.3 Procedure for Determining the Attenuation Coefficients \( e_1 \) and \( e_2 \)

Define the following values of light intensity, determined after the instrument has been recovered:

(a) Before the lenses are cleaned (\( e_1, e_2 \ll 1 \))

(i) \( I_T^{(1)} \), the clear water (\( \theta = 0 \)) reading with the instrument screened from ambient light (\( I_A = 0 \)).

Hence, from equation (68)

\[ I_T^{(1)} = e_1 e_2 I_E \]  \hspace{1cm} (69)

(ii) \( I_T^{(2)} \), the clear water (\( \theta = 0 \)) reading with sensor screened from the source (\( I_E = 0 \)) but exposed to ambient light (\( I_A = I_{A_0} \), the ambient light intensity at the surface in clear water).

Hence from equation (68)

\[ I_T^{(2)} = e_2 I_{A_0} \]  \hspace{1cm} (70)

(b) After cleaning the lenses (\( e_1 = e_2 = 1 \)), repeat the above measurements to give \( I_T^{(3)} \) and \( I_T^{(4)} \) respectively where from equation (68)

\[ I_T^{(3)} = I_E \]  \hspace{1cm} (71)
\[ I_T^{(4)} = I_{Ao} \]  \hspace{1cm} (72)

The attenuation coefficients \( e_1 \) and \( e_2 \) can now be obtained from equations (69) to (72) as
\[ e_1 = \frac{I_T^{(1)}}{I_T^{(2)}} \frac{I_T^{(4)}}{I_T^{(3)}} \]  \hspace{1cm} (73)
\[ e_2 = \frac{I_T^{(2)}}{I_T^{(4)}} \]  \hspace{1cm} (74)

Using a suitable equation for the sensor response characteristics discussed in section 2.3, equations (73) and (74) can be rewritten in terms of the EOTM output voltage and the coefficients \( e_1 \) and \( e_2 \) obtained.

4.4 Depth of Penetration of Ambient Light

Ambient light intensity varies with depth due to its attenuation by the suspended solids in the water column. Hence, in the near surface waters an EOTM responds to the attenuation of the emitted light plus the local ambient light level, \( I_A \). This is less than the ambient light intensity at the surface by a factor of \( \exp \left\{ -k_o^{(1)} \bar{z} \bar{\phi} \right\} \), where \( \bar{z} \) is the depth below the surface and \( \bar{\phi} \) is the mean solids volume fraction above the depth \( \bar{z} \). In the absence of lens contamination effects, the light intensity measured by an EOTM at a depth \( \bar{z} \) and local solids concentration \( \phi(\bar{z}) \) is given by
\[ \frac{I_T}{I_E} = \frac{I_{Ao}}{I_E} \exp \left\{ -k_o^{(1)} \bar{z} \bar{\phi} \right\} + \exp \left\{ -k_o^{(1)} L \phi \right\} \]  \hspace{1cm} (75)

where \( \bar{\phi} = \frac{1}{\bar{z}} \int_{0}^{\bar{z}} \phi(\bar{z}) \, d\bar{z} \)

If the attenuation of the light beam is assumed to be due to the suspended solids alone, then the apparent solids volume concentration at depth \( \bar{z} \), \( \phi \), can be obtained from equation (75) as
\[ \phi = \bar{\phi} - \frac{1}{k_o^{(1)} L} \ln \left[ 1 + \frac{I_{Ao}}{I_E} \exp \left\{ -k_o^{(1)} \left( \bar{z} \bar{\phi} - L \phi \right) \right\} \right] \]  \hspace{1cm} (76)
Hence, the effect of ambient light is to cause the observed concentration to be less than the actual solids concentration as the second term on the RHS of equation (76) is always positive.

Equation (76) can also be used to determine the depth $\zeta$, below which ambient light effects will be negligible. In general, $I_{A0} < I_E$ and hence ambient light effects will be negligible for $\exp \{ -k_o^{(1)} (2 \tilde{\phi} - 1 \tilde{\phi}) \} \approx 0.01$. In near surface waters, $\tilde{\phi} \approx \phi$ and, with the exception of high concentrations ($\phi > 0.001$ for silt/clay particles) $\zeta >> 1$. Thus, the depth below which ambient light effects are negligible is given by

$$\zeta \approx \frac{4.6}{k_o^{(1)} \phi}$$

with $k_o^{(1)}$ being obtained from the EOTM calibration.

5. CONCLUSIONS

The theoretical response of an electro-optical turbidity meter to cohesive sediments has been discussed. To aid this discussion, a simple model for the size and volume fraction of flocs in a turbulent shear flow was developed. The effect of ambient light and lens contamination on the performance of an EOTM has also been considered. A method of minimising and correcting for these potential errors when using a single beam system has been proposed. Finally, the potential depth of ambient light penetration has been determined.

On the basis of the above analysis the following conclusions have been reached:

(i) The calibration of an EOTM for flocculated sediments is sensitive to local hydrodynamic conditions and therefore, must be calibrated in-situ.

(ii) A single beam EOTM is, in principle, a suitable instrument for determining suspended cohesive sediment concentrations in marine systems.

(iii) The magnitude of all the potential systematic errors in the system can be established and, if necessary, suitable corrections can be made.

(iv) There is an insignificant variation in EOTM calibration within the water column due to the variation in hydrodynamic shear rate, such that rapid continuous vertical turbidity profiles record variations in suspended solids concentration only.

ACKNOWLEDGEMENT

I wish to thank Dr R Kirby and Dr W R Parker for their encouragement and advice without which this work could not have been completed.
REFERENCES


Figure 1: Schematic diagram of an electro-optical turbidity meter

Figure 2: Schematic diagram of an electro-optical turbidity meter including lens contamination and ambient light effects
Table 1.

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Comparison between theoretical response and observed response of EOTM to flocculated sediments at constant shear rate.