British Society for Research into Learning Mathematics
Geometry Working Group

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DEVELOPING A NEW PEDAGOGY FOR GEOMETRY

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Major improvements in the teaching and learning of geometry will only come, argues a recent report from the Royal Society and Joint Mathematical Council, through the development of a completely new pedagogy for geometry. This report examines existing models of pedagogy for geometry and considers what research might have to contribute to the development of new approaches. New pedagogic approaches for geometry need to give greater emphasis to work in 3-D, incorporate the effective use of computer technology, especially dynamic geometry, and focus on discursive methods of engagement and methods of assessment so that the pressure on pupils is not solely to rote learn.

INTRODUCTION

The recent report on the teaching and learning of geometry by the Royal Society and Joint Mathematical Council (2001) argues that "the most significant contribution to improvements in geometry teaching will be made by the development of good models of pedagogy, supported by carefully designed activities and resources" (p19). The report suggests that "in many respects, we need to develop a completely new pedagogy in geometry" (p11). This paper examines existing models of pedagogy for geometry and considers what research might have to contribute to the development of new pedagogic approaches.

MODELS OF PEDAGOGY FOR GEOMETRY

That the development of effective teaching methods for geometry has been the subject of debate for some considerable time is demonstrated by the numerous inquiries into the teaching and learning of geometry. These inquiries range from those carried out by UK Mathematical Association (for example, in 1923 and 1938) to international studies (for instance, the recent ICMI study overseen by Mammana and Villani, 1998). Indeed, the forerunner, in the UK, of the Mathematical Association was the Association for the Improvement of Geometry Teaching, instigated as long ago as 1871.

The reasons for these inquiries are numerous but revolve around the lack of success in teaching geometry and the difficulties in designing a suitable geometry curriculum for schools (for an extended discussion of these problems, see Jones, 2000a or 2001). A particular problem is that the school geometry curriculum, for so long dominated by proofs in the Euclidean tradition, has been found to be wanting. Despite the efforts of people like Harold Fawcett (1938) in the 1930s, and many others, the general situation, as Howson (2000) attests, has been that “'Euclid-style' geometry [is] found extremely difficult (and often uninteresting) by most [school] students”. Indeed, research studies carried out at a time when proofs in the Euclidean fashion dominated school geometry (such as those by Williams, 1980, and by Senk, 1985) provide evidence across a wide range of schools of how little those pupils who followed such a geometry curriculum could do at the end of their course.

The reasons for this lack of success in teaching geometry, particular when the geometry curriculum is dominated by proofs in the Euclidean tradition, are plentiful. For example, with respect to the teaching of proof the cumulative research evidence suggests that students fail to see a need for proof because all too often they are asked to prove things that are obvious to them (see recent reviews, see Jones and Rodd, 2001, and/or Dreyfus, 1999). Another major problem, as identified by the ICMI study, is that, unlike in number and algebra, “a simple, clear, ‘hierarchical’ path from first beginnings to the more advanced achievements of geometry …. has not yet been found - and perhaps does not exist at all” (Mammana & Villani, 1998, p337). This means that the relations between intuitive, inductive and deductive approaches to geometrical objects, the use of practical experiments, and the age at which geometrical concepts should be introduced, are far from clear.

As the ICMI study details, the main consequence of these problems has been that many countries have tried to bypass the obstacles by cutting down the amount of geometry taught or resorting to pedagogical approaches that rely heavily on memorisation. As a result there is not much in the way of a base of good practice on which to base development. This is why the Royal Society and Joint Mathematical Council (2001) report argues that there is a further problem: “We believe that there are many teachers who have been taught geometry through styles of teaching which we would not advocate as appropriate” (p19). This means that existing teachers have little in the way of their own experience on which to base or develop their practice.

In the next section we look at what might be learnt from one example of an attempt to spread good practice in the past

LEARNING FROM PAST GOOD PRACTICE

A major reform of mathematics teaching in general, and of the teaching of geometry in particular, occurred in the UK, and subsequently more widely, in the years around, and just following, 1901. In 1903, a textbook entitled *Elementary Geometry* and written by Godfrey and Siddons was published. The book was an instant success and is now consider something of a classic text (see, Howson, 1982, or Quadling, 1996).

Although this particular textbook did endeavour to cover much of the Euclidean content, it did so by including a large number of experimental and practical tasks. For example, it included the following, now very familiar and oft used, task,

> Cut out a paper triangle; mark its angles; tear off the corners and fit them together with their vertices at one point. What relation between the angles of a triangle is suggested by this experiment?

Fujita (2001a) summarises the objectives of such tasks as; a) making students familiar with geometrical instruments and figures; b) leading students to discover geometrical facts; c) applying the theorems to practical problems; and d) justifying geometrical facts and theorems through experimental tasks. Fujita's analysis illustrates that the aims of developing both geometrical intuition and what Godfrey and Siddons call the 'geometrical eye' - "the power of seeing geometrical properties detach themselves from a figure" (Godfrey, 1910, p. 197) - were central to Godfrey and Siddons' pedagogy and that these aims remains of crucial importance in developing new pedagogies for geometry.

More recent texts, which maintain the emphasis on practical and experimental tasks but which take a different tack on where and how to introduce theorems and their proofs (and have been criticised for such different approaches) include Michael Serra's *Discovering Geometry* (1997) and the Everyday Learning Corporation's *Connected Geometry* (2000). Since the time of Godfrey and Siddons there have been developments, not only in geometry but also in pedagogic tools and in some of the wider influences on schooling. These new developments are outlined in the next section in an attempt to understand what factors influence the development of new pedagogies for geometry and what research might have to offer.

THE DEVELOPMENT OF NEW PEDAGOGIES

As noted above, the failure of existing pedagogic models for geometry means that across many countries important aspects of geometry (such as work in 3-D) are omitted, there is an over-reliance on teaching methods that rely solely on memorisation, and there is little experience of new pedagogic tools, especially recently developed computer software such as dynamic geometry.

The recent report on the teaching and learning of geometry by the Royal Society and Joint Mathematical Council (2001) argues for further development of the curriculum and of teaching methods especially with respect to work in 3-D and the use of computer technology. As the report observes, while it is simplistic just to note that we live in a 3-D world, there is a great need to be able to "develop the geometrical skills to represent 3-D objects and to solve problems involving them" (p10). The report goes on the say that "clearly 3-D modelling is of great importance in a wide range of disciplines, such as science, engineering and design. We now come into contact with a much wider range of 2-D representations of 3-D objects than was previously the case. Spatial awareness, powers of visualisation and realistic means of applying geometry cannot be developed successfully without paying greater attention to work in 3-D". Examples of 3-D geometry included in a detailed appendix of the report include perspective, regular solids, tetrahedra, spherical triangles, and knots and links. How such topics can be taught effectively and how pupils' experience of such topics can be integrated with their other experiences of geometry remain open questions. A set of very useful examples of integrated approaches to geometry teaching (the examples include integrating various aspects of geometry within a particular theme, integrating geometry with other areas of mathematics such as algebra and handling data, and integrating geometry with other subjects such as science, history and art) is given in another detailed appendix of the report but again research evidence of how successful such approaches might be is sorely needed.

In terms of the use of computer technology, there is some current research that indicates positive ways forward. For example, Mogetta, Olivero and Jones (1999) examine how to design tasks for use with dynamic geometry software which foster the idea of justification in geometry and, in this way, should encourage students to go on to prove their ideas. Research by Jones (2000b) with lower secondary school pupils indicates that when using dynamic geometry software, students' mathematical explanations about geometrical shapes can evolve from imprecise, 'everyday' expressions to mathematical explanations of the geometric situation that transcend the particular tool being used. This latter stage, Jones suggests, should help to provide a foundation on which to build further notions of deductive reasoning in mathematics. Further evidence of the benefits of appropriate use of dynamic geometry software comes from Japan, a country that has attempted to teach a theoretically orientated geometry curriculum to *all* pupils (see Howson, 2000). While recent Japanese research has reported that only about 20% of students are able to solve geometrical proof problems as a result of being taught a curriculum which specifies a considerable amount of work on geometrical proofs, a detailed study by Nomura and Nohda (1999) of students using dynamic geometry software found three major effects: through using such software the students could visualise the geometrical character of a figure more clearly, they had a better understanding of the meaning of the theorem, and were clearer about what they should be proving.

Such research on the use of computer technology, especially dynamic geometry, needs replication and amplification. In particular, research could usefully focus on the nature of the tasks that students are expected to tackle, the form of teacher input during such tasks and the influence of the classroom environment and culture. For teachers in particular, that something works is one thing - further examples of how it can be made to work in the variety of classrooms are crucial.

CONCLUDING COMMENTS

In addition to classroom experiments focusing on pedagogy, research is also needed on the relations between intuitive, inductive and deductive approaches to geometrical objects, the role and impact of practical experiments, and the age at which geometrical concepts should be introduced. A specific issue in teaching geometry is what theorems to include and in what order they should be taught (see Fujita, 2001b, for an analysis of what we might learn about this issue from the developments in textbooks in the early 20th Century). Furthermore, and as Jones and Rodd conclude from their consideration of the teaching of proof in geometry, unless teaching methods can be developed to engage all, there is a real danger of returning to the situation of non-comprehension to which Howson (*ibid*), quoted above, refers.

There is growing evidence of effective teaching practices in general (see, for example, Muijs and Reynolds, 2001) and, to some extent, for mathematics in particular (see, for instance, Stigler and Hiebert, 1999), coupled with a recognition that it not known to what extent effective practices are specific to particular school subjects. Developing new teaching methods for geometry must mean avoiding a repeat of past failures and this will certainly require new pedagogical approaches that are likely to involve the use of a range of resources, as well as discursive methods of engagement and methods of assessment that reduce the pressure to rote learn.

REFERENCES

Dreyfus, T.: 1999, Why Johnny can't prove, *Educational Studies in Mathematics* **38**(1-3), 85-109.

Everyday Learning Corporation: 2000, *Connected Geometry*. Chicago, Ill.: Everyday Learning.

Fawcett, H. P.: 1938, *The Nature of Proof: a description and evaluation of certain procedures used in a senior high school to develop an understanding of the nature of proof*. NCTM Year Book. New York, Columbia University Teachers College.

Fujita, T.: 2001a, The Study of *Elementary Geometry* (1903) by Godfrey and Siddons (1): Roles of Experimental Tasks in the Teaching of Geometry. *Hiroshima Journal of Mathematics Education*, **9**, 11-19.

Fujita, T.: 2001b, The Order of Theorems in the Teaching of Euclidean Geometry: Learning from developments in textbooks in the early 20th Century. *International Reviews on Mathematical Education*, **33**(6), 196-203.

Godfrey, C.: 1910, The Board of Education Circular on the Teaching of Geometry. *Mathematical Gazette*, Vol. V, pp.195-200.

Godfrey, C. and Siddons, A. W.: 1903, *Elementary Geometry: practical and theoretical*. Cambridge, Cambridge University Press.

Howson, A. G.: 1982, *A History of Mathematics Education in England*. Cambridge, Cambridge University Press.

Howson, G. 2000, *Geometry 1950-1970*. Plenary presentation at the International Symposium: One Hundred Years of *L'Enseignement Mathématique*: Moments of Mathematics Education in the 20th Century, Geneva, 20-22 October 2000. To appear in a *Monograph de l'Enseignement Mathématique*.

Jones, K.: 2000a, Critical Issues in the Design of the Geometry Curriculum. In Bill Barton (Ed), *Readings in Mathematics Education*. Auckland: University of Auckland, pp 75-91.

Jones, K.: 2000b, Providing a Foundation for Deductive Reasoning: Students' interpretation when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics*, **44**(1&2), pp 55-85.

Jones, K.: 2001, Issues in the Teaching and Learning of Geometry. In Linda Haggarty (Ed), *Aspects of Teaching Secondary Mathematics: Perspectives on Practice,* London: Routledge.

Jones, K. and Rodd, M. M.: 2001, Geometry and Proof, *Proceedings of the British Society for Research into Learning Mathematics*, **21**(1), 95-100.

Mammana, C. and Villani, V. (Eds.): 1998, *Perspectives on the Teaching of Geometry for the 21st Century: an ICMI study*. Dordrecht, Kluwer.

Mathematical Association: 1923, *Teaching of geometry in schools*, Glasgow University Press, Glasgow.

Mathematical Association: 1938, *A Second Report on the Teaching Geometry in Schools*. London, Bell.

Mogetta, C., Olivero, F. and Jones, K.: 1999, Providing the Motivation to Prove in a Dynamic Geometry Environment. *Proc. of the British Society for Research into Learning Mathematics*, St Martin's University College, Lancaster, 1999, pp91-6.

Muijs, R.D. and Reynolds, D.: 2001, *Effective Teaching: Evidence and practice*. London: Paul Chapman.

Nomura, T. and Nohda, N.: 1999, The Effects of Cabri-geometry for Exploring Geometry in the Classroom. *Proceedings of the Fourth Asian Technology Conference on Mathematics*. Guangzhou, China.

Quadling, D.: 1996, A Century of Textbooks. *Mathematical Gazette.* **487,** 119-126.

Royal Society and Joint Mathematical Council: 2001, *Teaching and Learning Geometry Pre-19,* London, Royal Society/Joint Mathematical Council.

Senk S. L. 1985, How well do students write geometry proofs? *Mathematics Teacher,* **78**(6), 448-456.

Serra, M.: 1997, *Discovering Geometry: an inductive approach*. Emeryville, C., Key Curriculum Press. Second edition.

Stigler, J. W. and Hiebert, J.: 1999, *The Teaching Gap: best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Williams, E.: 1980, An investigation of senior high school students' understanding of the nature of mathematical proof, *Journal for Research in Mathematics Education*, **11**, 165-166.

BSRLM GEOMETRY WORKING GROUP

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions that could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

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