I.O.S.

PARAMETERIZING EXTREME STILL WATER LEVELS AND WAVES IN DESIGN LEVEL STUDIES

BY

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BIDSTON

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"Neither can the floods drown it."

Songs of Solomon 8:7

CONTENTS

1. INTRODUCTION

The trend in all design studies of onshore and offshore structures is to work with system responses (of walls, rigs etc.) to joint events (swl-waves, currents-waves) of the oceanographic variables.

In the U.K., post-1953 sea defence design was based on a specified extreme still water level (often the 1953 observed level), and waves were allowed for by adding a certain additional free-board. Following the series of coastal floods during 1976-78, more modern design thinking requires the consideration of the effect of combinations of still water levels and waves, since it is recognised that critical extreme floods are not necessarily due to extreme events of either variable alone.

The design of proposed sea defences or the assessment of existing ones therefore depends on estimating the probabilities of extreme still water levels and of waves. The problem is to determine the degree of swl-wave correlation and to present it in such a way that it is useful in the engineering design of sea defences.

This report is concerned with the state of the art in parameterising and estimating extreme still water level, extreme waves and their combination. It is intended as a background document for the MAFF commissioned work by the IOS. However it may also be useful as a more general introduction to the subject. The report is based on an extensive literature review and on discussions with MAFF engineers, several U.K. Water Authorities, and the Dutch Rijkswaterstaat.

2. STILL WATER LEVEL

This section opens with a brief discussion of the definition of still water level (swl) and its component variables. Sub-section 2.2 is concerned with the definition and use of swl parameters in the existing UK and European Guidance Notes or Codes of Practice for the design of on- or off-shore installations. It also describes the present use by various authorities of swl design values. Sub-section 2.3 describes and discusses the two main methods of estimating extreme swl by either analysing extreme values or by statistically combining joint probabilities of tide and surge. Finally, in sub-section 2.4 there is a discussion of the problem of interpolating between extreme swl estimates at specified sites in order to obtain estimates for intermediate sites along the coast.

2.1 Basic variables

Still water level (swl) is defined here as the observed water level at a location when waves have been averaged out. It contains contributions due to astronomical tides, meteorologically induced surges, and mean sea level. A further contribution due to tide-surge interaction may be important in some shallow water areas and its significance is discussed where appropriate.

The basic statistics of tide, surge and msl, and their variations are described and discussed in Pugh and Faull (1983). Useful statistics for extreme level studies are Highest and Lowest Astronomical Tide (HAT and LAT), which can be determined as the definite upper and lower limit of the frequency distribution of predicted tide levels (usually computed over a 18/19y period to include nodal variations (Pugh and Vassie 1980)). The frequency distribution of observed surge levels has no theoretical limits but those from UK ports generally have a slight positive skewness and a longer "tail" than the classic Gaussion distribution (Pugh and Vassie 1980). A contribution to the surge level may be a steady mean wave set-up due to any wave activity during the period of observation. This may be an important factor in the swl reached at the defence site, the set-up at the shorelines on beaches is about one-fifth of the significant wave height offshore (James, 1983). Wave set-up effects may also raise the level in harbours relative to that outside (Thompson 1983), and may modulate the tidal signal at the tide gauge site (James 1983).

Mean sea level can be defined as the level at which the sea would lie if the effects of tides, surges and waves were removed. Mean sea level can be computed over any time scale although monthly and yearly values are usually quoted. Mean levels measured over less than a year will be affected by the seasonal cycle in msl which has a typical amplitude of 0.07m around the British Isles.

2.2 Design parameters

Guidance notes or Codes of practice for the design of on-or off-shore installations usually use the concept of a "return-period" still water level. The return period of events which exceed a particular level ($h>H$) is the average time between such events and is therefore related to the probability (p) and frequency (f) of occurrence of the observed events, such that

$$rp(h>H) = (p.f)^{-1} \tag{1a}$$

Note that if the probability (p) is defined in terms of a level (H) not being exceeded, then

$$rp(h > H) = [(1-p).f]^{-1}. \tag{1b}$$

Thus an event with a probability of 0.0001 of occurring once in a particular year has a return period = (0.0001×1) =10000y; with the same probability of occurrence once an hour, it has a return period of 10000h or 10000/8766 = 1.1y. In the case of High Water (HW) levels, if the probability of exceedance of a particular HW level is 0.0001, then its return period is (0.0001×705) = 14.2y, because there are 705 HWs per year.

Note that the return period is the average time between occurrences of an event, and that there is a finite risk that one such event will occur during a period equal to the return period. This risk (ri) is related to the return period (rp) and design life of the structure (L) by

$$ri = 1 - (1 - 1/rp)^{L}. \tag{2}$$

Tabulations of ri, rp and L are given in the Flood Studies Report (1975). If L = rp, then ri = 0.63, i.e. there is a 63% probability that the return period event will occur during the life time of the structure; the risk can be reduced by choosing a return period greater than the effective lifetime of the structure.

If L = 1, then ri = $(rp)^{-1}$, i.e. the probability that the event concerned will be equalled or exceeded in any one year is the reciprocal of the return period - this is defined as the exceedance probability.

The Department of Energy (DEn) Guidance Notes on offshore installations (1977) contain a map of equal mean spring tidal range; the highest annual spring range is found, to an acceptable degree of accuracy, by multiplying the mean values by 1.30. A map of surge levels is also included which gives the expected wind induced surge component in the North Sea with a 50y return period. The surge map has been published in MIAS Bulletin No. 6. Both maps were revised in 1972 and are currently being updated by the IOS. The Guidance Notes advise that the design 50y surge level at the specified location should be computed by adding the wind induced component to a component of 600mm due to atmospheric pressure reduction during the 50y return period storm.

The Code of Practice for fixed offshore structures issued by the British Standards Institute (1982) contains the identical maps of highest annual spring range and expected wind induced surge in the North Sea produced by the IOS in 1972, but also a more detailed map of the latter produced by the IOS in 1978 (with significant differences).

Bureau Veritas have not yet issued any Code of Practice but notes provided by them (I.J. Day, personal communication, 1982) indicate that the DEn Guidance Notes are taken into account in their verification and certification of offshore installations, except that a return period of 100y is favoured.

The Guidance Notes issued by the Danish Energy Agency (1980) also make a distinction between variations in water level due to tide and storm surge. The tidal variation is defined as the (HAT - LAT) range at the location in question, obtained from predicted tide tables, mean water level is defined as the average of HAT and LAT. The surge is defined as consisting of components due to wind and barometric pressure and is considered to be a statistical phenomenon for which "the usual statistical parameters can be derived using generally acknowledged statistical methods". (There are no guidelines as to what these methods are). It is considered desirable to correlate measurements of water level at the specified location with long term statistics from other locations, so that an extrapolation to an annual probability of exceedance of 0.02 (return period = 50y) can be made. The statement is made that water level variations are rather small in the Danish sector of the North Sea, and that the probability that the extreme water variation exceeds 1.2m is 0.02.

The Guidelines issued by Det norske Veritas (DnV) (1977) also define the tidal range as (HAT - LAT), the mean water level as the average of HAT and LAT, and storm surges as including wind-induced and pressure- induced effects. Extreme swl at a location is defined as "the highest astronomical tide including storm surge" : an accompanying figure indicates that this means HAT plus storm surge level. No guidelines on actual values are given, although other parameters are based on a return period of 100y.

The UK "Waverley Committee" Report on coastal flooding (Home Office, 1954) recommended that the maximum standard of protection afforded by public authorities should be that sufficient to withstand the water level (tide plus surge) caused by the flood of January 1953. Following the storm surges of 1977 and 1978, the Flood Protection Review Committee (FPRC) reviewed the standard in 1979 (B. Trafford personal communication, 1983) and recommended substituting the phrase "worst recorded storm tide" (i.e. tide plus surge) for the Waverley Committee's 1953 flood standard, with the proviso that this recommendation should not be rigidly applied when return periods exceed about 1000y. A further recommendation was that the revised standard should not be considered exclusively in terms of "static" (still) water levels, but should take into account wave effects, and this is discussed in sub-section 4.1.

Advice issued by the Ministry of Agriculture, Fisheries and Food (MAFF) to River Authorities in the 1960's (K.C. Noble, personal communication, 1970) suggested the standard of defence should, relative to conditions 50y ahead,

a) protect against a tide having a return period of 200 to 300y without permitting any noteworthy flooding,

and b) protect against a tide with a 1000y return period without disastrous flooding.

There is a distinction between a) "noteworthy" and b) "disastrous" flooding, which could be characterised as

a) due to the combined level of swl and waves being above the defence level,

and b) due to the swl itself being above the defence level.

The Anglian Water Authority use design values of swl based either on the 1953 observed level or on return period levels based on computations, using annual maxima, by Suthons (1963), Lennon (1963) or Graff (1981), see sub-section 2.3.1.

The Sussex Division of the Southern Water Authority (SWA) use return period levels estimated by Blackman and Graff (1978) using annual maxima. The Kent Division of the SWA use estimates for the Deal-Sandwich coast produced by Binnie and Partners (1980), based on Blackman's and Graff's estimates and their own analysis of the joint frequency of HW and corresponding surges.

The Wessex Water Authority (1979) use the 100y return period swl at Avonmouth, estimated using annual maxima, as the basis for their sea defence bank designs, and transpose this level to other sites using differences of HAT. The Severn-Trent Water Authority use design levels based on a HW profile computed by a 1-dimensional mathematical model developed by the Hydraulic Research Station Ltd. (HRS). This uses a 30y return period swl at Avonmouth estimated using annual maxima (HRS 1981a).

Dutch coastal defences are designed to a 10,000y return period swl, based on frequency of exceedance curves using annual maxima from 1888-1956 (Delta Committee 1982), see next sub-section.

2.3 Estimation of extreme still water levels

2.3.1 Analysis of extreme values

The method involves fitting the cumulative frequency distribution of a series of observed sea levels, usually annual maxima, by a distribution and extrapolating to rarer events to obtain the required low probability and long return period value. Tide, surge and msl components are treated together as one single observed level. Various distributions have been used to fit the data using various methods such as graphical fitting and fitting by moments and maximum likelihood (NERC 1975). The estimates obtained depend critically on the choice of distribution and fitting method and the length of data used.

Suthons (1963) and Lennon (1963) applied the method to annual maxima data from U.K. east and south coast ports and west coast ports respectively. Suthons used methods due to Gumbell and Jenkinson to fit the data, in addition Lennon used methods due to Baricelli and also the linear log, normal frequency and adjusted normal frequency distributions. Lennon's estimates for Avonmouth ranged from 8.41 to 8.90m for the 1 in 100y swl, using the 6 different methods on 31 annual maxima. HRS used the adjusted normal distribution to fit and extrapolate 49 annual maxima, recomputed their estimates using updated and corrected data and the normal distribution (HRS, 1981a), and finally recomputed their estimates after the 1981 Bristol Channel surge (HRS, 1981a, Appendix); their 1 in 100y estimates were 8.60, 8.56 and 8.64m respectively.

Graff (1981) has applied the technique to annual maxima obtained from 67 ports around the British Isles, and his work is based on the Jenkinson method used by Lennon (1963) and Suthons (1963). A series of N annual maxima, $h = H1, H2 \ldots HN$ are ranked in ascending order of magnitude and the cumulative frequency of the mth value found from

$$P = (2m-1)/2n. \tag{3}$$

The cumulative frequency distribution is fitted by one of a family of extreme value distributions, described by the two-parameter General Extreme Value (GEV) distribution (Jenkinson 1955)

$$h = a\left(1 - e^{-ky}\right), \tag{4}$$

where a and k are conditional parameters of the distribution and y is the reduced variate

$$y = -\ln\left(-\ln P\right), \tag{5}$$

a and k are calculated from the mean annual maximum, and the standard deviations of the annual and biennial maxima.

The curves are classified as Fisher-Tippet types 1, 2, 3 (Fisher-Tippet 1928) depending on the curvature, and hence the value of k since

$$dy/dh = (1/ak)\exp ky. \tag{6}$$

Hence

$k = 0$, Fisher-Tippet Type 1, h has neither an upper nor lower asymptotic limit,

$k < 0$, Fisher-Tippet Type 2, h has a lower asymptotic limit,

$k > 0$, Fisher-Tippet Type 3, h has a higher asymptotic limit. (see Figure 1a). (See Appendix A for further definitions of the distributions).

Frequency distribution curves of height, h, against reduced variate, y, or return period are drawn (Figure 1b), and the value of h for any return period can be read off, the curve being extrapolated if necessary, since, for annual maxima,

$$(rp)^{-1} = 1 - P = 1 - \exp\left(-e^{-y}\right), \tag{7}$$

noting that the probability, P, is the observed probability of an annual maximum $< h$.

Graff (1981) found that the estimates of the extreme levels were unstable, depending critically on the length of data analysed and on the inclusion or exclusion of particular values, see Figure 2. For example, following the Bristol Channel floods of 13th December 1981, a reanalysis of the Avonmouth data, using 6 extra annual maxima either unavailable to Graff or rejected by him, increased the estimate of the 100y return period level by 0.35m and the 250y return period level by 0.44m (D. Blackman, personal communication, 1982). This lack of stability makes extrapolation to probabilities less than 0.01y (return period $>$ 100y for annual events) very undesirable using this method.

The Dutch Delta Committee (1962) produced a frequency of exceedance curve of swl as a criterion for the design of the Delta Plan works. This curve is based on data from 1888 - 1956 corrected for influences due to the Delta works, and is given by

$$P(h_m > h) = \exp\left(\frac{2.94 - h}{0.3026}\right), \tag{8}$$

where h_m is the highest swl (in metres to Normaal Amsterdams Peil (NAP)) during a storm. This distribution is the Weibull distribution whose upper "tail" can be fitted by a F-T 1 curve.

De Young and Pfafflin (1975) applied the GEV method to data from USA east coast stations, using a cumulative frequency $m/(n+1)$ and a distribution of the form

$$P = \exp\left(-\exp - a \ln h/h_f\right), \tag{9}$$

where h was recorded tide height (i.e. tide + surge), h_f the modal value, and "a" an empirical constant. Three extreme value distributions were evaluated for each station, using all monthly highest tides above a threshold value, the monthly maxima, and the annual maxima. The assumption is made that the surge level occurs simultaneously with the astronomical tide level, and therefore the probability of occurrence of the "abnormal" high tide is the product of the probability of occurrence of the astronomical tide, ($1-p_1$), and the probability that the wind generated portion will be equalled, ($1-p_2$). Then

$$rp = [f.(1-p_1).(1-p_2)]^{-1}, \tag{10}$$

where f is 12 or 1 for the monthly or yearly data respectively.

The choice of value of p_1 is the main problem and they take the point at which the straight line plot of L_n (cumulative frequency) against observed height deviates from a straight line, as the astronomical tides are assumed to follow a log - normal distribution. As an example of the results, the 3 extreme value distributions for The Battery, New York, gave a return period of 38.8, 38.0 and 33.3y for a level of 13.3ft for the monthly highest tides, monthly maxima and annual maxima respectively. De Young and Pfafflin note that the assumption of the surge occurring simultaneously with the high astronomical tide does not allow for the possibility of the highest level occurring due to a combination of a higher surge level and a lower astronomical tide level.

Walden and Prescott (1980) (and in greater detail, Walden 1982) derived confidence intervals for frequency distribution curves for Portsmouth, Immingham and Newlyn using a maximum likelihood procedure. They showed that the size of the confidence region depends very much on the parameter estimates, a and k, as well of the length of the series of annual maxima levels available. For example, using 59 and 61 annual maxima from Immingham and Newlyn respectively, then the confidence interval for the 100y return period at Immingham (0.60m) was about 4 x that for Newlyn; due to the k parameter for Immingham being smaller than that for Newlyn. It was also twice as large as the difference between the 10y and 100y estimated return period levels at Immingham!

The use of calendar years as sampling years for the annual maxima method means that there is the possibility that two annual maxima from adjacent years could come from the same storm event in December-January. DeYoung and Pfafflin (1975) have applied the extreme value method to monthly maxima, but two maxima from adjacent months could come from the same storm event. Also, use of only one value does not allow for the possibility of two or more occurrences of the maximum level in any one year. DeYoung and Pfafflin have fitted extreme value distributions to monthly highest tides above a threshold value and this is the nearest approach to the "peaks over threshold" (POT) method used in hydrological flood studies (NERC 1975).

In the simplest POT model, the number of exceedances per year of flood flows (q) greater than a threshold flow (q_0) is treated as a Poisson variate whose parameter (λ) is estimated by

$$\hat{\lambda} = M/N \tag{11}$$

where M is the number of exceedances in N years of record. The magnitudes of the exceedances are treated as an exponential distribution whose parameter (β) is estimated by

$$\hat{\beta} = \bar{q} - q_0 = \sum_{i=1}^{M} \frac{q_i}{M} - q_0 \tag{12}$$

where $\bar{q}$ is the mean of the exceedance flows. Then the flood with return period of R years may be estimated from

$$Q(R) = q_o + \hat{\beta} \ln \hat{\lambda} + \hat{\beta} \ln T. \tag{13}$$

2.3.2 Joint- or Combined- probability method

This method is based on the separation of hourly values of swl into msl, tide and surge components. Separate probability distributions are computed for tide and surge components and the probabilities of obtaining tide and surge levels combined together to obtain the probability of a particular swl, and hence return period levels. The return period levels can be adjusted for any trends in msl if these are identifiable for the specified location.

The statistical method assumes that tide and surge are independent events; in practice, tide-surge interaction is only important in extreme shallow water areas, but the basic method of statistical combination may be adapted to allow for this. It is also assumed that both tide and surge are stationary processes, and although this is satisfactory for tides (Pugh and Faull 1982), surges are not randomly distributed in time because of seasonal and storm-surge meteorological effects. Lamb (1982) states that an increasing frequency of north-westerly and northerly winds over the British Isles has occurred during the period 1960 - 1980, and that this seems to account for the increasing roughness of the North Sea as observed and reported by the German Navy. He states that river-gauge observations in the Elbe at Cuxhaven and Hamburg show that the North Sea storm flood frequency in the winter of 1972-73 was the greatest since 1792-93.

Ackers and Ruxton (1974) used the joint probability method but only applied it to predicted HW levels and corresponding surges at Southend.

HW levels for 1969-73 were enumerated into 0.5ft intervals and probabilities of occurrence assigned to each range. The frequency distribution of surge residuals in 1m intervals was determined by counting the average number of occurrences per year. Combined frequencies of predicted tides and surges were found by computing the number of occasions per year of combinations of predicted tide and surge by multiplying the probability of tide interval and surge interval.

The return period of a particular level was obtained by adding together all the probabilities of combination of HW plus surge which gave a level greater than, or equal to, the particular level. Therefore their method assumes that an extreme swl will occur at the maximum of the predicted tide. A similar method was used by them for Deal and Sheppey (Binnie and Partners 1980).

Pugh and Vassie (1979, 1980) applied the method to tide and surge distributions obtained from hourly tide gauge records from 7 U.K. ports, and computed the probability of obtaining any of the surges at any state of the tide. For each port they analysed all the available data to give a set of harmonic constituents, from which a 19y set of tides could be predicted and used to compute the tide frequency distribution. From the observations and predictions, the hourly values of surge residuals were computed and the surge frequency distribution determined. Figure 3 shows the tide and surge frequency distributions for Aberdeen, which are typical of those obtained from all 7 ports.

Probabilities of tide and surge were combined to produce a total probability of occurrence of a particular swl, h :

$$P(h) = \int_{-\infty}^{\infty} P_T(h-y) \cdot P_s(y)\, dy, \tag{14}$$

where P_T, P_s are probability density functions (pdf) for tide and surge respectively. e.g.

$$P(h=4m) = P_T(T=4m) \times P_s(s=0) + \ldots + P_T(T=0) \times P_s(s=4m) \tag{15}$$

From $P(h)$, the probability of exceeding a particular level, H, is the cumulative distribution function:

$$Q(H) = \int_{H}^{\infty} P(h)\, dh, \tag{16}$$

and the probability of exposure of a level is given by

$$R(H) = \int_{-\infty}^{H} P(h)\, dh. \tag{17}$$

Note that the method yields probabilities of extreme low levels as well as extreme high levels and that a flooding exposure index may be computed from the cumulative distributions (Figure 3). Figure 4 shows an example of the probability curves obtained.

Pugh and Vassie investigated the problem of converting probabilities of instantaneous values into yearly return periods when the samples are not independent, as with hourly swl observations, due to correlation of the surge residuals. They found that the necessary adjustment for correlation according to the formula

$$r_p = [8766 \times Q(H)]^{-1} \quad \text{or} \quad [8766 \times R(H)]^{-1} \tag{18}$$

is so small compared with the uncertainity associated with statistical sampling that, in view of the accuracy required, in practice it is unnecessary. The return period may be taken to be the inverse of the probability, in hours.

The stability of the method was investigated by computing estimates from several shorter blocks of the available data, and Pugh and Vassie found, for example, that the standard deviation of the estimates of the

100y level at Aberdeen from 9 one year blocks was only 0.09m. To investigate the effect of additional extreme surges, they added 3 artifical surges to the tide and surge statistics and found that the 100y return period level was changed by less than 0.01m. The stability of the method derives from the independence of the physical forcing factors of mean sea level, tide and surge; namely geological/climatic, tidal and weather.

The method assumes the independence of tide and surge and this was investigated by studying the variance of the surge distribution as a function of tidal level. Pugh and Vassie found that, except for Southend, any interaction effects in the two levels were of secondary importance and that tide and surge could be treated as statistically independent variables. The extreme levels at Southend were computed using separate surge probability density functions for different parts of the tidal range; surge data being grouped into 17, 9, 5, 3 and 1 tidal division. Comparison of the average of the 9 and 5 division cases with the single division case showed that the interactive effects which influence the swl observations at Southend reduced the maximum 100y return period level by 0.5m and increased the minimum level by 0.6m.

Comparison of the 1 in 100y extreme level estimates at 5 of the 7 ports obtained using the joint probability method and the extreme value method showed the former were higher by 0.10 - 0.25m. This is probably because the joint probability method assumes that any possible combination of tide and surge could occur, including extreme tide with extreme surge, whereas extreme swl is statistically more likely to occur with a large tide and a large surge together (Pugh and Vassie 1980).

Walden et al (1982) used a joint probability method which is an adaption of the approach of Tayfun (1979). The probability distribution function of the maximum swl for a surge-tide combination is developed by taking the transient nature of the surge event into account in terms of an effective duration; its amplitude being replaced with an equivalent magnitude defined as an average over the effective duration interval, τ. Suppose

$$Z = \max_{t_0 \le t \le t_0+\tau} \left[A(t) + S(t) \right], \tag{19}$$

where $A(t)$ is the predicted tide relative to msl and $S(t)$ is a postive surge commencing at any time, to, in a tidal cycle and having duration τ. If the variation in $S(t)$ is gradual enough to allow it to be approximated by an equivalent constant intensity X over the interval $(t_0, t_0+\tau)$, then

$$Z = \max_{t_0 \le t \le t_0+\tau} \left[A(t) + X \right]. \tag{20}$$

They applied the method to hourly observations at Portsmouth during 1962 and 1964-74, and considered a positive surge to be represented by a series of consecutive hourly residuals all greater than 0.05m, and with a duration, τ, taken as the number of complete hours for which the residual series remained above this level. For surges with $\tau \leqslant 13$h. the equivalent intensity X was determined by equating $X.\tau$ to the area under the surge. For $\tau > 13$h, the highest equivalent intensity over a 13h interval was considered. Distributions of X for τ = 2(1) 50h and for $\tau > 50$h were obtained.

The conditional distribution function of the tidal component was estimated, for given "a" and τ, by measuring the proportion of the total time that the condition $A(t) \le a$ for any interval $(t_0, t_0+\tau)$ was satisfied, by moving an interval of length τ through the $A(t)$ series of 19y predicted tidal values for 1960-1978, and dividing by the total number of hours in 19y. The procedure was repeated for a series of values of "a" for each surge length $\tau = 1, 2, \ldots, 13$, and the distribution functions $P_{A|\tau}(a;\tau)$ so obtained were then numerically differentiated to give the conditional densities $p_{A|\tau}(a;\tau)$.

The conditional distribution function of the maximum swl, Z, for given τ was determined by numerical integration using

$$P_{Z|\tau}(z|\tau) = 1 - \int_{a_s}^{a_{max}} \left[1 - P(X \leq z - a|\tau)\right] p_{A|\tau}(a;\tau)\, da \qquad (21)$$

where a_s was any level in (a_{min}, a_{max}) - the lowest and highest predicted tide respectively.

The 12y of non-tidal residuals were next used to determine the distribution of the surge duration $P_\tau(\tau)$ and the frequency of occurrence f. The marginal distribution of Z was obtained by weighting each $P_{Z|\tau}(Z/\tau)$ by $p_\tau(\tau)$ and summing over τ to give $P_Z(z)$ and hence the 50, 100 and 250y return period levels from

$$rp = \left[f \cdot (1 - P_Z(z)\right]^{-1}. \qquad (22)$$

They compared these values with estimates computed by themselves using all available annual maxima (104), and the 12 annual maxima from their data set, and with Pugh and Vassie's results. The latter values were higher than the annual maxima estimates, while those from the adapted Tayfun approach fell between those from the 104y and 12y annual maxima analyses. The extreme swl estimates from both joint probability methods fell within the approximate 95% confidence intervals computed from the 104y annual maxima sample for Portsmouth (Walden and Prescott 1980).

2.4 Interpolation of extreme swl estimates

Once the extreme levels have been determined at specified sites by the appropriate method, values may need to be obtained for intermediate points. Lennon (1963) suggested a method based on relating given extreme tide levels H_N to Mean High Water Spring levels (MHWS) and Mean Low Water Spring levels (MLWS)

$$F_N = \frac{H_N - MHWS}{MHWS - MLWS} . \tag{23}$$

If the value of this ratio is approximately constant for a given return period level along a stretch of coast, then the extreme level can be found at points where MHWS and MLWS are known. Graff (1981) has computed this ratio for 32 ports around Britain, using the 1 in 100y levels obtained from the GEV method. He obtained a fairly systematic pattern in the distribution of this ratio, with greatest values and variability in the NW and SE regions, and minimum levels of intensity and variation in the SW region. However, the ratio is unstable near tidal amphidromes and in regions of extensive shallow water, e.g. East Anglia. Binnie and Partners (1980) used this method to transpose extreme swl estimates from Dover to Deal, Ramsgate and Sandwich.

The Wessex Water Authority use a simpler method to transfer extreme levels along their coastline from Avonmouth to Clevedon by assuming that the difference in extreme levels between Avonmouth and the location is the same as the differences in HAT values (Wessex Water Authority 1979).

Within IOS, Pugh and Vassie (personal communication, 1983) are currently computing alternative empirical formulae based on their joint-probability estimates.

3. WAVES

The first sub-section is concerned with the variables and parameters commonly used to describe wave sea-states. The present parameterization of the design wave used in engineering studies is then described in sub-section 3.2. The next sub-section deals with the estimation of extreme values of wave heights using methods of analysing wave or wind data. The estimation of extreme values of wave periods is discussed in sub-section 3.4. Finally, the modification of off-shore wave data by shallow effects is discussed in sub-section 3.5.

3.1 Basic variables

The most widely used variables are significant wave height H_s, maximum wave in a given duration $H_{max}(dur)$, mean zero up-crossing wave period T_z, period of the most likely highest wave in a given duration $T_{max}(dur)$, and wave steepness S. It is assumed that the sea-state is stationary over a given duration (usually 3h) and that it can be represented by 1 sample of the wave record during that period - the sample duration is usually 10 to 20 minutes. These "short-term" statistical parameters are calculated from the short record and considered to be representative of the longer period. "Long-term" statistical parameters are estimated by analysing the distribution of the short term parameters over long periods of time.

H_s is defined as the mean height of the highest 1/3 of the waves in the period. It is determined by the direct counting of the waves defined by zero up-crossing, or approximately from the area of the wave spectrum,

$$H_s = 4\, m_0^{1/2}, \tag{24}$$

where M_o, the zero order spectral moment, is the area under the energy-frequency wave spectrum and hence represents the variance of the sea surface elevation. Generally, the spectral moments of a wave record are given by

$$m_i = \int_0^\infty f^i \, E(f) \, df, \tag{25}$$

where E is the surface elevation spectrum (see below). The spectrum- or band-width parameter, ϵ , is a measure of the range of frequencies present in the wave spectrum, and is usually given by

$$\epsilon^2 = (m_0 m_4 - m_2^2) / m_0 m_4 \, . \tag{26a}$$

ϵ varies between 0 (very narrow spectrum with regular waves) and 1 (broad spectrum with an irregular wave pattern and many different wave periods present). However, for some practical purposes this parameter is inconvenient since m_4 may depend rather critically on the behaviour of the spectrum at high frequencies. Longuet-Higgins (1983) has derived a theoretical probability density for the joint distribution of wave periods and heights which has a spectral width defined by

$$\nu^2 = m_0 m_2 / m_1^2 - 1 \, . \tag{26b}$$

and is therefore independent of higher order moments.

H_{max} (d_w) depends upon the number of waves occurring during the period and upon the wave distribution. Assuming a Rayleigh density function for the wave heights (Longuet-Higgins 1980) then

$$H_{max}(d_w) \simeq H_s \left(\ln N_z / 2 \right)^{1/2}, \tag{27}$$

where N_z is the number of waves in the sample, i.e. $(d_w)/T_z$. For practical purposes, $H_{max}(3h)$ may be taken to be 1.9 H_s (Carter and Challenor 1981b) for a duration of 3 hours. The probability density function (pdf) of the surface elevation in a random sea is considered to be Gaussian or Normal.

T_z is estimated by taking the mean of the wave periods of the given wave record. In terms of spectral moments,

$$T_z = (m_0/m_2)^{1/2}. \tag{28}$$

T_{max} ($\bar{T}_{w}$) is often obtained indirectly from T_z using an empirical relationship, or from H_{max} and an assumption about wave steepness.

Steepness is defined as the ratio of the wave's height to its length, $S = H/L$, and can be calculated either from those variables of an individual wave or from the wave statistics H_s and T_z, since (Battjes 1970),

$$S = 2\pi H_s / g T_z^2. \tag{29}$$

Tann (1976) describes in detail the present IOS techniques for estimating the short-term statistics from wave records.

The representation of the sea surface by a linear superposition of sinusoidal waves results in the concept of a wave spectrum. A 1-dimensional wave spectrum represents the distribution of energy with frequency and can be extended to include wave direction.

The Pierson-Moskowitz energy spectrum (P-M) is given in terms of the peak frequency of the spectrum, f_p, and the wind speed at 19.5m, $U_{19.5}$,

$$_{PM}S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right], \tag{30}$$

where $\alpha = 8.1 \times 10^{-3}$ and $f_p = 0.8772 \ (g/2\pi U_{19.5})$. It is a fully-developed wave spectrum, applying to waves generated by constant winds over long fetches for long times, and is based on data from shipborne wave recorders in the NE Atlantic Ocean. The P-M spectrum leads to a H_s and T_z for the peak of the spectrum given by (Carter 1982),

$$H_s = 0.02466\, U^2$$
$$T_z = 0.558\, U, \qquad (31)$$

where U is the wind speed at the 10m level.

A commonly used fetch-limited spectrum, the JONSWAP spectrum, is based on measurements in the North Sea and was derived by Hasselman et al (1973) by multiplying the P-M spectrum by a peak enhancement factor

$${}_J S(f) = {}_{PM}S(f) \cdot \gamma^{\exp\left[-(f-f_p)^2/2\sigma^2 f_p^2\right]}, \qquad (32)$$

where $\sigma = 0.07$ for $f < f_p$

$= 0.09 \quad f \geq f_p$,

and $\gamma = 3.3$ for the mean JONSWAP spectrum. This gives (Carter 1982) a H_s and T_z corresponding to the peak frequency of

$$H_s = 0.02013\, X^{0.55}\, U^{0.40}$$

$$T_z = 0.470\, X^{0.33}\, U^{0.34}, \qquad (33)$$

where X is the fetch (in km) and U is wind speed at the 10m level.

There is evidence (Saetre 1974 and Carter 1982) that severe storm wave spectra, especially in confined seas like the North Sea, are more accurately represented by sharply-peaked spectral forms similar to those given by JONSWAP spectra rather than by the broader spectral characteristics of fully-developed conditions. Carter (1982) has compared predictions of wave height using formulae based on the P-M and JONSWAP spectra, and also using formulae derived by Darbyshire and Draper (1963) and Bretschneider (1973) (also known as the Sverdrup, Munk and Bretschneider (SMB) method). He proposes that the formulae based on the JONSWAP spectrum be used for the prediction of fetch- and duration-limited waves formed under the action of the local wind field.

Thorn and Roberts (1981) advise use of the Darbyshire and Draper method for the east coast of England, but consider that the SMB prediction method, as presented in the Shore Protection Manual (1977) is more suitable for a fetch across a shallow estuary. HRS (1981b) use the SMB method for calculating onshore wave heights at sea walls, as it is the only one to include the effect of water depth in reducing wave heights.

Computer programmes are available within IOS for the estimation of wave height and period from wind speed and fetch or duration using formulae based on the Darbyshire and Draper, SMB, and JONSWAP methods (D. Carter, personal communication, 1982, and Carter 1982).

The short-term statistics derived from the wave data are usually presented in standard diagrams and tables. The percentage occurrence of H_s and T_z are shown on histograms and the percentage exceedance of and $H_{max}(d_w)$ drawn, (Figure 5 a). A scatter plot of H_s and T_z shows the number of wave records having particular combinations of values of H_s and T_z, and points of equal occurrences are joined by contour lines to give an indication of the bivariate probability distribution of H_s and T_z, and to illustrate the correlation between them. Lines of constant steepness can be added ; theory predicts that progressive waves greater than 1:7 break but most waves do not even approach this steepness and there are generally fewer than 1:10. (See Figure 5b for an example).

3.2 Design parameters of waves

The "design wave" used in the engineering design of coastal or offshore structures is usually characterized by the most probable height and period of the highest wave with a certain return period (usually 50 or 100y). Thus the 50y wave parameter is that which is exceeded on average once in 50y, i.e. a wave having a probability of 0.02 of being exceeded in any one year. The return period is related to the cumulative probability of the parameter being exceeded (or not exceeded) and the sampling interval by equations 1a or 1b. Thus if the wave height is estimated from 3h records,

$$r_p(y) = \left[(1-p)\times(1\times 365.25\times 8)\right]^{-1}, \tag{34}$$

where p is the probabilty of exceedance defined in terms of the height not being exceeded. Either H_s or H_{max} is used as the design wave height, T_z is usually used as the design wave period, if required.

The UK DEn Guidance Notes (1977) recommend use of a minimum design wave based on the most likely highest wave height, H_{max}, and associated period, with a 50y return period, occurring in a fully developed storm lasting 12h. The guidelines contain maps of these statistics estimated from wind records and wave observations by IOS in 1977 (and currently being revised). Identical maps and advice are contained in the BSI Code of Practice (1982).

Det norske Veritas (1977) use a similar parameter but with a 100y return period. The 100y return period is also recommended by Bureau Veritas (I.J. Day, personal communication, 1982) but details of the design wave parameters are not given, only that the wave "is defined from sea observation data in the concerned region, such data are generally provided by the designer and obtained from measurements in-situ". The Danish Energy Authority (1980) uses H_s and define a spectral peak period, T_p , as

$$(13H_s)^{1/2} < T_p < (28H_s)^{1/2}. \tag{35}$$

A 50y return period, estimated from a fit to the data by the Weibull distribution is recommended.

3.3 Estimation of extreme values of wave heights

There are two main approaches to estimating extreme conditions: extrapolation of measured wave statistics or use of wind-speed statistics and application of them to a wave forecasting technique.

3.3.1 Analysis of wave data

Once the values of H_s or H_{max} have been estimated from each record, a distribution is fitted to the cumulative distribution of all the available data and the "tail" extrapolated to the wave height which has a probability corresponding to the required return period. Various distributions and fitting methods have been used and as there is no theoretical justification for chosing any of them (Carter and Challenor, 1981b), the one which appears to give the best straight line fit to the plotted data is used. Considerable ingenuity has been taken in constructing probability graph paper whose parameters enable straight line fits to be obtained. (See Appendix A for definitions of distributions).

The return period heights obtained depend critically on the distribution chosen to fit the data, e.g. log-normal, Weibull, F-T 1, 2, or 3; and the way in which the best fit is obtained e.g. by eye, by graphical fitting using least-squares fitted straight lines, or by moments and maximum likelihood methods (Carter and Challenor 1983). For example, Carter and Challenor (1981b) tabulate the estimates of the 50y H_s values obtained from 3h data from MV "Famita" obtained by Draper and Driver (1971) using log-normal, Saetre (1974) using Weibull and F-T 1, and Fortnum (1978) using log-normal, Weibull, F-T 1 and 3; they range from 14.4 to 16.3m. Figures 6a to 6c show the fitting and extrapolation of wave data from "Famita" by different distributions (Fortnum 1978).

The above method can be applied to monthly, seasonal or annual maxima of the wave height, and a distribution fitted and extrapolated to obtain the long return period value required. Theoretically, use of the

F-T 1, 2 or 3 extreme value distributions is justified as Fisher and Tippett (1928) have shown that the distribution of maxima of samples from an identically distributed population tends to one of the F-T distributions whatever the population distribution. The F-T 1 distribution is the most widely used of the three in estimating extreme waves from maxima values; unlike the other two, it is unbounded and therefore gives higher and hence safer results (Carter 1983). Fortnum and Tann (1977) fitted F-T 1 to 5 annual maxima of data from "Seven Stones" LV, whereas Carter and Challenor (1978) fitted F-T 1 to both 7 annual maxima and 7 March maxima at the same site. Carter and Challenor (1981a) have fitted F-T 1 to the winter maxima of data at "Famita".

The extreme value method assumes that the data are independent and identically distributed, and as this is more likely to be true for monthly maxima than for annual maxima, Carter and Challenor (1978 and 1981b) analysed monthly maxima separately and combined the resulting distributions to obtain extreme values based on the full 12 months, using the F-T 1 distribution. Assuming the monthly maxima are independent, then the return value of wave height, based on measurements in any particular year, can be obtained from

$$P(x < x) = \prod_{i=1}^{12} \exp\{-\exp[-(x-A_i)/B_i)]\}, \quad (36)$$

where A_i and B_i are the F-T 1 scale and location parameters obtained from each month.

Challenor (1982a) has extended the method by replacing the monthly values of the location parameter by a sine curve representative of the seasonal variation.

Carter and Challenor (1981b) consider that at least 5y of data is required for the extreme value method to be used, and 10-15y required to produce confidence limits of reasonable proportions. They have therefore suggested an alternative approach which can be applied to only one year's wave height data. A distribution is fitted to the upper "tail" of the wave height distribution and extrapolated to give low probabilities of occurrence. The method has a firm theoretical basis because there are only three limiting distributions for the upper tails of distribution functions, corresponding to the F-T 1,2,3 distributions respectively. The type 1 tail distribution is the -ve exponential distribution and has been fitted by Carter and Challenor to "Seven Stones" and "Famita" data, using various methods to determine where in the tail of the data the fit is to be made. However because of the between-year variation in wave distribution, they consider it highly doubtful that this method will give a satisfactory estimate of 50y wave height from 1 year of data.

Challenor (1982b) has fitted a F-T 1 distribution to the 10 highest of the 240 Hs values in each month of data at "Seven Stones" Light Vessel during 1968-1977, hence obtaining estimates of the 50 or 100 return value based on each month's data separately.

Estimates of 50 and 100y return values differ significantly from those obtained by analyses of monthly, seasonal or annual maximas and Challenor states that patterns in the discrepancies raise the possibility of some non-random year to year variation in the distribution of extreme wave heights. If this is the case, he states that it would invalidate the analysis of extremes, or any other analysis using more than one year's data, and make the results of any one year's data difficult to interpret and of limited application.

3.3.2 Analysis of wind data

Several methods have been developed for deriving return values of wave height from wind speed data: the application of general formulae to derive corresponding wave heights from return period wind speed, the analysis of joint observations of winds and waves to derive direct empirical relationships, and the use of a wave model to generate hindcast wave heights which can then be analysed.

In the first method, if the estimated return value of wind speed can be calculated, then one of the formulae for estimating wave height given wind speed, fetch and duration can be applied to give the corresponding return value of wave height. The U.K. Meteorological Office has produced a map (reproduced in DOE's guidelines (1977) and the BSI code of practice (1982)) of the 50y return values of hourly mean wind speed over U.K. waters. The choice of formula used, e.g. that due to Darbyshire and Draper (1963), Bretschneider (1973) or JONSWAP (see Carter 1982), can lead to quite different results - see Tables 5.1 and 5.2 of Carter and Challenor (1981b), and estimates of the 50y return value of wind speed over the open ocean are themselves questionable.

The method can be applied to a specific location by considering the different sectors of wind direction, and hence fetch, to which the location is exposed; and calculating the probability of wave heights given the frequency and duration of appropriate wind speeds. It was used in this way by the Wessex Water Authority (Bown 1983) to examine the wave climate at Kingston Seymour, Somerset following the December 1981 flood events. Waves were hindcast assuming fetch limited waves with a Force 9 wind from 262.5°, using the Shore Protection Manual curves based on the SMB formula. HRS (1981b) have used the same method and formula to hindcast waves, at the Sheppey sea walls, as a function of effective fetch lengths for the main wind directions to which the walls are exposed.

The second method is to use joint observations of wind speed and wave height to obtain a relationship between them, and thus calculate and analyse wave heights from a longer wind record. Binnie and Partners (1980) used this method as part of their studies on flood defences for the Southern Water Authority. They used wind data at Shoeburyness and wave data at "Tongue" and "Varne" Light Vessels to determine relationships between the expected value, E, and standard deviation, σ, of H_s and maximum hourly mean wind speed. The observed wave heights were then standardised,

$$Y = (H_s - E)/\sigma, \tag{37}$$

and empirical cumulative frequency distributions prepared for each class of wind speed in which there were more than 50 events. For each site, the frequency curves were averaged and a fourth order equation fitted to the data:

$$Y = a + bP + cP^2 + dP^3 - eP^4, \tag{38}$$

where "a" to "e" are empirical constants and P is the cumulative probability. The conditional probability, $P(H_s > x|u)$, was then calculated, i.e. the probability of H_s exceeding any given value X for any given daily maximum hourly wind speed u, by calculating E and σ

from U and hence the appropriate Y value, and reading the corresponding probability off the averaged cumulative probability curve.

The method has also been applied by Hogben and Miller (1980) to i) open ocean sites using data from Ocean Weather Ship "India" and "Seven Stones" LV and ii) more sheltered sites using data from "Shambles" "Varne", "Owers" and "Mersey Bar" LVs. They first derived expressions for the mean wave height, $\bar{H}$, (either computed significant wave height or visually observed values) and variance, σ, of wave height in terms of the wind speed :

$$\bar{H}^2 = \bar{H}_{sea}^2 + \bar{H}_{swell}^2 = [(aU^n)^2 + \bar{H}_{swell}^2]^{1/2} \tag{39}$$

$$\sigma = \bar{H}_{swell}\,(b + cU), \tag{40}$$

where $\bar{H}$swell, a, b, c are empirical constants ($\bar{H}$swell = 2m and 1m for i) and ii)). Secondly, they assumed that the conditional distribution of wave height, given the wind speed, is a gamma distribution, such that

$$p(x) = c^{b+1} x^b \exp[-(x)/\Gamma(b+1)], \tag{41}$$

where the two parameters are determined from the mean and variance,

$$\begin{aligned} \text{mean}(x) &= (b+1)/c \\ \text{var}(x) &= (b+1)/c^2. \end{aligned} \tag{42}$$

Therefore, given any wind speed distribution histogram, the distribution of wave height for each wind speed class is summed to give a marginal wave height histogram. This histogram is itself fitted by a distribution and extrapolated to determine the 50y return value. Carter and Challenor (1981b) state that there is little evidence that the use of equations (39) and (40) and the gamma function are valid at very high wind speeds and wave heights, so the derived estimates of 50y return value wave heights are of questionable validity, and the method may be of more use in describing the general wave climate than for estimating extremes. This method is presently being evaluated by the National Maritime Institute Ltd., in co-operation with the Meteorological Office.

The third method is to use a wave model with meteorological storm data to generate hindcast wave heights, which are then analysed to give return values. IOS, HRS, and the Max Planck Institute (Ewing et al 1979) have developed a North Sea Wave Model (NORSWAM) using wind fields determined from synoptic weather charts during 42 storms between 1966-76. The model computed wind and swell waves via wind-wave and wave-wave interactions, and the maximum values of H_s determined at each grid point. These were then analysed, using the F-T 1 distribution, to derive estimates of the 50y return values. Results are 1 to 2m higher than H_s values equivalent to the H_{max} values given in DOE guidelines (1977).

The Meteorological Office have a wind-wave model (Golding 1977) which is used for routine wave prediction. The model uses a parametric technique to predict the wind sea and a discrete spectral model for the swell. Shallow water effects are included by representations of shoaling, refraction and bottom friction. Comparisons of the Met Office NORSWAM wave models with measured wave data collected during March 1980 are given in Ewing et al (1981) - the standard errors of model estimates vary from 0.6m to 1.4m. The model's output has recently been compared with an extensive set of South Uist data. The correlation was disappointing but the statistics agree well, even to the few events per year level. This may be due to the model getting the timing wrong (M.J. Tucker, personal communication, 1983).

The Dutch Meteorological Institute (Sanders 1976) has developed a wave model to forecast waves at the harbour approaches to Rotterdam. It is based on the deep water wave model of the Norwegian Meteorological Institute (Haug 1968) but incorporates wave growth, energy dissipation and wave refraction effects due to shallow water.

A catalogue of 23 existing wave prediction models has recently been compiled by Draper (1983).

3.4 Estimation of extreme values of wave periods

The first available method is to extrapolate the cumulative frequency distribution of all the available measurements of T_z. Bouws (1978) has done this, using the Weibull distribution, to wave data recorded between April 1973 and February 1976 off the Dutch coast. A similar method is to extrapolate the upper and lower limiting values of T_z taken from the H_s - T_z scatter diagram, by plotting pairs of values of H_s and T_z on linear axes, drawing smooth curves by eye and extrapolating to the required extreme value of H_s.

Periods associated with return values of H_s and H_{max}(3h) can be estimated using a method based on Battjes' range of steepness values:

$$S = 2\pi H_s / g T_z^2. \tag{43}$$

From analysis of wave data recorded at various OWS and Light Vessels, Battjes (1970) suggested that the highest waves have maximum steepness of between 1/16 and 1/20, with most values near 1/18. Given the estimated extreme value of H_s, the upper, lower and mean values of T_z can be estimated using the range of steepness values, and equivalent values of T_{max} obtained using relationships derived by Bell (1971) from an analysis of North Sea wave spectra,

$$\begin{aligned} \text{Mean } T_{max} &= 1.28\ T_z \\ \text{Max } T_{max} &= 1.42\ T_z \\ \text{Min } T_{max} &= 1.14\ T_z. \end{aligned} \tag{44}$$

Estimates of wave period may be directly made for given wind speed, duration and fetch using hindcast relationships derived by Darbyshire and Draper (1963) and Bretschneider (1966). These give T_s, the mean of the periods associated with H_s, and hence T_z from, Goda (1979),

$$T_z = 0.82\ T_s. \tag{45}$$

A further method is to theoretically model the joint height and period distribution of individual wave heights and periods (Bretschneider 1966, Cavane et al 1976, Longuet-Higgins 1983). In each case the model consists of an expression for the joint probability density as a function of the spectral width parameter ϵ or ν and height and period variables non-dimensionalised using lower order spectral moments. The probability distribution of individual wave periods for a given H_{max} can be calculated if the corresponding values of H_s, T_z and ϵ or ν known. The range of periods likely to be associated with the given H_{max} can be calculated from specified percentiles and the most likely period, T_{max}, obtained from the mode of the distribution. The two latter models have the property of an asymmetric distribution in accordance with observations of wave spectra with a broad bandwidth (Goda 1978). However, Longuet-Higgins' model depends only on the three lowest spectral moments of the spectral density function and therefore avoids the problem of the critical dependence of m_4 on the behaviour of the spectrum at high frequencies.

3.5 Modification of off-shore data

Any computational procedure should consider the effects of shallow water in modifying observed or hindcast offshore wave data, and therefore determining the wave height on-shore. These effects may be caused by refraction and breaking due to variations in bathymetry or by tidal or other currents, (Muir Wood and Fleming 1981, Holmes 1983, Peregrine et al 1983). Hedges (1978) discusses the effects of measurements and analysis of currents on the measurement and analysis of waves. Breaking and attenuation of waves will also be dependent on tidal depth, particularly over off-shore banks (Tucker et al 1983).

Because of these shallow water effects, the short- and long-term statistics derived from observed or hindcast data at an off-shore location may not be fully representative of an adjacent on-shore site.

4. COMBINED STILL WATER LEVEL AND WAVES

The first sub-section is concerned with the definition of the basic variables and parameters used in combining swl and waves and also with the present design practices used. Methods of estimating the probability of occurrence or exceedance of combined swl and wave events are then described in sub-section 4.2. The final sub-section contains a discussion of the use and estimation of the parameter of overtopping discharge of a wall.

4.1 Basic variables and design parameters

Advice issued by MAFF to River Authorities in the 1960's distinguished between "noteworthy" and "disastrous" flooding, the former due to the combined level of swl and waves being above the defence level, the latter due to the swl itself being above the defence level.

A common design practice for combined levels was to base design levels on a certain return period swl (often the 1953 or highest observed level) and to add a certain suitable freeboard for waves calculated on the basis of local knowledge and experience. The Anglian Water Authority used a freeboard of 0.4m above design swl on estuarine banks and 1.0m on sea frontage banks to allow for wave action and run-up in the Wash, Nene estuary and the Welland estuary . (P. Stott personal communication, 1982). A minimum allowance of 0.5m has been used by the Seven-Trent Water Authority, with larger allowances at more exposed places along the Severn Estuary (D En 1981).

The present-day design criterion should be based on a statistical estimate of the return period of an extreme event due to a combination of swl and wave levels. Since both storm surges and wave activity are generated by meteorological forces, there may be correlation between swl and waves at the defence site. The problem is therefore to identify the degree of correlation between them and then to calculate the probability of occurrence of a joint swl-wave event from their separate probabilities of occurrence; this is discussed in sub-section 4.2.

The recognition that a relatively low swl combined with severe wave action might be a more critical design condition than the maximum swl plus more modest wave action, has led to the consideration of the quantity of water allowed to flow over the defence as the major design parameter. The parameter used is the overtopping discharge, Q, defined as the volume of water, per unit length of wall, discharging over the wall in unit time. It depends both on the sea wall characteristics and environmental parameters (HRS 1980). The MAFF revised sea defence standards (B. Trafford, personal communication, 1983) state that the design of a defence should consider the effect of a wide spectrum of combinations of water levels and wave heights in terms of overtopping and possible breaching.

The design criterion used is the "overtopping discharge" or the "overtopping level" (crest elevation) of the wall, and one of two design standards is usually adopted: either that the overtopping rate should not equal or exceed a permissible return period value, or that the adopted crest elevation of the wall should be such that only a certain percentage of waves overtop it during the design storm. Dutch practice is to use a 2% of overtopping waves criterion, with the proviso that the overtopping discharge does not exceed 2×10^{-3} m^{3} s^{-1} m^{-1} (2 litres s^{-1}

m^{-1}) during any specific storm event. HRS (1980) favour use of the overtopping discharge rather than a certain percentage of overtopping waves as this gives direct information on the volume of water overtopping the wall, which is of direct use in the assessments necessary before adoption of the seawall design level. The Essex River Division of the Anglian Water Authority used an overtopping design value of 1×10^{-2} m^3 s^{-1} m^{-1} for their Thames side defences (D. Shipman, personal communication, 1982).

There are no published guidelines in the U.K. for allowable overtopping rates. MAFF's revised standard states that "the performance of the proposed design will be assessed on the pattern of overtopping discharges to be expected, the economic criterion will be that the present value of the expected flood damage avoided over the life of the structure will exceed the construction costs". In order to carry out the cost-benefit analysis required, most authorities responsible for coast protection adopt different standards of defence, usually based on the land use behind the defence. For example, the Southern Water Authority have three standards (Thorn and Roberts 1981):

1 in 1000y standard - flooding of extensive residential and valuable industrial areas leading to serious damage, major risk to human life on a large scale;

1 in 250y - flooding would affect residential and industrial areas or large areas of valuable agricultural land;

1 in 50y - elsewhere.

HRS's handbook of seawall design (1980) contains a discussion of the environmental, physical, and economic assessments necessary before adoption of a seawall design.

4.2 Estimation of combined swl and wave events

The occurrence of waves is likely to be at least partially correlated with swl, since both waves and storm surges are generated by meteorological forces. The degree of correlation will vary from one coastal site to another depending on the exposure of the site to different wind directions. If the meteorological system generating the surges causes strong winds at the site from a direction having a long fetch length, then swl and waves are likely to be strongly dependent. If the local winds from the system are off-shore, then the swl and waves are likely to be fairly independent.

Even adjacent defences at the same site may be open to exposure to different wind directions and therefore have significantly different swl-wave correlation. For example, calculations by the Southern Water Authority (G. Setterfield, personal communication, 1982) gave a difference of 0.7m in the 1 in 100y estimates of extreme swl-wave levels for waves generated by north-easterly or north-westerly winds at Sheerness.

A degree of dependence of waves on swl will also occur at sites where the wave height reaching the seawall is always limited by breaking, since a higher swl will allow greater wave heights to reach the wall.

The two extreme cases of swl-wave dependence is either that they are statistically fully dependent or fully independent. In the first case, of perfect correlation, swl and wave heights having the same probabilities, and hence return period, are added together to find the joint swl-wave event with the same probability of occurrence. In the second case, of complete independence, then the joint probability of a swl-wave event is the product of their separate probabilities of occurrence,

$$p_j(h, H_s) = p_{swl}(h) \times p_w(H_s). \tag{46}$$

If there is any degree of correlation of swl and wave height, then use of the fully dependent method will produce the most conservative design level.

The first case, of full dependence and perfect correlation, has been used by HRS in studies on design levels for the Wessex Water Authority (WWA), with the conclusion that the likely 1 in 100y joint event would be due to a combination of the 1 in 100y swl and the effect of the 1 in 100y wave. Subsequent studies, by the WWA, of waves and swl during the December 1981 surge event have produced a final 1 in 100y design criteria for the Kingston Seymour defences of the 1 in 100y swl plus 1 in 5y wave height (Bown 1983).

Two computational schemes for estimating the probability of a joint swl-wave event, assuming the second extreme case of statistical independence, are given in Thorn and Roberts (1981). In the first method, the number of times a specific swl (HW level) is equalled or exceeded in 1000y is computed and tabulated from observations. The number of times that a specific onshore wind strength is equalled or exceeded in 1000y is also computed and tabulated from observations. Then the number of times in a million years that any combination of swl and wind strenth is equalled or exceeded is calculated by multiplying their separate frequencies together. Hence the return period of a joint swl-wind event can be found and application of a wind-wave relationship yields the joint swl-wave return period.

In the second method the probabilities of swl being equalled or exceeded are tabulated, together with the probabilities of a range of wave heights being equalled or exceeded. Application of equation (46) then gives the combined probability of the swl being equalled or exceeded with the wave height within the corresponding wave height range.

The method can be extended (HRS 1978) to consider ranges of swl, then the probability density of joint occurrence can be written:

$$q_j(h, H_s)\, dh\, dH_s = q_{sw}(h)\; q_w(H_s)\; dh\; dH_s, \tag{47}$$

where $q_{swl}(h)dh$ is the probability that the swl is between h and dh, and $q_w(H_s)\,dH_s$ is the probability that the significant wave height is between H_s and $H_s + dH_s$. q_{swl} and q_w are found from the known probabilities P_{swl} and P_w since

$$P_{swl} = \int_h^\infty q_{swl}(h)\; dh \tag{48}$$

and

$$P_w = \int_{H_s}^\infty q_w(H_s)\; dH_s,$$

hence for any interval Δh and ΔH_s,

$$q_{swl}(h)\,\Delta h = P_{swl}(h+\Delta h) - P_{swl}(h)$$

$$q_w(H_s)\,\Delta H_s = P_w(H_s+\Delta H_s) - P_w(H_s). \tag{49}$$

HRS used both assumptions of extreme swl-wave dependence in their seawall design studies at Llanddulas, Fleetwood and Sheerness (HRS 1978, 1979, 1981b). Overtopping discharge probabilities were calculated twice (see sub-section 4.3), on the assumptions of perfect correlation and perfect independence of swl and waves (Tables 1 and 2). For the same specified overtopping discharge and the same specified return period, the difference in design height of both seawalls was about 2m. The method involves the extrapolation of both observed swl and waves to long return periods, and this was done by fitting and extrapolating a Weibull distribution.

It is not likely that there is complete dependence or independence between swl and wave height, but that there is a degree of correlation between them. Ackers and Ruxton (1974) attempted to allow for the correlation between swl and windspeed (and hence wave height) at Southend by classifying the observed maximum mean hourly wind speed at Shoeburyness according to whether there was a surge residual at HW on the same day which was greater than 3ft, less than 2ft, or otherwise.

The frequencies of occurrence of surge-wind events were computed and a computational scheme, allowing for shallowing water and for fetch, used to estimate the range of wave heights at Southend for each wind speed. Therefore the surge-wind frequencies were translated into surge-wave frequencies and expressed as the probability that the specific wave height occurred on any day of the specified surge. Previously determined frequency of occurrence of surge heights with swl, expressed in terms of the number of occasions in 100y that the surge occured with the particular swl, enabled the joint probability of a swl-wave event, expressed as the number of occasions in 1000y, to be evaluated by multiplying swl-surge frequency by surge-wave probability (Table 3).

The overall frequency of occurrence per 1000y of a joint event with both a particular swl and wave height being exceeded was then found by summing all the frequencies of events giving the desired combination, and the return period calculated. Final results were plotted to show return period curves of events due to swl alone and swl-wave combined (see Figure 7).

The approach of Ackers and Ruxton was one of using the conditional probabilities of tide, surge and wind (and hence wave) to obtain the final joint probability of swl-wave events. A similar approach is taken by Vrijling et al (1983) in extreme and real-time predictions of wave and swl conditions at the Eastern Schelde storm surge barrier. A set of mathematical models are used in real-time predictions of the tide, surge and wave effects, and they have also been run to predict the extreme swl-wave conditions for design studies of the barrier.

The first model evaluates the joint probability density function of the swl and the local wind speed (Figure 8). The swl is modelled as a linear superposition of a random storm surge due to wind set-up and a random astronomical tide. It is assumed that the highest storm surge level occurs at or very near astronomical HW because the wind set-up has a duration which is much longer than the period of the astronomical tide. For extreme conditions, the maximum storm surge is considered to be generated by a north-westerly wind with a duration of 9 hours.

The second model calculates the wave spectrum near the barrier given the sea state (wave spectrum) in the North Sea (Sanders 1976), the swl and the local windspeed. It contains all the shallow water dissipative (breaking, bottom friction), generative (wave growth by wind) and distributive (diffraction and refraction by depth and current) effects. When run under extreme conditions, the model is based on the assumption that there are two sources of wave energy, from the open sea (low frequency) and local wind fields (high frequency).

For extreme conditions, the two models are linked by taking the joint pdf derived using north-westerly winds, feeding it into the wave model and obtaining the pdf of maximum swl and wave height (Figure 9). In this figure the small conditional probabilities for high values of the significant wave height are caused by the effect of breaking and the lower boundary shows the effect of the minimum wind speed which is needed to reach the swl.

Recently, HRS (1981b) obtained an estimate of the joint probability of occurrence of waves and swl at Sheerness, using observed swl, surge residuals, wind speed and direction, all at the time of HW for the period 1963 to 1979. For each class interval of swl, the probability of occurrence of the windspeed and direction was evaluated from the data. For each windspeed/direction the resulting wave height was then calculated, using the SMB method described in the Shore Protection Manual (1977), and the probability of occurrence of a given waveheight at this swl obtained by summation over all possible windspeeds/direction giving this waveheight.

This process was then repeated for each water level, and the probability of occurrence of each water level also calculated. Finally, by multiplying the probability of occurrence of each swl by the probability of occurrence of each wave height at that swl, the observed joint probability distribution for swl-wave height in combination was obtained (Figure 11a). This can then be compared with any joint probability distribution calculated using total dependence or independence of wave and swl (Figure 11b).

4.3 Estimation of overtopping discharge

HRS (1980) have issued a handbook for the design or assessment of seawalls allowing for wave overtopping, based on physical model tests carried out on seawalls with varying values of crest elevation, berm width and slope, and at various discrete values of swl, wave height and wave period. The empirical reltionship between these design and environmental parameters and the ensuing overtopping discharge for a particular wall is expressed in terms of a curve of dimensionless discharge, Q_* , versus dimensionless freeboard R_*. Q_* and R_* are defined as

$$Q_* = \bar{Q} / g \, H_s \, T_z \tag{50}$$

$$R_* = R / T_z (g H_s)^{1/2}, \tag{51}$$

where $\bar{Q}$ is the mean overtopping discharge per metre length, g is acceleration due to gravity, H_s is significant wave height at the wall (i.e. after breaking), T_z is mean wave period, and R is the difference between swl and the wall crest level.

For all seawall geometries and wave climates tested by HRS, the plots of Q_* versus R_* (Figure 10) show a relationship of the type

$$Q_* = A \, e^{-BR_*}, \tag{52}$$

where A and B are constants depending on the seawall geometry.

Hence for any assumed value of profile crest level, combined swl and wave height, and wave period R_* can be calculated and the corresponding value of Q_* found from equation (52), and hence $\bar{Q}$ calculated from equation (50). T_z must be related to H_s before the dimensionless discharge curve can be used to predict the discharge and this is usually done from H_s and Battjes' steepness parameter, S (equation 43). The overtopping discharge for any specified crest level therefore depends on the parameter S and the statistical interdependence of swl and wave-height.

If it is assumed that the swl and waves are statistically fully dependent, then the overtopping discharge due to a swl-wave combination has the same return period as each of the swl and wave events, e.g. the 100y overtopping discharge is generated by the 100y wave occurring simultaneously with the 100y swl.

If it is assumed that the swl and waves are statistically fully independent, then the procedure is to calculate the overtopping discharge for each possible combination of swl and wave height, and attach the probability occurrence of that swl-wave event to it. The overall probability of occurrence of a particular discharge is then the sum of all the probabilities of all swl-wave events that give that discharge.

Overtopping discharge is then plotted as a function of return period for both the dependent and independent assumptions of swl-wave correlation and usually for different steepness values (Figure 12) - although S = 0.05 is usually accepted as the value for the peak of a storm.

Present HRS practice (HRS 1978, 1979, 1981b) is to calculate the return period for overtopping discharge using both of the extreme assumptions of swl-wave correlation, to make a judgement on the relative dependence of swl and waves at the site, and then to adopt whichever of the extreme assumptions is more justifiable on the evidence of the degree of swl-wave correlation. At Llanddulus, HRS (1978) considered that swl and wave heights be assumed independent as they stated that the weather systems associated with exceptionally high water levels do not necessarily lead to strong onshore winds along the North Wales coast. The same assumption was made for Fleetwood and Cleveleys.

At Sheerness (HRS 1981b), the theoretical joint probability distribution of swl and waves was calculated assuming full independence and compared with the recorded distribution obtained from surge and wind observations during the period 1963 to 1979. For the northern wall, there was very little difference between the calculated and recorded number of occurrences, (Figures 11a, 11b) and so the assumption of total independence of swl and wave height was considered reasonable. For the western wall, overall agreement was reasonably good but two observed joint events had return periods of 680 and 1 million years if full independence was assumed, suggesting a partial dependence of wave height on swl. Therefore the overtopping discharge for various return periods was considered to lie within the range of values calculated from the extreme assumptions of swl-wave correlation.

There is usually an engineering requirement to know the total volume of water which overtops the seawall at the time of an extreme event during a complete tidal cycle, since this determines the degree of flooding behind the sea defences.

In the HRS proposed method (HRS 1980), a curve of overtopping discharge against swl for the design wave conditions is derived by calculating the overtopping discharge, $\bar{Q}$, over a range of still water levels from equations 50 to 52. The "typical" tidal curve is then examined to determine over what period the water level is high enough to give a significant overtopping discharge i.e. not less than 1% of the peak overtopping discharge.

The tide levels are then read off at appropriate fixed time intervals within this period and the overtopping discharge for each water level estimated using the previously prepared graph. The overtopping volume is then calculated from $\bar{Q}\Delta t$ where Δt is the selected time interval, and the cumulative volume of overtopping during the tidal cycle is then $\sum \bar{Q}\Delta t$ or $\Delta t \sum \bar{Q}$ if a constant time step is used.

5. SUMMARY

The estimates of extreme swl computed using an analysis of extreme values (usually annual maxima) depend critically on the quality and length of the data series, the inclusion or exclusion of particular values, and the choice of distribution and method of fitting it to the data (Lennon 1963, Graff 1981, HRS 1981a). Walden and Prescott (1980) have shown that the confidence limits of estimates computed using annual maxima data depend on the parameters a and k as well as the series length. As these parameters are calculated from the mean annual maximum value and the standard deviations of the annual and biennial maxima, then this means that the method is sensitive to the environmental parameters of the data site.

The annual maxima value may have a surge component which is not the highest in that year's surge population, and so the method does not allow for the possibility of future extreme swl events being due to a combination of a higher surge level and a lower astronomical tide level. This disadvantage is overcome in the joint-or combined-probability method which separates swl into msl, tide and surge residual components. Any trends in these components can be identified and incorporated in the predicted estimates, whereas with the annual maxima method only trends in the total swl can be detected, making it difficult to determine the underlying physical causes.

The method has been applied to HW data (Ackers and Ruxton 1974) and hourly data (Pugh and Vassie 1979, 1980) and produces stable estimates of extreme high levels (and low levels if hourly data is used). This is due to the independence of the physical forcing factors of msl, tide and surge; namely geological/climatic, tidal and weather, and means that reliable estimates can be obtained from only a few years of good quality data. In contrast, the annual maxima method needs at least 25 years of data. The assumed independence of tide and surge must be tested for by examining the variance of the surge distribution as a function of tidal level. Pugh and Vassie found that any interaction effects were of secondary importance except for Southend, and that the interaction could be treated by grouping the surge data according to different tidal levels.

One problem in the joint-probability method is the correlation of hourly surge data, and this can be overcome using the adapted Tayfun approach of Walden et al (1982), in which the surge is represented as a single event with an intensity dependent on its amplitude and duration. The method is still a joint-probability approach which treats tide and surge as independent but hourly surge residuals are not considered to be uncorrelated.

Of the three methods, the unmodified joint-probability approach gives the most conservative estimates, especially for a location where tide-surge interaction has not been allowed for, but this may be preferable for design purposes. An investigation should be carried out on the feasibility of applying the "peaks over threshold" (POT) method to swl data.

Direct transfer of extreme levels along a coastline using extrapolated differences between HAT values is of questionable reliability, especially if HAT itself has been estimated from extrapolated MHWS differences. The concept of an index based on the ratio between extreme swl and more easily computed statistics (such as MHWS and MLWS) is attractive if the index has wide spread regional stability. Lennon's ratio exhibits a fairly systematic pattern around Great Britain but has a large variability in some regions, particularly near amphidromes and in very shallow water, e.g. East Anglia (Graff 1981). Further investigation of regional factors is desirable in order to try to obtain a smoothly varying function which could be applied to specific coastlines.

As annual maxima swl data is available at many more ports than have hourly swl data, comparisons of estimates should be made at ports with both data sets. The feasibility of generally correlating annual maxima estimates at ports with those from joint-probability estimates from a "reference" port in the same region could then be investigated.

The main problem in the prediction of extreme wave statistics is the lack of observed wave data. Indirect methods of computing either the short or long term wave statistics are therefore usually necessary and have their own limitations and drawbacks.

Use of relationships derived from simultaneous wind-wave observations and subsequent scaling to longer periods using long wind records (Binnie and Partners 1980, Hogben and Miller 1980) is questionable as the relationships may not themselves be valid at very high wind speeds or wave heights.

If no wave data is available at all, short or long term wave statistics can be forecast or hindcast from one of many formulae available if the wind speed and duration and effective fetch is forecast or known. Choice of the "correct" formula is critical as the values of H_s and T_z obtained depend significantly on it (Carter and Challenor 1981b). Formulae based on the JONSWAP spectrum (Carter 1982) are now being accepted as appropriate for fetch-limited wave generation offshore. Onshore wave heights at sea defence sites are probably best calculated using the SMB method as it is the only one to include the effect of wave depth in reducing wave heights. Refraction and breaking effects due to topography and/or tidal or other currents may be dominant in coastal regions and any computational scheme should ideally model such effects.

Sophisticated wave models have been developed (Ewing et al 1979), Golding (1977), Sanders (1976) which include wave growth offshore, energy dissipation, refraction and breaking processes but they are expensive to run and require much computational effort. Comparisons of two of the models produced H_s estimates differing, for the same conditions, by 0.6 to 1.4m (Ewing et al 1981).

Estimates of the long term statistics of wave height and period computed from either observed or derived short term statistics of wave or wind depend critically on the distribution chosen and the method of obtaining the best-fit to the data (Carter and Challenor 1981b).

There is usually considerable seasonal variation in the statistics and therefore it may be necessary to analyse the data on a monthly basis (Carter and Challenor 1978, 1981a). Recent work by Challenor (1982b), using the 10 largest values of H_s in each month, has suggested that there may be differences in the distribution of extreme wave heights from year to year which are greater than would arise by chance, i.e. that there is some non-random year to year variation. If this is so, then, as Challenor states, it would invalidate the analysis of extremes or any other analysis using more than one year's data. It would also make the results of any analysis of only one year's data difficult to interpret and of limited application. However there is not at present a consensus of views within IOS on this topic and it remains an active area of study.

This suggests that methods of directly combining independently obtained extreme swl and wave estimates may be unreliable, especially if they are derived from different periods. The only feasible approach would therefore seem to be the analysis of simultaneous observed swl and observed, or hind-cast, wave data.

If direct swl-wave observations are unavailable, then there are indirect methods of obtaining the joint frequency distribution of swl and wave by combining the joint frequency distribution of swl and wind with the conditional probability of wave height given wind (Binnie and Partners 1980, Vrijling et al 1983). These methods will introduce uncertainties into the estimates as wave-wind formulae have to be applied to obtain the wave statistics from the wind data. As the onshore wave height at the defence is an important variable, a computational scheme which takes into account the refraction and breaking of the deep-water waves is necessary. Sophisticated wave models have been used (Sanders 1976, Vrijling et al 1983), but the simplest method would be to use the SMB approach (Shore Protection Manual 1977).

Upper and lower bounds on design levels can be computed by assuming that the swl-wave correlation is either statistically fully independent or fully dependent - the latter assumption will produce the most conservative design level for any particular return period. Computation of both these extreme cases is straight forward (see HRS 1981b) but the observed swl and wave data have to be extrapolated to give values at the long return periods, unless the joint-probability method has been used for the swl estimates (Ackers and Ruxton 1973, Binnie and Partners 1980).

Results based on the fully independent or fully dependent assumptions can be presented semi-graphically as probabilities (Tables 1, 2), or graphically as return period curves due to extreme swl events alone or combined swl-wave events (Figure 7). Curves could also be constructed of all swl-wave combinations giving an event with a certain return period, (Figure 13).

The return period concept is best applied to an univariate parameter rather than a bivariate swl-wave distribution. A suitable parameter is the overtopping discharge which defines the volume of water overtopping the sea defence. This design parameter also has the attraction of linking the environmental and structural parameters since it is a function of both. Methods are available (HRS 1980, 1981b) for computing the overtopping discharge assuming either of the extreme cases of swl-wave correlation (Figure 12); the difficulty is then in deciding which is more appropriate to adopt depending on the actual swl-wave correlation, and this may be a subjective decision. The actual determination of the swl-wave correlation is therefore of fundamental importance. As the correlation will be unique to each defence site because of its specific exposure to wind, surge and tide, extrapolation of the results to adjacent sites will be difficult and may not be feasible.

Estimation of the cumulative overtopping is important in order to determine the depth of flooding behind the defences. "Typical" tide curves have been used (HRS 1981b) but consideration should be given to using the swl probability distribution generated from the observations.

As there is only limited swl or wave data available and even less simultaneous data, then the main concern is the development of methods which enable the optimum statistical analyses to be made.

As a first step, IOS propose to use simultaneous U.K. swl-wave data to study the ways in which the two parameters may be treated together and the results usefully presented.

The degree of correlation between swl and waves will be computed and expressed as an observed joint probability distribution. Return period curves of events due to swl alone and observed swl-wave combinations will be computed and compared with estimates calculated assuming either full dependence or full independence. Curves will also be computed which indicate all swl-wave combinations giving an event with a specified return period.

The overtopping discharge will be calculated for the swl-wave combinations, to yield overtopping discharge return period curves. Estimates of cumulative overtopping could then be made using the swl probability distribution generated from the observations.

However, the available data sets are very short and there is a need to take simultaneous water level and wave measurements over long periods. This is probably best done at off-shore locations and further work needs to be done on producing models which can adequately transform the off-shore wave data to specific on-shore sites.

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8. FIGURES

1a The three classes of extreme value distribution curves, from Graff (1981).

1b Frequency distribution curves for south-east U.K. ports using the annual maxima method, from Graff (1981).

2 The range of estimates for the 1 in 100y frequency level for ports in Figure 1b, from Graff (1981). The shaded region covers estimates based on analysis of cumulative subsets ordered to the most recent year of observations, the unshaded area covers estimates from subsets ordered to the earliest year.

3 Tide, surge and sea level frequency distributions and flood-exposure index for Aberdeen, from Pugh and Vassie (1979).

4 Probabilities of exceeding high levels and of falling below low levels at (a) Aberdeen, 1964-73 and (b) Newlyn, 1951-69,

of Hydraulics Research, Wallingford, England.

9. TABLES

1. HW levels and Hs for fully dependent return periods at Sheerness, after HRS (1981b).

2. Combined probabilities of HW and Hs at Sheerness assuming independence, after HRS (1981b).

3. Combined tide, surge and wave probabilities at Southend, after Ackers and Ruxton (1975).

4. Joint frequency of swl and Hs at Isle of Sheppey, after Binnie and Partners (1980). The values shown are the number of days in 1 million years when the swl exceeds the indicated level and Hs falls in the indicated interval.

Appendix A Probability Distributions

Note that F(x) is the distribution function of the probability distribution, and is the probability that the variate value of a unit drawn at random from the population is less than x, i.e. $F(x) = P(X < x)$. Its derivative, f(x), is the probability density function (pdf), $f(x) = dF(x)/dx$.

(i) Normal or Gaussian Distribution

$$f(x) = \exp(-(x-\alpha)^2/2\beta^2)/\sqrt{2\pi}\,\beta \qquad -\infty<x<\infty$$

$$\text{mean} = \alpha$$
$$\text{variance} = \beta^2$$

(ii) Log-normal Distribution

$$f(x) = [x\sqrt{2\pi}\,\beta]^{-1}\exp[-(\log(x) - \alpha)^2/2\beta^2] \qquad x>0$$
$$= 0 \qquad \text{otherwise}$$

$$\text{mean} = \exp(\alpha+\beta^2/2)$$
$$\text{variance} = \exp(2\alpha)\exp(\beta^2)[\exp(\beta^2)-1]$$

The three-parameter log-normal is produced by replacing x by $x-\theta$ and is defined on $[\theta,\infty)$.

(iii) Gamma Distribution

$$f(x) = x^{\alpha-1}\exp(-x/\beta)/[\beta^{\alpha}\Gamma(\alpha)] \qquad \alpha,\beta,x>0$$
$$= 0 \qquad \text{otherwise}$$

$$\text{mean} = \alpha\beta$$
$$\text{variance} = \alpha\beta^2$$

The three-parameter gamma is produced by replacing x by $x-\theta$ and is defined on $[\theta,\infty)$.

(iv) Weibull Distribution

$$f(x) = \beta(x/\alpha)^{\beta-1}\exp[-(x/\alpha)^{\beta}]/\alpha \qquad \alpha,\beta,x>0$$
$$= 0 \qquad \text{otherwise}$$

$$F(x) = 0 \qquad x<0$$
$$= 1-\exp[-(x/\alpha)^{\beta}] \qquad x>0$$

mean $= \alpha\Gamma[\beta^{-1}+1]$
variance$= \alpha^2[\Gamma(2\beta^{-1}+1) - \{\Gamma(\beta^{-1}+1)\}^2]$

The three-parameter Weibull is produced by replacing x by $x-\theta$ and is defined on $[\theta,\infty)$.

(v) Exponential Distribution

$f(x)$	$= \exp(-x/\alpha)/\alpha$	$\alpha, x>0$
	$= 0$	otherwise
$F(x)$	$= 0$	$x<0$
	$= 1-\exp(-x/\alpha)$	$x>0$

mean $= \alpha$
variance $= \alpha^2$

The two-parameter exponential is produced by replacing x by $x-\theta$ and is defined on $[\theta,\infty)$.

The exponential distribution is a special case of the Gamma (α=1) and Weibull (β=1) distributions.

(vi) Rayleigh Distribution

$f(x)$	$= x\exp(-x^2/2\alpha^2)/$	$\alpha, x>0$
	$= 0$	otherwise
$F(x)$	$= 0$	$x<0$
	$= 1-\exp(-x^2/2\alpha^2)$	$x>0$

mean $= \sqrt{\pi}\alpha$
variance $= 2\alpha^2(1-\pi^2/2)$

The two-parameter Rayleigh is produced by replacing x by $x-\theta$ and is defined on $[\theta,\infty)$.

The Rayleigh distribution is a special case of the Weibull distribution ($\beta=2, \alpha=\sqrt{2}\alpha$).

(vii) Generalised Extreme Value Distribution (GEV)

$F(x)$	$= 0$	$x<\alpha+\beta/\theta$ and $\theta<0$
	$= \exp\{-[1-\theta(x-\alpha)/\beta]^{1/\theta}\}$	
	$= 1$	$x>\alpha+\beta/\theta$ and $\theta>0$

mean $= \alpha + \beta \{1 - \Gamma(1+\theta)\}/\theta$ $\theta \neq 0$
$= \alpha + \gamma\beta$ (γ = Euler's constant =0.5772...) $\theta = 0$

(viii) Fisher-Tippett Type I or Gumbel Distribution

$f(x) = \exp\{-(x-\alpha)/\beta - \exp[-(x-\alpha)/\beta]\}/\beta$ $\beta > 0$

$F(x) = \exp\{-\exp[-(x-\alpha)/\beta]\}$

mean $= \alpha + \gamma\beta$ (γ = Euler's constant =0.5772...)
variance$= \pi^2 \beta^2/6$

The FT-I is a special case of the GEV (θ=0).

(ix) Fisher-Tippet Type II or Frechet Distribution

$f(x) = \alpha^{1/\beta} \beta^{-1} (x-\theta)^{-(1+1/\beta)} \exp[-\{(x-\theta)/\alpha\}^{-1/\beta}]$ $x > \theta,\ \beta > 0$
$F(x) = \exp[-\{(x-\theta)/\alpha\}^{-1/\beta}]$
mean $= \alpha\Gamma(1-\beta) + \theta$
variance $= \alpha^2\{\Gamma(1-2\beta) - [\Gamma(1-\beta)]^2\}$

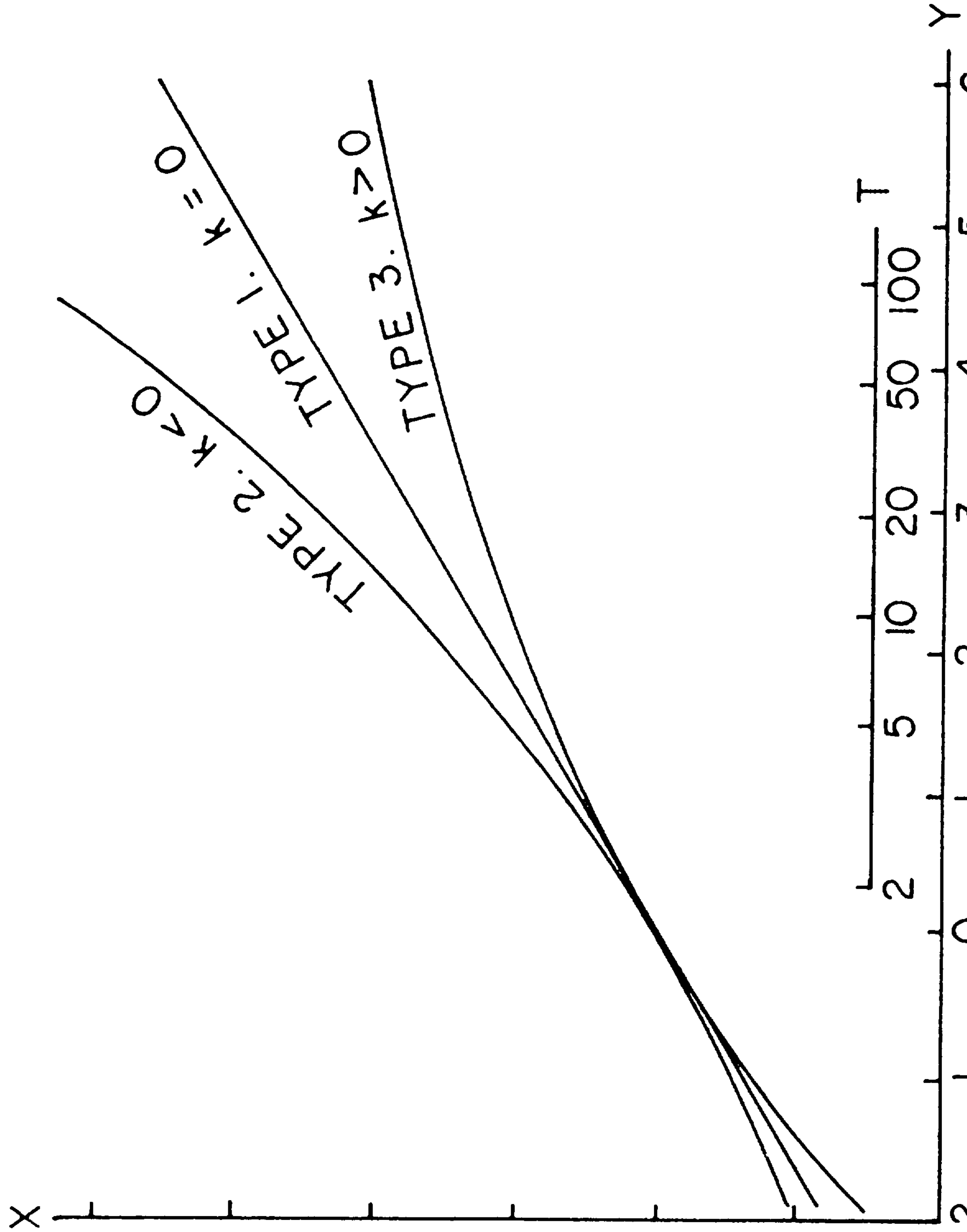
(x) Fisher-Tippett type III Distribution

$f(x) = \beta[(\theta-x)/\alpha]^{\beta-1}\exp\{-[(\theta-x)/\alpha]^{\beta}\}/\alpha$ $x < \theta$
$= 0$ otherwise

$F(x) = \exp\{-[(\theta-x)/\alpha]^{\beta}\}$ $x < \theta$
$= 1$ $x > \theta$

mean $= \theta - \alpha\Gamma(\beta^{-1}+1)$
variance$= \alpha^2[\Gamma(2\beta^{-1}+1) - \{\Gamma(\beta^{-1}+1)\}^2]$

The FT-III distribution is a special case of the GEV (-<0).
If the transformation X=-X is applied to the FT-III a three-parameter Weibull distribution is obtained.



X
30
25
20
15
10
5
0
-2
-1
0
1
2
3
4
5
6
Y
2
5
10
20
50
100
T
TYPE 2. k<0
TYPE 1. k = 0
TYPE 3. k>0

Figure Ia

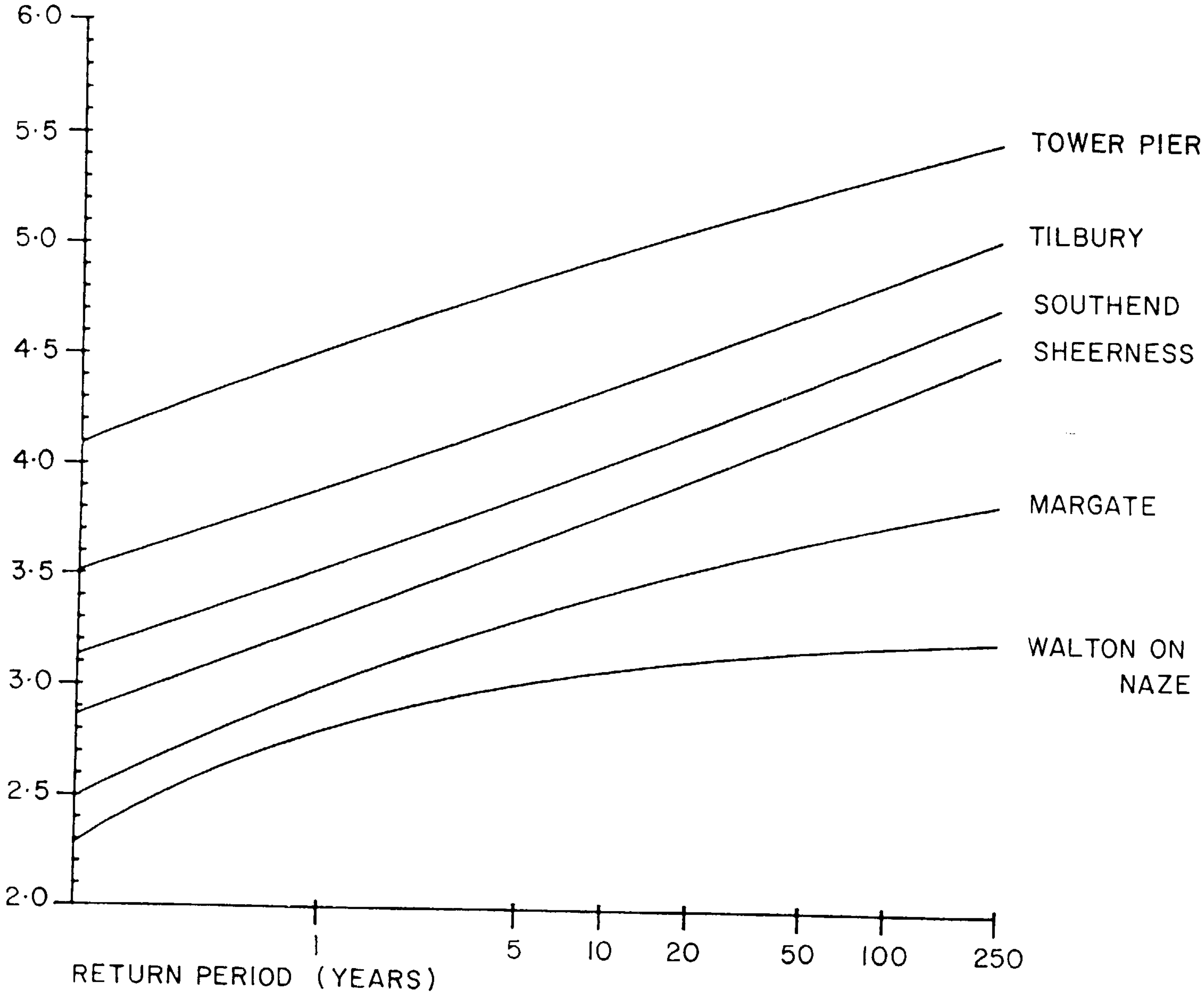

6·0
5·5
5·0
4·5
4·0
3·5
3·0
2·5
2·0
TOWER PIER
TILBURY
SOUTHEND
SHEERNESS
MARGATE
WALTON ON NAZE
1
5
10
20
50
100
250
RETURN PERIOD (YEARS)

Figure 1b

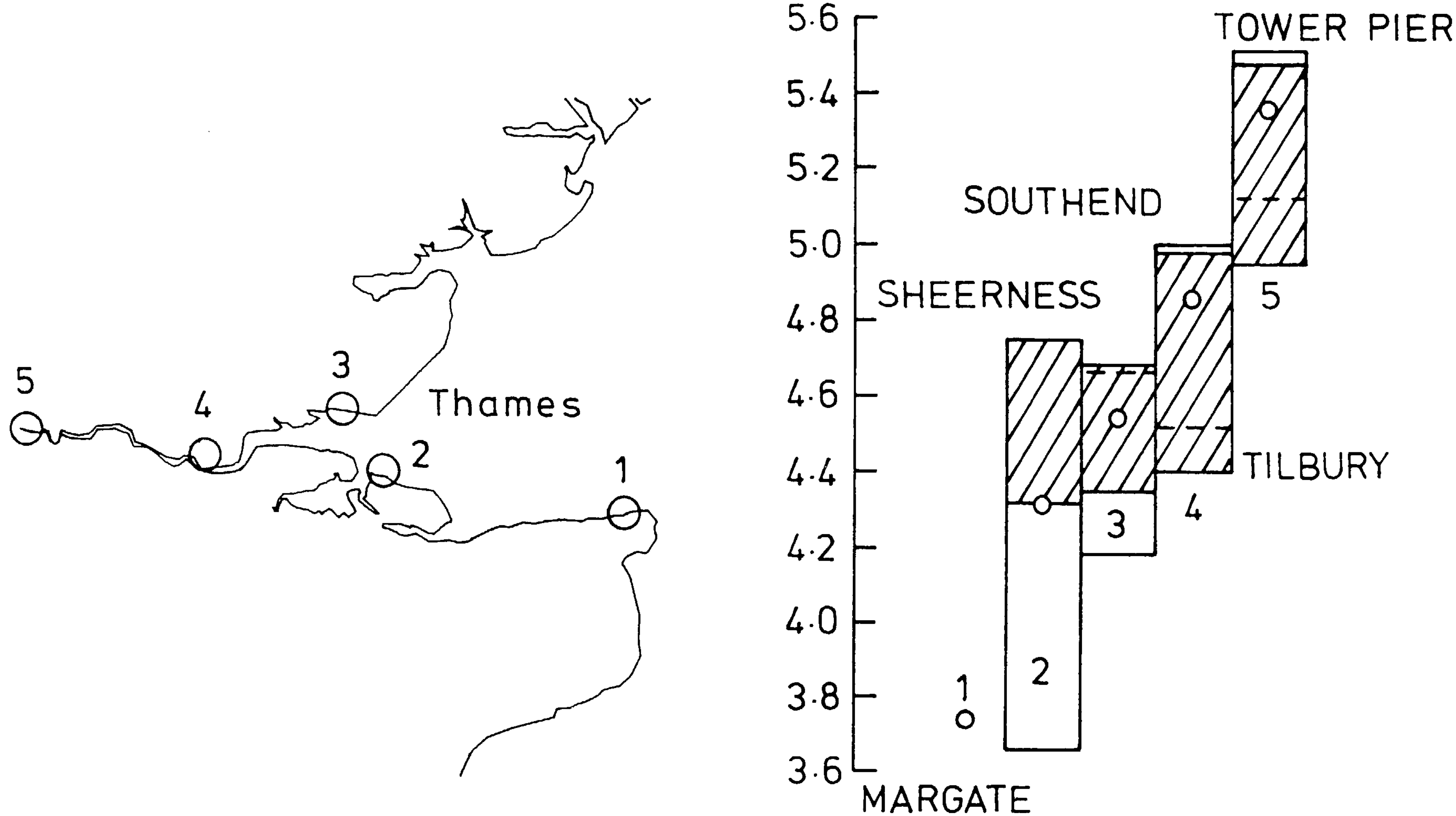
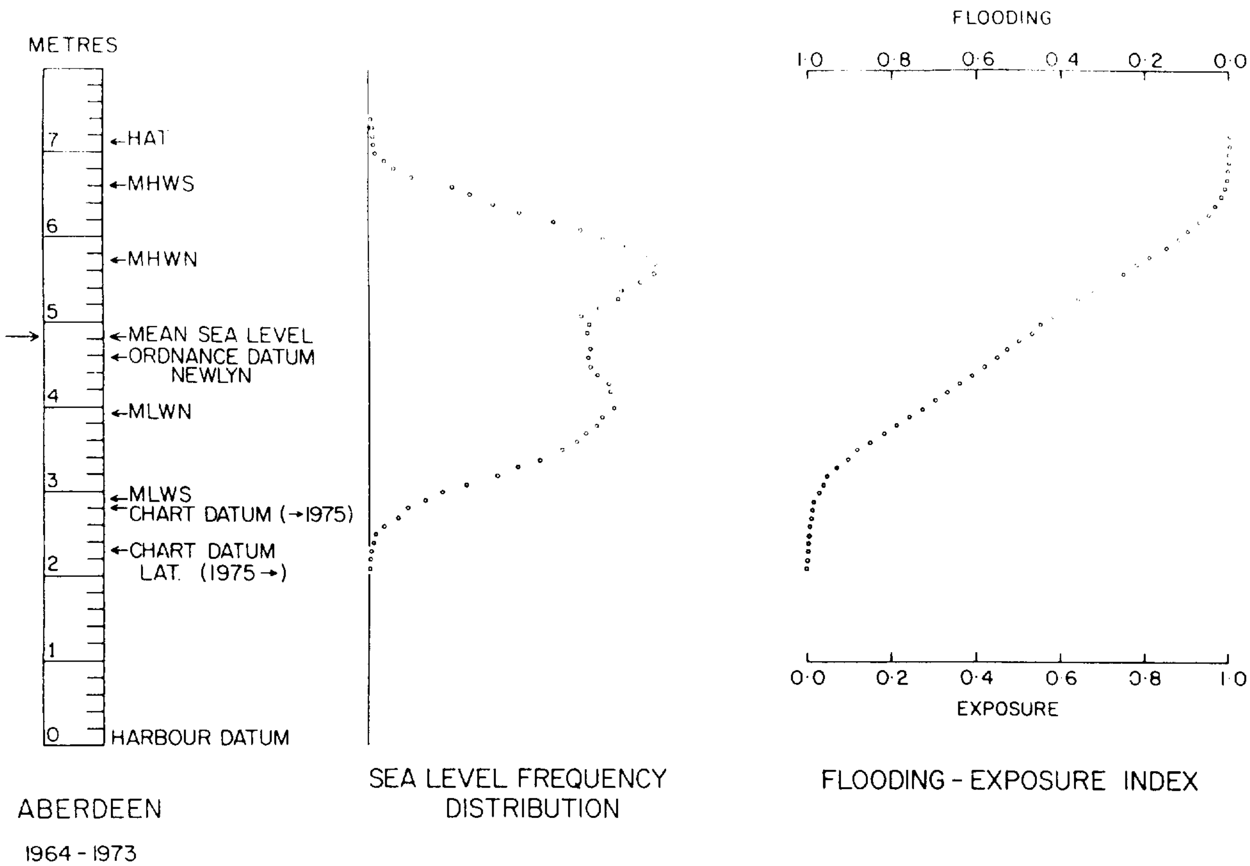
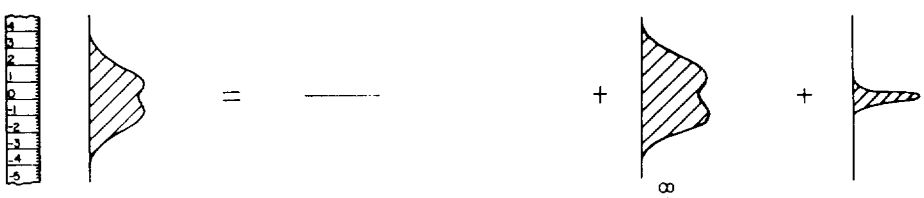

5
4
3
2
1
Thames
5·6
5·4
5·2
5·0
4·8
4·6
4·4
4·2
4·0
3·8
3·6
TOWER PIER
SOUTHEND
SHEERNESS
TILBURY
MARGATE
5
4
3
2
1

Figure 2



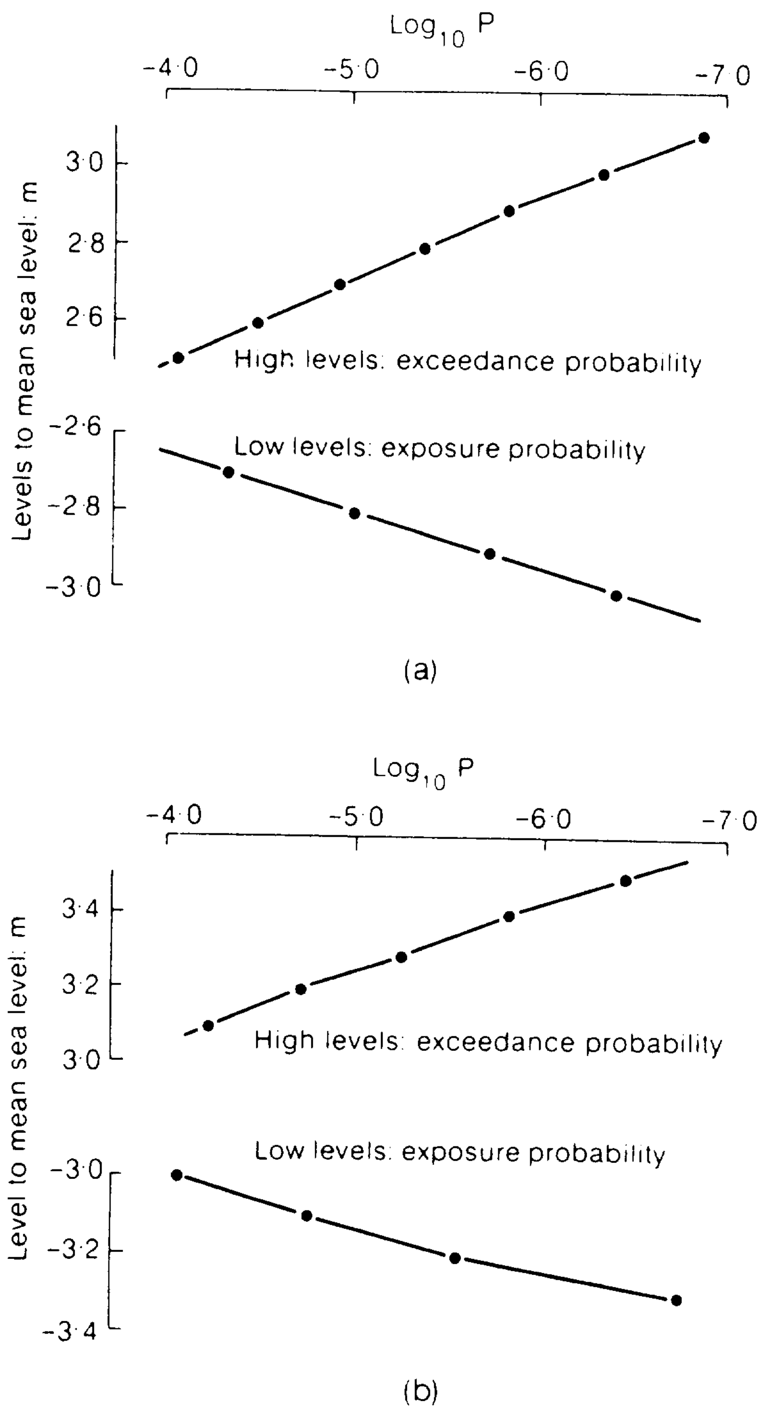
OBSERVED	Sea level	=	MEAN	Sea level	+	TIDAL level	+	RESIDUAL
	current component	=		current component	+	current	+	
	current speed	=		current speed	+	speed	+	
Variance (Aberdeen, 1964 - 1973)	100 %		——			97 % Predictable		3 % Not predictable
	ζ_0	=	z_0		+	ζ_T	+	ζ_R



$$P_{ts}(\zeta) = \text{——} + \int_{-\infty}^{\infty} P_t(\zeta - y)\, P_s(y)\, dy$$

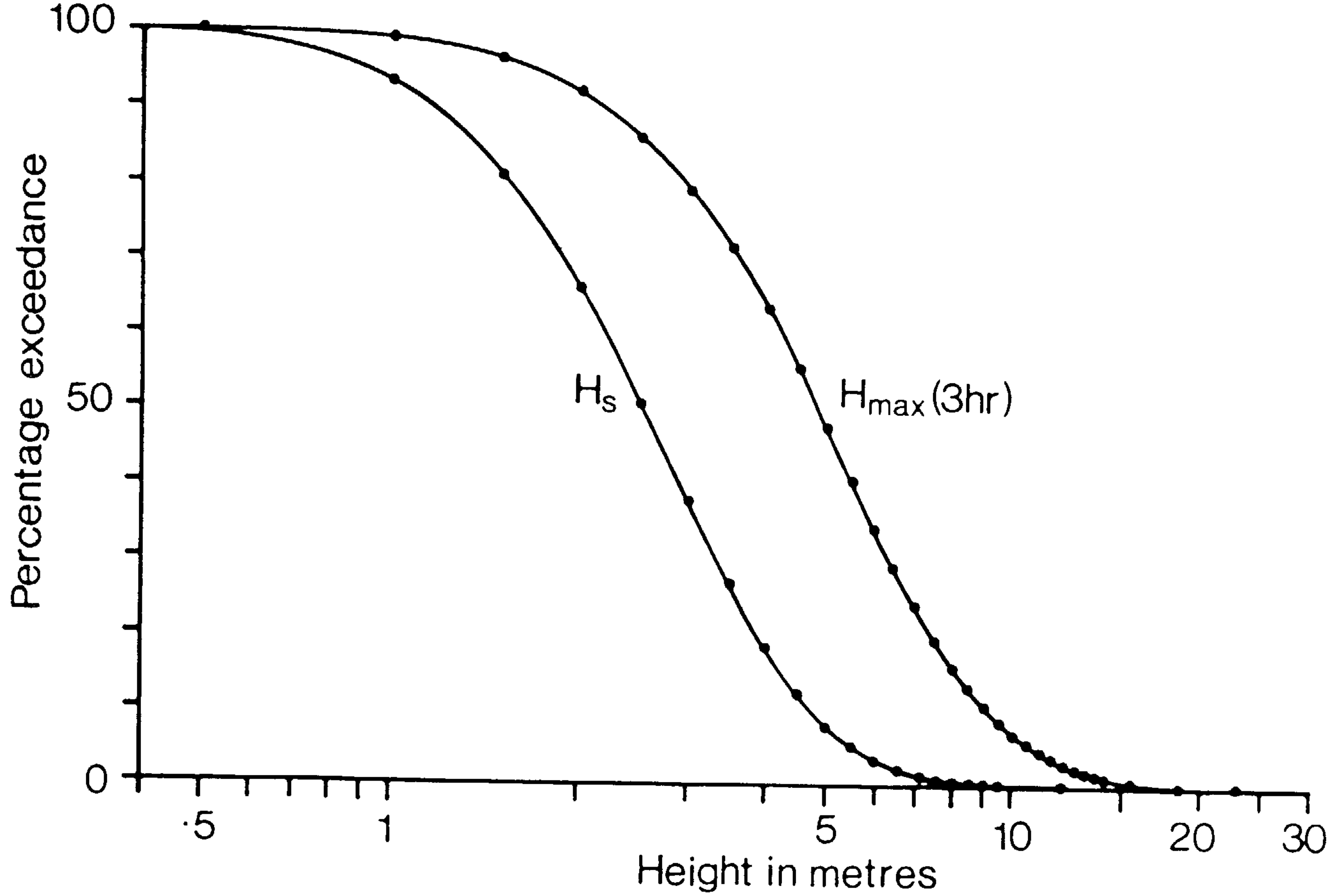
where P is probability density

Figure 3



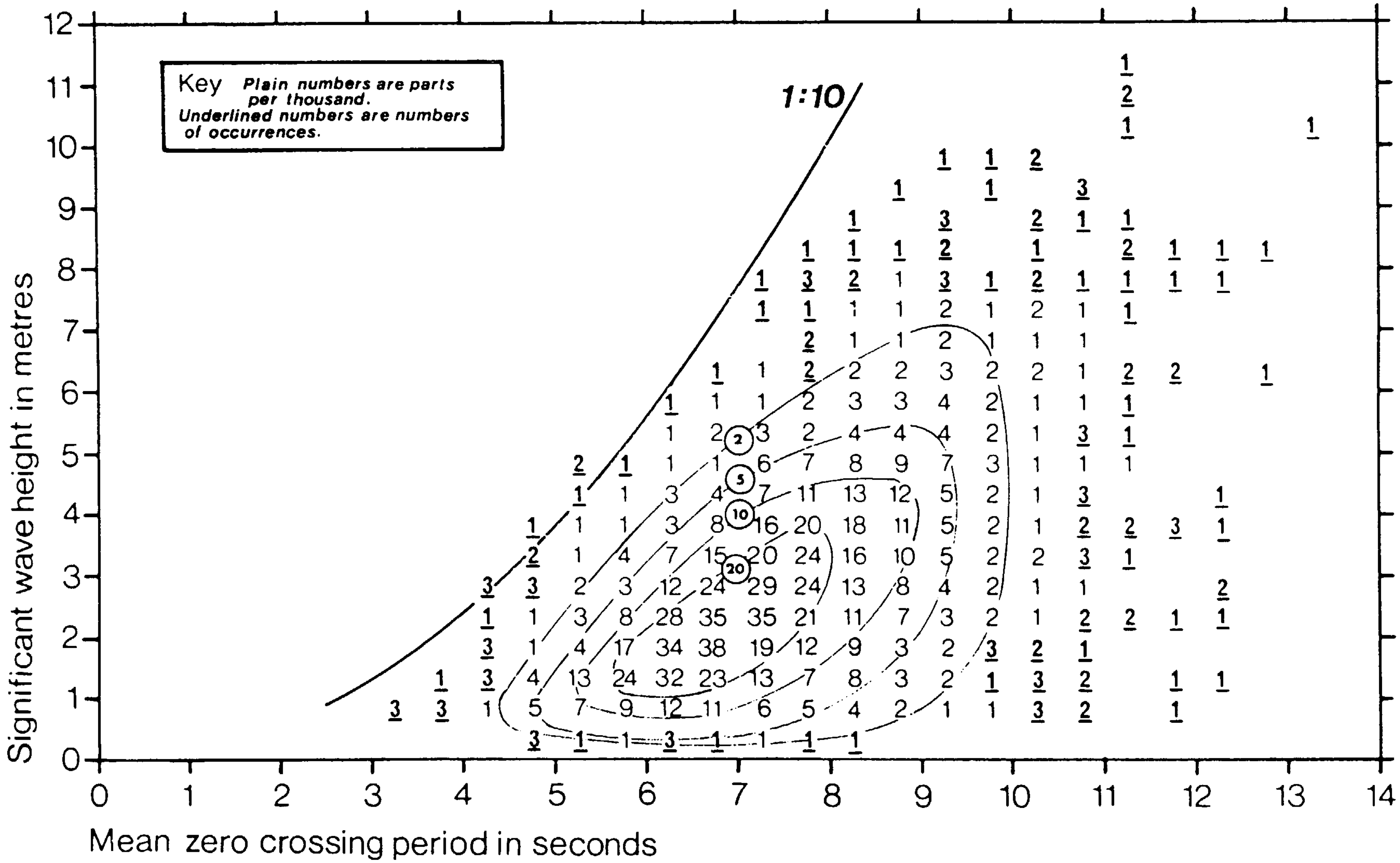
Probabilities of exceeding high levels and of falling below low levels at: Aberdeen, 1964–73; (b) Newlyn, 1951–69

Figure 4



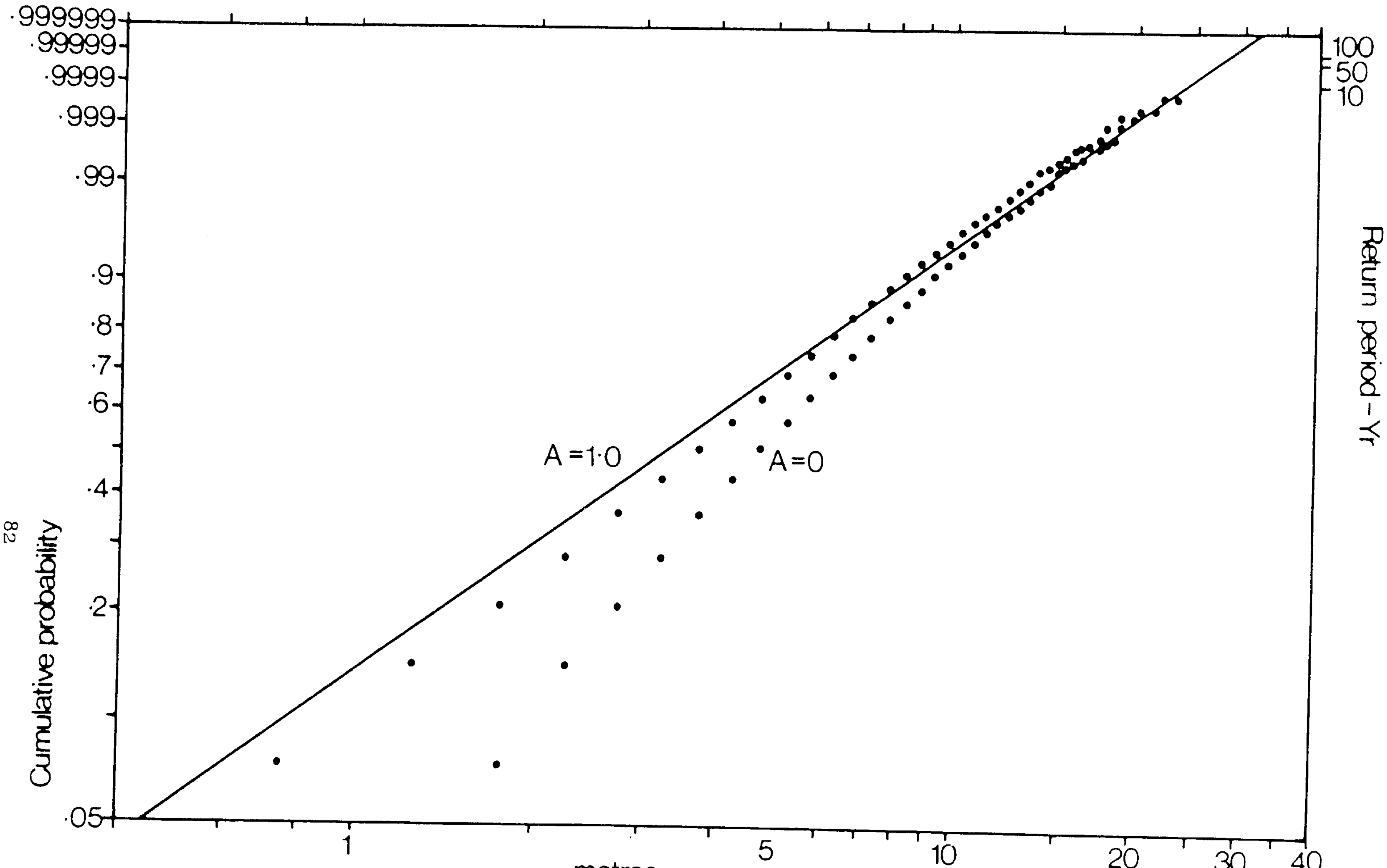
Percentage exceedance of H_s and H_{max}(3hr).

Figure 5a



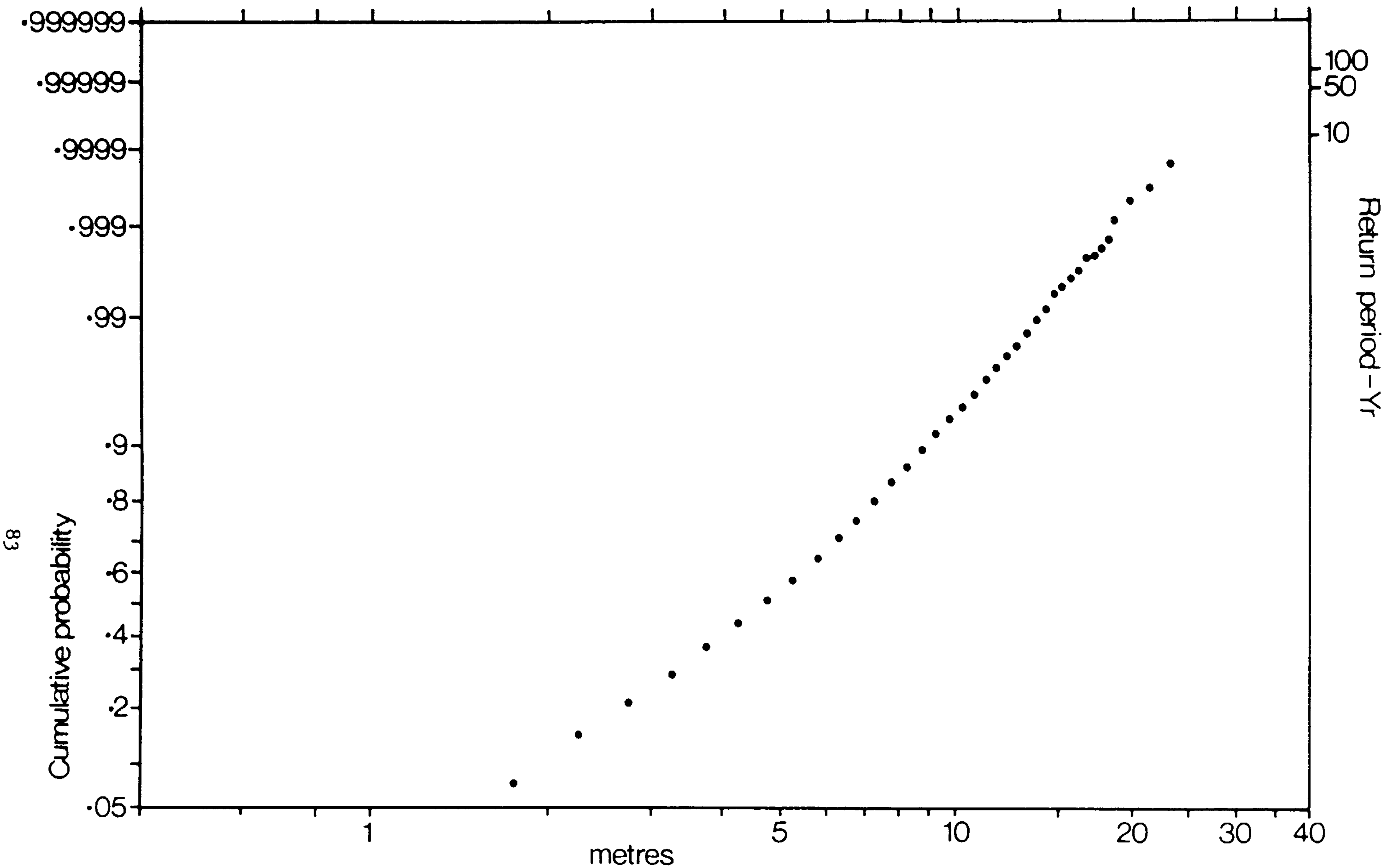
Scatter diagram in parts per thousand.

Figure 5b



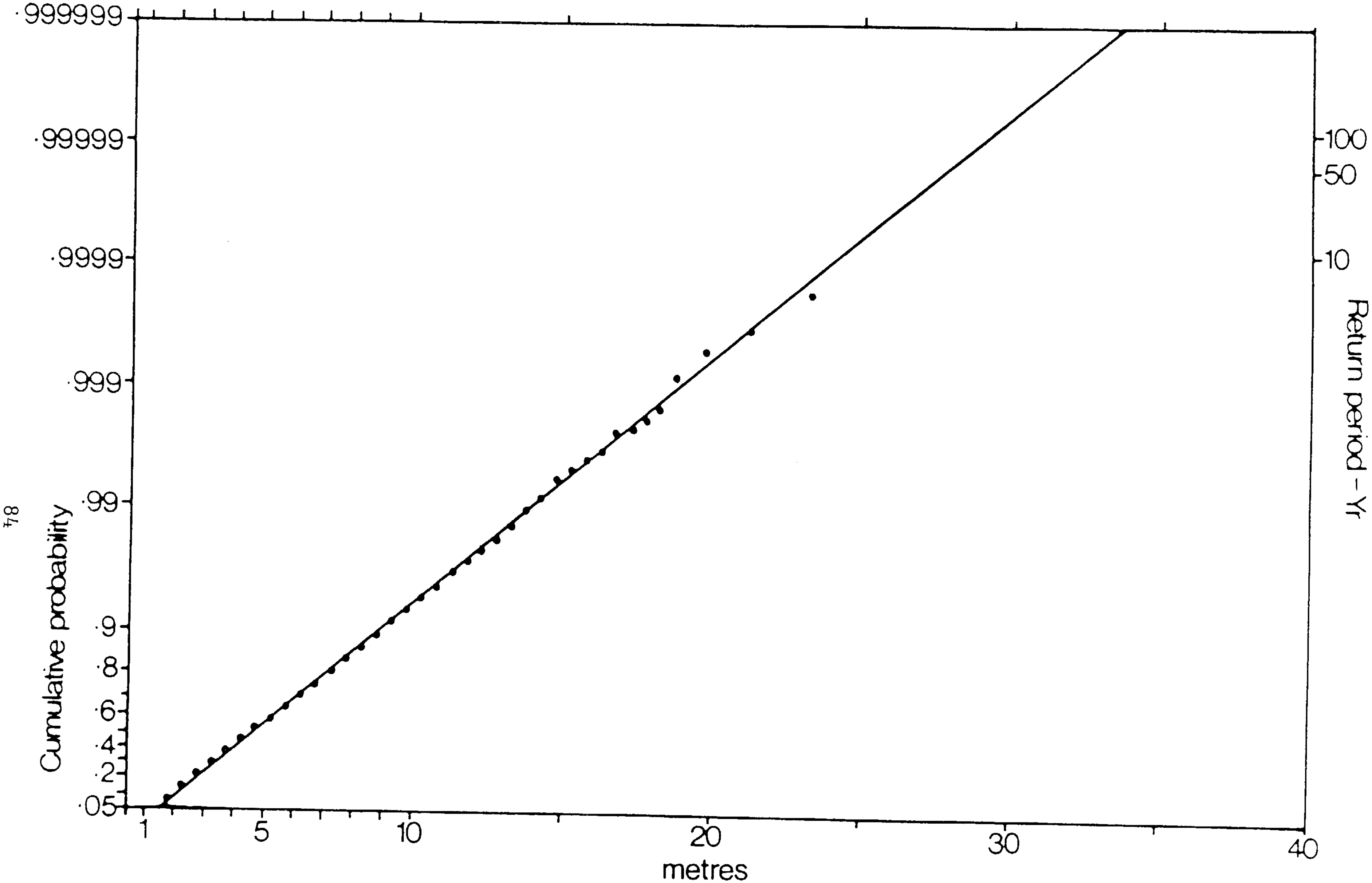
Cumulative distribution of H_{max} (3 hr) Weibull scale.

Figure 6a



Cumulative distribution of H_{max} (3 hr) Log normal scale.

Figure 6b



Cumulative distribution of H_{max} (3hr) F – T I scale.

Figure 6c

METEOROLOGICAL SURGES

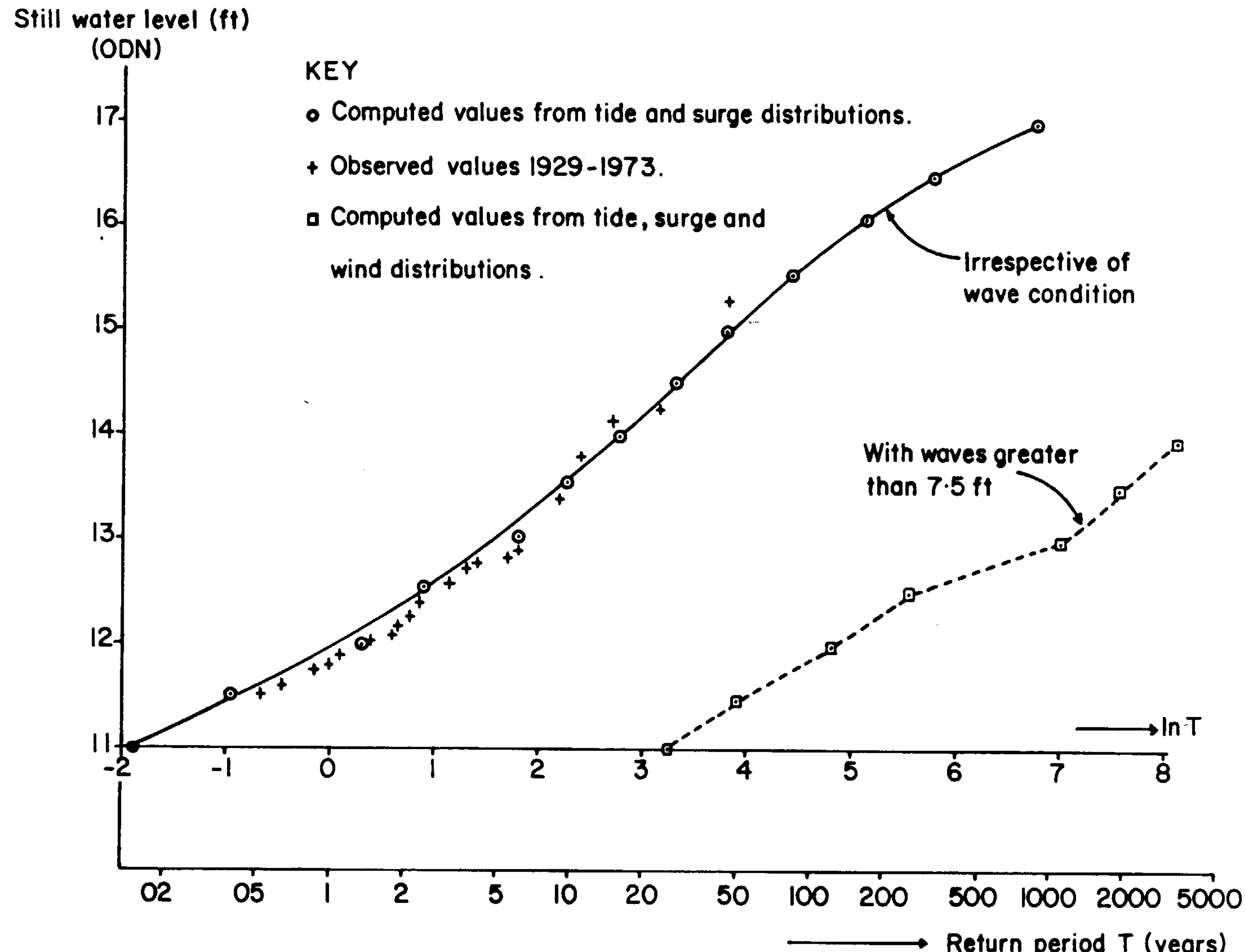


Figure 7

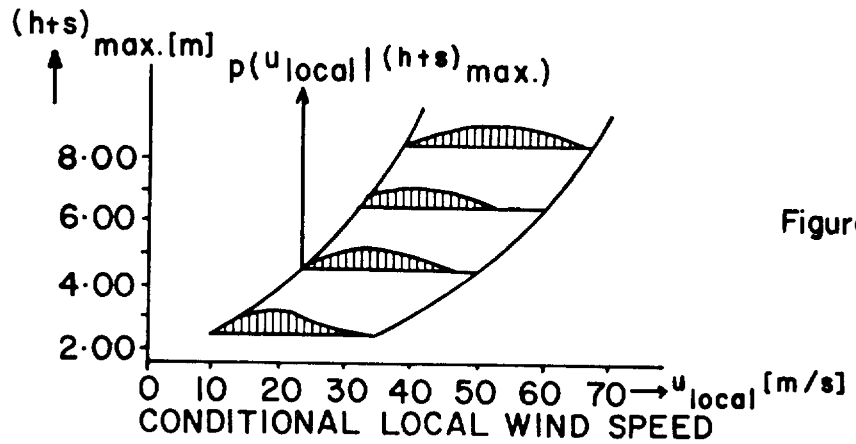


Figure 8

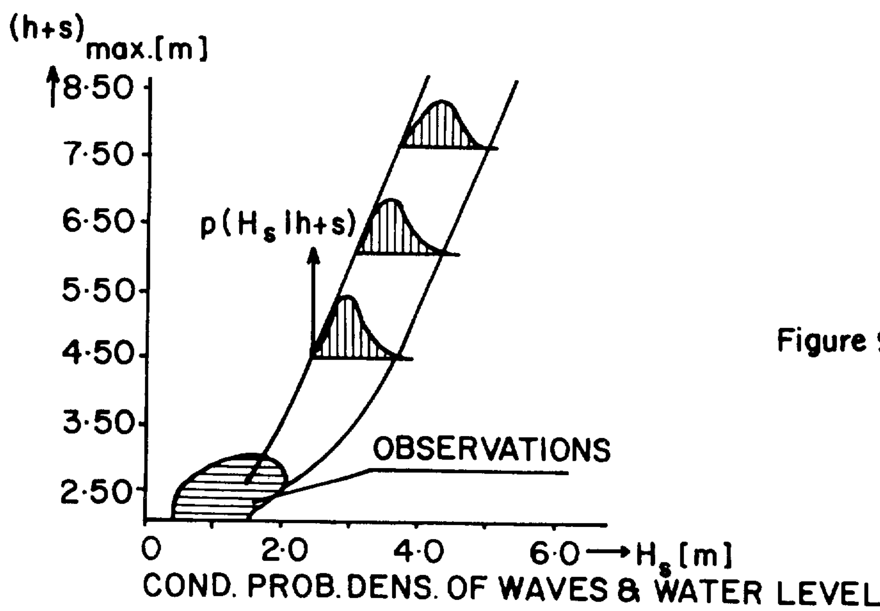
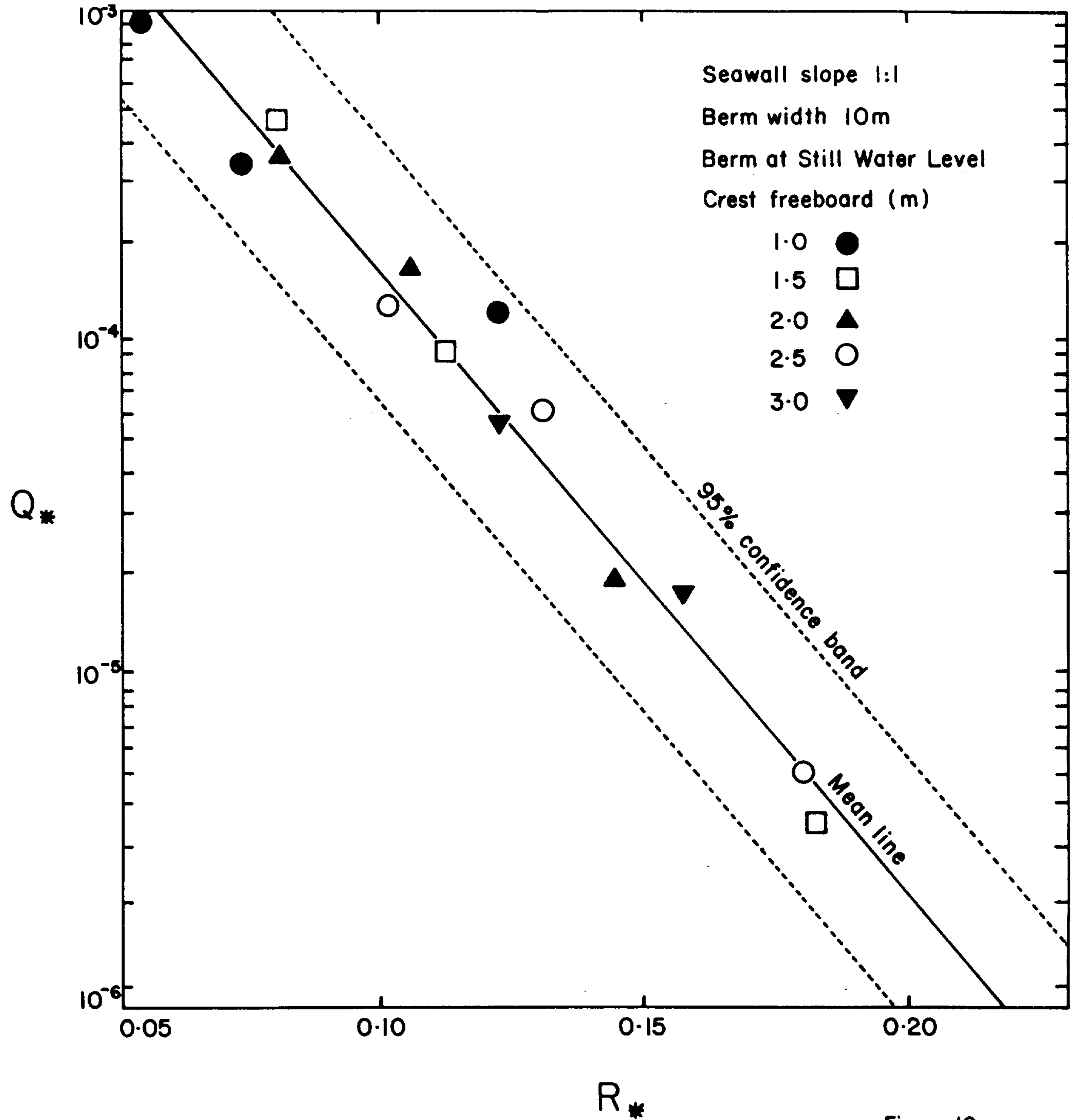


Figure 9


Seawall slope 1:1
Berm width 10m
Berm at Still Water Level
Crest freeboard (m)
1·0
1·5
2·0
2·5
3·0
95% confidence band
Mean line
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Q_*
0·05
0·10
0·15
0·20
R_*
Figure 10

Significant wave height : m	0·0–0·5	0·5–0·75	0·75–1·0	1·0–1·25	1·25–1·5	1·5–1·75	1·75–2·0	2·0–2·25	2·25–2·5	2·5–2·75	2·75–3·0	3·0–3·25	3·25–3·5	3·5–3·75	3·75–4·0	4·0–4·25	4·25–4·5
1·4–1·6																	
1·2–1·4					2		2	1									
1·0–1·2				2		3	3	2	6	1	2	2					
0·8–1·0										2		1	1				
0·6–0·8				4	8	10	21	32	42	32	17	15	1	1			1
0·4–0·6				4	25	67	130	187	191	170	129	44	3				
0·0–0·4	6	3	13	100	368	756	1293	1834	2085	1818	1112	446	80	19	3		2

High water level : m ODN

Recorded number of joint occurrences of tides and waves,
Sheerness northern walls, 1963 - 1979 .

Figure 11a

Significant wave height : m	0·0–0·5	0·5–0·75	0·75–1·0	1·0–1·25	1·25–1·5	1·5–1·75	1·75–2·0	2·0–2·25	2·25–2·5	2·5–2·75	2·75–3·0	3·0–3·25	3·25–3·5	3·5–3·75	3·75–4·0	4·0–4·25	4·25–4·5
1·4–1·6																	
1·2–1·4								1	1	1							
1·0–1·2						1	1	2	2	2	1						
0·8–1·0						3	5	8	8	7	4	1					
0·6–0·8				1	6	12	21	32	36	29	15	6	2				
0·4–0·6	1		1	8	36	72	124	193	212	172	91	35	9	2			
0·0–0·4	5	3	11	85	378	752	1288	1999	2199	1777	943	364	97	23	6	2	1

High water level : m ODN

Calculated number of joint occurrences of tides and waves, Sheerness northern walls, 1963-1979.

Figure 11b

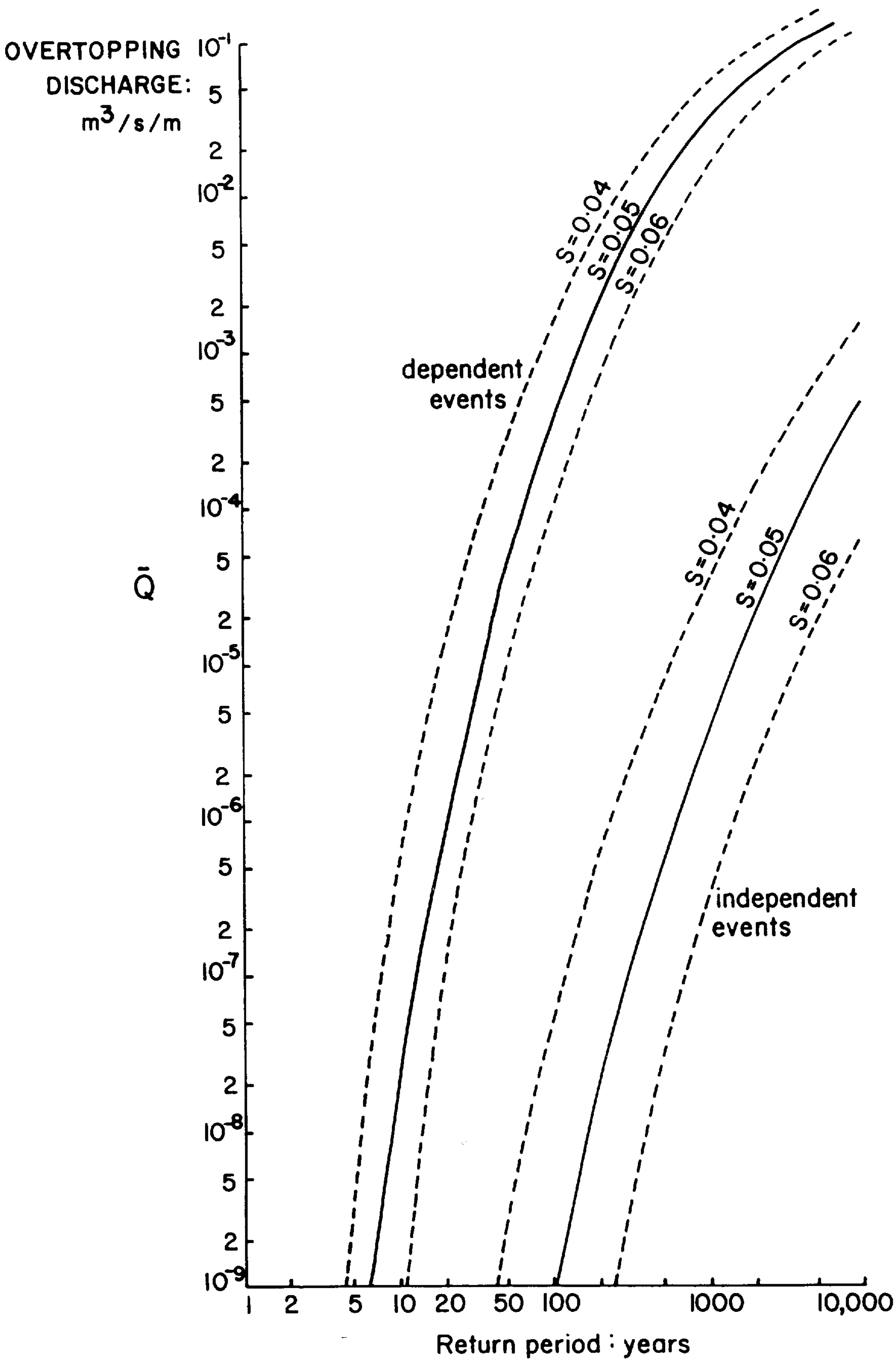

OVERTOPPING DISCHARGE: $m^3/s/m$
$\bar{Q}$
10^{-1}
5
2
10^{-2}
5
2
10^{-3}
5
2
10^{-4}
5
2
10^{-5}
5
2
10^{-6}
5
2
10^{-7}
5
2
10^{-8}
5
2
10^{-9}
1 2 5 10 20 50 100 1000 10,000
Return period : years
S=0·04
S=0·05
S=0·06
dependent events
S=0·04
S=0·05
S=0·06
independent events

Figure 12

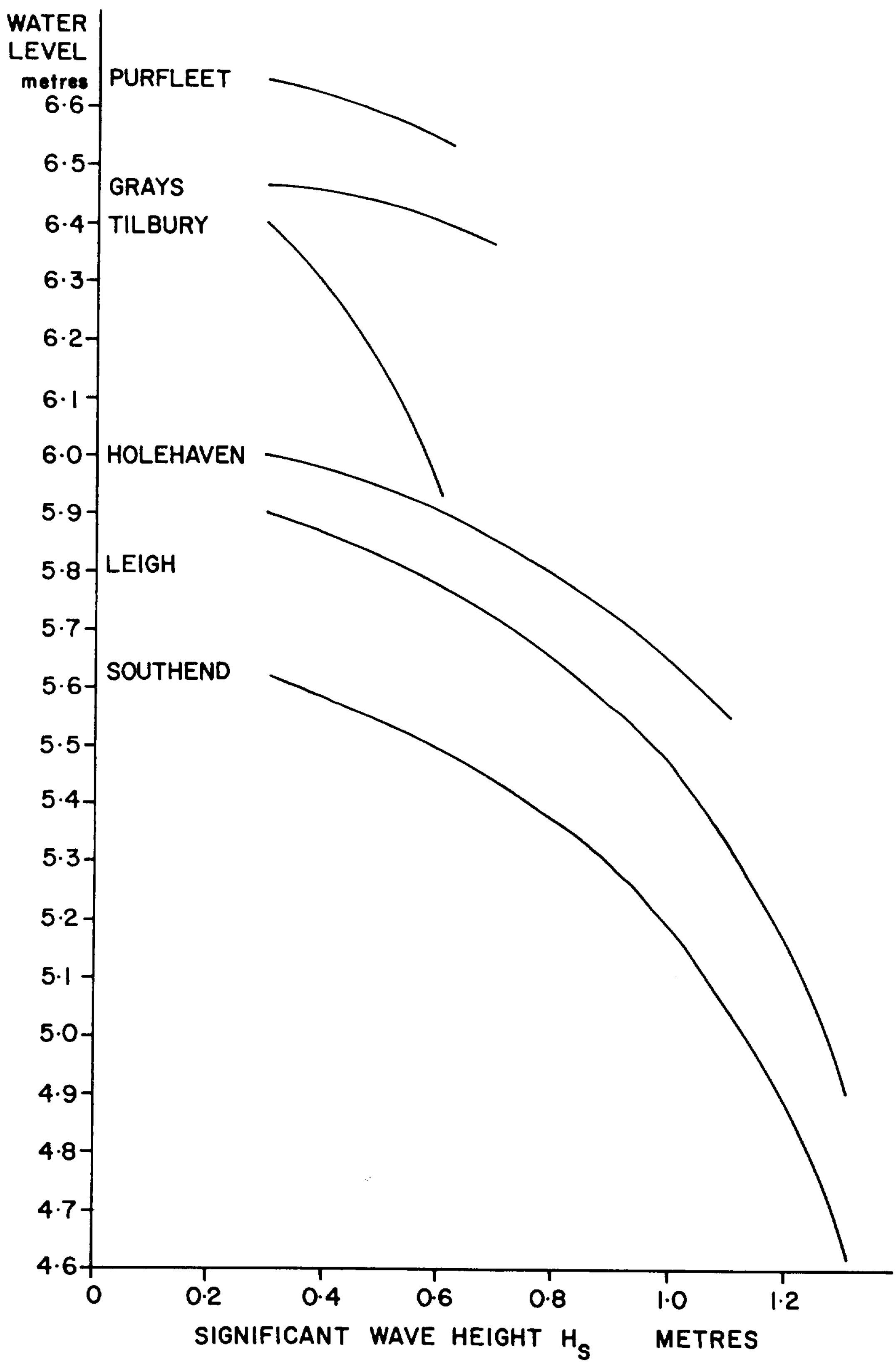

WATER LEVEL metres
PURFLEET
GRAYS
TILBURY
HOLEHAVEN
LEIGH
SOUTHEND
6·6
6·5
6·4
6·3
6·2
6·1
6·0
5·9
5·8
5·7
5·6
5·5
5·4
5·3
5·2
5·1
5·0
4·9
4·8
4·7
4·6
0
0·2
0·4
0·6
0·8
1·0
1·2
SIGNIFICANT WAVE HEIGHT H_S METRES

Figure 13

TABLE 1

HIGH WATER LEVELS AND WAVE HEIGHTS FOR FULLY DEPENDENT RETURN PERIODS

Return Period	High Water level m ODN	Significant wave height Sheerness m	Significant wave height Western walls m
1	3.62	1.00	0.49
2	3.76	1.12	0.53
5	3.99	1.29	0.57
10	4.14	1.42	0.61
20	4.32	1.57	0.65
50	4.50	1.76	0.69
100	4.59	1.91	0.73
200	4.77	2.06	0.76
500	4.99	2.28	0.81
1000	5.15	2.43	0.84
2000	5.29	2.60	0.88
5000	5.48	2.84	0.92
10000	5.61	3.02	0.96

TABLE 2

WESTERN SEAWALLS – COMBINED PROBABILITIES OF HIGH WATER AND WAVE HEIGHTS : %

HW Level	WAVE HEIGHT (m) 0	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	4.183×10^{-2}	1.073×10^{-2}	2.035×10^{-3}	3.460×10^{-4}	5.176×10^{-5}	8.910×10^{-6}	1.210×10^{-6}	1.628×10^{-7}	2.327×10^{-8}	3.366×10^{-9}
0.5										
	2.282×10^{-2}	5.583×10^{-3}	$1.\ 11 \times 10^{-3}$	1.887×10^{-4}	2.823×10^{-5}	4.860×10^{-6}	6.600×10^{-7}	8.880×10^{-8}	1.269×10^{-8}	1.836×10^{-9}
0.75										
	8.670×10^{-2}	2.224×10^{-2}	4.218×10^{-3}	7.171×10^{-4}	1.073×10^{-4}	1.847×10^{-5}	2.508×10^{-6}	3.374×10^{-7}	4.822×10^{-8}	6.977×10^{-9}
1.0										
	6.487×10^{-1}	1.664×10^{-1}	3.156×10^{-2}	5.365×10^{-3}	8.027×10^{-4}	1.382×10^{-4}	1.877×10^{-5}	2.525×10^{-6}	3.608×10^{-7}	5.220×10^{-8}
1.25										
	2.896	7.429×10^{-1}	1.409×10^{-1}	2.395×10^{-2}	2.892×10^{-3}	6.169×10^{-4}	8.378×10^{-5}	1.127×10^{-5}	1.611×10^{-6}	2.330×10^{-7}
1.5										
	5.749	1.475	2.797×10^{-1}	4.755×10^{-2}	7.114×10^{-3}	1.225×10^{-3}	1.663×10^{-4}	2.238×10^{-5}	3.198×10^{-6}	4.627×10^{-7}
1.75										
	9.826	2.5207	4.780×10^{-1}	8.127×10^{-2}	1.216×10^{-2}	2.093×10^{-3}	2.842×10^{-4}	3.824×10^{-5}	5.465×10^{-6}	7.907×10^{-7}
2.0										
	15.32	3.929	7.452×10^{-1}	1.267×10^{-1}	1.895×10^{-2}	3.263×10^{-3}	4.431×10^{-4}	5.961×10^{-5}	8.519×10^{-6}	1.233×10^{-6}
2.25										
	16.83	4.318	8.188×10^{-1}	1.392×10^{-1}	2.082×10^{-2}	3.585×10^{-3}	4.869×10^{-4}	6.550×10^{-5}	9.361×10^{-6}	1.354×10^{-6}
2.5										
	13.64	3.500	6.638×10^{-1}	1.128×10^{-1}	1.688×10^{-2}	2.906×10^{-3}	3.947×10^{-4}	5.310×10^{-5}	7.589×10^{-6}	1.098×10^{-6}
2.75										
	7.219	1.852	3.512×10^{-1}	5.971×10^{-2}	8.932×10^{-3}	1.538×10^{-3}	2.088×10^{-4}	2.810×10^{-5}	4.015×10^{-6}	5.809×10^{-7}
3.00										
	2.788	7.152×10^{-1}	1.356×10^{-1}	2.306×10^{-2}	3.450×10^{-3}	5.939×10^{-4}	8.065×10^{-5}	1.085×10^{-5}	1.551×10^{-6}	2.244×10^{-7}
3.25										
	7.416×10^{-1}	1.903×10^{-1}	3.608×10^{-2}	6.134×10^{-3}	9.177×10^{-4}	1.580×10^{-4}	2.145×10^{-5}	2.887×10^{-6}	4.125×10^{-7}	5.968×10^{-8}
3.5										
	1.735×10^{-1}	4.452×10^{-2}	8.443×10^{-3}	1.435×10^{-3}	2.147×10^{-4}	3.697×10^{-5}	5.020×10^{-6}	6.755×10^{-7}	9.653×10^{-8}	1.397×10^{-8}
3.75										
	4.749×10^{-2}	1.218×10^{-2}	2.311×10^{-3}	3.928×10^{-4}	5.877×10^{-5}	1.012×10^{-5}	1.374×10^{-6}	1.849×10^{-7}	2.642×10^{-8}	3.822×10^{-9}
4.0										
	1.29×10^{-2}	3.323×10^{-3}	6.301×10^{-4}	1.071×10^{-4}	1.603×10^{-5}	2.759×10^{-6}	3.747×10^{-7}	5.041×10^{-8}	7.204×10^{-9}	1.042×10^{-9}
4.25										
	4.828×10^{-3}	1.239×10^{-3}	2.349×10^{-4}	3.994×10^{-5}	5.974×10^{-6}	1.029×10^{-6}	1.397×10^{-7}	1.879×10^{-8}	2.686×10^{-9}	3.886×10^{-10}
4.5										
	1.497×10^{-3}	3.842×10^{-4}	7.285×10^{-5}	1.239×10^{-5}	1.853×10^{-6}	3.190×10^{-7}	4.332×10^{-8}	5.828×10^{-9}	8.329×10^{-10}	1.205×10^{-10}
4.75										
	4.197×10^{-4}	1.077×10^{-4}	2.042×10^{-5}	3.472×10^{-6}	5.193×10^{-7}	8.041×10^{-8}	1.214×10^{-8}	1.634×10^{-9}	2.334×10^{-10}	3.378×10^{-11}
5.0										
	1.416×10^{-4}	3.633×10^{-5}	6.889×10^{-6}	1.171×10^{-6}	1.752×10^{-7}	3.016×10^{-8}	4.096×10^{-9}	5.512×10^{-10}	7.876×10^{-11}	1.140×10^{-11}
5.25										
	4.752×10^{-5}	1.219×10^{-5}	2.312×10^{-6}	3.931×10^{-7}	5.880×10^{-8}	1.012×10^{-8}	1.375×10^{-9}	1.850×10^{-10}	2.643×10^{-11}	3.824×10^{-12}
5.5										

TABLE 3 COMBINED TIDE, SURGE AND WAVE PROBABILITIES AT SOUTHEND

LEVEL EXCEEDED								WAVE HEIGHT (ft)			
Level in ft ODN		Number of occasions level exceeded in 1000 years						7.5 – 9.0	9.1 – 10.4	10.5 – 12.0	>7.5
						Probability that wave occurs on a day of stated surge	a) >3ft surge	2.55 x 10^{-3}	–	2.55 x 10^{-3}	
							b) 2-3ft surge	12.6 x 10^{-3}	2.1 x 10^{-3}	1.05 x 10^{-3}	
							c) <2ft surge	2.88 x 10^{-3}	0.3 x 10^{-3}	–	
year 1971*	year 2000*	TOTAL	with <2ft surges	with 2-3ft surges	with >3ft surges						
17.0	17.3	1	–	–	1			2.55 x 10^{-3}	–	2.55 x 10^{-3}	0.005
16.5	16.8	3	–	–	3			7.65 x 10^{-3}	–	7.65 x 10^{-3}	0.015
16.0	16.3	6	–	–	6			15.3 x 10^{-3}	–	15.3 x 10^{-3}	0.031
15.5	15.8	12	–	–	12			30.6 x 10^{-3}	–	30.6 x 10^{-3}	0.061
15.0	15.3	22	–	–	22			56 x 10^{-3}	–	56 x 10^{-3}	0.112
14.5	14.8	37	–	–	37			94 x 10^{-3}	–	94 x 10^{-3}	0.184
14.0	14.3	61	–	–	61			0.155	–	0.155	0.31
13.5	13.8	101	–	–	101			0.258	–	0.258	0.52
13.0	13.3	164	–	7	157			0.40) 0.09) 0.49	-) 0.014) 0.014	0.40) 0.007) 0.407	0.91
12.5	12.8	405	–	169	236			0.60) 2.13) 2.73	-) 0.355) 0.355	0.60) 0.177) 0.777	3.86
12.0	12.3	800	60	413	327			0.835) 5.2) 6.208 0.173)	-) 0.867) 0.885 0.018)	0.835) 0.434) 1.269 -)	8.36
11.5	11.8	2620	1380	828	412			1.05) 10.45) 15.48 3.98)	-) 1.74) 2.15 0.41)	1.05) 0.87) 1.92 -)	19.55
11.0	11.3	6478	4630	1376	472			1.20) 17.35) 31.90 13.35)	-) 2.89) 4.28 1.39)	1.20) 1.44) 2.64 -)	38.82

*Rise in sea level of 0.011ft/year assumed up to these dates: no rise subsequently.

TABLE 4 JOINT FREQUENCY OF STILL WATER LEVEL AND WAVE HEIGHT: ISLE OF SHEPPEY

SWL 2038 mODN	Deep water significant wave height (m)								
	4.0–4.5	3.5–4.0	3.0–3.5	2.5–3.0	2.0–2.5	1.5–2.0	1.0–1.5	0.5–1.0	0.0–0.5
5.20	0.151	0.442	1.91	7.72	27.5	81.0	186	318	376
5.00	0.360	1.05	4.54	18.4	65.4	193	444	758	896
4.80	0.857	2.51	10.8	43.8	156	459	1057	1805	2135
4.60	2.09	6.10	26.3	107	379	1117	2573	4394	5196
4.40	5.25	15.3	66.2	268	954	2809	6469	11049	13065
4.20	13.2	38.6	167	675	2403	7075	16294	27828	32906
4.00	32.3	100	435	1748	6246	18763	44480	76629	82567
3.80	63.6	263	1163	4542	16452	53498	141837	253011	177170
3.60	98.7	490	2322	9066					
3.40	142	752	4052	16166					
3.20	385	1967	13405	55766					
3.00	1092	5281	42912	183143					
2.80	1796	8563	72419	310673					
2.60	2537	12002	103558	445377			No data available		
2.40	2982	14058	122308	526564					
2.20	3268	15378	134371	578806					
2.00	3423	16091	140929	607240					
1.80	3479	16345	143268	617377					
1.60	3494	16412	143888	620066					
1.40	3495	16417	143931	620254					