International Journal of Control
Vol. 00, No. 00, Month 200x, 1–9

RESEARCH ARTICLE

Application of Discrete Time Sliding Mode Control to a Spacecraft in 6DoF

N.K. Lincoln[†]and S.M. Veres[†‡]

[†]*School of Engineering Sciences, University of Southampton, UK*

(*Received 00 Month 200x; final version received 00 Month 200x*)

Keywords: discrete sliding mode control; spacecraft control; six degree of freedom; autonomous control

This paper presents the application of two separate discrete time sliding mode controllers, developed in conjunction with a potential function guidance method, to provide control in both position and attitude for a rigid, holonomic spacecraft body. The system is demonstrated in MATLAB^{TM} simulations to illustrate effectiveness under realistic actuator constraints.

1 Introduction

There is a substantial amount of literature within the field of spacecraft control which deals with the separate issues of position and attitude control; comparatively little work has been carried out on the joint nonlinear problem of spacecraft position *and* attitude control. (R.Pongvthithum etal 2005, H.Wong etal 2001, S.M.Veres etal 2002) provide control solutions for spacecraft position control, where the task is to follow a leader spacecraft. (R.Pongvthithum etal 2005) applies the universal adaptive control approach and (H.Wong etal 2001) uses a continuous time Lyapunov stability based approach in conjunction with disturbance estimates to facilitate tracking of a reference trajectory with unknown spacecraft mass. (S.M.Veres etal 2002) explores a discrete time constrained control method for stabilization and control of two nano-satellites using time optimal control but, similarly to (H.Wong etal 2001), is only applied to position control. (O.Hegrenaes etal 2005) uses quadratic programming to solve a constrained, linear model predictive control problem in a discrete time environment. Within the analysis, the author makes use of Euler angles for attitude specification; which although may be more intuitive for visualization, does not offer the robust benefit of the quaternion method implemented within (J.T.Wen and K.Kreutz-Delgado 1991) and (F.Lizarralde and J.T.Wen 1995). (F.Terui 1998) presents a combined sliding mode controller for an individual satellite, controlling both attitude and position in the presence of zero disturbance, without guidance and within a continuous time environment. A related paper (O.Junge etal 2006) presents a scheme for both attitude and position control applied to a group of satellites operating in Lagrange point conditions, using a trajectory optimization process based on minimizing a fuel cost function across the satellite group. Alike (F.Terui 1998), the control scheme is based within a continuous time environment.

Integration of potential function guidance methodologies with continuous time sliding mode control has been proven suitable for distributed control of large scale satellite swarms: this method was initially conceived by (V.Gazi 2003, Gazi 2003) and subsequently expanded within

[‡]Corresponding author. Email: latex.helpdesk@tandf.co.uk

ISSN: 0020-7179 print/ISSN 1366-5820 online

DOI: 10.1080/0020717YYxxxxxxxx
http://www.informaworld.com

(D.Izzo and L.Pettazzi 2007, Izzo and Pettazzi 2005, 2006), wherein the potential function methods were developed to provide a behavioral guidance response; conceptually the behavioral development is similar to that developed for the robotic system presented within (Mataric 1992). The continuous time methods presented within these papers, although applied to scenarios of swarm aggregation to form large structures, do not consider the inherent requirement of combined position and attitude control.

As has been highlighted, the majority of publications deal with continuous time systems. Although this leads to strong theoretical solutions, the issue of implementing a continuous time control system using digital hardware in a real sampled data system is never addressed. There are currently no published, practically implementable and robust solutions to the joint attitude and position control problem of a spacecraft: this paper is intended to address this gap. Physical testing of control methods in the space industry is still in its infancy, though facilities exist where it is possible to test control methods in a 5DoF ground based environment(S.M.Veres etal 2007). The authors' personal laboratory experiments lead to the conclusion that methods published so far are unsatisfactory for practical implementation. In searching for a practically implementable solution, the details for discrete time sliding mode control for the joint position and attitude control of a spacecraft with guidance, has been completed and global stability proven.

This paper presents a complete robust control solution, including guidance, for the joint position and attitude control problem for discrete time implementation. A novel aspect of the guidance methodology implemented is the use of potential functions for both attitude and position guidance. Two discrete time controllers are presented, the second of which does not require knowledge of the coupling parameters between position and attitude and provides robustness against model uncertainty.

2 Dynamics

Within this note the dynamics and kinematics will be based upon a single spacecraft, considered holonomic with respect to control. In addition, the notation will be as follows:

- Bold face capitals are matrices, e.g. $\mathbf{A}$
- Bold face lower case letters are vectors, e.g. $\mathbf{a}$
- Italic lower case letters are scalars, e.g. a
- m is total mass of spacecraft (s/c),
- $\mathbf{J}$ is inertial matrix of s/c
- $\mathbf{d}$ is position of mass center of s/c in body frame, $[d_x, d_y, d_z]^T$
- $\mathbf{v}$ is velocity of mass center of s/c in body frame, $[v_x, v_y, v_z]^T$
- $\mathbf{q}$ is quaternion vector of s/c, $[q_{(1-3)}, q_4]^T$
- ω is angular velocity of s/c in body frame, $[\omega_x, \omega_y, \omega_z]^T$
- $\mathbf{f}$ is total force of thrusters, $[f_x, f_y, f_z]^T$
- τ is total torque of thrusters, $[\tau_x, \tau_y, \tau_z]^T$

A well known (M.J.Sidi 2002) complete continuous time representation of the dynamics and kinematics for a single rigid spacecraft is given by a 13-dimensional state vector $\mathbf{x} = [\mathbf{d}^T, \mathbf{v}^T, \mathbf{q}^T, \boldsymbol{\omega}^T]^T$ as

$$\frac{d}{dt}\begin{bmatrix}\mathbf{d}\\ \mathbf{v}\\ \mathbf{q}\\ \omega\end{bmatrix} = \begin{bmatrix}-\boldsymbol{\Omega}\mathbf{d}+\mathbf{v}\\ -\boldsymbol{\Omega}\mathbf{v}\\ \frac{1}{2}\widetilde{\boldsymbol{\Omega}}\mathbf{q}\\ -\mathbf{J}^{-1}\widetilde{\boldsymbol{\Omega}}\mathbf{J}\omega\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{(3,3)} & \mathbf{0}_{(3,3)}\\ m^{-1}\mathbf{I}_{(3,3)} & \mathbf{0}_{(3,3)}\\ \mathbf{0}_{(4,3)} & \mathbf{0}_{(4,3)}\\ \mathbf{0}_{(3,3)} & \mathbf{J}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{f}\\ \tau\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{(3,3)} & \mathbf{0}_{(3,3)}\\ m^{-1}\mathbf{I}_{(3,3)} & \mathbf{0}_{(3,3)}\\ \mathbf{0}_{(4,3)} & \mathbf{0}_{(4,3)}\\ \mathbf{0}_{(3,3)} & \mathbf{J}^{-1}_{(3,3)}\end{bmatrix}\begin{bmatrix}\mathbf{f}_d\\ \tau_d\end{bmatrix}$$

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, t) + \mathbf{Bu}(t) + \mathbf{Cu}_d(t) \tag{1}$$

where the skew symmetric matrix $\boldsymbol{\Omega}$ represents the angular velocity cross product matrix and

$$\widetilde{\boldsymbol{\Omega}} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

In this notation τ represent the effect of disturbance and residual control torques, a consequence of actuator misalignment. $f(\mathbf{x}, t)$ is a nonlinear function which couples the translational and rotational dynamics and kinematics of the rigid body. $\mathbf{u}$ is the vector of total applied control and $\mathbf{u}_d$ is a vector representing external force and torque disturbances.

2.1 *Time Discretisation*

The nonlinear discrete system which we wish to use is of the form:

$$\mathbf{x}\,(k+1) = f(\mathbf{x}(k), \mathbf{u}(k), \mathbf{u}_d(k))$$

where k is the sampling instance. Whilst time discretisation of this nonlinear model is not possible, time propagation of the given nonlinear system (1) can be approximated by Euler's explicit method as

$$\hat{\mathbf{x}}(k+1) = \mathbf{x}(k) + h{\cdot}\dot{\mathbf{x}}(k) \tag{2}$$

where $\dot{\mathbf{x}}(k) = \frac{d\mathbf{x}}{dt}\mid_{t=kh}$ can be evaluated from the continuous time model (1) based on the current system state which has been extracted at the current discrete time instant kh.

Greater accuracies in time propagation, with minimal increase in computational expense, may be achieved by using higher order methods such as RK4. These numerical methods may be implemented in real time hardware; a critical consideration for a practical system.

3 Guidance and Control

Guidance for the desired spacecraft state will be provided through the use of potential functions in a similar manner to the method in (D.Izzo and L.Pettazzi 2007), but with the added novelty of including potential functions for attitude guidance. Separate potential functions will be used to prescribe desired velocity vectors for position and attitude in the inertial system, $\boldsymbol{\Psi}\mathbf{v}_d$ and $\boldsymbol{\Psi}\omega_d$, respectively, where $\boldsymbol{\Psi}$ is the inertial to body conversion matrix. A feedback controller will be constructed based upon the velocity vector errors:

$$\begin{aligned} \mathbf{x}_e(k) &= \begin{bmatrix} 0_{(3,3)} & I_{(3,3)} & 0_{(3,4)} & 0_{(3,3)} \\ 0_{(3,3)} & 0_{(3,3)} & 0_{(3,4)} & I_{(3,3)} \end{bmatrix} \mathbf{x}(k) - \begin{bmatrix} \boldsymbol{\Psi}\mathbf{v}_d(k) \\ \boldsymbol{\Psi}\omega_d(k) \end{bmatrix} \\ &= \mathbf{Gx}(k) - \boldsymbol{\Psi}'\mathbf{x}_d(k) \end{aligned}$$

which will be used within the sliding mode control regime presented in Section 3.2.

3.1 *Guidance Law- Potential Functions*

Similar to (D.Izzo and L.Pettazzi 2007), the potential function used as position guidance will be constructed as a function of the spacecraft distance from a desired point in space, referred to as a sink location. Complex behaviors can be achieved by forming compound potentials through introducing n attractors towards the n sinks, $\boldsymbol{\epsilon}_j$. In addition, repulsive terms for avoidance constraints can be introduced, though this will not be considered here. A general expression for the potential field gradient for a gathering behavior of spacecraft located at $\mathbf{d}_i$ to sink at location $\boldsymbol{\epsilon}_j$ is

$$\boldsymbol{\lambda}_{ij}^{gather}(k) = \|(\boldsymbol{\epsilon}_j(k) - \mathbf{d}_i(k))\| \cdot (\boldsymbol{\epsilon}_j(k) - \mathbf{d}_i(k))$$

The potential function used as guidance for rotation will be constructed as a function of the spacecraft orientation error from a desired orientation in space, using the quaternion notation (J.B.Kuipers 1999). The error quaternion, $\mathbf{q}^e = [\sin(\alpha_e/2)\mathbf{q}_e^a, \cos(\alpha_e/2)]^T$, is given as

$$\mathbf{q}^e = \mathbf{Q}^d \left[-q_1, -q_2, -q_3, q_4\right]^T \tag{3}$$

where $\mathbf{Q}^d$ is the matrix multiplication of the desired quaternion vector, constructed by using the appropriate components of $\mathbf{q}^d = [\sin(\alpha_d/2)\mathbf{q}_d^a, \cos(\alpha_d/2)]^T = [q_1^d, q_2^d, q_3^d, q_4^d]^T$:

$$\mathbf{Q}^d = \begin{bmatrix} q_4^d & q_3^d & -q_2^d & q_1^d \\ -q_3^d & q_4^d & -q_1^d & q_2^d \\ q_2^d & -q_1^d & q_4^d & q_3^d \\ -q_1^d & -q_2^d & -q_3^d & q_4^d \end{bmatrix}$$

There exists a one-to-one equivalence between the direction cosine matrix elements and the elements of the quaternion vector and so a suitable output potential function for a desired orientation can be represented in the form as defined here.

Definition 3.1: Using the attitude reference $\mathbf{q}_j^d$ as a sink, an attitude guidance vector in terms of $\boldsymbol{\omega}_i$ for spacecraft i is defined as

$$\boldsymbol{\lambda}_{ij}^{orient}(k) = q_4^e(k) \cdot \begin{bmatrix} q_1^e(k) \\ q_2^e(k) \\ q_3^e(k) \end{bmatrix},$$

where $[q_{1_{ij}}^e(k), q_{2_{ij}}^e(k), q_{3_{ij}}^e(k), q_{4_{ij}}^e(k)]^T = \mathbf{Q}_j^d(k)\mathbf{q}_i(k)$.

In a similar manner to the potential function for locations within the cluster, the attitude guidance vector for a required attitude can consist of multiple weighted components of attitude guidance vectors, indexed by j, to result in a more complex behavior. Typically only the acquisition of a specified attitude is considered with a single attitude sink. However, the avoidance of thruster impingement of one spacecraft by another necessitates the use of attitude guidance with multiple weighted components.

The overall position and angular velocity reference for the spacecraft in the presented scenario can be defined as the weighted sum of all partial contributions

$$\mathbf{x}_d(k) = \begin{bmatrix} \mathbf{v}_d(k) \\ \boldsymbol{\omega}_d(k) \end{bmatrix} = \begin{bmatrix} k_v \cdot \boldsymbol{\lambda}_{ij}^{gather}(k) \\ k_a \cdot \boldsymbol{\lambda}_{ij}^{orient}(k) \end{bmatrix} \tag{4}$$

where k_v and k_a represent fixed scalar gains, which are used to scale the gradients of the potential function outputs to keep thruster force requirements within their physical limits.

3.2 *Discrete Time Sliding Mode Control Development*

A combined position *and* attitude controller implies the necessity of two sliding surfaces relating to the separate issues of translational and rotational motion. Using a discrete time notation, appropriate sliding surfaces for spacecraft control in six degrees of freedom in vector form are given as a concatenation of two 3-vectors, $\sigma_1(k)$ and $\sigma_2(k)$, which relate to position and attitude respectively.

$$\begin{aligned} \boldsymbol{\sigma}(k) &= [\boldsymbol{\sigma}_1(k)|\boldsymbol{\sigma}_2(k)]^T \\ &= \mathbf{x}_e(k) \\ &= \mathbf{G}\mathbf{x}(k) - \boldsymbol{\Psi}'\mathbf{x}_d(k) \end{aligned} \tag{5}$$

Numerous reaching conditions have been presented for quantifying what represents system motion for 'sliding modes in discrete time' and indeed this is a highly contentious issue (A.J.Koshkouei and A.S.I.Zinober 2000, V.Utkin 1998, B.Bandyopadhyay and S.Janardhanan 2006, W.Gao etal 1995). Whilst there is no absolute agreement, the notion of a system exhibiting sliding motion on a scalar surface $s(k)$, implicity requires that at the very least $|s(k+1)| < |s(k)|$ with the desire to achieve $s(k+1) = 0$ as given within(A.J.Koshkouei and A.S.I.Zinober 2000, V.Utkin 1998, B.Bandyopadhyay and S.Janardhanan 2006). Whilst in (W.Gao etal 1995) it is stated that for a system to be in discrete sliding motion the $s(k) = 0$ boundary must be crossed and crossed infinitely often thereafter, for the remainder of this paper it is not assumed that this is a requirement. Indeed, within all applications of sliding mode control, methods are actively sought to eliminate the chattering associated with $s(k) = 0$ boundary crossing, as opposed to seeking to achieve this instance.

Extending the requirements of discrete time sliding motion to exhibit $s(k+1) = 0$ or $|s(k+1)| < |s(k)|$ to the vector sliding surface $\boldsymbol{\sigma}$, given in (5), entails the satisfaction of

$$\boldsymbol{\sigma}(k+1) = [\mathbf{G}\hat{\mathbf{x}}(k+1) - \boldsymbol{\Psi}' \cdot \mathbf{x}_d(k+1)] = 0 \tag{6}$$

or

$$\|\boldsymbol{\sigma}(k+1)\| \; < \; \|\boldsymbol{\sigma}(k)\| \tag{7}$$

For the presented scenario, $\mathbf{x}_d$ is a constant hence $\mathbf{x}_d(k+n) = \mathbf{x}_d(k)$,$\forall n \in \Re$, and the satisfaction of (6) reduces to attaining $\mathbf{G}\hat{\mathbf{x}}(k+1) = \boldsymbol{\Psi}\cdot\mathbf{x}_d$. Using (1) and (2) this requirement expands to

$$\mathbf{G}\mathbf{x}(k) + h\left[\mathbf{G}f(\mathbf{x},k) + \mathbf{G}\mathbf{B}\mathbf{u}(k) + \mathbf{G}\mathbf{C}\mathbf{u}_d(k)\right] = \boldsymbol{\Psi}'\mathbf{x}_d(k) \tag{8}$$

from which the desired control at the time instant k can be formed as

$$\mathbf{u}(k) = -(\mathbf{G}\mathbf{B})^{-1}[\boldsymbol{\sigma}(k)h^{-1} + \mathbf{G}f(\mathbf{x},k) + \mathbf{G}\mathbf{C}\mathbf{u_d}(k)] \tag{9}$$

Where $(\mathbf{GB})$ is an invertible $(6,6)$ matrix. This function will tend towards negative infinity as $h \to 0$, due to the $-(h\mathbf{GB})^{-1}\boldsymbol{\sigma}(k)$ term. However, $-(\mathbf{GB})^{-1}[\mathbf{G}f(k) + \mathbf{GC}\mathbf{u_d}(k)]$ takes finite values. This implies that the bounds for control should be taken into account.

Definition 3.2: Assume that the control signal available may vary within the domain $\|\mathbf{u}(k)\| \le u_B$, where u_B is a scalar constant representing the maximum norm of the control output. We will say that the *sliding mode controllability* criterion is satisfied if

$$u_B \ge \|(\mathbf{GB})^{-1}(\mathbf{G}f(\mathbf{x},k) + \mathbf{GC}\mathbf{u}_d(k))\| + \frac{\delta}{h} \tag{10}$$

holds, where $\delta > 0$.

Proposition 3.3: *Assume that (10) holds. Define a controller by:*

$$\mathbf{u}(k) = \begin{cases} \mathbf{u}(k) & \text{if } \|\mathbf{u}(k)\| \leq u_B \\ u_B \frac{\mathbf{u}(k)}{\|\mathbf{u}(k)\|} & \text{if } \|\mathbf{u}(k)\| > u_B \end{cases} \tag{11}$$

Then, in a domain around the sliding surface, (11) provides a stabilizing sliding mode controller to achieve the sliding mode (7).

Proof Considering the situation in which $\|\mathbf{u}(k)\| > u_B$, we have:

$$\boldsymbol{\sigma}(k+1) = \left[\mathbf{G}\mathbf{x}(k) + h\mathbf{G}\dot{\mathbf{x}}(k) - \boldsymbol{\Psi}'\mathbf{x}_d\right]$$

which can also be represented as

$$\boldsymbol{\sigma}(k+1) = \boldsymbol{\sigma}(k) + h\left(\mathbf{G}f(\mathbf{x},k) + \mathbf{G}\mathbf{B}\mathbf{u}(k) + \mathbf{G}\mathbf{C}\mathbf{u}_d(k)\right)$$

hence

$$\boldsymbol{\sigma}(k+1) = \left[\boldsymbol{\sigma}(k) + h\left(\mathbf{G}f(\mathbf{x},k) + \mathbf{G}\mathbf{C}\mathbf{u}_d(k)\right)\right] \cdot \left[1 - \frac{u_B}{\|\mathbf{u}(k)\|}\right]$$

Using (9) and (10) it follows that

$$\|\boldsymbol{\sigma}(k+1)\| \leq \|\boldsymbol{\sigma}(k)\| - \delta$$

therefore $\boldsymbol{\sigma}(k)$ decreases and after a finite number of steps, $\mathbf{u}(k)$ will be within the admissible domain of $\|\mathbf{u}(k)\| < u_B$. □

3.3 *Sliding Mode Control With Unknown Coupling*

The sliding mode controller presented in the previous section assumes the use of the dynamic coupling terms between translational and rotational movements, namely $f(\mathbf{x},k)$. An alternative method, based upon the existing sliding surfaces $\boldsymbol{\sigma}(k)$, in which $f(\mathbf{x},k)$ is not assumed to be known, shall now be presented.

Proposition 3.4: *By implementing the control law:*

$$u(k) = \begin{cases} -(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k) & \text{if } \|(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\| \leq u_B \\ -u_B \frac{(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)}{\|(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\|} & \text{if } \|(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\| > u_B \end{cases} \tag{12}$$

sliding motion will be achieved.

Proof considering the case in which $\|(h\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\| > u_B$, we have:

$$\begin{aligned}\boldsymbol{\sigma}(k+1) &= \boldsymbol{\sigma}(k) + h\mathbf{G}f(\mathbf{x},k) + h\mathbf{G}\mathbf{B}\left(\frac{-\mathbf{u}_B(\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)}{\|(\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\|}\right) \\ &\quad + h\mathbf{G}\mathbf{C}\mathbf{u}_d \\ &= \boldsymbol{\sigma}(k)\left(1 - \frac{h\mathbf{u}_B}{\|(\mathbf{G}\mathbf{B})^{-1}\boldsymbol{\sigma}(k)\|}\right) \\ &\quad + h\left(\mathbf{G}f(x,k) + \mathbf{G}\mathbf{C}\mathbf{u}_d\right)\end{aligned}$$

which, using (10) and (12), reduces to

$$\|\boldsymbol{\sigma}(k+1)\| \leq \|\boldsymbol{\sigma}(k)\| - \delta$$

hence, in a comparative manner to the discrete time controller developed with Section 3.2, $\boldsymbol{\sigma}(k)$ decreases to the point where the control demand enters the admissible domain and $\boldsymbol{\sigma}(k)$ will continue to decrease thereafter; discrete time sliding motion is achieved without knowledge of $f(\mathbf{x}, k)$. □

Although the control law presented within (12) does not use knowledge of the $f(\mathbf{x}, k)$ dynamic coupling, it is assumed that the model parameters encapsulated within the $\mathbf{B}$ matrix are known. These internal model parameters can be estimated and updated during control in order to refine the control process, thus implementing an adaptive control element within the scheme, as completed within (V.Utkin 1998) and (N.K.Lincoln and S.M.Veres 2006).

4 Simulation

Within the simulations, actuator thrust and torque limitations of 0.2N and 0.01Nm were enforced for a spacecraft of mass 10kg. The k_v and k_a constants were both set to an empirically determined value of 0.1 for the control scheme with knowledge of the coupling, whilst values of 0.02 and 1.1 were used for scheme (12). For the presented simulation output, a time step of 0.1 seconds was used with the desire to achieve a translation of $[5, 2, -6]$ meters and Euler rotations of $[10, 15, -12]$ degrees about the body axes. Note that although Euler rotations have been mentioned, this was primarily for the benefit of the reader; these Euler rotations were translated into a quaternion rotation requirement and throughout all simulations, the quaternion notation was maintained.

Simulation output is presented within Figures 1 to 4. Within Figures 1 and 2, a time evolution of the Euclidean norm for spacecraft position, velocity and angular velocity errors, relating to the action of controllers (9) and (12) respectively, is provided. Figure 3 shows the time response of the sliding surfaces of the spacecraft response using both controllers. Figure 4 shows the respective control action provided by both controllers over the same time period.

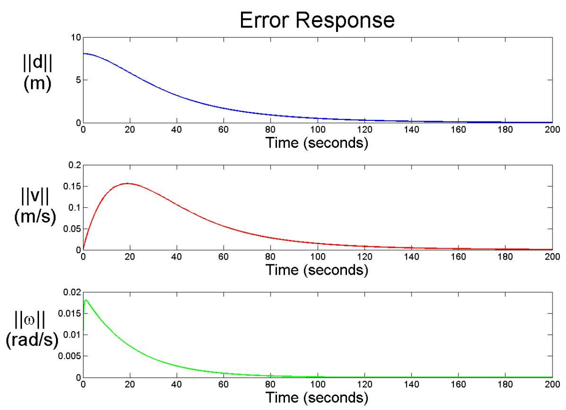


Figure 1. Euclidean norm of position, velocity and angular velocity error response for controller with parameter knowledge, (9), $h = 0.1$s

5 Conclusions

Two joined discrete time sliding mode controllers have been developed for the purpose of controlling a spacecraft in six degrees of freedom and applied to a simulation environment. As is evident from Figures 1 and 2, both controllers achieve regulation to a specified state; however, the manner in which this is achieved differs for the controllers.

Whilst not strikingly evident from Figures 1 to 3, inspection of Figure 4 reveals a difference in actuator usage: the controller with unknown parameters is more conservative with regards to control action. Within the current implementation, where reasonably strict actuator bounds are enforced, the difference in actuator usage is almost negligible. It has however been observed within simulation that with greater permissable actuator forces, this difference increases

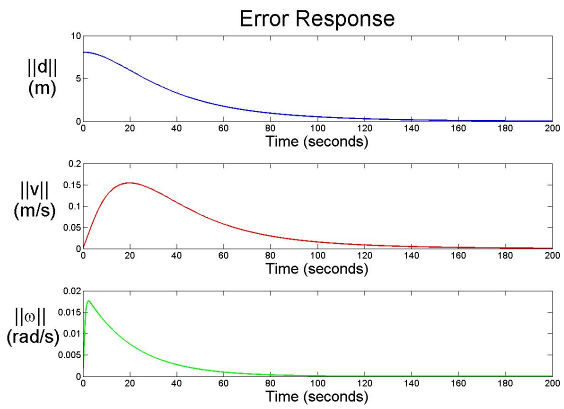
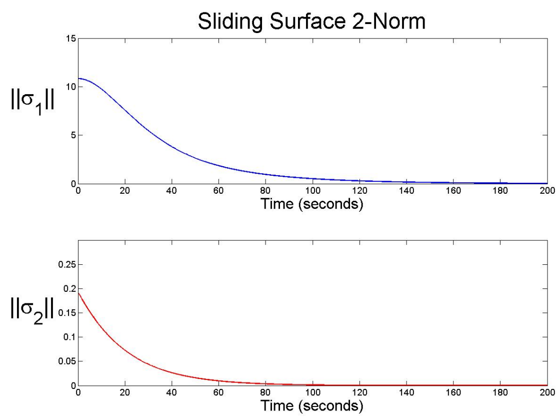
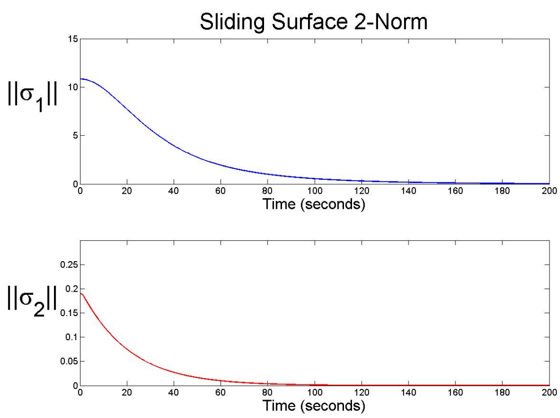


Figure 2. Euclidean norm of position, velocity and angular velocity error response for controller without parameter knowledge, (12), $h = 0.1$s



(a) Known parameters, controller (9)



(b) Unknown parameters, controller (12)

Figure 3. Euclidean norm sliding surface evolution for controllers with and without parameter knowledge, $h = 0.1$s

markedly.

Whilst both controllers require knowledge of the the spacecraft inertial matrix and mass parameters, encapsulated within the $\mathbf{B}$ matrix of (1), controller (12) works without knowledge of the non-linear coupling within $f(\mathbf{x}, t)$ of (1) and as such could be considered superior to the model predictive form presented within (9).

6 Future Work

Both discrete time controllers assume knowledge of the spacecraft mass and inertial matrix. In many cases, accurate knowledge of these parameters is not possible; indeed in virtually all spacecraft applications these parameters are time varying as propellent is consumed. Future work is to involve introducing an adaptive regime to the controller thus enabling continual refinement of internal parameters and expanding the discrete control method to a group of spacecraft to achieve a multi-agent approach to spacecraft formation flying.

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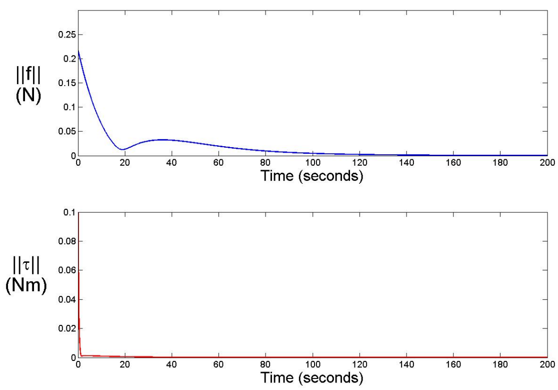
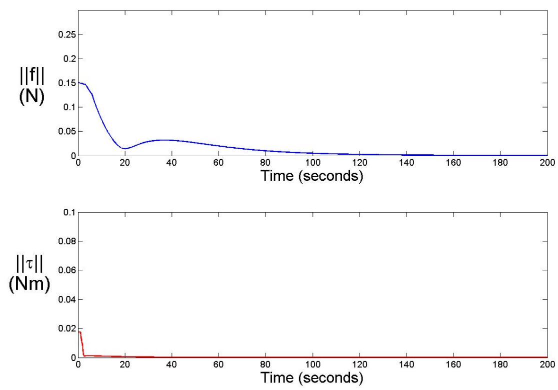


(a) Known parameters, controller (9)



(b) Unknown parameters, controller (12)

Figure 4. Euclidean norm of control action (force and torque) evolution for controllers with and without parameter knowledge, $h = 0.1$s

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