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Interpretations of National Curricula: the case of geometry in popular textbooks in Japan and the UK

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Abstract

This paper presents an analysis of how the geometry component of the National Curricula for mathematics in Japan and in one selected country of the UK, specifically Scotland, is interpreted by textbook writers. While, of course, textbooks are not the only critical influences on student learning, such texts, as analyses of data from the Trends in International Mathematics and Science Study (TIMSS) confirm, do have a major impact and are thus important subjects for study. Our analysis is framed by the procedures derived from the work associated with TIMSS. The results presented focus on identifying features of geometry, and approaches to geometry learning, privileged in the textbooks, together with a discussion of how these designs might influence students' performance in geometry. Our analysis indicates that, following the specification of the mathematics curriculum in these countries, Japanese textbooks set out to develop students' deductive reasoning skills through the explicit teaching of proof in geometry, whereas comparative Scottish textbooks tend, at this level, to concentrate on measuring, drawing, finding angles, and so on, coupled with a modicum of opportunities for conjecturing and inductive reasoning. The available research suggests that each approach has its own strengths and weaknesses. Finding ways of capitalising on the strengths and mitigating the weaknesses could prove helpful in formulating new curricular models and designing new student textbooks. An emerging issue is how the design of textbooks might either build on, or neglect, students' intuitive skills when they tackle geometrical problems.

Introduction

While comparative education has traditionally been concerned with school outcomes, there has been a new interest in the detailed study of the content of schooling and with the internal workings of the school. One reason for this trend has been the realisation that it is necessary to go beyond the headline-grabbing publication of league tables of the educational prowess of nations to investigate the reasons for differential achievement in international comparisons. This is even the case in respect of the teaching and learning of mathematics; a subject which has been the subject of numerous comparative studies perhaps because it is sometimes perceived as a universal subject and where some, such as Reynolds and Farrell (1996, p2), feel that they can claim that " the effects of the educational system outweigh the effects of home background in determining achievement".

In the light of this interest in more detailed studies, a recent comparative study of geometry curricula (Hoyles, Foxman and Küchemann, 2002) found considerable variation in current approaches to the design of the school geometry curriculum across a range of countries around the world. Thus, the study found, a 'realistic' or practical approach is apparent in Holland, while a theoretical approach is evident in France and Japan. In most countries, although not all, the study found that elements of proof and proving were included in the curricula specifications for geometry. Yet even here there are variations too, with some countries favouring an approach with congruence as a central element, while other used similarity and transformations. The study concludes by noting "there is evidence of a state of flux in the geometry curriculum, with most countries looking to change" (*op cit* p. 121).

This state of flux means that there are opportunities to improve the specification of the curriculum for geometry, yet, as Pepin (2001, p158) observes, we need to refine our understandings of the teaching and learning cultures of mathematics in different countries in order to avoid being "pulled in inappropriate and ill-judged directions by policy makers intent on short-term, measurable outcomes of performance improvements in a narrow range of areas". This entails, amongst other things, Pepin argues, refining our understandings of the materials used for learning, such as textbooks, each of which is influenced, and in some cases determined, by the educational and cultural traditions of the particular country in which the teaching and learning takes place.

This paper presents an analysis of how the geometry component of the National Curricula in Japan and in a selected country of the UK, specifically Scotland, is interpreted by textbook writers in those countries. The intention is to focus on identifying features of geometry, and approaches to geometry learning, privileged in the textbooks, and to discuss how these designs might influence students' performance in geometry.

Mathematics Curriculum and Textbooks in the UK and Japan

In Scotland, there is no statutory national curriculum; rather there are national 'guidelines' for the teaching and learning of mathematics for students aged 5-14 (see, Scottish Executive, 1991; Métais *et al*, 2001). In geometrical 'Shape, position and movement', the guidelines stipulate that as an outcome of teaching, 'The pupil recognises, understands, uses and applies concepts, facts and techniques associated with properties of two and three dimensional shapes, and properties of position and movement (Scottish Office, 1991, p. 37). The guidelines emphasise the importance of "adopting an investigative approach to learning concepts, skills and techniques" (Scottish Office, 1991, p. 48) but make little explicit mention of deductive reasoning.

For Scotland we chose for our analysis the revised *New Maths in Action S1*^{2} and *S2*^{2} books (designed for students aged 12-14) which mainly cover the content corresponding to the level D/E and E/F of the national guidelines for mathematics for Scotland (also see Scottish Office, 1991, pp. 37-41 and Scotland Office, 1999, pp. 15-6). For example, *New Maths in Action S2*^{2} consists of 18 chapters (442 pages, 24 cm. x 17 cm.). Of these 18 chapters, 8 are related to geometry [chapter 3, Angles; chapter 7, Scale and coordinates; chapter 9, Calculating distance; chapter 10, Transformation; chapter 11, Area; chapter 13, The triangle; chapter 16, Two-dimensional shapes; and chapter 17, Three dimensions]. In each chapter, there are 7-10 units . For example, chapter 3 (on angles) of *New Maths in Action S2*^{2} consists of 9 units [unit 1, Looking back; unit 2, Some relations; unit 3, Vertically opposite angles; unit 4, Angles and parallel lines; unit 5, Corresponding angles; unit 6, Alternate angles; unit 7, Mixed examples; unit 8, Interior and exterior angles; and unit 9, Check-up]. Every chapter has a similar structure, i.e. it starts from 'Looking back', provides the main content, and finishes with 'Check-up' at the end.

The specification of the mathematics curriculum for Japan can be found in *Mathematics Programme in Japan* (edition in English published by the Japanese Society of Mathematics Education, 2000). The curriculum states that, in geometry for students aged 13-14, they must be taught to "understand the significance and methodology of proof" (JSME, 2000, p. 24). Textbooks for use in schools must follow the content prescribed in the *Mathematics Programme in Japan*. The textbooks selected for analysis in this study are the latest editions of *New Mathematics (Atarashii Suugaku)* for lower secondary school published by Tokyo Shoseki (2001), one of the major Japanese publishers. In comparison to the British textbooks, these textbooks~~is~~ are smaller (21 cm. x 15 cm.). In the first grade textbook *New Mathematics 1* (designed for students aged 12-13, 197 pages), there are 7 chapters in total, and the content of geometry appears in chapters 5 (the study of symmetry and geometrical constructions, 30 pages) and 6 (the study of 3-D shapes including their volume and surface areas, 27 pages). In the second grade textbook *New Mathematics 2* (for students aged 13-14, 190 pages), there are 6 chapters in total, and geometry is studied in chapters 4 (angles, parallel lines, and congruency, 30 pages) and 5 (triangles, quadrilaterals and circles, 38 pages). At this grade level, the principles of how to proceed with mathematical proof are explained in detail, including the explanations of 'definition' and 'mathematical proof'. In the third grade *New Mathematics 3* (aged 14-5, 195 pages), there are 7 chapters in total, and geometry is studied in chapters 4 (similarity, 28 pages) and 5 (the Pythagorean theorem, 20 pages).

Analysis Procedure

In this paper, we focus on 'Chapter 4 Parallel and congruency' and 'Chapter 5 Properties of shape' in Japanese 2^{nd} grade text *New Mathematics 2*, and 'Chapter 3 Angles' (23 pages), 'Chapter 13 The triangle' (22 pages) and 'Chapter 16 Two-dimensional shapes' (26 pages) in Scottish *New Maths in Action S2*^{2}, because both texts are studied by similar aged students (ages from 13 to 14). These chapters include the studies of the basic properties of angles, triangles and quadrilaterals with mathematical proof, i.e. we focus a part of *Maths in Action S2*^{2} and the whole content of geometry in *New Mathematics 2*.

Our analysis is framed by the following procedure, which is derived from the study by Valverde *et al* (2002):

- division of the geometry parts of textbooks into 'units' and 'blocks';
- coding of each 'block' in terms of content, performance expectations and perspectives (Table 1, also see in Valverde *et al*; 2002, pp. 184-7);

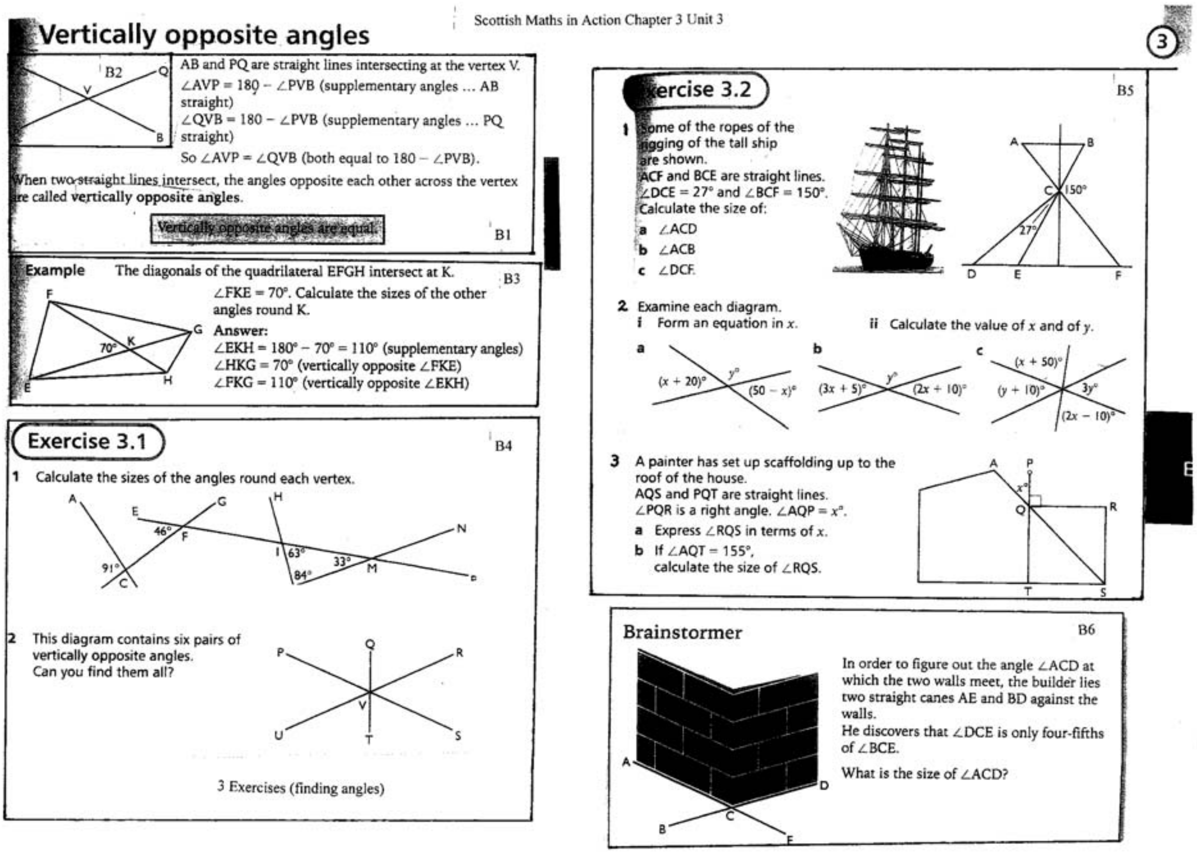
- identifying features of geometry in the textbooks;

The discussion that follows this analysis focuses on how these designs might influence students' performance in geometry, including their intuitive skills.

Table 1. Codes used for the analysis

Block type	Content (subject matter topic)	Performance Expectations	Perspective
1 Central instructional narrative 2 Related instructional narrative 3 Unrelated instructional narrative 4 Graphic (those directly related narrative) 5 Graphic (those not directly related narrative) 6 Question 7 Exercise Set 8 Suggested activities 9 Worked examples 10 Others	1.1. Geometry: Position, visualisation, and shape 1.1.1. Two-dimensional geometry: Co-ordinate geometry 1.1.2. Two-dimensional geometry: Basics (point, line, and angles) 1.1.3. Two-dimensional geometry: Polygons and circles 1.1.4. Three-dimensional geometry 1.1.5. Vectors 1.2. Geometry: Symmetry, congruence, and similarity 1.2.1. Transformation 1.2.2. Symmetry 1.2.3. Congruence 1.2.4. Similarity 1.2.5. Constructions using straightedge and compass 1.3. Measurement 1.3.1. Perimeter, area, and volume 1.3.2. Angle and bearing	2.1. Knowing 2.1.1. Representing 2.1.2. Recognising equivalents 2.1.3. Recalling properties and theorems 2.1.4. Consolidating notation and vocabulary 2.1.5. Recognising aims of lessons 2.2. Using routine procedures 2.2.1. Using equipment 2.2.2. Performing routine procedures 2.2.3. Using more complex procedures 2.3. Investigating and problem solving 2.3.1. Formulating and clarifying problems 2.3.2. Developing strategy 2.3.3. Solving 2.3.4. Predicting 2.3.5. Verifying 2.4. Mathematical reasoning 2.4.1. Developing notation and vocabulary (proof) 2.4.2. Developing algorithms 2.4.3. Generalising 2.4.4. Conjecturing and discovering 2.4.5. Justifying and proving 2.4.6. Axiomatising 2.5. Communicating 2.5.1. Using vocabulary and notation 2.5.2. Relating representations 2.5.3. Describing/discussion 2.5.4. Critiquing	3.1. Attitude toward science, mathematics, and technology 3.2. Careers involving in science, mathematics, and technology 3.2.1. Promoting careers in science, mathematics, and technology 3.2.2. Promoting the importance of science, mathematics, and technology in non-technical careers 3.3. Participation in science and mathematics by underrepresented groups 3.4. Science, mathematics and technology to increase interest 3.5. Scientific and mathematical habits of mind

For example, 'Unit 3 Vertically opposite angles' in 'Chapter 3 Angles' (*Action S2*[2]) mainly includes the study of basic angle properties and measurement, is coded as follows. First, we divide pages 63-5 into the 'block', and they are numbered as 'B1', 'B2', ... (see the figure 1). The narrative block 'B1' contains another graphical block 'B2'. The clear intent of these two blocks is for the students to recall of the mathematical properties in vertically opposite angles, to learn the term 'vertically opposite angles', and to justify the statement. The block 'B3' is an 'example' block, in this case showing the routines to find the angles. The blocks 'B4' (Exercise 3.1) and 'B5' (Exercise 3.2.) are sets of exercises, which expect students to perform routine procedures (mainly 'B4'), and complicated procedures (mainly 'B5'). This unit includes a suggested activity 'B6' as 'Barnstormer', a problem solving activity. It should be noted that these exercise blocks consist of more than one exercises, and therefore the number of blocks *does not* represent a precise quantitative aspect of these textbooks. Each 'block' is then coded in terms of content, performance expectations and perspectives (Table 2). The coding procedure is carefully undertaken, and the preliminary results are discussed between the authors.


Scottish Maths in Action Chapter 3 Unit 3

3

Vertically opposite angles

B2

AB and PQ are straight lines intersecting at the vertex V.
∠AVP = 180 − ∠PVB (supplementary angles … AB straight)
∠QVB = 180 − ∠PVB (supplementary angles … PQ straight)
So ∠AVP = ∠QVB (both equal to 180 − ∠PVB).
When two straight lines intersect, the angles opposite each other across the vertex are called vertically opposite angles.
Vertically opposite angles are equal.
B1

Example The diagonals of the quadrilateral EFGH intersect at K.
B3
∠FKE = 70°. Calculate the sizes of the other angles round K.
Answer:
∠EKH = 180° − 70° = 110° (supplementary angles)
∠HKG = 70° (vertically opposite ∠FKE)
∠FKG = 110° (vertically opposite ∠EKH)

Exercise 3.1
B4
1 Calculate the sizes of the angles round each vertex.
2 This diagram contains six pairs of vertically opposite angles. Can you find them all?
3 Exercises (finding angles)

Exercise 3.2
B5
1 Some of the ropes of the rigging of the tall ship are shown.
ACF and BCE are straight lines.
∠DCE = 27° and ∠BCF = 150°.
Calculate the size of:
a ∠ACD
b ∠ACB
c ∠DCF.
2 Examine each diagram.
i Form an equation in x.
ii Calculate the value of x and of y.
3 A painter has set up scaffolding up to the roof of the house.
AQS and PQT are straight lines.
∠PQR is a right angle. ∠AQP = x°.
a Express ∠RQS in terms of x.
b If ∠AQT = 155°, calculate the size of ∠RQS.

Brainstormer
B6
In order to figure out the angle ∠ACD at which the two walls meet, the builder lies two straight canes AE and BD against the walls.
He discovers that ∠DCE is only four-fifths of ∠BCE.
What is the size of ∠ACD?

Figure 1: *Maths Action S2*[2] Chapter 3 Unit 3

Table 2: *Maths Action S2*[2] Chapter 3 Unit 3

Block Number	B1	B2	B3	B4	B5	B6
Block Type	1	4	9	7	7	8
Content	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.
Performance Exp.	2.1.3. 2.1.4. 2.4.5.	2.1.3. 2.1.4. 2.4.5.	2.2.2.	2.2.2.	2.2.3.	2.3.3.
Perspective	0	0	0	0	0	0

Japanese textbook, *New Mathematics 2* are also coded by the same procedure given above. First, each of the units in the chapters is divided into 'lessons' by referring to the guide for teacher and students. Then each lesson is coded. For example, a lesson in 'Unit 1.2' in 'Chapter 4 Parallel and congruency' (2nd grade) is coded as follows (see in p. 6 of the textbook). This lesson starts from a narrative block, 'B1', which tell the students the aim of the unit. The next box 'B2' is a question block, which would lead students to discover (or conjecture) that vertically opposite angles are equal. Blocks 'B3', 'B5' and 'B8' are narrative blocks, and 'B5' includes a justification of the statement. These blocks include graphical blocks 'B4', 'B6', and 'B9', which would help students to learn the statement and vocabulary.

Blocks ‘B7’ and ‘B10’ are exercise blocks which confirm the statement by the justification used in ‘B5’, and involve finding angles. Block ‘B11’ is another narrative block which introduce new terms which can be extended from the study of ‘vertically opposite angles’. In the footnote in p. 86, there is a block ‘B13’ which explains a symbol (∠) which represents angles.

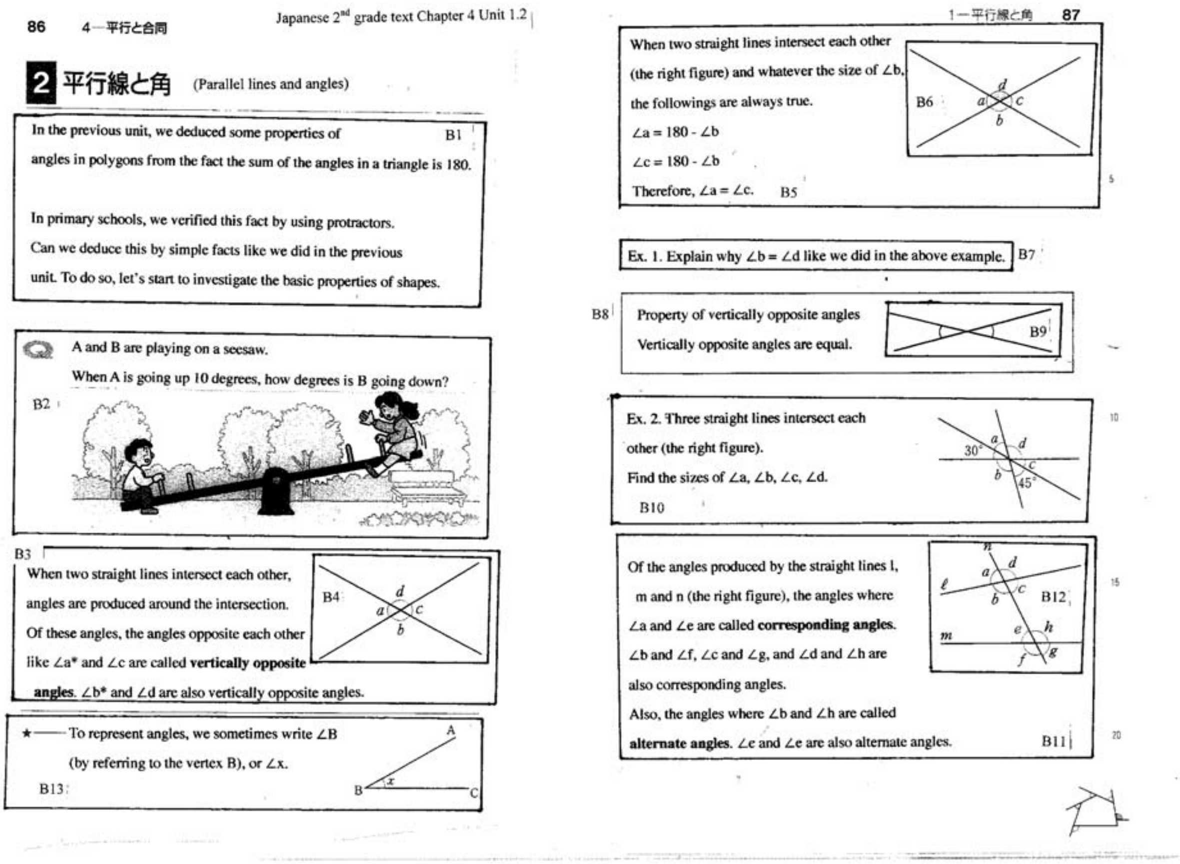


Figure 2: 2nd Grade Chapter 4 Unit 1.2.

Table 3: 2nd Grade Chapter 4 Unit 1.2.

Block Number	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13
Block Type	1	6	1	4	1	4	7	1	4	7	1	4	10
Content	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.	1.1.2. 1.3.2.
Performance Exp.	2.1.5.	2.3.4.	2.4.1.	2.4.1.	2.4.5.	2.4.5.	2.4.5.	2.1.3.	2.1.3.	2.2.2.	2.4.1.	2.4.1.	2.1.1..
Perspective	0	0.	0	0	0	0	0	3.4.	0	0	0	0	0

Findings: Identified Features Of Textbooks

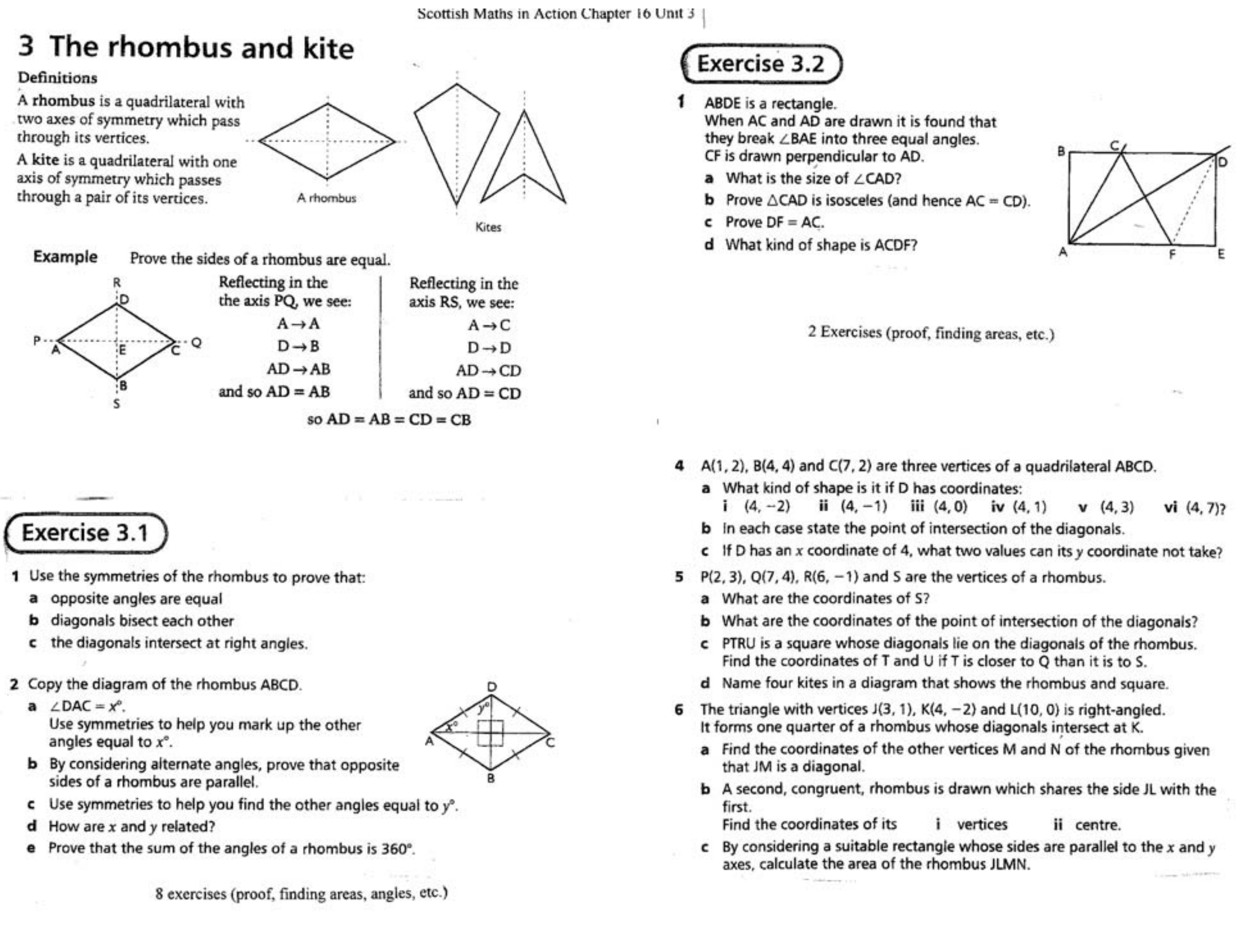
This section shows our preliminary findings from the analysis undertaken by the procedures described above. Since we have not completed the whole analysis of the textbooks, the findings tell only some aspects of the approaches in geometry in each country. Nevertheless, the results suggest interesting features about the teaching and learning of geometry in the two countries.

The design of approaches in geometry

The design of *Maths in Action S2*[2] contrasts with Japanese *New Mathematics 2*. Whereas almost all units in *Maths in Action S2*[2] start from a narrative block, with examples and then exercises and suggested activities following, various approaches are adopted in *New Mathematics 2*. Sometimes, for example, this Japanese textbook starts from a problem solving situation (about 37% of lessons in this grade begin with problem solving situations), and a narrative block which recalls some facts and theorems comes later with less exercises. In fact, the figures in the previous pages, the study of vertically opposite angles, are typical examples which tell us the differences between these textbooks. In the Japanese textbook, the principles of how to proceed with mathematical proof are explained in detail, including the explanations of 'definitions' and 'mathematical proof'. On the other hand, no systematic explanations of proof are presented in *Maths in Action S2*[2].

Content of geometry

In both the units we analysed, both the Japanese and Scottish textbooks concentrate on 1 or 2 topics in each unit (for example, 2-D basic, 2-D polygons and circles, symmetry, construction, measurement of areas and angles, etc). However, it is interesting to notice that *Maths in Action S2*[2] sometimes provides a wider content, in particular in the exercise blocks. For example, 'Unit 3 the rhombus and kite' in Chapter 16 starts from the definitions of a rhombus and kite, and the example shows how to prove 'the sides of a rhombus are equal' (p. 372). Therefore, the main theme in this unit is justifications and proof. Nevertheless, the exercise blocks in this unit also include calculating angles, areas of rhombuses and kites, and the study of figures ~~on~~ using co-ordinates (pp. 373-6). In contrast, the similar unit 'The properties of parallelograms' (pp. 124-8) in *New Mathematics 2*, just concentrates on proving various statements concerning with properties of parallelograms (see the figure3 below).


Scottish Maths in Action Chapter 16 Unit 3

3 The rhombus and kite

Definitions

A **rhombus** is a quadrilateral with two axes of symmetry which pass through its vertices.

A **kite** is a quadrilateral with one axis of symmetry which passes through a pair of its vertices.

A rhombus

Kites

Example Prove the sides of a rhombus are equal.

Reflecting in the the axis PQ, we see: A→A, D→B, AD→AB, and so AD = AB

Reflecting in the axis RS, we see: A→C, D→D, AD→CD, and so AD = CD

so AD = AB = CD = CB

Exercise 3.1

1 Use the symmetries of the rhombus to prove that:
a opposite angles are equal
b diagonals bisect each other
c the diagonals intersect at right angles.

2 Copy the diagram of the rhombus ABCD.
a ∠DAC = $x°$. Use symmetries to help you mark up the other angles equal to $x°$.
b By considering alternate angles, prove that opposite sides of a rhombus are parallel.
c Use symmetries to help you find the other angles equal to $y°$.
d How are x and y related?
e Prove that the sum of the angles of a rhombus is 360°.

8 exercises (proof, finding areas, angles, etc.)

Exercise 3.2

1 ABDE is a rectangle. When AC and AD are drawn it is found that they break ∠BAE into three equal angles. CF is drawn perpendicular to AD.
a What is the size of ∠CAD?
b Prove △CAD is isosceles (and hence AC = CD).
c Prove DF = AC.
d What kind of shape is ACDF?

2 Exercises (proof, finding areas, etc.)

4 A(1, 2), B(4, 4) and C(7, 2) are three vertices of a quadrilateral ABCD.
a What kind of shape is it if D has coordinates:
i (4, −2) ii (4, −1) iii (4, 0) iv (4, 1) v (4, 3) vi (4, 7)?
b In each case state the point of intersection of the diagonals.
c If D has an x coordinate of 4, what two values can its y coordinate not take?

5 P(2, 3), Q(7, 4), R(6, −1) and S are the vertices of a rhombus.
a What are the coordinates of S?
b What are the coordinates of the point of intersection of the diagonals?
c PTRU is a square whose diagonals lie on the diagonals of the rhombus. Find the coordinates of T and U if T is closer to Q than it is to S.
d Name four kites in a diagram that shows the rhombus and square.

6 The triangle with vertices J(3, 1), K(4, −2) and L(10, 0) is right-angled. It forms one quarter of a rhombus whose diagonals intersect at K.
a Find the coordinates of the other vertices M and N of the rhombus given that JM is a diagonal.
b A second, congruent, rhombus is drawn which shares the side JL with the first. Find the coordinates of its i vertices ii centre.
c By considering a suitable rectangle whose sides are parallel to the x and y axes, calculate the area of the rhombus JLMN.

Figure 3: The study of quadrilaterals

Performance expectations
From the national curriculum specifications in Japan and Scotland, it is expected that deductive reasoning would be very prominent in *New Mathematics 2* (and indeed it is with 32 of 36 lessons contain 'justifying and proving' geometrical facts), whereas consolidating facts and vocabulary, problem solving, and routine procedures would be foremost in *Maths in Action S2*[2] (as is the case). However, Scottish textbooks do include some proofs of geometrical facts (10 of the 23 units contain 'justifying and proving'). Interestingly, the approaches to proving in both in textbooks are very different. Whereas Japanese textbook mainly use congruency to prove various geometrical facts, symmetry is used in the Scottish textbooks. For example, the statement 'opposite sides of a parallelogram are equal' is proved by using rotational symmetry in *Maths in Action S2*[2] (thus the students are reminded of the property 'the parallelogram possesses half-turn symmetry', p. 377), whereas congruent triangles are used in *New Mathematics 2* (pp. 124-5).

Discussion

Given the differences in the curricular specifications in Japan and the UK (specifically Scotland), it is important not just to consider which approach might be better suited to developing students' reasoning skills, but to focus on how these different approaches might influence or shape such skills in geometry. An interesting difference in the overall designs of the textbooks is that the geometrical facts to be learnt always come first in the Scottish textbooks, and later in the Japanese ones. In relation to this, Shimizu (1999) reported an interesting feature of Japanese mathematics lessons. For Japanese teachers, the 'summing up' stage, which summarises facts learnt in a lesson, is very important, and by the time that students reach this stage, they have spent considerable time investigating or thinking through the facts for themselves and that often this is through, for example, a problem solving situation rather than performing routine procedures (Shimizu, 1999, p. 192). Thus, the geometrical facts studied in lessons often do not come first, but they are shown after students fully understand them. On the one hand, this approach to lessons, incorporated into the design of Japanese textbooks might build up students' view of mathematics that an important thing in learning mathematics is to understand, consider and justify statements. On the other hand, the design in Scottish textbook might encourage students to use and apply the facts to various problems and show routine procedures, while justifications of various statements are not completely neglected in the textbook.

As we have seen, in Scottish textbook, the wide range of content is studied, e.g. proof, drawing, measurement etc., whereas each unit in Japanese textbook often concentrates on one theme, e.g. proof (this is exemplified in the study of parallelograms described above). Overall, and while it should also be noted that we have not yet analysed other chapters in *Maths in Action S2*[2] (which include the studies of similarity, tessellations, areas, 3-D figures etc), in general Scottish 14 year old students study much broader content in geometry than do their Japanese contemporaries. In Japan, instead of providing students with such broad content, the manner of mathematical proof is carefully built up through proving various geometrical statements. These different approaches might have influences on, again, shaping the skills in geometry as well as the view of geometry of students; Japanese students might see geometry as a very formal subject for study and therefore with no need any practical approaches, whereas Scottish students might see geometry as both practical and formal from a wide range of contexts in mathematics.

A major study by Healy and Hoyles (1998; 1999) reports that in the U.K even high-attaining 14-15 year-olds show a consistent pattern of poor performance in constructing proofs. In fact, students in the UK 'are likely to focus on measurement, calculation and the production of specific (usually numerical) results, with little appreciation of the mathematical structures and properties, the vocabulary to describe them, or the simple inferences that can be made from them' (Healy and Hoyles, 1999, p. 166). Yet Healy and Hoyles also found evidence that students could respond positively to the challenge of attempting more rigorous and formal proofs alongside informal argumentation. In Japan, the teaching and learning of deductive reasoning remains a major problem. Despite the design of the textbooks, research indicates that while most 14-15 year-old students (Japanese secondary 3rd grade) can write down a proof, around 70% cannot understand why proofs are needed (Miyazaki, 1999; Kunimune, 2000). A similar result (ie the ability to write a proof but not understand why proofs are needed) is reported in Healy and Hoyles (*op cit*, p. 166) in the case of a student in the UK recently arrived from Hong Kong where the geometry curriculum is similar to that in Japan,.

Thus the approaches to deductive reasoning and proof evident in the textbooks in both the UK and in Japan have their own strengths and weaknesses. In this way, the textbooks, as Pepin (2001, p162) observes, reflect a nation's cultural values and have embedded in them, and will legitimise, the different cultural educational values present in the particular country. For example, in the UK, students appear to complete lower secondary school with good skills in conjecturing and inductive reasoning but with little idea of deductive reasoning. Nevertheless, they can respond positively when challenged to produce deductive proofs. The Scottish textbooks analysed for this study reinforce the former and fail to exploit this latter potential. In Japan, for all the efforts evident in their textbooks to instil the notion of proof, a majority of lower secondary school students still fail to gain the sort of understanding of proof specified in the Japanese national curriculum.

The final section of this paper looks at how we might capitalise on the strengths and mitigate the weaknesses in current textbooks, as this should prove helpful in formulating new curricular models and designing new student textbooks.

Improving the Teaching of Geometry

One of problems in geometry is related to students' intuitive skills in that some students appear to be unable to 'see' geometrical properties, or decide where to start, when they solve exercises in geometry (Nakanishi, 1987). As we report in a previous paper (Fujita and Jones, 2002; also see Fujita and Jones, in press), in the early 20th Century in England, Charles Godfrey, a leading mathematics educator at that time, insisted that geometry could not be undertaken only by logic. Godfrey proposed that the 'geometrical eye', the ability "to see geometrical properties detach themselves from a figure" (Godfrey, 1910, p. 197), would be essential to solve geometrical problems. He also stated that we could develop learners' geometrical eye through experimental tasks (*op cit*, p. 197). Godfrey and Siddons endeavoured to implement this pedagogical consideration in the design of the geometry textbooks they produced. For example, the numerous experimental exercises they included were carefully chosen and designed, leading to showing and requiring a proof. Using this design, the aim of Godfrey and Siddons was to develop in students what they called the geometrical eye. Although Godfrey's idea of the geometrical eye was suggested about 100 years ago, recent educators have also discussed similar aspects, e.g. mathematisation, the mental process which produce mathematics (Wheeler; 2001, p. 50); figural concepts, that a geometrical shape is a spatial representation and a concept, and that successful reasoning in geometry may be related to the harmony between figural and conceptual constrains (Fischbein and Nachlieli, 1998). Godfrey's geometrical eye can be considered as a more specialised version of this mathematisation, and this can be interpreted as a sort of intuitive skill in geometry.

Further research is needed to examine whether it would be possible to define more clearly the notion of the *geometrical eye*, what the relationships are between difficulties of proof in geometry and the *geometrical eye*, and how (or whether) it would be possible to develop students' *geometrical eye* though practical tasks. Such research could make an important contribution to providing a firmer theoretical basis for formulating new curricular models for geometry and designing new student textbooks.

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