Cite as: Fujita, T. and Jones, K. (2002), Opportunities for the Development of Geometrical Reasoning in Current Textbooks in the UK and Japan, *Proceedings of the British Society for Research into Learning Mathematics*, **22**(3), 79-84.

British Society for Research into Learning Mathematics
Geometry Working Group

Convenors: Keith Jones, University of Southampton, and Taro Fujita, University of Glasgow

OPPORTUNITIES FOR THE DEVELOPMENT OF GEOMETRICAL REASONING IN CURRENT TEXTBOOKS IN THE UK AND JAPAN

A report based on the meeting at the University of Nottingham, 16th November 2002

by

Taro Fujita* and Keith Jones**

*Curriculum Studies, University of Glasgow

**Research and Graduate School of Education, University of Southampton

Developing a good model of the school geometry curriculum continues to be one of the most important tasks in curricular design in mathematics. This paper reports on an initial analysis of current best-selling textbooks used in lower secondary schools in Japan and the UK (specifically England and Scotland). The analysis indicates that, following the specification of the mathematics curriculum in these countries, Japanese textbooks set out to develop students' deductive reasoning skills through the explicit teaching of proof in geometry, whereas comparative UK textbooks tend, at this level, to concentrate on finding angles, measurement, drawing, and so on, coupled with a modicum of opportunities for conjecturing and inductive reasoning. The available research suggests that each approach has its own strengths and weaknesses. Finding ways of capitalising on the strengths and mitigating the weaknesses could prove helpful in formulating new curricular models and designing new student textbooks.

INTRODUCTION

Geometry is one of the most important components of the school mathematics curriculum yet designing a suitable geometry curriculum remains a difficult task (see Clausen-May, Jones, McLean, and Rollands, 2000). Amongst the many difficult issues to get right is the approach to the teaching and learning of deductive reasoning and proof (Jones, 2000; 2002). While agreement about the importance of deductive reasoning in geometry teaching is widespread (Royal Society/Joint Mathematical Council; 2001), there are considerable problems in implementing this successfully in school mathematics curricula. A range of research across a number of countries has documented that, even after considerable teaching input, many students fail to see a need for deductive proving and/or are unable to distinguish between different forms of mathematical reasoning such as explanation, argument, verification and proof (for reviews of this research, see Hanna and Jahnke, 1996; Dreyfus, 1999).

A recent comparative study of geometry curricula found considerable variation in current approaches to the design of the school geometry curriculum (Hoyles, Foxman and Küchemann, 2002). For example, a 'realistic' or practical approach is apparent in Holland, while a theoretical approach is evident in France and Japan. Most countries, although not all, include elements of proof and proving in their curricula specifications. Here there are variations too, with some countries favouring an approach with congruence as a central element, while other used similarity and transformations. The review concludes by noting "there is evidence of a state of flux in the geometry curriculum, with most countries looking to change" (*ibid* p.121).

One way of informing such change is to evaluate the influence of different curricular models on what students experience in the classroom and compare this to what they are able to do once they have been taught. In general, the curricular model adopted by different countries is experienced by pupils through the textbooks that are used in their classrooms (and for homework). While, of course, textbooks and curriculum guides are not the only critical influences on student learning, such texts, as the Third International Mathematics and Science Study confirms (see, for example, Foxman, 1999), do have a major impact and are thus worthy of study.

This paper reports on an initial analysis of current best-selling textbooks used in lower secondary schools in Japan and in the UK (specifically England and Scotland). This textbook analysis is complemented by reviewing the current research on what is known about student capability in proof and proving with a view to suggested how curricula and textbook design might be improved.

TEXTBOOK ANALYSIS AS CRITICAL REVIEW OF CURRICULA

In this paper, we report on an initial analysis of the textbooks in use in Japan and the UK because these countries provide interesting and contrasting approaches to school geometry. For example, as Hoyles *et al* (2002) report, while there are many similarities between the curricula in these two countries, such as their spiral nature, a core of Euclidean plane geometry, some transformation geometry, a similar approach to mensuration, and a commitment to fostering logical thinking and inference, Japan is one of the few countries that tries to teach *deductive reasoning* (proof) in geometry to *all* students in lower secondary school (students aged from 13 to 15) in mixed-attainment classes. In contrast, there is greater emphasis in the curriculum in the UK on conjecturing and inductive reasoning, with proofs involving logical argument likely to be only encountered by students *after* extensive experience both of inductive reasoning and of investigations where conjectures have to be explained.

The specifications of the curricula for Japan and England can be found in *Mathematics Programme in Japan* (edition in English published by the Japanese Society of Mathematics Education, 2000) and *Mathematics: the National Curriculum for England* (Department for Education and Employment, 1999), respectively. In Scotland, there is no statutory national curriculum; rather there are national 'guidelines' for the teaching and learning of mathematics for students aged 5-14 in Scottish schools (see, Scottish Executive, 1991).

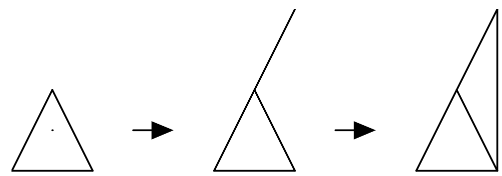
Comparing these curricula specifications and guidelines we find the following: in Japan, for students aged 13-14 the curriculum states that, in geometry, pupils must be taught to "understand the significance and methodology of proof" (JSME, 2000, p24); in England, students in the 11-14 age-range need to be taught to "distinguish between practical demonstration, proof, conventions, facts, definitions and derived properties", to "explain and justify inferences and deductions using mathematical reasoning", and to "show step-by-step deduction in solving a geometrical problem" (DfEE, 1999, p36). For Scotland, the current curriculum guidelines for mathematics (dating from 1991) make little mention of deductive reasoning, although "adopting an investigative approach to learning concepts, skills and techniques" (Scottish Office, 1991, p48) is emphasised.

The textbooks chosen for analysis are reportedly amongst the best-selling texts in the UK and Japan. The Japanese textbooks are the latest editions for lower secondary school published by Tokyo Shoseki (2001), one of the major Japanese publishers. For Scotland we chose the *Maths in Action* series now published by Nelson Thornes (the existing series, not the recently revised "New Maths in Action for S1 and S2"). For England we choose *Key Maths* series also now published by Nelson Thornes (again, the existing series, not the recent "Key Maths - Revised"). The recommended age-ranges of the books are 13-15 in Japan, 12-16 in Scotland, and 11-16 in England.

From the national curricula prescriptions, and following Hoyles (1997), it is expected that deductive reasoning and proof in geometry are dominant in Japanese textbooks and, indeed, this is the case. After studying geometrical constructions at age 12-13, the basic properties of lines, triangles, parallelograms and circles are introduced to 13-14 year olds. The principles of how to proceed with mathematical proof are explained in detail, including the explanations of 'geometrical constructions', 'definition' and 'mathematical proof'. For pupils aged 14-15, the properties of similar figures and the Pythagorean theorem are studied. About one-third of the contents of the textbooks are devoted to geometry. In contrast, much of the Scottish and English mathematics textbooks chosen for analysis concentrate on finding angles, lengths, or areas, drawing and measurement of geometrical figures.

Various approaches are used in Japanese textbooks. For example, the study of geometry often starts from problem-solving situation. Thus, while the statement 'the base angles of an isosceles triangle are equal' is proved (in the textbook for 13-14 year olds), prior to this proof, a task to make a right angle by using isosceles triangles (see Figure 1) is introduced, which encourages students to think of the properties that have to be used in the proof. After proving such statements, theoretical exercises are presented which require the application of the statements which the students have learnt, e.g. 'In a parallelogram ABCD, the angle bisectors of ∠B=∠C meet AD and BC at E and F respectively. Prove BE//FD'.

Figure 1: Making a right angle



In the current UK textbooks analysed for this study, and unlike Japanese textbooks, facts are usually given first (generally in a box at the top of the relevant page), and the exercises follow. The exercises invariably involve finding angles, measurement, drawing, and so on. Occasionally, exercises such as 'prove that vertically opposite angles are always equal, ...' (*Maths in Action 2*, p. 20) and 'Draw a parallelogram ABCD, and join A to C. Explain why the sum of the angles of the parallelogram is 360' (*Maths in Action 2*, p. 170) appear. Some opportunities for conjecturing and inductive reasoning are evident. In general, however, systematic explanations of proof and geometrical constructions are not present in the editions of the textbooks analysed. Whether the new editions of these textbooks are any different in this respect will be reported in a planned extension to this research project.

STUDENTS' UNDERSTANDING OF PROOF

The analysis conducted indicates that current editions of UK mathematics textbooks are designed around a set of exercises with mathematical theorems merely stated rather than developed or proved. In contrast, in Japan, textbooks attempt to develop students' deductive reasoning through teaching 'proof' using various approaches. The issue addressed in this section of this paper is the impact this has on student learning.

In the UK, a major study by Healy and Hoyles (1998; 1999) reports that even high-attaining 14-15 year-olds show a consistent pattern of poor performance in constructing proofs. In fact, students in the UK 'are likely to focus on measurement, calculation and the production of specific (usually numerical) results, with little appreciation of the mathematical structures and properties, the vocabulary to describe them, or the simple inferences that can be made from them' (Healy and Hoyles, 1999, p. 166). Yet Healy and Hoyles also found evidence that students could respond positively to the challenge of attempting more rigorous and formal proofs alongside informal argumentation.

In Japan, the teaching and learning of deductive reasoning remains a major problem. Despite the design of the textbooks, research indicates that while most 14-15 year-old students (Japanese secondary 3rd grade) can write down a proof, around 70% cannot understand why proofs are needed (Miyazaki, 1999; Kunimune, 2000). Similar results with a student who was educated in Hong Kong, where the geometry curriculum is similar to that in Japan, are reported in Healy and Hoyles (*op cit*, p. 166).

Thus the approaches to deductive reasoning and proof evident in the textbooks in both the UK and in Japan have their own strengths and weaknesses. In the UK, students appear to complete lower secondary school with good skills in conjecturing and inductive reasoning but with little idea of deductive reasoning. Nevertheless, they can respond positively when challenged to produce deductive proofs. The current textbooks analysed for this study fail to exploit this potential. In Japan, for all the efforts evident in their textbooks to instil the notion of proof, a majority of lower secondary school students still fail to gain the sort of understanding of proof specified in the Japanese national curriculum.

The final section of this paper looks at how we might capitalise on the strengths and mitigate the weaknesses in current textbooks, as this should prove helpful in formulating new curricular models and designing new student textbooks.

THE IMPROVEMENT OF GEOMETRY TEACHING AND GEOMETRICAL EYE

One of problems in geometry is related to students' intuitive skills in that some students appear to be unable to 'see' geometrical properties, or decide where to start, when they solve exercises in geometry (Nakanishi, 1987). As we report in a previous paper (Fujita and Jones, 2002), in the early 20th Century in England, Charles Godfrey, a leading mathematics educator at that time, insisted that geometry could not be undertaken only by logic. Godfrey proposed that the 'geometrical eye', the ability "to see geometrical properties detach themselves from a figure" (Godfrey, 1910, p. 197), would be essential to solve geometrical problems. He also stated that we could develop learners' geometrical eye through experimental tasks (*op cit*, p. 197). Godfrey and Siddons endeavoured to implement this pedagogical consideration in the design of the geometry textbooks they produced. For example, the numerous experimental exercises they included were carefully chosen and designed, leading to showing and requiring a proof. Using this design, the aim of Godfrey and Siddons was to develop in students what they called the geometrical eye.

Further research is needed to examine whether it would be possible to define more clearly the notion of the *geometrical eye*, what the relationships are between difficulties of proof in geometry and the *geometrical eye,* and how (or whether) it would be possible to develop students' *geometrical eye* though practical tasks. Such research could make an important contribution to providing a firmer theoretical basis for formulating new curricular models for geometry and designing new student textbooks.

REFERENCES

Clausen-May, T., Jones, K., McLean, A. and Rollands, S.: 2000, Perspectives on the Design of the Geometry Curriculum, *Proceedings of the British Society for Research into Learning Mathematics*, **20** (1&2), 34-41.

Department for Education and Employment: 1999, *Mathematics: the National Curriculum for England*. London: DfEE/QCA.

Dreyfus, T.: 1999, Why Johnny can't prove, *Educational Studies in Mathematics* **38** (1–3), 85–109.

Foxman, D.: 1999, *Mathematics Textbooks Across the World: some evidence from the third international mathematics and science study*. Slough: NFER.

Fujita, T. and Jones, K.: 2002, The Bridge between Practical and Deductive Geometry: developing the 'geometrical eye'. In: A. D. Cockburn and E. Nardi (Eds), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, UEA, Vol 2, 384-391.

Godfrey, C.: 1910, The Board of Education Circular on the Teaching of Geometry. Mathematical Association. *Mathematical Gazette*, **5**, 195-200.

Hanna, G. and Jahnke, H.N.: 1996, Proof and proving. In A. J. Bishop (Ed.), *International Handbook on Mathematics Education*, Kluwer, Dordrecht.

Healy, L. & Hoyles, C.: 1998, *Justifying and Proving in School Mathematics*, London: University of London, Institute of Education. Technical Report to the ESRC.

Healy, L. & Hoyles, C.: 1999, Students' performance in proving: competence or curriculum? In: Inge Schwank (Ed), *European Research in Mathematics Education*. Osnabrueck, Germany: Forschungsinstitut für Mathematikdidaktik, 153-167.

Hoyles C.: 1997, The curricular shaping of students' approaches to proof. *For the Learning of Mathematics*, **17** (1), 7-16.

Hoyles, C., Foxman, D. and Küchemann, D.: 2002, *A Comparative Study of Geometry Curricula*. London: Qualifications and Curriculum Authority.

Japanese Society of Mathematics Education: 2000, *Mathematics Programme in Japan*. Tokyo, JSME.

Jones, K.: 2000, Critical Issues in the Design of the Geometry Curriculum. In: Bill Barton (Ed), *Readings in Mathematics Education*. Auckland, New Zealand: University of Auckland. pp75-91.

Jones, K.: 2002, Issues in the Teaching and Learning of Geometry. In: Linda Haggarty (Ed), *Aspects of Teaching Secondary Mathematics*. London: RoutledgeFalmer. pp121-139.

Kunimune, S.: 2000, A Change in Understanding with Demonstration in Geometry. *Journal of Japan Society of Mathematics Education*, **82**(3), 66-76 (in Japanese).

Miyazaki, M.: 1999, The Status of Proof in the Geometry Curriculum of Lower Secondary School of Japan: focusing on the meaning and arguments of proof in textbooks for 8th graders. In: *Proceeding of the first ICMI-East Asia Regional Conference on Mathematics Education*. Cheongju, Korea: Korea National University of Education. Vol. 2, 135-152.

Nakanishi, C: 1987, Analysis of the Difficulties of Proof. In K. Koseki (Ed), *The Teaching of Demonstration in Geometry*. Tokyo: Meiji Publishing, 129-158 (in Japanese).

Royal Society and Joint Mathematical Council: 2001, *Teaching and Learning Geometry 11-19*, London, Royal Society/Joint Mathematical Council.

Scottish Executive: 1991, *Mathematics: 5-14 National Guidelines*. Edinburgh: Learning and Teaching Scotland.

BSRLM GEOMETRY WORKING GROUP

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. Suggestions of topics for discussion are always welcome. The group is open to all.