THE EFFECTS OF WIND WAVES ON SEA LEVEL AT THE COAST

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BIDSTON

The effects of wind waves on sea level at the coast

by

I.D. James

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CONTENTS

SUMMARY

The influences of wind waves on coastal sea levels range in frequency from that of the waves themselves to steady mean set-up, including possibly strong surf beat at 30s to several minute periods. These wave effects have generally been neglected in surge forecasts because their high spatial variability along the coast makes them difficult to predict. They can be important : set-up on beaches amounts to about a fifth of the offshore significant wave height, and surf beat oscillations can be comparable with the wave height. Set-up not only affects the measurement of mean sea level, but through its dependence on depth can modulate the tidal signal, even inside a harbour.

Recent developments in theory and observation include the application of non-linear wave theory to set-up, prediction of set-up and run-up due to random waves (given an offshore wave spectrum), estimation of set-up in harbours, numerical modelling of set-up and nearshore currents, field measurements of set-up and swash spectra and interpretation of surf beat in terms of edge waves and a time-varying break point.

factors listed above. The wavelengths associated with tides and storm surges are large compared with irregularities in the coastline and sea bottom, but the much shorter wind waves are subject to considerable refraction and diffraction near the coast, which can concentrate wave energy at one point while leaving a sheltered area nearby. When these waves reach the shore their behaviour will depend on the beach profile, which depends on the beach material and the waves themselves, and on the presence of man-made structures and sea defences. All these factors introduce local effects, and a measurement of sea level unaffected by them requires a careful siting of the tide gauge. In this connection, it may be noted that harbours are not immune from wave set-up (Thompson and Hamon 1980). Miyata and Groves (1968) suggest the effect of surf as an explanation of differences in sea level at two nearby stations.

The study of coastal sea levels, apart from its intrinsic scientific interest, is usually motivated by the wish to avoid, or at least predict, flooding in low-lying inhabited areas. Whether a certain sea level will cause flooding depends not only on its magnitude but also its duration: a wave added to a storm surge may reach a height sufficient to overtop a sea wall, but drainage may be adequate to prevent serious flooding. For a given volume flux of water over the wall, local conditions on the landward side of the sea defences will determine whether it is acceptable. The spectrum of wave-induced sea levels varies from the incoming wave frequencies to the steady mean set-up and may include considerable energy at surf beat frequencies (periods of 30s to a few minutes). The whole frequency band will be considered here, since energy at any point within it may have consequences for sea defences, but emphasis will be placed on set-up because of its relatively long duration.

In view of the apparent complexity and variety of nearshore conditions, the different beach shapes (morphologies), breaker types, surf zone widths and circulation patterns, it is useful to characterise and classify a stretch of coast under given wave conditions in terms of a few simple dimensionless parameters. Although a simplification, this helps to sort out which of the concepts to be discussed here are likely to be relevant in any particular setting.

If a monochromatic wave train approaches a plane beach, dimensional quantities are the deep-water wave height H_o (or, alternatively, the breaker height H_b), the wave period T, gravitational acceleration g and water density ρ. Non-dimensional quantities are the beach angle α (giving a slope $\tan\alpha$) and breaker angle θ_b (the angle between wave crests at the breaker line and the longshore direction, see Figure 1). A dimensional quantity influencing beach shape is the sediment fall velocity for the beach material, w_s.

It turns out that there is a 'surf-similarity parameter'

$$\varepsilon = H\sigma^2/g\tan^2\alpha \tag{1}$$

(where H is some measure of wave height and $\sigma = 2\pi/T$ is the wave frequency) which appears to govern many surf zone phenomena. This was noted by Bowen et al. (1968), Battjes (1974, 1975) and Huntley et al. (1977). Different parameters ε_o, ε_b and ε_T are defined by taking H

equal to H_c, H_b or the wave height at the toe of the beach, H_T, in the case of a beach levelling to a constant depth offshore. Since $\sigma^2 = gk$ for small amplitude waves in deep water, where k is the wavenumber, ε_o is proportional to the deep water wave steepness.

In general, high values of ε correspond to dissipative beaches, with wide surf zones and spilling breakers. As ε decreases, the breaker type changes to plunging, then collapsing and then surging, and the number of waves within the surf zone decreases (Figure 2). The proportion of wave energy reflected increases until at a critical value of ε complete reflection occurs with no breaking.

According to Carrier and Greenspan (1958) and Munk and Wimbush (1969) the critical value of ε_b at which complete reflection occurs is near 2, if H_b is now taken as the wave height at the shoreline (since no breaking occurs). It is very rare to have perfect reflection of wind waves: as Meyer and Taylor (1972) point out, for a slope as large as 1/30 and a wave period as long as 10s, the maximum run-up height without breaking is less than 2.5 cm.

The transition between different breaker types is somewhat subjective. Both Galvin (1972) and Battjes (1974) quote parameter values for the surge-plunge transition which are near to the above critical value (Galvin remarks that this transition is <u>expected</u> to occur at the onset of turbulent breaking), although Wright et al. (1982) quote a value equivalent to $\varepsilon_b = 5$. Battjes (1974) and Wright et al. (1982) put the plunge-spill transition at $\varepsilon_b = 40$. Highly dissipative beaches may have values of ε_b in the hundreds.

As ε_b increases the width of the surf zone increases. Battjes (1974) estimates a surf zone width of about $0.3\,\varepsilon_b^{\frac{1}{2}}$ wavelengths. Hence with spilling breakers there are at least two breaking waves in the surf zone at any time. Battjes (1974) also points out that the breaker height to depth ratio H_b/d_b has a (weak) dependence on ε_o. This ratio tends to increase from about 0.8 for spilling breakers (near the theoretical maximum height to depth ratio for a solitary wave) to 1.1 for plunging and 1.2 for collapsing breakers (Galvin 1968).

An empirical formula for the maximum run-up height R for breaking waves was given by Hunt (1959), and written in terms of ε_o is

$$R = H_o\,(2\pi/\varepsilon_o)^{\frac{1}{2}} \tag{2}$$

This was well supported by the experiments of Bowen et al. (1968): the steady set-up formed the greater part of R. The experiments of Battjes and Roos (Battjes 1974) show a difference between run-up height and run down height

$$H_r = 2{\cdot}5\,H_o/\varepsilon_o \tag{3}$$

hence $\varepsilon_r = 2.5$, a constant, where ε_r is defined by (1) with $H = H_r$. Guza and Bowen (1976), however, found $\varepsilon_r = 6\pm2$, and Van Dorn (1978) 4.0 ± 0.6, the discrepancy probably due to the ill-defined nature of run-down (according to Guza and Thornton (1982)). The constant value of ε_r represents saturation : increasing the incident wave height increases the steady set-up but not the amplitude of the shoreline oscillations. This conclusion is not true when there is a full spectrum of incoming waves rather than a monochromatic wave train

(Huntley et al. (1977), Guza and Thornton (1982)); then surf beat oscillations increase with increasing incident wave energy (see section 5).

The contrast between spectra (of elevation and current components) obtained from reflective and dissipative beaches is shown by Wright (1981) and Wright et al. (1982). On reflective beaches there is likely to be strong subharmonic resonance; by the mechanism of Guza and Davis (1974) energy may be transferred from the standing wave component of the incident wave to subharmonic edge waves. Those at half the frequency of the incident wave grow most rapidly. These were observed by Huntley and Bowen (1973, 1975) and they also suggest (1975) that interaction between a breaker and the swash from a previous wave may introduce a strong first subharmonic ($\sigma/2$) oscillation. Run-up may become dominated by the subharmonic frequency, and erosion may take the form of beach cusps with spacing controlled by the edge wave wavelength. Edge waves of the same frequency as the incoming waves may also be found (Bowen and Inman 1969).

On highly dissipative beaches the spectra show no evidence of subharmonic oscillations, but have a large amount of energy at infragravity (surf beat) frequencies. These may take the form of edge waves, growing by resonant interaction between incoming waves (Gallagher (1971), Huntley (1976), Bowen and Guza (1978), Huntley et al. (1981)), or long waves generated by break point and set-up variations at the group frequency (Symonds et al. 1982). The latter mechanism has been suggested as the explanation of Tucker's (1950) offshore surf beat observations. Erosion may take place by bores superimposed on large shoreline oscillations at surf beat frequencies. Offshore bars may be produced at the nodes of progressive edge waves.

Several beach states intermediate between highly reflective and highly dissipative have been identified (Wright et al. (1979, 1982)). These have irregular or rhythmic bars and troughs, and may be in a more mobile state than the extreme types. Holman and Bowen (1982) have shown how the interaction of edge waves may produce such topographic features.

More details on edge waves and surf beat will be found in section 4.

A parameter relating wave conditions and beach material to beach state (Dean (1973), Dalrymple and Thompson (1977), Wright et al. (1982)) is

$$\Omega = H_b / \omega_s T \tag{4}$$

The change from reflective to intermediate beaches is found at $\Omega \sim 1$, while that from intermediate to dissipative beaches is found at $\Omega \sim 6$. Beaches may therefore be expected to change state with differing wave conditions, given enough time, if these threshold values are crossed. If two parameters, ε and Ω (equations 1 and 4) are taken to represent beach state, this suggests a connection between equilibrium beach slope and fall velocity through a parameter such as $\omega_s/(gH_b)^{\frac{1}{2}} \tan\alpha$. A similar combination does appear in the suspended load part of Bagnold's (1963) sediment transport equation, where $(gH_b)^{\frac{1}{2}}$ is replaced by a steady flow velocity in the slope direction. Bailard (1981) gives an equation for the equilibrium beach slope $\tan\alpha$ in terms of wave velocities, ω_s and also including bed load.

A real beach does not have a constant slope, nor is it composed of uniform material. Therefore a significant tidal range may mean that the beach state is different for different states of the tide. The wave refraction and diffraction patterns will also change tidally. The presence of an additional surge level may change the beach state parameters (as well as the wave refraction patterns) from those customary for a given state of the tide. It may be necessary to recall, when considering the statistics of water levels on a beach, that the characteristic behaviour of the waves may be different for different values of the mean sea level offshore. The waves may also be affected by currents, including tidal currents. In general, then, it would not be true to say that the part of the sea level due to waves is independent of that due to tide and surge.

In the following sections, the theoretical and observational background to some of the concepts introduced here will be treated in more detail. Some conclusions will be found in section 6.

2. CLASSICAL THEORY OF SET-UP AND OTHER EFFECTS OF WAVE MOMENTUM FLUX

Wind waves generated by storms may travel great distances across the oceans with little attenuation. They transport mass, energy and momentum. The effects of these transports are concentrated in the nearshore zone, where waves rapidly lose their energy by breaking.

Successful theories of nearshore wave effects have been based on small-amplitude, sinusoidal (Airy) wave theory, (see, for example, Phillips (1966)), despite its apparent unsuitability for large-amplitude, steep, breaking waves. Some comments on the use of other wave theories will be made later (section 3).

For sinusoidal waves travelling in the x direction,

surface displacement $\zeta = a\cos(kx - \sigma t)$ (5)

the dispersion relation is $\sigma^2 = gk\tanh kh$ (6)

the velocity field $(u, w) = \frac{a\sigma}{\sinh kh}\Big(\cosh k(z+h)\cos(kx-\sigma t),$
$\sinh k(z+h)\sin(kx-\sigma t)\Big)$ (7)

and the pressure $p = -\rho g z + \rho g a \cosh k(z+h)\cos(kx-\sigma t)/\cosh kh$ (8)

where a = H/2 is the amplitude, h is the still-water depth (replaced by the mean depth d if $\bar{\zeta}$/h is not small) and z is measured upwards from the still water level (see Figure 1).

The mean energy per unit area $E = \frac{1}{2}\rho g a^2$, (9)

and the wave energy travels with the group velocity

$$c_g = \partial\sigma/\partial k = \tfrac{1}{2}c\left(1 + 2kh/\sinh 2kh\right) \quad (10)$$

where $c = \sigma/k$ is the phase velocity.

The momentum per unit area m = E/c, (11)

and the radiation stress tensor (i.e. the excess momentum flux due to the waves, see Longuet-Higgins and Stewart (1964) and Phillips (1966)) is

$$S_{xx} = E\left(2c_g/c - \tfrac{1}{2}\right),\ S_{yy} = E\left(c_g/c - \tfrac{1}{2}\right),\ S_{xy} = S_{yx} = 0 \quad (12)$$

In line with the use of sinusoidal waves, these quantities have been calculated to terms in second order (in ak).

Longuet-Higgins and Stewart (1962, 1963, 1964) show that for a wave train approaching normally to a beach, changes in the wave momentum flux are balanced by slopes in the mean sea level $z = \bar{\zeta}$ so that

$$\partial S_{xx}/\partial x + \rho g(\bar{\zeta}+h)\partial\bar{\zeta}/\partial x = 0 \tag{13}$$

Outside the surf zone, energy flux Ec_g is constant. Longuet-Higgins and Stewart assume $\bar{\zeta} << h$, so $\partial\bar{\zeta}/\partial x = -(\partial S_{xx}/\partial x)/\rho g h$, and hence

$$\begin{aligned}\bar{\zeta} &= -\tfrac{1}{2}a^2 k/\sinh 2kh \\ &= -a^2/4h \quad \text{in shallow water, where } kh << 1\end{aligned} \tag{14}$$

inside the surf zone, Bowen et al (1968) assume

$$H = \gamma(\bar{\zeta}+h) = \gamma d \tag{15}$$

with γ constant, and shallow water waves, so $S_{xx} = 3\rho g H^2/16$. From equation (13) directly,

$$\partial\bar{\zeta}/\partial x = -(\partial h/\partial x)/(1+8/3\gamma^2) \tag{16}$$

Hence the water level slope is proportional to beach slope. If γ = 0.8 (the approximate value for spilling breakers), the set-up slope is 0.2 times the beach slope.

The maximum height of the water ($\bar{\zeta}_{max}$, the value of $\bar{\zeta}$ at the shoreline) can be calculated by integrating (16) from the breaker point (Battjes 1974):

$$\bar{\zeta}_{max} - \bar{\zeta}_b = (\bar{\zeta}_{max} + h_b)/(1+8/3\gamma^2) \tag{17}$$

So using equation (14), $$\bar{\zeta}_{max} = 0{\cdot}3\gamma H_b \tag{18}$$

Hence the maximum rise in sea level due to wave set-up is a significant fraction (a quarter if $\gamma = 0{\cdot}8$) of the breaker height.

If breaker height varies with position along the beach, the set-up also varies: this is able to drive a steady circulation in the nearshore zone. Bowen (1969a) assumes a sinusoidal longshore variation in wave height, and shows that the forcing terms in a transport stream function equation are zero outside the breaker line and sinusoidal alongshore within the surf zone, with the result that circulation cells are formed with seaward flowing rip currents at the positions of low waves and longshore flow away from the positions of high waves. Bowen and Inman (1969) consider observations of rip currents where the longshore modulation in wave height is caused by the interaction of the incoming waves with resonant edge waves. For edge waves of the same frequency as the incoming waves, rip currents occurred at alternate antinodes of the edge wave displacement at the shoreline. For edge waves at half the frequency of the incoming waves, no circulation patterns were produced. This is in agreement with the requirement that there should be a steady, rather than oscillatory, forcing term for the nearshore circulation to exist, since edge waves of the same frequency reinforce successive incoming wave crests in the same way while those of half frequency have the opposite effect on successive incoming waves. Bowen and Inman point out that the same-frequency edge waves produce a steady forcing whether they are standing or progressive alongshore, since in either case incoming wave crests and edge wave crests coincide at the same points at intervals of one wave period. More on edge waves will be found in section 4.

Dalrymple (1975) gives another mechanism for the generation of rip currents, namely the interaction between wave trains of the same frequency arriving from different directions. The rip current spacing has a wavenumber equal to the difference between the two longshore wavenumbers (each of the form $k \sin\theta$ where θ is the angle between wavecrest and the shoreline) of the incoming wave trains. By Snell's law, $k \sin\theta$ remains constant as the waves approach the shore. Such pairs of wave trains could be produced by reflection of a single wave train, at a breakwater for example. The rip current spacing by this mechanism has no upper limit (it has a minimum of half the deepwater wavelength λ_0) whereas the edge wave wavelengths are limited for a given frequency (to a maximum λ_0 , for edge waves of the incoming wave frequency). If the two wave trains have slightly different frequencies, the rip current system would move slowly alongshore.

Dalrymple (1975) calculates the set-down outside the breaker line from the Bernoulli equation of Mei (1973):

$$g\overline{\zeta} = \left\{ \overline{\zeta \partial w/\partial t} + \tfrac{1}{2}\left(\overline{u^2} + \overline{v^2} + \overline{w^2} \right) \right\}_{z=0} , \tag{19}$$

where ζ, u, v, w are the total wave surface displacement and velocities and the overbar denotes time average. This gives the same result as (14) for a single wave train. Equation (19) cannot, of course, be used when there is dissipation by breaking. Mei (1973) also points out that the neglect of dynamic bottom pressure in the derivation of (13) is true only for small bottom slopes - this was noted also by James (1974).

Miller and Barcilon (1978) examine the instability of the basic set-up, as another method for producing circulation cells on a beach. Here the potential energy stored in the set-up provides the energy for the circulation.

The steady circulation cells may be obscured by the addition of a mean longshore current, which is produced when the waves arrive at an oblique angle θ (Figure 1a), even if this angle is small (as it usually is near the shoreline, by refraction). This longshore current was determined in terms of the radiation stress by Bowen (1969b) and Longuet- Higgins (1970a,b). If we retain the coordinates x onshore, y alongshore, z upwards the longshore current is due to the shoreward gradient of the mean flux of y-momentum carried across planes x =constant: this flux is S_{xy}, which, by transformation of the radiation stress tensor, is $E(c_g/c) \sin\theta \cos\theta$. Outside the surf zone the flux of energy towards the shore, equal to $E c_g \cos\theta$, is constant, and so is $\sin\theta/c$ by Snell's law, hence S_{xy} is constant and there is no forcing for a longshore current. Inside the surf zone, $\partial S_{xy}/\partial x$ is non-zero and there is a forcing term for the longshore current, which may be balanced by frictional terms to give a steady current.

Longuet-Higgins (1970a) shows that the predicted longshore current is then

$$v = 5\pi\gamma\,(g/h_b)^{\frac{1}{2}}\,h \tan\alpha \sin\theta_b / 16C \qquad (20)$$

inside the surf zone, and zero outside, where C is a friction coefficient of the order of 0.01. In practice, this distribution of current is smoothed out by horizontal eddy viscosity (Longuet-Higgins 1970b) and because breaking would not occur at the same point for every wave in the case of random waves (Battjes 1974). Currents of the order of 1 ms^{-1} are possible (Putnam et al. 1949).

3. DEVELOPMENTS IN THE THEORY AND OBSERVATION OF SET-UP

In the classical theory of set-up described in the previous section drastic assumptions were made: a monochromatic wave train was envisaged approaching normally to a beach with a monotonic slope (a plane beach in experimental tests), and momentum fluxes were calculated from small-amplitude sinusoidal wave theory without much justification (apart from simplicity). The consequences of each of these assumptions have been examined theoretically.

The problem of a full spectrum of random waves approaching the shore has been tackled by Collins (1972) and Battjes (1973,1974). Collins calculates the expected value of the radiation stress as the sum of those due to a number of monochromatic waves, weighted by the probability of occurrence of each combination of wave height, period and angle of approach in deep water. For each component wave, the height H, wavelength λ and angle of approach θ may be calculated for each value of x (the onshore coordinate) from Snell's law, the dispersion relation (6) and the conservation of energy flux $E c_g \cos\theta$ if the wave is not breaking. (Collins assumes $h=h(x)$; a more complex refraction-diffraction calculation would be necessary with two-dimensional beach topography). At any point, breaking is taken into account by a criterion giving a maximum value of H; such a criterion covering deep water as well as shallow water, due to Miche (1951) is

$$H/\lambda < 0.14 \tanh kh \tag{21}$$

In this way, the wave parameters for each component wave can be found numerically for arbitrary beach profiles by stepping shorewards from deep water. Hence the expected values of S_{xx} needed to calculate set-up from (13) (and those quantities required for a longshore current calculation) can be found, when those for each component wave are weighted by using probability density functions: those used by Collins (1972) are the Rayleigh distribution

$$p(H_o) = \left(2H_o / \overline{H_o^2}\right) \exp\left(-H_o^2 / \overline{H_o^2}\right) \tag{22}$$

$$p(\lambda_o) = \left(2\lambda_o / \overline{\lambda_o^2}\right) \exp\left(-\lambda_o^2 / \overline{\lambda_o^2}\right) \tag{23}$$

and

$$p(\theta_o) = (8/3\pi) \cos^4 (\theta_o - \overline{\theta}_o) \tag{24}$$

Each component wave is supposed to represent a finite element

$\Delta H_o \Delta \lambda_o \Delta \theta_o$ of $(H_o, \lambda_o, \theta_o)$ space, the probability of occurrence of this element being calculable from (22)-(24).

The results of Collins (1972) show that the effect of random waves is to smooth out the set-up and longshore current profiles. In particular, relative to monochromatic waves with the same total energy, the wave set-down is much reduced, set-up at the shoreline is reduced slightly and there is no discontinuity in longshore current (even without horizontal mixing) because all waves do not break at the same point. The Rayleigh distribution in deep water is shown to be changed in shallow water because of the cut-off at the limiting wave height.

Battjes (1974) criticises Collins' approach on the grounds that certain non-linearities are neglected. For example, the waves give rise to a set-up which in turn affects the waves: the significantly increased depth near the shoreline produced by some of the component waves affects the other component waves. In the longshore current calculations, bottom friction should be nonlinear. Therefore the transformation of each component wave cannot be considered entirely separately. Battjes prefers to estimate the wave spectrum at each point first and then calculate the radiation stress from the spectrum. However, his calculations are still based on linear wave theory, in which the dispersion relation (6) holds, and the spectrum at a given position and time can be expressed as a function of frequency and direction, $G(\sigma,\theta)$ such that the total variance of the sea surface displacement is $\int_0^\infty\int_0^{2\pi} G(\sigma,\theta)\,d\theta\,d\sigma$.

If there is no energy dissipation, and retaining linear dynamics, it can be shown (Longuet-Higgins (1957), Battjes (1974), Le Méhauté and Wang (1982)) that $c_g G(\sigma,\theta)/k$ is constant along a wave ray. Hence, if the bottom contours are parallel, the spectrum at any point can be found from deep-water values (denoted by suffix zero) from

$$G(\sigma,\theta) = k c_{g0} G_0\left(\sigma, \sin^{-1}\left[k \sin\theta / k_0\right]\right) / k_0 c_g \quad . \tag{25}$$

Battjes (1974) shows that the radiation stress tensor , with random sinusoidal waves, becomes

$$S_{ij} = \rho g \int_0^\infty \int_0^{2\pi} \left(n e_i e_j + \left(n - \tfrac{1}{2}\right)\delta_{ij}\right) G(\sigma,\theta)\, d\theta\, d\sigma \quad , \tag{26}$$

where $n = c_g/c$, $c = \sigma/k$ (phase velocity), $e_1 = \cos\theta$ and $e_2 = \sin\theta$.

This is of the same form as before except for the integral and multiplication by the factor G. The time means used for the calculation of S_{ij} are supposed to be long compared with the surf beat periods produced by interaction between different frequencies in the spectrum.

Inside the surf zone, where there is wave breaking, (25) and (26) cannot be used directly. Battjes (1974) first calculates a "fictitious" spectrum $G_f(\sigma, \theta)$, which would be the spectrum without breaking. He adopts a breaking criterion adapted from Miche (1951), namely

$$H < H_b = 0{\cdot}14\lambda \tanh\left(\gamma d / 0{\cdot}14\lambda\right) \tag{27}$$

In (27), λ is calculated from (6), where σ is taken to be a constant, mean frequency (where a narrow spectrum is being considered). In any case, (27) gives $H_b = \gamma d$ in shallow water. The spectrum G_f leads to a fictitious energy

$$E_f = \rho g \int_0^\infty \int_0^{2\pi} G_f(\sigma, \theta)\, d\theta\, d\sigma \tag{28}$$

and mean square wave height $\overline{H_f^2} = 8E_f/\rho g$ (29)

The fictitious wave height is supposed to be Rayleigh distributed, while the actual wave height has the same distribution except that it is clipped at $H = H_b$:

$$P(H \leq \hat{H}) = 0 \qquad \text{for} \quad \hat{H} < 0 \tag{30}$$

$$
\left.\begin{aligned} &= 1-\exp\left(-\hat{H}^2/\overline{H_f^2}\right) \quad \text{for } 0 \le \hat{H} < H_b \\ &= 1 \quad \text{for } \hat{H} \ge H_b \end{aligned}\right\} \tag{30}
$$

Hence, $$\overline{H^2} = \left(1 - Q_f(H_b)\right)\overline{H_f^2} \tag{31}$$

where $Q_f(H) = P(H_f > H) = \exp\left(-H^2/\overline{H_f^2}\right)$ for $H \ge 0$. (32)

$Q_f(H_b)$ may be regarded as the fraction of breaking waves.

A fictitious radiation stress, S_{ijf} , may be calculated using C_f , from (26). The true radiation stress S_{ij} in the presence of wave breaking is taken by Battjes to be given by

$$S_{ij} = \left(1 - Q_f(H_b)\right) S_{ijf} \tag{33}$$

This is reasonable since in shallow water the factor

$(ne_ie_j + (n-\frac{1}{2})\delta_{ij})$ in (26) is not frequency-dependent and the radiation stresses may be assumed to be reduced by breaking in proportion with the total energy. Where $Q_f(H_b) \to 1$ nearly all the waves are breaking and $\overline{H^2} \to H_b^2$.

For a narrow spectrum,

$$S_{ij} = \rho g \left(ne_ie_j + \left(n-\tfrac{1}{2}\right)\delta_{ij}\right)\overline{H^2}/8 , \tag{34}$$

where the bracket is evaluated for the mean σ and θ_o . This is the same expression as for a monochromatic wave, except that root-mean-square wave height $H_{rms} = \left(\overline{H^2}\right)^{\frac{1}{2}}$ is used. This is related to the "significant wave height" H_s (the mean value of the highest third of the waves) in the case of the Rayleigh distribution by

$$H_s = \sqrt{2}\, H_{rms} \tag{35}$$

Set-up can be calculated from (13), given the deep water wave spectrum, with the boundary condition $\bar{\zeta} = 0$ in deep water. Since S_{xx} depends on the mean depth $d = h + \bar{\zeta}$, this would seem to require an iterative method. However, if the depth decreases monotonically shorewards (this may not be true if there are bars), (13) can be rewritten

$$\partial S_{xx} / \partial(d) + \rho g d\, \partial\bar{\zeta} / \partial(d) = 0 \tag{36}$$

and solved with d as the independent variable.

Battjes (1974) considers two types of deep-water spectra. One is narrow, for which only the total energy E_o , mean frequency $\bar{\sigma}_o$ and mean direction $\bar{\theta}_o$ need be specified. Then a deep water wave steepness can be defined. The other is a wide spectrum, $G(\sigma, \theta_o) = S_o(\sigma) D_o(\theta_o)$ where

$$S_o(\sigma) = H_{ro}^2\, \bar{\sigma}_o^4\, \sigma^{-5} \exp\left\{ -(\bar{\sigma}_o/\sigma)^4/\pi \right\} / 2\pi\ , \tag{37}$$

in which H_{r_o} is the deep water r.m.s. wave height and $\bar{\sigma}_o$ is calculated from the mean period in deep water, is the Pierson-Moskowitz spectrum (a predecessor of the JONSWAP spectrum (Hasselmann et al. 1973)) and

$$\left.\begin{aligned} D_o(\theta_o) &= 2\cos^2(\theta_o - \bar{\theta}_o)/\pi \quad \text{for } |\theta_o - \bar{\theta}_o| \le \tfrac{\pi}{2} \\ &= 0 \text{ otherwise} \end{aligned}\right\} \tag{38}$$

The waves travelling in offshore directions should be excluded from the calculations. Battjes (1974) displays set-up profiles for the narrow spectrum for various deep water wave steepnesses (0.005 to 0.04) at fixed $\overline{\theta}_o = 15^\circ$ and $\gamma = 0.8$, reproduced in Figure 3. They show slowly varying negative values for the larger depths and a fairly steep rise in shallow water, with a gradual transition between the two. As the shoreline is approached, the set-up gradient approaches a constant value such that $\partial\overline{\zeta}/\partial h \to -1/(1+8/3\gamma^2)$. The values of $\overline{\zeta}_{max}/H_{rb}$ increase from 0.19 to 0.31 as the deep water wave steepness decreases from 0.04 to 0.005. Hence $\overline{\zeta}_{max}$ is a significant fraction of the deep water significant wave height.

These results for the narrow spectrum are in fact close to (though smaller than) those which would be obtained from monochromatic wave theory: if it is assumed that energy flux Ec_g is constant from deep water to the breaker line, this gives (if the waves break in shallow water)

$$\frac{1}{2} H_o^2 (g/k_o)^{\frac{1}{2}} = H_b^2 (gh_b)^{\frac{1}{2}} \tag{39}$$

Hence, if $H_b = \gamma h_b$,

$$H_b = (\gamma/8\pi)^{\frac{1}{2}} (\lambda_o/H_o)^{\frac{1}{5}} H_o \, . \tag{40}$$

And so, from (18), if $\gamma = 0.8$, $\overline{\zeta}_{max}/H_o = 0.23$ if $H_o/\lambda_o = 0.04$ and $= 0.35$ if $H_o/\lambda_o = 0.005$. Equations (40) and (18) together show that the maximum set-up is strongly dependent on offshore wave height and only weakly dependent on offshore wave steepness.

Battjes (1974) does not present set-up results for the wide spectrum: longshore current calculations showed half the current obtained with the narrow spectrum for the same total energy, in line with the reduction in the total stress on the surf zone. Also, deep water values of S_{xx} (where x is in the mean wave direction) are reduced by a quarter (Battjes 1972).

Battjes and Janssen (1978) describe a development of this model in which H_{rms} is calculated from an energy balance equation, in which wave energy dissipation is estimated from that in a bore of the same height. They applied it to a bar-trough beach profile, showing good agreement with experimental wave heights and reasonable agreement with experimental set-up profiles, though the results depend on the specification of an arbitrary (of order one) dissipation parameter, as well as γ .

The experimental comparisons of Battjes (1974) show the theory in that report to overestimate set-up levels in the surf zone, though gradients are fairly well predicted. He gives several possible reasons for the discrepancy, one of which is the use of linear wave theory, which clearly does not describe the waves accurately in the surf zone. A numerical solution of (13), in which S_{xx} was prescribed from nonlinear wave theories, was given by James (1974). Third order Stokes waves were assumed offshore, and third order hyperbolic waves (an approximation to cnoidal waves which essentially treats them as a train of solitary waves) nearshore, with the transition between them given by a condition of continuous energy flux. This was intended to give a reasonably realistic model of spilling breakers. Results were given in terms of a parameter $P=T\sqrt{g/d_b}$, where d_b is the mean depth at the break point. $P\to\infty$ corresponds to the arrival of a single solitary wave, while P=0 for waves which are so steep that they break in deep water. It was shown that momentum fluxes at the break point are of the same order as for linear wave theory for a wide range of typical values of P. However, the ratio of set-up slope to beach slope ($1/(1+8/3\,\gamma^2)$ on linear theory, from (16)) was smaller than linear theory would give, and decreases as P increases. The set-up slope was found not to be constant with position within the surf zone but to decrease as the shoreline was approached, and the maximum set-down occurred <u>before</u> the break point. Both these features were found experimentally by Stive and Wind (1982), as shown in Figure 4. The maximum set-down before breaking may also be seen in the observations of Saville (1961) and Galvin and Eagleson (1965). The 'convex upward' set-up profile was also found by Hwang and Divoky (1970) in a cnoidal wave decay model and was seen in some of Saville's (1961) results, although the experiments of Bowen et al. (1968) suggested that the set-up became tangential to the beach as $d\to 0$. The latter effect may possibly be due to run-up and backwash, in a region not well modelled by the set-up theories.

Stive and Wind (1982) compare the James (1974) theory with one based on Cokelet's nearly exact irrotational theory for steep waves, assuming breakers can be modelled by non-breaking waves of maximum energy flux. This theory, applied to wave shoaling, is described by Sakai and Battjes (1980). The two theories give closely comparable results, and both have the properties of maximum set-down before the break point and a convex upward set-up profile. They are nearer to the experimental results than the linear theory for both tests shown in the paper. In each test breakers on the beach took the form of spilling breakers or bores, but in one, shown in Figure 4, for which the prediction of maximum set-up by the nonlinear theories is particularly close to that measured, the initial breaking was described as spilling and in the other plunging (values of $\mathcal{E}_b$ were 358 and 162 respectively). It was found that the linear theory could be adjusted to give radiation stress values in the surf zone (and hence set-up magnitudes) nearer to those of the nonlinear theories by reducing γ from 0.8 to 0.6. The radiation stress was calculated directly from the velocity field (measured by laser doppler velocimeter) and surface elevations (measured by resistance-type wave gauges) and compared with that calculated from the mean water level and the various theories, as shown in Figure 4a. Equation (13) was well verified. The radiation stress magnitude at the breakpoint was fairly well predicted by the nonlinear theories (much better than by the linear theory), though inaccuracies in the prediction of the position of the break point prevented an even better prediction of the set-up. The criterion used in the James theory, that breaking occurs when the height to trough depth ratio reaches 0.85, seemed in fact to give somewhat better results for break point than that of maximum energy flux used with the Cokelet theory. The Cokelet theory is exact only for waves on water of constant depth, and the discrepancy may be explained by the effects of the finite bottom slope: all the theories described assume that these effects are negligible, and that the waves behave locally as if they were in constant depth.

The effect of an oblique angle of incidence on the set-up is likely to be small, since refraction ensures that on most beaches $\sin\theta_b$ is small. Jonsson and Jacobsen (1973) show on the basis of linear theory that the maximum set-up varies as $(\cos\theta_0)^{2/5}$ and this agrees well with Collins' (1972) and Battjes' (1974) calculations for random waves.

The effect of a non-planar beach profile on linear theory was considered by McDougal and Hudspeth (1981), who found that for a concave-upwards Bruun profile $h \propto (-x)^{2/3}$ the mean level is also concave-upwards, with mean depth $d \propto (-x)^{1/2}$. In many real cases, the beach slope is not even monotonic, and there are offshore bars and troughs. In the field measurements of Sonu (1972), where the waves break on an offshore bar, the lowest level was found at the shoreline. Arbitrary bottom profiles have been considered by Leontyev (1980) and the presence of bars is allowed in the models of Collins (1972) and Battjes and Janssen (1978). Longuet-Higgins (1967) derived expressions for the difference in level between the two sides of a submerged bar or breakwater when a non-breaking wave train passes over it. An extreme form of bar-like profile is the coral reef considered by Tait (1972): here a lagoon is separated from the ocean by a reef with a flat top at sea level (the still water level with no waves). Wave breaking takes place on the sloping oceanward side of the reef, and then the waves are assumed to pass into the lagoon with no further decay. Then (16) may be integrated from the breaker point to the reef flat, similarly to the integration leading to (17), to give the sea level at the edge of the reef flat (where still water depth h =0)

$$\bar{\zeta}_{reef} = \left(-\gamma^2/16 + 1/\left[1 + 8/3\gamma^2\right]\right) h_b \tag{41}$$

This is taken as the increase in sea level in the lagoon (at least, near the reef) compared with the ocean. If γ=0.8, it is 0.2 times the breaker height. This calculation can be modified if the still-water level changes tidally.

Gerritsen (1980) has also considered wave set-up on a coastal reef. He adds a bottom shear stress term to the left hand side of (13), and calculates the energy dissipation due to bottom friction and wave breaking (from the bore analogy, as in Battjes and Janssen (1978)). Gerritsen modifies the results of Battjes and Janssen, which use a truncated Rayleigh distribution for wave heights in the surf zone, since he considers a Weibull distribution to be more realistic. This distribution is continuous rather than truncated. Good agreement between predicted and observed H_{rms} was shown for a line of observations on a Hawaiian reef, with a suitable choice of dissipation coefficients. It was suggested by Gerritsen that the bottom friction term (adjusted to give good agreement with observed set-up) was necessary in the set-up equation, otherwise the set-up (calculated from radiation stresses based on the linear theory relationship between S_{xx} and E) was too large. However James (1974) and Stive and Wind (1982) found the bottom friction term to be small, and the discrepancy may probably be better explained by the deficiencies of linear wave theory.

In the case of the reef and lagoon there is a small connection in the vertical (that is, a shallow zone over the reef top) between the ocean and the nearshore waters, which may have different levels because of wave set-up. In the case of a harbour there is a narrow connection in the horizontal. Thompson and Hamon (1980) have shown that the difference in levels between the harbour and the open sea produced by set-up can be significant. They argue that as the waves spread out into the harbour from the narrow entrance their height decreases from that at the harbour mouth, S_{xx} decreases and so the mean level in the harbour increases such that along a wave ray

$$S_{xx} + \rho g h \bar{\zeta} = \text{constant} \qquad (42)$$

Hence, with $S_{xx} = 3\rho g H_e^2/16$ at the harbour entrance (where H_e is the wave height at the entrance), the water level inside the harbour will be up to $3H_e^2/16h$ higher. With h=10m and H_e=6m this gives a set-up of 68 cm. Since there must be a net increase in the amount of water in the harbour it will take time to achieve this set-up: calculated from the mass flux brought in by the waves, this is a few hours for a typical harbour. If the mass flux results from the Stokes velocity, $\frac{1}{8}(H_e/h)^2 c$ for shallow water (this implies that no compensating return flow has been generated), the time scale is $3A/2W(gh)^{\frac{1}{2}}$, where A is the area of the harbour and W the width of the entrance. In fact, a compensating flow must be produced as the level is built up, and (opposing this effect as far as the time scale estimate is concerned) the increase in level is not uniform throughout the harbour. The set-up profile as a function of distance from the harbour mouth was calculated by McDougal and Slotta (1981). If the wave crests spread out in semicircular arcs from the harbour mouth, energy conservation gives $H^2 x$= constant, where x is the distance from the mouth. Hence, from (42), $\bar{\zeta}$ inside the harbour increases asymptotically to $3H_e^2/16h$. This value is reached in a distance of a few mouth widths , so the level is raised to this maximum amount over most of the harbour.

McDougal and Slotta (1981) also point out that since the harbour set-up depends on depth h it changes with the state of the tide. Since the set-up is greater for smaller depths, this acts to reduce the amplitude of the tide as measured in the harbour. The tidal signal in the harbour is actually modulated by the presence of waves. For the example cited before, a 2m tidal range would be reduced by about 6%. In general, the wave set-up in the harbour changes tidally by a multiple $3H_e^2/16h_1h_2$ of the tidal range, where h_1 and h_2 are the depths at low and high tide.

It is clear from these studies of wave set-up effects in lagoons and harbours that both mean sea level <u>and</u> tidal measurements may be affected by waves, depending on the siting of the tide gauge. Even on a plane beach, the tidal signal at any position is modulated by set-up, since the position of the sensor relative to the shoreline is changing with the tide. At high tide it may be at a position of low set-up (or even set-down) and at low tide it may be at a position, near the shoreline, of high set-up. Thus the measured tidal amplitude would be reduced with most reduction at times of high waves.

Although several laboratory studies of set-up have been mentioned, measurements in the field have been rather few. The large values of set-up (up to 1.3m) in the figure from Saville (1961) reproduced by Longuet- Higgins and Stewart (1964) are actually scaled up from model tests. Dorrestein's (1961) field measurements at various points across a surf zone were shown by Battjes (1974) to agree with his theoretical model (better, in fact, than Battjes' laboratory data), but Guza and Thornton (1981) point out that Dorrestein's 72s averaging time may not have been long enough to filter out surf beat, which may have contaminated the results. More recent field measurements have been made by Hansen (1978,1979) on the the west coast of Sylt, West Germany, by Gerritsen (1981) on a Hawaiian reef (already mentioned) and by Guza and Thornton (1981) on Torrey Pines Beach, California.

Guza and Thornton measured the mean shoreline position with a resistance wire run-up meter supported about 3cm above the beach (therefore the run-up location was the most shoreward point with depth at least 3cm). The mean shoreline position was then taken as a 4096s average of the instantaneous shoreline position, and the height was referred to an offshore pressure sensor well outside the surf zone where set-up effects were supposed to be absent. The results, shown in Figure 5, gave a best fit relation between maximum set-up $\overline{\zeta}_{max}$ and offshore significant wave height H_{so} of $\overline{\zeta}_{max} = 0.17 H_{so}$. This equals $0.24 H_{rc}$, by (35), so appears closely comparable with Battjes' (1974) results and those following equation (40), based on linear theory. The data was based on values of H_{so} between 0.6 and 1.6m with spectral maxima between 0.1 and 0.065 Hz on a beach with $\tan\alpha$ = 0.02, but no results were given which might relate the scatter to variations in offshore wave steepness. They find a large discrepancy between their results for $\overline{\zeta}_{max}$ and the set-up slope calculated from direct measurements of radiation stress. These direct measurements were obtained from current meter and/or pressure sensor data, using (6), (10) and (12) to give $S_{xx}(\sigma)$, the radiation stress as a function of frequency (the radiation stress spectrum) and then integrating over σ to give the total, S_{xx}^{T}. Therefore formulae appropriate to linear waves were used: Guza and Thornton (1980) found on the same beach that a measurement of p, u or ζ allowed a reasonably close prediction of the spectra of the other two variables at that location from the linear theory relationships even within the surf zone, while linear shoaling theory gave a reasonably good prediction outside the breaker zone, given an input spectrum at 10m depth. The direct measurements in the surf zone then gave

$$S_{xx}^{T} = 3\rho g H_{rms}^{2}/16 = 3\rho g H_{s}^{2}/32 = 3\rho g \gamma'^{2}(\bar{\zeta}+h)^{2}/32 \qquad (43)$$

with $\gamma'=0.4$. This gives a set-up slope considerably less than previous predictions: it is equivalent to putting $\gamma=0.28$ in equation (16), giving $\bar{\zeta}_{max}$ from (18) nearly an order of magnitude smaller than with $\gamma=0.8$ for the same value of h_b, and substantially less than the measured values. Guza and Thornton (1981) attempt to explain the large discrepancy by citing the increase in set-up slope very near the shoreline seen in some of the laboratory results of Bowen et al. (1968) and Van Dorn (1976), but this does not account for an order of magnitude.

Hansen (1977,1978) measured set-up on the coast of Sylt using a line of pressure cells, with offshore measurements of waves and the tide, which gave the still water level. He reports somewhat higher values of set-up than Guza and Thornton (1981), namely $\bar{\zeta}_{max}=0.3\,H_{so}$

and $\overline{\zeta}_{max}$=0.5H_{sb}, where H_{sb} is the significant wave height at breaking. The implication that H_{sb} =0.6H_{sc} would be unexpected from monochromatic wave theory, for equation (40) would give a deep water wave steepness well above the maximum possible (which is 0.142, according to Michell (1893)). Unusually, Hansen's results also give a positive set-up at the break point on all occasions when there were spilling breakers, and the sea level profiles shown all have a maximum offshore, then a minimum in the breaker zone before rising to a value of $\overline{\zeta}_{max}$ at the shoreline.

An attempt to predict wave set-up effects in a real area of complex topography must rely on a numerical model, similar to those used for storm surge prediction, but scaled down to resolve the length scale of changes in radiation stress. Such a model has been described by Birkemeier and Dalrymple (1975,1976). Radiation stress is then taken as an addition to wind stress in the (vertically integrated) equation for acceleration of the mean flow $U_\alpha, \alpha=1,2$, to give (in the linearised form, see also Phillips (1966))

$$\partial U_\alpha/\partial t = -g\partial\overline{\zeta}/\partial x_\alpha - \left(\partial S_{\alpha\beta}/\partial x_\beta + \tau_{b\alpha} - \tau_{s\alpha}\right)/\rho(\overline{\zeta}+h) \quad , \qquad (44)$$

where $\tau_{b\alpha}$ and $\tau_{s\alpha}$ are the bottom stress and surface wind stress. As we have seen, fairly good estimates of $S_{\alpha\beta}$ may be obtained from linear theory once the wave field has been calculated from a wave refraction and diffraction program, including wave-current interaction, given the offshore wave conditions. The mean water level is found from the mass conservation equation

$$\partial\bar{\zeta}/\partial t + \partial\left(U_\alpha\left(\bar{\zeta}+h\right)\right)/\partial x_\alpha = 0 \qquad (45)$$

Birkemeier and Dalrymple (1975) show results of the model for a one-dimensional wave channel with a plane beach and for a two-dimensional open coast with regular contours (periodic alongshore). An example of the channel with monochromatic input waves agreed well with an experimental run of Bowen et al. (1968) and when the input wave height was made to vary with time (giving the effect of a wave group) the set-up was shown to vary, with a time lag. This excited large oscillations if the group (or surf beat) period was near to the natural period of the channel.

Backhaus et al. (1982) have incorporated radiation stress in a similar way into a storm-surge model of the German Bight, using measured wave data. They show that the effect of radiation stress becomes important in comparison with wind stress only in places with shallow depths and significant bottom slope. In an example (for a wind speed of $10ms^{-1}$ and H_S about 2.4m offshore) a rise in level due to wave set-up of about 2 to 5cm is given near the coast (mostly confined to the grid squares next to the coast) with about 1cm offshore in the central German Bight. The set-up in the surf zone is additional to this, since the beach scale is very much smaller than the model grid size. They also find in this case that currents near the coast are changed by up to $10cms^{-1}$ (with tidal currents of the order of $50cms^{-1}$ and wind driven currents of $20\text{-}30cms^{-1}$)by the inclusion of radiation stress terms. Of course, tuning in most successful storm surge models, by adjustment of surface and bottom stress coefficients to give good agreement with data, may crudely account for some of the radiation stress effects: the wind-stress coefficient may include a factor for wave stress. However, this cannot account for the wave set-up distribution in space (which is dependent on bottom topography, and is on small scales) or time (for example, swell may arrive when there is little wind). Once the wind-stress and wave-stress terms in a surge model are separated, it becomes important to consider exactly how wind momentum is transferred to waves and steady currents. Donelan (1979) and Hsu et al.(1982) examine what fraction of the momentum which is transferred across the air-sea interface goes into wave momentum. Part of the wave momentum may be transmitted from the storm area where it is generated to be converted to steady currents and/or set-up in other places at later times.

4. SURF BEAT AND EDGE WAVES

Since waves often arrive at the shore in groups (that is, there is an amplitude modulation of the incoming wave train) it seems reasonable to expect the set-up to oscillate at the group (surf beat) frequency, as in the numerical experiments of Birkemeier and Dalrymple (1975), mentioned in the previous section, and the two- frequency theory of McReynolds (1977). Furthermore, the theoretical existence of free edge waves, which are trapped to the coast, having an amplitude decreasing rapidly away from the shoreline as does the set-up amplitude, raises the possibility that they are resonantly excited by certain combinations of incoming waves. In this way, the predominant surf beat oscillations may take the form of edge waves.

The consequence of wave groupiness on the set-up, ignoring edge waves, was examined in a two-dimensional surf zone model by Symonds et al. (1982). They use equations (44) and (45), without the stress terms , with a radiation stress forcing term, in the surf zone only, varying with time because of a time-varying breakpoint. Long waves at the group period and its harmonics are radiated from the forcing region both shoreward and seaward, the shoreward ones assumed to be totally reflected at the shoreline. It was shown that in certain circumstances the outgoing seaward progressive wave may dominate the locally forced wave, which is 180° out of phase with the incident wave group and travels with the group (hence at the group velocity) and which was described by Longuet- Higgins and Stewart (1964). This would help to explain Tucker's (1950) offshore surf beat observations, in which the surf beat lagged the incident wave groups by about the time required for the forced wave to reach the shore and the free wave to travel back. Symonds et al. point out that if there is a longshore variation in the time-dependent breakpoint position there would be a mechanism for edge wave generation additional to those considered previously.

For a plane beach $h=-x\tan\alpha$, Eckart (1951) found, from shallow water theory, edge wave solutions with a dispersion relation

$$\sigma_n^2 = gk\left(2n+1\right)\tan\alpha \tag{46}$$

where k is the longshore wavenumber of the edge wave (it is proportional to $\cos(ky-\sigma t)$) and n is the mode number. The $n=0$ mode decays exponentially offshore like $exp(kx)$, while the other modes have

n zero crossings (nodal lines) of the amplitude parallel to the coast, this oscillatory variation being superimposed on the same exponential decay (Figure 6). Ursell (1952) found, without using shallow water theory,

$$\sigma_n^2 = gk \sin(2n+1)\alpha \tag{47}$$

with the condition $(2n+1)\alpha \leq \pi/2$, (48)

therefore there is a maximum mode number, which is high if α is small. This is the "cut-off mode". Equations (46) and (47) coincide for low modes and small slopes.

Ball (1967) found edge wave solutions for an exponential beach profile $h = h_o(1-\exp(\alpha' x))$. This levels off to a depth h_o far offshore, so is more realistic than the constant slope profile. The dispersion relation is

$$\sigma^2 = \frac{1}{2} g \alpha'^2 h_o \left[(2n+1)\left(1 + 4[k/\alpha']^2\right)^{\frac{1}{2}} - \left(2n^2 + 2n + 1\right) \right] \tag{49}$$

and if the waves are to be trapped to the coastline,

$$k^2 > n(n+1)\alpha'^2 \quad \text{and} \quad \sigma^2 > n(n+1)\alpha'^2 g h_o \tag{50}$$

for $n>0$ (the $n=0$ mode has no cut-off). Examples of these dispersion curves are given by Huntley and Bowen (1975).

Progressive edge waves may travel in either direction along the coast, and two oppositely travelling waves may be added to give a standing edge wave. Natural features such as headlands may determine the possible wavelengths of standing edge waves (Bowen (1973)), but such features are not apparently necessary for the production of regular sedimentary features by edge waves (Huntley and Bowen 1975). The common existence of regular features suggests that a particular edge wave mode often dominates, despite the wide range of wavelengths and modal numbers which may be allowed to exist for each frequency. A beach between two headlands a distance L apart is resonant if L is a multiple of half a wavelength, so from (47) the resonant frequencies are giver by

$$\sigma^2 = (g\pi m/L)\sin(2n+1)\alpha \tag{51}$$

For typical values of L and α , m and n can be chosen to give frequencies in the surf beat range.

Huntley (1976) reported nearshore velocity measurements in Hell's Mouth Bay, North Wales which indicated progressive edge waves in the surf beat range with discrete peaks in the spectra, and suggested that the cut-off mode (the lowest frequency given by (50)) was preferentially excited. Gallagher (1971) examined the interaction between two incoming waves, showing that edge waves could grow resonantly, but only included the three lowest edge wave modes. Bowen and Guza (1978) examined all the possible resonances and tested the results with a laboratory experiment for the case of a two-frequency swell arriving from a single direction. If two wave trains approach the shore (denoted by suffices 1 and 2) the longshore wave numbers $k_1\sin\theta_1$ and $k_2\sin\theta_2$ are constant during refraction and resonance occurs if the frequency $\sigma=\sigma_1\pm\sigma_2$ and wavenumber $k=k_1\sin\theta_1\pm k_2\sin\theta_2$ satisfy the edge wave dispersion relation. It follows that only the difference frequency can occur. In Bowen and Guza's laboratory experiments (for which the conditions for resonant excitation of a mode 1 edge wave were satisfied) it was found that set-up variations were insignificant in comparison with the displacements associated with the resonant edge wave. The resonance continued to occur if the incident waves were breaking : it was suggested by Bowen and Guza that surf zone damping discriminates against edge wave scales which are small in comparison with the surf zone width.

The observations of Huntley et al. (1981) on Torrey Pines Beach, California showed clear evidence of low mode progressive edge waves in the longshore currents at surf beat periods, while there were other sources of low frequency energy in the on/offshore currents, possibly standing edge waves or reflected incident waves. Holman (1981) describes infragravity energy in a surf zone subject to a broad directional spread of incident storm waves, a case where a large number of edge wave modes may be generated. This energy increased linearly with the incident energy, calculated from the significant wave height.

5. RUN-UP AND OVERTOPPING, BORES AND SWASH

Run-up may be defined as the sum of a mean (set-up) and the fluctuations about that mean (swash). In the previous section it was shown that swash at surf-beat periods may be important, particularly if there is resonant excitation of edge waves. On reflective beaches, edge waves at the incoming wave frequency or subharmonic frequencies may dominate the run-up.

As noted in the introduction (equation (3)), the swash for monochromatic waves becomes saturated, and increasing the incident wave height increases the set-up but not the amplitude of the swash. From (3) and (1),

$$H_r^2 = \varepsilon_r^2 g^2 \tan^4\alpha \, \sigma^{-4} \tag{52}$$

with ε_r constant, so one would expect swash spectra at wind wave frequencies to be proportional to σ^{-4}, with the constant of proportionality dependent on beach slope. This was verified by Huntley et al. (1977) for beach run-up data. Guza and Thornton (1982), on Torrey Pines Beach, confirmed saturation in this frequency range, but found a σ^{-3} dependence. At surf beat periods, they found the run-up energy to increase linearly with the incident wave energy, with $\overline{H}_r \sim 0.7 H_{so}$, which is considerably greater than the monochromatic result (3). This is also significantly larger than the mean shoreline set-up $\overline{\zeta}_{max} \sim 0.17 H_{so}$, found by Guza and Thornton (1981) on the same beach. Therefore with large incident waves the shoreline is dominated by surf beat, at least on dissipative beaches.

From Hunt's formula (2) it can be seen that the total run-up height for each wave (neglecting the possibility of surf beat or subharmonic interactions) can be considerably greater than the set-up for the smaller values of ε_o only, that is, only for reflective beaches and for sea walls and dikes. If the dike crest is lower than the total run-up height which would be reached if the dike were of unrestricted height there will be overtopping.

Battjes (1974) has calculated the run-up of irregular waves, assuming the individual waves have a run-up given by Hunt's formula, and that H_o and λ_o have a bivariate Rayleigh distribution. He gives a value for the run-up level exceeded for 2% of the waves (a typical design height for a wall allowing a certain amount of overtopping) equal to

$$R_2 = B\overline{T}\left(gH_{so}\right)^{\frac{1}{2}} \tan\alpha \qquad (53)$$

where B varies from 0.59 to 0.74 depending on the correlation between wave height and wavelength in the assumed bivariate distribution. A similar result, with B dependent on spectral width, was obtained experimentally by van Oorschot and d'Angremond (1968). Battjes (1974) also calculated the expected overtopping volume per unit length of dike for each wave.

These calculations are for normally incident breaking waves, and do not hold for waves which have such small values of ε_o that they are totally reflected without breaking, for which Miche (1944) gives

$$R = H_o\left(\pi/2\alpha\right)^{\frac{1}{2}} \qquad (54)$$

(see also Keller (1963) and Meyer and Taylor (1972)). Waves of this type may occur on beaches as a result of tsunamis (which are apparently strongly amplified for small α) or on very steep slopes. Equation (54) gives $R=H_o$ for a vertical wall, as expected for a standing wave in constant depth: H_o is the incident wave height, not the total height ($2H_o$) of the offshore standing wave. Keller (1963) shows that (54) holds for non-linear as well as linear theory, and also that the amplification is less than this when the bottom is not uniformly sloping. This result is relevant to tsunamis, which are not necessarily deep water waves far offshore. For the extreme case of very shallow water far from shore ($k_o h_c \to 0$) the amplification factor for wave amplitude tends to 2, as for a shallow water standing wave, the beach having effectively degenerated to a vertical wall.

The Hunt formula for run-up (2) is only empirical and not all experimenters support it (Van Dorn 1976). Theoretical study of the run-up region involves a combination of the finite-amplitude shallow water equations and bores for regions of large surface slope. Mass and momentum flux are conserved across the bore. Models of this type were summarised by Meyer and Taylor (1972), and results of a numerical model of beach bores, including a periodic bore, were given by Hibberd and Peregrine (1977). Theory indicates that when the bore reaches the shoreline, with speed u_o, it continues to climb the beach in such a way that the shoreline now behaves just like a frictionless block. That is, it has an initial velocity u_o and a downslope acceleration $g\sin\alpha$. If the motion of the bore is periodic, so that it returns to its initial position in time T, this implies that $T = 2u_o/g\sin\alpha$, and also that the vertical excursion

$$H_r = gT^2\sin^2\alpha/8 \tag{55}$$

This may be compared with (3): it is equal to $(4\pi^2/8)(H_o/\varepsilon_o)$ if α is small. Van Dorn (1976) finds good experimental support for (55), though most data points fall below the theoretical value, possibly because of friction. As Hibberd and Peregrine (1977) point out, in the thin sheet of water formed behind the run-up tip viscosity, porosity, surface tension and beach roughness may all become important.

It is not clear how the theoretical set-up and run-up values may be combined to give a result for R as simple as Hunt's empirical formula (2). Since run-up range is saturated, and so is independent of H_0, and set-up increases with H_0 they behave quite differently.

6. CONCLUSIONS

(a) Wind waves give an important contribution to sea level at the coast, both in the mean (set-up) and at a wide range of frequencies, including those of surf beat (30s to a few minutes period).

(b) Wave set-up at the shoreline on beaches is about a fifth of the significant wave height offshore. It is weakly dependent on the offshore wave steepness. The mean level dips below the offshore (still-water) level just outside the breaker line and rises in the surf zone to reach the maximum set-up at the shoreline.

(c) Wave set-up effects may raise the level in harbours relative to that outside, and may modulate the tidal signal in the harbour.

(d) Because of refraction and diffraction, the spatial scale of set-up variability is small compared with the scale of other sea-level variations. Wave set-up may be very different at nearby places on the coast, as it depends on the local topography.

(e) Both mean sea level and tidal measurements may be affected by wave set-up, depending on the siting of the tide gauge.

(f) Swash (the oscillation of the shoreline) at the wave frequency is saturated, that is, independent of offshore wave height, while set-up increases with wave height.

(g) On dissipative beaches (characterised by wide surf zones and spilling breakers) there may be strong surf beat oscillations with magnitude comparable with the offshore wave height. On reflective beaches (characterised by narrow surf zones and plunging or surging breakers) there may be strong subharmonic oscillations. These oscillations (both surf beat and subharmonic) may take the form of progressive or standing edge waves trapped to the coast.

7. ACKNOWLEDGEMENT

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9. NOTATION

a wave amplitude

A area of harbour

B run-up coefficient (equ. 53)

c phase velocity

c_g group velocity

C friction coefficient

d mean depth

d_b mean depth at breaker line

$D_o(\theta_o)$ offshore directional distribution

e_1 $\cos\theta$

e_2 $\sin\theta$

E energy per unit area

f as suffix, 'fictitious' values (without breaking)

g gravitational acceleration

$G(\sigma,\theta)$ directional spectrum

h still-water depth

h_o value of h far offshore

H wave height

H_o deep water wave height

H_b wave height at breaker line

H_e wave height at harbour entrance

H_r difference between run-up height and run-down height

H_{rms} root mean square wave height

H_{ro} deep water value of H_{rms}

H_s significant wave height

H_{so} deep water value of H_s

H_{sb} value of H_s at breaking

H_T wave height at toe of beach

k wavenumber = $2\pi/\lambda$

L distance between headlands

m mean momentum per unit area <u>or</u> longshore mode number

n = c_g/c <u>or</u> edge wave mode number

p pressure

P = $T\,(g/d_b)^{\frac{1}{2}}$

$Q_f(H_b)$ fraction of breaking waves

R maximum run-up height (elevation above still water level)

R_2 run-up level exceeded by 2% of waves

S_{ij} radiation stress tensor

S_{ij}^T integral over frequency of $S_{ij}(\sigma)$, the radiation stress spectrum.

$S_o(\sigma)$ offshore frequency spectrum

t time

T wave period

(u, v, w) velocity components

U_1, U_2 mean horizontal velocity components

u_o initial velocity of bore face as it reaches shoreline

w_s sediment fall velocity

x coordinate in direction of wave travel <u>or</u> shoreward direction

y coordinate in longshore direction

z coordinate in upwards direction (measured from still water level)

α angle between beach and horizontal

α' exponential beach profile parameter

γ ratio of wave height to depth (H/d) for breaking waves

γ' $= H_S/d$

$\varepsilon, \varepsilon_o, \varepsilon_b, \varepsilon_T$ surf similarity parameters (equ.1)

ε_r value of ε calculated from H_r

ζ surface elevation above still water level

θ angle between wave crests and the longshore direction

θ_o value of θ in deep water

θ_b value of θ at breaker line

λ wavelength

λ_o deep water wavelength

ρ water density

σ wave frequency = $2\pi/T$

$\overline{\sigma_o}$ mean frequency of incoming waves in deep water

τ_{b1}, τ_{b2} bottom stress components

τ_{s1}, τ_{s2} surface (wind) stress components

Ω beach state parameter (equ.4)

Overbar denotes mean, either in time <u>or</u> of a distribution.

Duplicated symbols are clear from the context.

10. FIGURES

(1) Definition sketch of the nearshore region, (a) view from above, (b) vertical section.

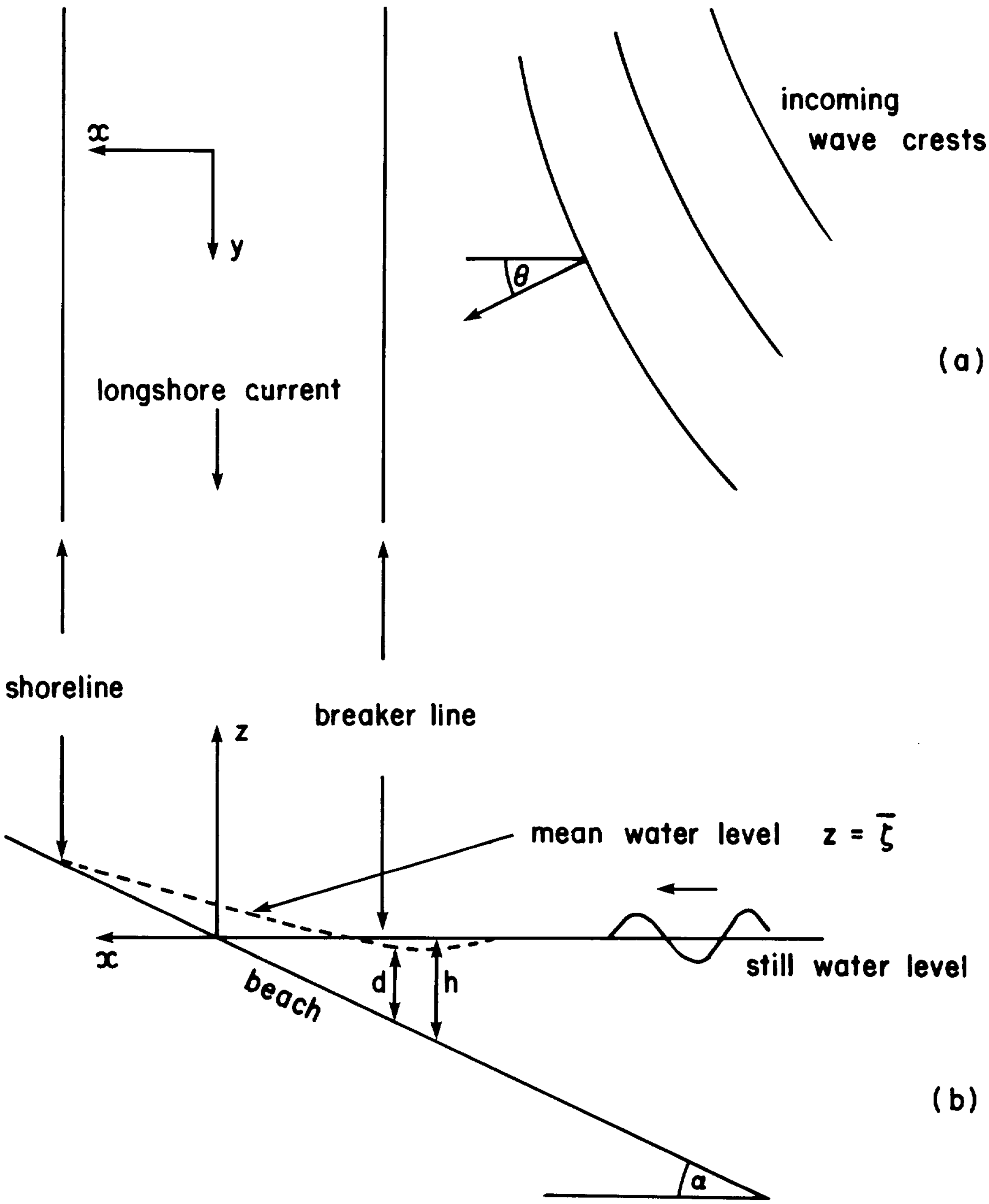
(2) Breaker types, after Battjes (1974), (a) surging, ξ = 0.25, (b) collapsing, ξ = 0.70, (c) plunging, ξ = 2.8, (d) plunging, ξ = 25, (e) spilling, ξ = 157.

(3) Calculated set-up curves from Battjes (1974), for a narrow spectrum with $\bar{\theta}_c$ =15 , γ=0.8 and various deep water wave steepnesses (= $H_{ro} T_c^2 / 2\pi g$)

(4) Laboratory measurements of (a) radiation stress and (b) set-up, from Stive and Wind (1982), test 1 (ξ_b=358, spilling breakers), and comparison with theories. Dots are measurements (in (a), dots with error bars are radiation stress calculated from measured velocity field and surface elevations, while the dashed line is the radiation stress calculated from the measured water level, using equation (13)). L1 is linear theory with γ=0.8, L2 is linear theory with γ=0.6, C is based on Cokelet's wave theory, J on hyperbolic waves (James 1974). Vertical arrows mark theoretical breakpoints, e the experimental breakpoint.

(5) Field data from Guza and Thornton (1981). Each circle represents a different day's data. The straight line is the best fit $\bar{\zeta}_{max}$ = 0.17H_{so} (the outlying point with a set-up of nearly 30cm was not included in the fit).

(6) Offshore dependence of edge wave modes n = 0 to 3, as a function of the dimensionless offshore coordinate X=$-\sigma^2 x / g\tan\alpha$, normalised to 1 at the shoreline.



incoming
wave crests
x
y
θ
(a)
longshore current
shoreline
breaker line
z
mean water level z = $\bar{\zeta}$
still water level
x
beach
d
h
(b)
α

Figure 1

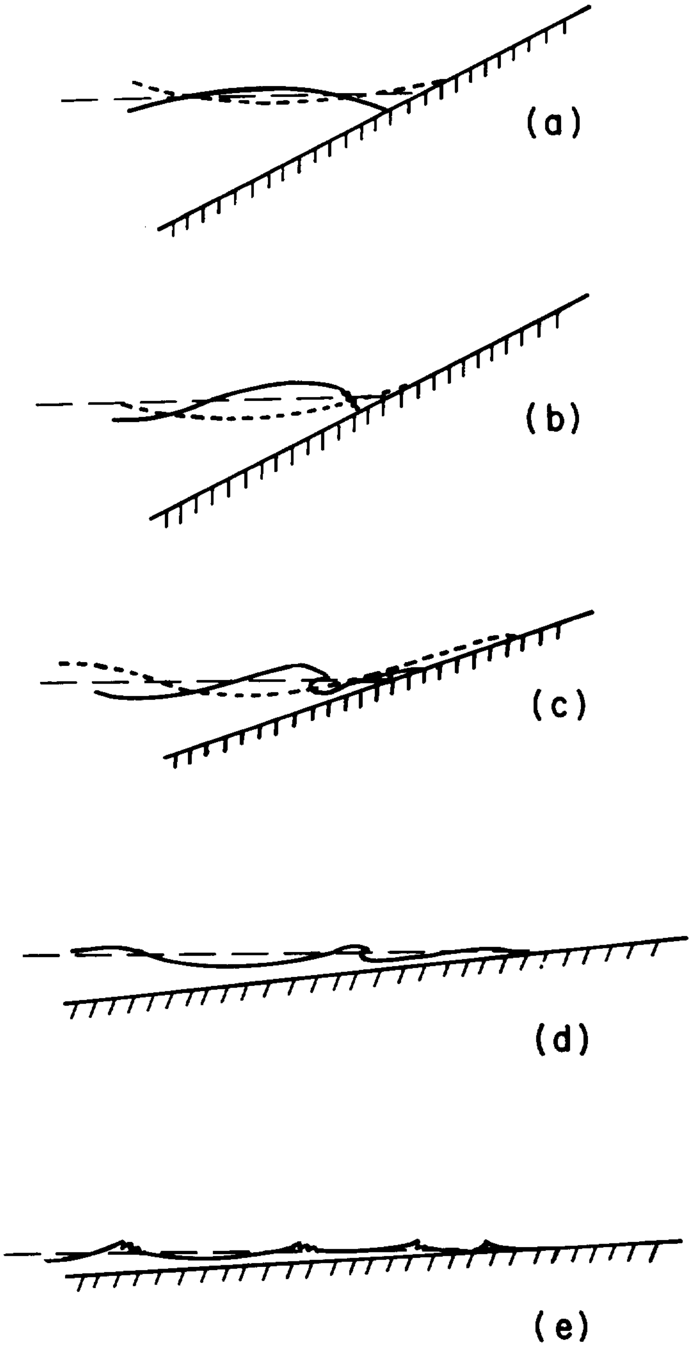


Figure 2

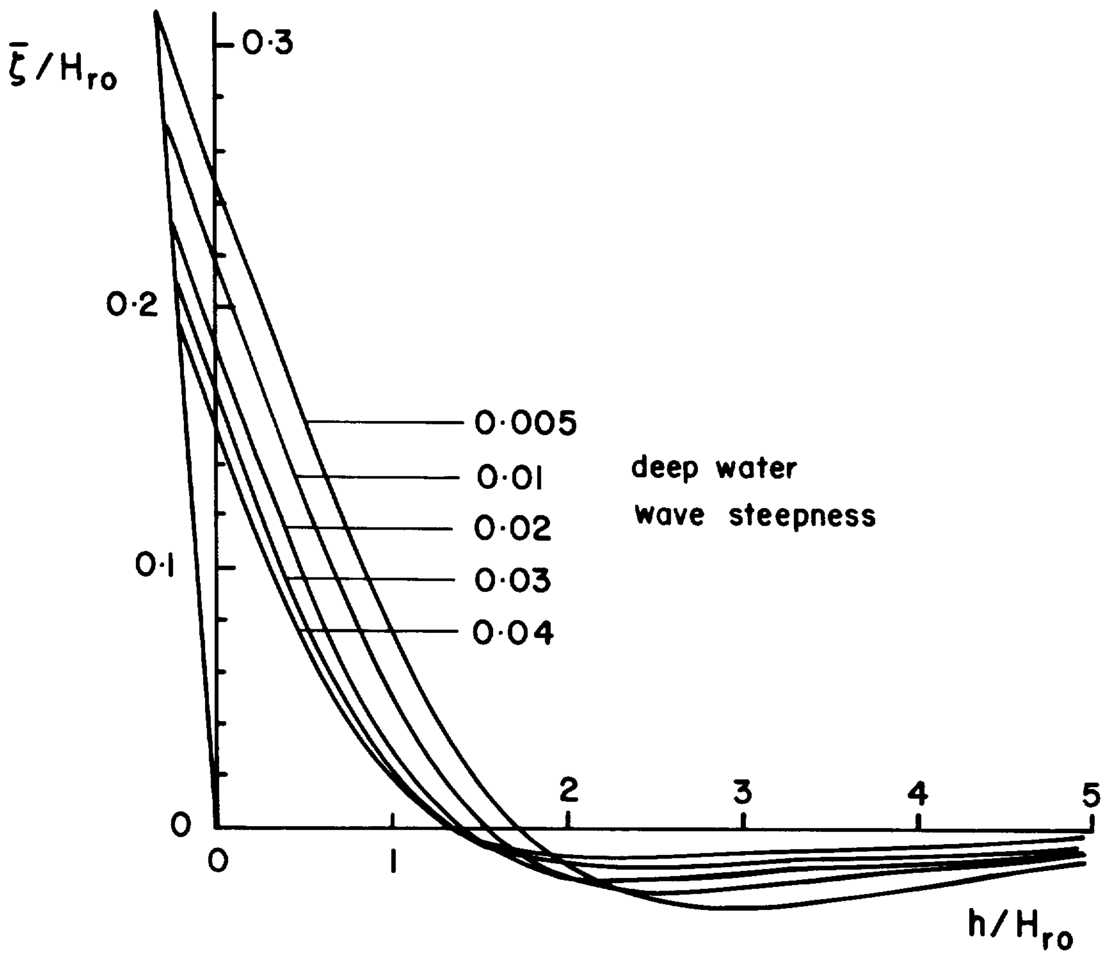
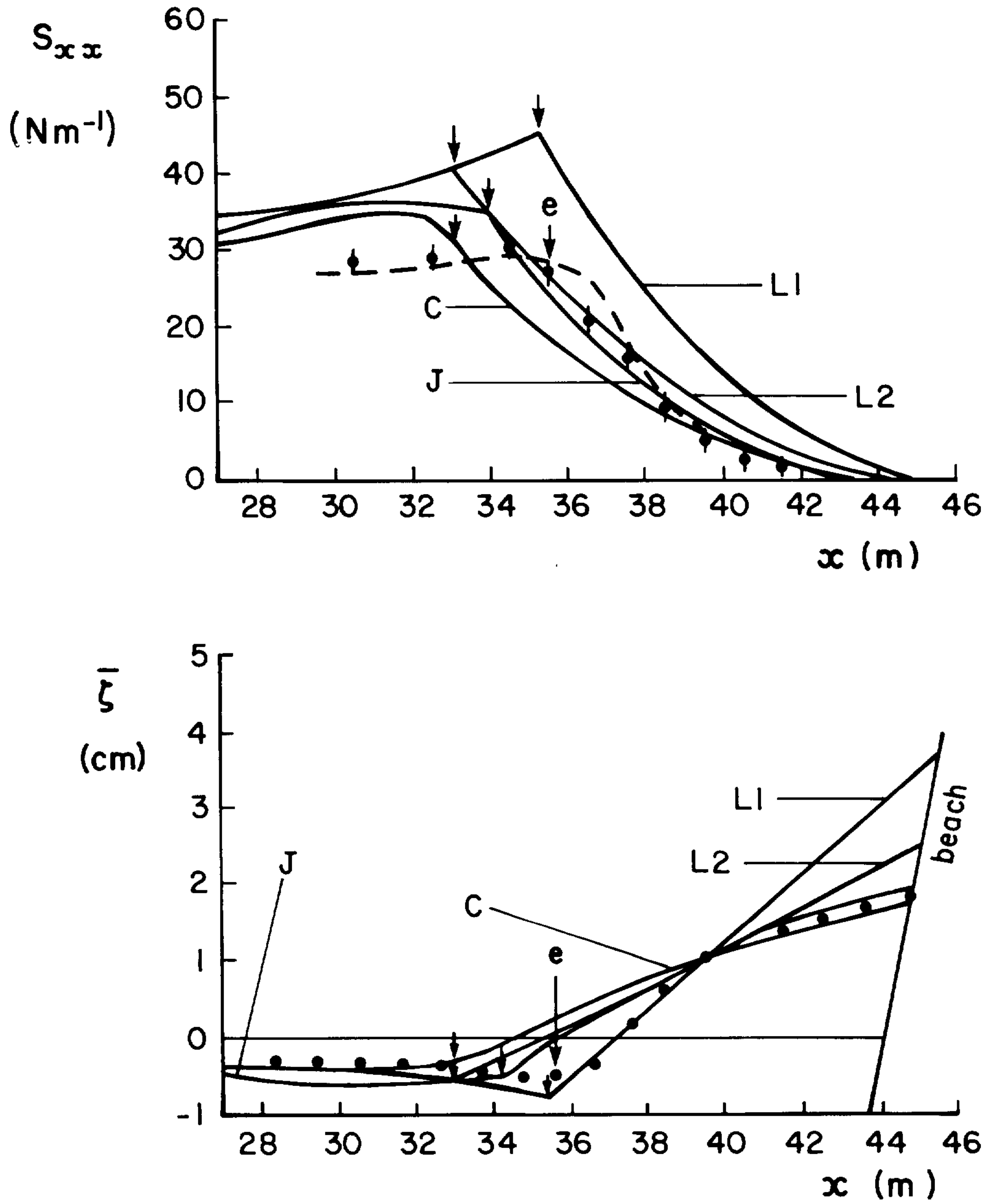

$\bar{\zeta}/H_{ro}$
0·3
0·2
0·1
0
0·005
0·01
0·02
0·03
0·04
deep water
wave steepness
0
1
2
3
4
5
h/H_{ro}

Figure 3



S_{xx}
(Nm^{-1})
60
50
40
30
20
10
0
e
L1
C
J
L2
28
30
32
34
36
38
40
42
44
46
x (m)
$\bar{\zeta}$
(cm)
5
4
3
2
1
0
-1
L1
L2
beach
J
C
e
28
30
32
34
36
38
40
42
44
46
x (m)

Figure 4

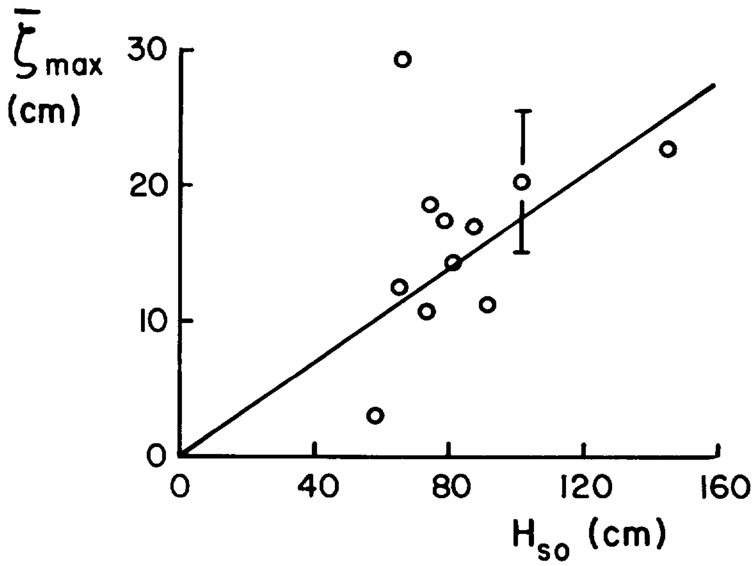


Figure 5

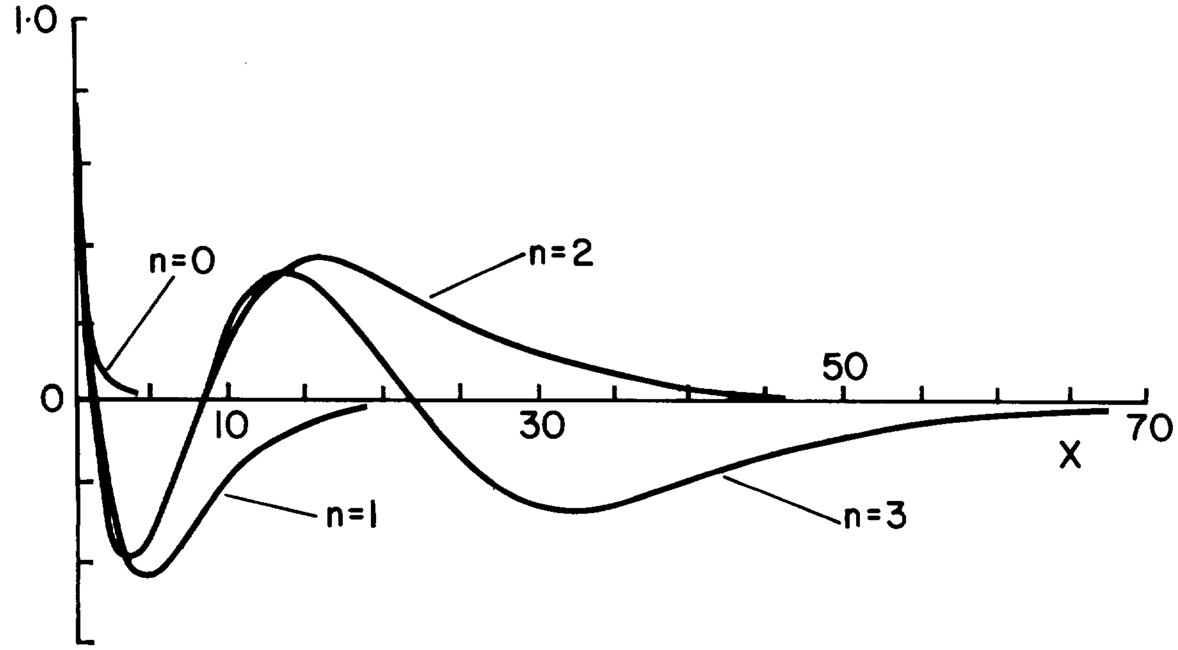


Figure 6