VERTICAL PROFILES OF WIND-INDUCED CURRENT

BY
N.S. HEAPS & J.E. JONES

REPORT NO. 238
1987
When citing this document in a bibliography the reference should be given as follows:—

Vertical profiles of wind-induced current

by

*N.S. Heaps and J.E. Jones

I.O.S. Report No. 238

March 1987

*Dr Norman S Heaps died on 26 June 1986
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2. Ekman theory</td>
<td>7</td>
</tr>
<tr>
<td>3. Vertical profiles</td>
<td>11</td>
</tr>
<tr>
<td>4. Frictional parameters</td>
<td>12</td>
</tr>
<tr>
<td>5. North Sea currents</td>
<td>13</td>
</tr>
<tr>
<td>6. Surface and bottom logarithmic boundary layers</td>
<td>15</td>
</tr>
<tr>
<td>7. Advice</td>
<td>18</td>
</tr>
<tr>
<td>References</td>
<td>19</td>
</tr>
<tr>
<td>Tables 1-9</td>
<td>20</td>
</tr>
<tr>
<td>Figures 1-8</td>
<td>29</td>
</tr>
</tbody>
</table>
ABSTRACT

Ekman-type current profiles are constructed for five positions in the North Sea and for a range of wind speeds between 5 and 40 ms\(^{-1}\). Their application to determine the vertical distribution of extreme surge current is described. The extent to which logarithmic surface and bottom boundary layers might influence the calculated profiles is investigated.
1. **Introduction**

The purpose of the present work is to provide some simple profiles of wind-induced current to enable designers of offshore structures in the North Sea to assess the likely variations of surge current through the vertical in extreme storm conditions. It is assumed that extreme values of depth-averaged surge current are available from the continental shelf model (Flather 1984).

The simplifying assumptions associated with Ekman's theory are made. In particular, vertical eddy viscosity is assumed to be a constant and its value is expressed in terms of wind speed by empirical formulae of generally uncertain validity. However, those formulae are probably among the best presently known. Allowance is made for the influence of the tides on the eddy viscosity, mean spring conditions being assumed. The profiles apply to steady-state flow and therefore dynamical effects which might occur in rapidly changing meteorological circumstances are excluded.

The influence of logarithmic surface and bottom boundary layers on the Ekman profiles is investigated. It turns out that the layers are relatively thin (of thickness less than one metre) for wind speeds greater than 15 ms\(^{-1}\). However, the uncertainty concerning the nature of the surface conditions is very considerable at the present time. More advanced theory is needed to link the surface and bottom boundary layer effects to the motion of the interior fluid in a unified rotating dynamical system. Regrettably, the parameters of such a unified system would again possess a significant degree of uncertainty.

It must be emphasised that the outlook of this paper is a practical one, aimed at providing some specific current profiles for use in engineering design. Full details of the derivation of the profiles are given in order to expose the various assumptions which have been made. The approach is a simple one by intention; more sophisticated analyses have, for the time being, been deferred. In any case, with the fundamental uncertainty about turbulence conditions through the vertical water column in storm conditions it seems unlikely that the use of more complicated mathematical formulations would at this stage lead to any more reliable results.
This work was carried out during the current revision of the 'Meteorological and Oceanographic design parameters' section of the Department of Energy publication 'Offshore installations: guidance on design and construction' and when the associated support document was being written. Alternative approaches to estimating current profiles ranging from power and logarithmic laws to three-dimensional modelling are contained within both documents.
2. **Ekman theory**

Let \( x, y, z \) denote Cartesian coordinates forming a left-handed set in which \( x, y \) are measured in the horizontal plane of the sea surface and \( z \) is depth below that surface (figure 1),

\( u, v \) the components of horizontal current at depth \( z \) in the directions of increasing \( x, y \),

\( \beta \) the geostrophic coefficient (equal to \( 2\omega \sin \lambda \) where \( \omega \) denotes the angular speed of the Earth's rotation and \( \lambda \) the latitude), regarded as a constant,

\( \mu \) the coefficient of vertical eddy viscosity, also regarded as a constant.

Then, after Ekman (1905), wind-induced currents through the vertical water column from sea surface to sea bed satisfy the dynamical equations

\[
\mu \frac{d^2 u}{dz^2} = -\beta v, \quad \mu \frac{d^2 v}{dz^2} = \beta u .
\]

These are derived assuming a homogeneous sea, horizontal uniformity, and a steady wind stress. They may be combined to give the well-known equation

\[
\frac{d^2 \omega}{dz^2} = \frac{\pi (1+i)}{\beta} \omega
\]

where horizontal current is expressed in the complex form:

\[
\omega = u + iv
\]

and

\[
D = \pi \left( \frac{\beta u}{\omega} \right)^{1/2}.
\]

Supposing that the wind stress is of magnitude \( T \) and acts in the \( y \)-direction, the surface stress condition is

\[
-\mu \frac{d \omega}{dz} = iT \quad \text{at} \quad z = 0 .
\]
At the sea bed, adopting a linear law of bottom friction, with a constant coefficient $k$, we have

$$- \rho \mu \frac{dw}{dz} = k \rho w \quad \text{at} \quad z = h. \quad (6)$$

Here $\rho$ denotes the density of the sea water and $h$ the total depth.

Solving (2), subject to (5) and (6), yields

$$w = \frac{\pi T}{gSD} \left[ \frac{a \alpha (1+i) \sinh \{ (1+i) \alpha \gamma \} + 2i \cosh \{ (1+i) \alpha \gamma \}}{a \cosh \{ (1+i) \alpha \} + (1+i) \sinh \{ (1+i) \alpha \gamma \}} \right] \quad (7)$$

where:

$$a = \frac{\pi D}{D}, \quad \alpha = \frac{2k}{\rho g} \quad \gamma = 1 - \frac{\rho h}{\rho g}. \quad (8)$$

Then, taking real and imaginary parts in (7), produces the horizontal components of current in the form:

$$u = \frac{\beta \gamma}{d}, \quad v = \frac{\beta \rho}{d} \quad (9)$$

in which

$$\beta = \frac{\pi T}{gSD} \quad (10)$$

$$d = a^2 \left( \cos^2 \alpha + \sin^2 \alpha \right)$$

$$+ 2a \left( \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \right)$$

$$+ 2 \left( \sin \alpha \cos \alpha + \cos \alpha \sin \alpha \right) \quad (11)$$
\[ p = \frac{k^2\pi}{a^2} \left\{ \sinh a \cos a \left( \cosh a + \sinh a \right) \\
- \cosh a \sinh a \left( \cosh a - \sinh a \right) \right\} \\
+ 2k \left\{ \sinh a \cos a \sinh a \cosh a + \cosh a \sinh a \cosh a \sinh a \\
- \sinh a \cosh a \cosh a \right\} \\
+ 2 \left\{ \cosh a \cosh a \left( \sinh a + \cosh a \sinh a \right) \\
- \sinh a \cosh a \left( \cosh a - \sinh a \cosh a \right) \right\} \\
\] (12)

and

\[ q = \frac{k^2\pi}{a^2} \left\{ \sinh a \cos a \left( \cosh a - \sinh a \right) \\
+ \cosh a \sinh a \left( \cosh a + \sinh a \right) \right\} \\
+ 2k \left\{ - \sinh a \cos a \cosh a \sinh a + \cosh a \sinh a \sinh a \cosh a \\
+ \cosh a \cos a \cosh a \cosh a + \sinh a \cosh a \sinh a \sinh a \right\} \\
+ 2 \left\{ \cosh a \cosh a \left( \sinh a + \cosh a \sinh a \right) \\
+ \sinh a \cosh a \left( \cosh a - \sinh a \cosh a \right) \right\} . \] (13)

Using (7), the depth-mean current comes out to

\[ \bar{v} = \bar{u} + i \bar{v} = \frac{1}{k} \int_0^h \omega dz \]

\[ = \pi T \frac{1}{SSD a} \left[ \frac{ak \left( \cosh \left\{ (1+i)a \right\} - 1 \right) + (1+i) \sinh \left\{ (1+i)a \right\}}{ak \cosh \left\{ (1+i)a \right\} + (1+i) \sinh \left\{ (1+i)a \right\}} \right] . \] (14)
Taking real and imaginary parts in (14) leads to the depth-mean components of current expressed in the form:

\[
\bar{u} = \frac{\beta \rho_1}{ad}, \quad \bar{v} = \frac{\beta \eta_1}{ad}
\]  

(15)

in which

\[
\rho_1 = a^2 \left( \cosh a \cos a + \sinh a \sin a - \cosh a \cos a \right) \\
+ ax \left( 2 \sinh a \cosh a - 2 \sinh a \cos a - \sinh a \cos a + \cosh a \sin a \right) \\
+ 2 \left( \sinh a \cos a + \cosh a \sin a \right)
\]

(16)

and

\[
\eta_1 = a^2 \sinh a \sin a + ax \left( \cosh a \sin a + \sinh a \cos a \right)
\]

(17)
3. **Vertical profiles**

At any location, equations (9) and (15) may be used to calculate the vertical profiles of

\[ u, \sigma \]  \hspace{1cm} (18)

and their departures from the mean:

\[ u - \bar{u}, \sigma - \bar{\sigma} \]  \hspace{1cm} (19)

Note that \( \sigma \) is the current component in the direction of the wind, with \( u \) to the right of it (figure 1).

Profiles may also be determined for the magnitude of the resultant current at any depth

\[ V = \left( u^2 + \sigma^2 \right)^{1/2} \]  \hspace{1cm} (20)

and the magnitude of the departure of this resultant from the depth-mean current:

\[ V_\delta = \left[ \left( u - \bar{u} \right)^2 + \left( \sigma - \bar{\sigma} \right)^2 \right]^{1/2} \]  \hspace{1cm} (21)

Wind stress in the calculations may be determined from a square law:

\[ T = C_D \frac{\varphi_A}{\rho} W^2 \]  \hspace{1cm} (22)

where \( W \) denotes the wind speed, \( \varphi_A \) the density of the air and \( C_D \) a drag coefficient. Then \( \beta \) from (10) may be evaluated using

\[ \beta = \frac{\pi C_D \varphi_A}{\varphi \delta D} W^2 \]  \hspace{1cm} (23)
4. Frictional parameters

Using a result due to Svensson (1979), and putting together arguments given by Csanady (1976) and Heaps (1984), for wind-driven flow we take

\[
\mu = \mu_w = \begin{cases} 
0.065 \, u_\star & \text{if } \lambda \leq \lambda_E \\
0.026 \, \frac{u_\star}{\sigma} & \text{if } \lambda > \lambda_E 
\end{cases}
\]

(24)

where

\[
u_\star = \left( \frac{T}{\pi} \right)^{1/4} = \left( \frac{c_0 f_a / \sigma}{W} \right)^{1/2}
\]

(25)

and

\[
\lambda_E = 0.4 \, \frac{u_\star}{\sigma} .
\]

(26)

For tidal flow we take

\[
\mu = \mu_T = \frac{1}{2} \, \kappa \left( \beta_T^2 + \epsilon_T^2 \right)
\]

(27)

where \( \alpha_T, \beta_T \) denote, respectively, the semi-major and semi-minor axes of the tidal current ellipse and \( \kappa = 0.2 \, s^{-1} \). Equation (27) is obtained by averaging over a tidal cycle the form for \( \mu \) used by Davies and Furnes (1980) when computing \( M_2 \) tidal currents in the North Sea, namely the form \( \mu = \kappa \left( \bar{u}_+^2 + \bar{v}_+^2 \right) \).

For a combined wind-driven and tidal flow it is tentatively assumed that

\[
\mu = \max \left( \mu_w, \mu_T \right)
\]

(28)

where \( \max \) indicates that the maximum of \( \mu_w \) and \( \mu_T \) is taken.

Setting \( k \mu_0 / \mu = 2 \) as in Heaps (1974), \( k \) is determined from

\[
k = \frac{2 \mu_0}{\mu}
\]

(29)
5. **North Sea currents**

Calculations of the vertical profiles associated with wind-induced currents were done for five positions in the North Sea: \(N_1, N_2, N_3, N_4, N_5\) marked in figure 2. The depths at these positions vary between 29m at \(N_1\), in the south, to 177m at \(N_5\), in the north. Table 1 lists the geographical coordinates, depth, and tidal parameters \(a_T, c_T\) and \(\mu_T\) at mean springs, \(M_2+S_2\). The values of \(\mu_T\) come from (27).

For the calculation of \(\mu_w\) at the various positions we took
\[
\begin{align*}
\rho &= 0.00125 \text{ gcm}^{-3} \\
\rho' &= 1.027 \text{ gcm}^{-3} \\
\sigma &= 1.18 \times 10^{-4} \text{ s}^{-1}
\end{align*}
\]
and, from Smith and Banke (1975),
\[
\sigma' = 10^{-3} (0.63 + 0.066 W)
\]
where the wind speed \(W\) is measured in \(\text{ms}^{-1}\). The geostrophic coefficient, \(\rho'\), corresponds to \(\lambda = 54^\circ\), a representative latitude for British waters. Then, from (25) and (26):
\[
\begin{align*}
\mu_\ast &= 0.11032 W (0.63 + 0.066 W)^{1/2} \\
\chi_e &= 33.9 \mu_\ast
\end{align*}
\]
where \(\mu_\ast\) is in \(\text{cms}^{-1}\) and \(\chi_e\) in metres. For the range of wind speeds:
\[
W = \begin{cases} 5 & (5) \ 40 \ \text{ms}^{-1} \\
\end{cases}
\]
the \(\mu_\ast\) and \(\chi_e\) are listed in Table 2. The \(\mu_w\) calculated using (24) are presented in Table 3 for the various positions and wind speeds. The corresponding \(\mu\) values, derived from (28) with \(\mu_T\) for mean springs given by Table 1, are presented in Table 4. Note that the tidal contribution to \(\mu\) is only important at \(N_1\) and to a lesser extent at \(N_3\). Values of \(\chi\), obtained using (29), are presented in Table 5.

Employing the preceding values in (9) and (15), the vertical profiles of
(a) \( u, \ \sigma, \ \sqrt{\sigma} = \left( u^2 + \sigma^2 \right)^{1/2} \)

(b) \( u - \bar{u} \)

(c) \( \sigma - \bar{\sigma} \)

(d) \[ V_s = \left[ (u - \bar{u})^2 + (\sigma - \bar{\sigma})^2 \right]^{1/2} \]

are plotted for each position \( N_1 \) to \( N_5 \) in figures 3 to 7 respectively. Each diagram shows curves for the range of wind speeds \( W = 5 \ (5) \ 40 \text{ ms}^{-1} \).
6. **Surface and bottom logarithmic boundary layers**

It seems important to estimate the thicknesses of the logarithmic boundary layers assumed to exist adjacent to the sea surface and sea bed. Within those layers, the profiles shown in figures 3 to 7 based on Ekman's theory will not apply.

The vertical distribution of horizontal current in a non-rotating surface logarithmic boundary layer (Bowden 1983, p. 134) may be written

\[
\begin{align*}
\mathbf{u} &= \mathbf{0} \\
\mathbf{r} &= \kappa \mathbf{w} - (u_*/\kappa_o) \log (z/z_o)
\end{align*}
\] (35)

where \( \kappa \) is the wind factor, \( z_o \) the roughness length, and \( \kappa_o \) Von Karman's constant. Note that the currents lie in the wind direction. The following values are taken:

\[
\begin{align*}
\kappa &= 0.03 \\
z_o &= 0.001 \text{ m} \\
\kappa_o &= 0.40
\end{align*}
\] (36)

Figure 8(a) illustrates how the logarithmic curve reduces from a surface value of \( V_s' \) ( \( \mathbf{r} = \kappa \mathbf{w} = V_s' \) at \( z = z_o \)) and cuts the Ekman profile \( V \) where \( z = z_e \), \( V = V_a \). This assumes that \( V_s > V_s \), where \( V = V_s \) at \( z = 0 \) on the Ekman profile.

Table 6 gives the values of \( V_s \), \( V_s' \), \( V_a \) and \( z_a \) determined (numerically) for the positions \( N_1 \) to \( N_5 \) and the various wind speeds from 10 to 40 ms\(^{-1}\). Gaps are left in the Table where \( V_s' < V_s \). It is evident that the thickness of the boundary layer, taken as \( z_a \), does not exceed 2.31m - its value at \( N_1 \) for \( \mathbf{w} = 10 \text{ ms}^{-1} \). For \( \mathbf{w} \geq 15 \text{ ms}^{-1} \), the thickness is less than 0.5m. Thus, for the higher wind speeds, the logarithmic surface boundary layer scarcely affects the Ekman profiles presented in this paper.

However, Csanady (1984) has suggested that \( z_o \) may be one to two orders of magnitude greater than the value allocated to it in (36). Taking \( z_o = 0.05 \text{ m} \) to exemplify such a possibility leads to the values of \( V_s \), \( V_s' \), \( V_a \) and \( z_a \) in Table 7. Considerably larger values of \( z_a \) now appear; in fact the surface boundary layer can be quite thick. For \( \mathbf{w} = 10, 15 \text{ ms}^{-1} \) the Ekman and
logarithmic curves do not even intersect, indicating that the logarithmic variation might affect the entire current profile from sea surface to sea bed. The uncertainty in \( \zeta \) at the sea surface, caused by the recent proposal by Csanady of a greatly enhanced value, clearly introduces an important uncertainty into the validity of the Ekman profiles of this paper.

For the logarithmic bottom boundary layer, we have

\[
V = \frac{u^*}{k_o} \log \left( \frac{Z}{Z_o} \right)
\]

where \( Z \) is distance measured vertically upwards from the bottom, \( Z_o \) a bottom roughness length, and

\[
u^* = \left( \frac{\tau_v}{k} \right)^{1/2}.
\]

Here, \( \tau_v \) denotes the bottom stress given in the present analysis by

\[
\tau_v = \frac{k}{k} \left( \frac{\sigma^*}{v^*} \right)^{1/2}
\]

where \( \sigma^* \) and \( v^* \) denote the values of \( u, \sigma \) at the sea bed. Combining (37), (38) and (39) yields

\[
V = \frac{k^{1/2}}{k_o} \left( \frac{\sigma^* + \sigma_v^*}{v^*} \right)^{1/4} \log \left( \frac{Z}{Z_o} \right).
\]

Using \( Z_o \) given in Table 8, estimated by R.L. Soulsby (private communication), the logarithmic curve (40) has been evaluated and its intersection point with the Ekman profile at \( Z = Z_e \), \( V = V_e \) determined. Figure 8(b) illustrates this behaviour; \( V = V_e \) at the sea bottom on the Ekman profile. Table 9 gives \( V_e, V_c, Z_e \) for the various positions \( N_1 \) to \( N_5 \) and the various wind speeds between 10 and 40 ms\(^{-1} \). It is evident that the thickness of the bottom boundary layer, taken as \( Z_e \), is everywhere less than about 0.1m and therefore the layer
has only a small perturbing influence on the Ekman profiles.

The dynamical inconsistency in the above discussion between the non-rotating logarithmic boundary layers and the rotating Ekman flow regime is recognised. However, the two concepts are regularly used and are retained together here because of that.

The roughness lengths $z_o$, $Z_o$ are clearly key parameters in determining how far the boundary influences penetrate vertically into the main body of the sea. The bottom roughness length $Z_o$ has been related to various types of sea-bed sedimentation (Soulsby 1983) and the values in Table 8 were derived on this basis. Estimation of the surface roughness length is much more uncertain. The value taken in (36) is consistent with the arguments given by Madsen (1977) and the laboratory experiments of Wu (1975).

Taking $\alpha = 0.03$ in (36) indicates that the surface current is assumed to be $3\%$ of the surface wind speed, a generally applicable result (Pearce and Cooper 1981).
7. **Advice**

The use of the wind-induced current profiles in figures 3 - 7 for the estimation of the distribution of extreme surge current through depth is now outlined.

A known extreme depth-integrated surge current may be incremented through depth by

\[
\mathbf{V} = \left( \mathbf{\bar{u}}^2 + \mathbf{\bar{v}}^2 \right)^{\frac{1}{2}}
\]

to yield estimates of the corresponding surge current extremes at various depths. The wind speed \( \mathbf{W} \) should be selected as the maximum expected value based on meteorological records. Here, \( \mathbf{V} \), \( \mathbf{\bar{u}} \), \( \mathbf{\bar{v}} \) come from figures 3(a) - 7(a).

Incrementing by \( V_s \) is also possible, tending however to yield overestimates of the extremes. The \( V_s \) comes from figures 3(d) - 7(d).

Known components of depth-integrated surge current may be incremented by \( u-\bar{u} \), \( v-\bar{v} \) (given by figures 3(b,c) - 7(b,c)) to yield estimates of the corresponding depth variations. Wind direction as well as magnitude is needed here to resolve the prescribed components in the wind direction and at right angles to it before incrementation. If the wind direction associated with the surge components is unknown, then a range of profiles relating to different wind directions could be evaluated.

Logarithmic boundary layers may exist at the sea surface and sea bed as explained in §6. For the higher wind speeds those layers are likely to be very thin - less than a metre thick for wind speeds greater than 15 ms\(^{-1}\). On these grounds, the Ekman current profiles may be considered to apply through most of the vertical water column in storm conditions. However, with uncertainties about the thickness of the surface layer, this has to be regarded as a tentative conclusion.
References


<table>
<thead>
<tr>
<th>Position</th>
<th>Coordinates</th>
<th>$h$ (m)</th>
<th>$a_T$ (cms$^{-1}$)</th>
<th>$\varphi_T$ (cms$^{-1}$)</th>
<th>$\mu_T$ (cm$^2$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$_1$</td>
<td>53° 10'N, 2° 15'E</td>
<td>29</td>
<td>86</td>
<td>17</td>
<td>768</td>
</tr>
<tr>
<td>N$_2$</td>
<td>56° 30'N, 2° 45'E</td>
<td>77</td>
<td>24</td>
<td>2</td>
<td>58</td>
</tr>
<tr>
<td>N$_3$</td>
<td>57° 50'N, 1° 15'E</td>
<td>97</td>
<td>32</td>
<td>5</td>
<td>105</td>
</tr>
<tr>
<td>N$_4$</td>
<td>60° 10'N, 2° 15'E</td>
<td>143</td>
<td>16</td>
<td>6</td>
<td>29</td>
</tr>
<tr>
<td>N$_5$</td>
<td>61° 30'N, 1° 15'E</td>
<td>177</td>
<td>8</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 1. Geographical positions, depth and tidal parameters at mean springs ($M_2 + S_2$) relating to N$_1$, N$_2$, N$_3$, N$_4$, N$_5$. 
<table>
<thead>
<tr>
<th>$W_{ms^{-1}}$</th>
<th>$u_*$ $_{cms^{-1}}$</th>
<th>$\kappa_E$ $_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.5405</td>
<td>18.3</td>
</tr>
<tr>
<td>10</td>
<td>1.253</td>
<td>42.5</td>
</tr>
<tr>
<td>15</td>
<td>2.106</td>
<td>71.4</td>
</tr>
<tr>
<td>20</td>
<td>3.081</td>
<td>104.4</td>
</tr>
<tr>
<td>25</td>
<td>4.164</td>
<td>141.2</td>
</tr>
<tr>
<td>30</td>
<td>5.347</td>
<td>181.3</td>
</tr>
<tr>
<td>35</td>
<td>6.621</td>
<td>224.4</td>
</tr>
<tr>
<td>40</td>
<td>7.980</td>
<td>270.5</td>
</tr>
</tbody>
</table>

Table 2. $u_*$ and $\kappa_E$ for various $W$. 
<table>
<thead>
<tr>
<th>$W_{(ms^{-1})}$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>236</td>
<td>346</td>
<td>346</td>
<td>346</td>
<td>346</td>
</tr>
<tr>
<td>15</td>
<td>397</td>
<td>977</td>
<td>977</td>
<td>977</td>
<td>977</td>
</tr>
<tr>
<td>20</td>
<td>581</td>
<td>1542</td>
<td>1943</td>
<td>2092</td>
<td>2092</td>
</tr>
<tr>
<td>25</td>
<td>785</td>
<td>2084</td>
<td>2626</td>
<td>3821</td>
<td>3821</td>
</tr>
<tr>
<td>30</td>
<td>1008</td>
<td>2676</td>
<td>3371</td>
<td>4970</td>
<td>6152</td>
</tr>
<tr>
<td>35</td>
<td>1248</td>
<td>3314</td>
<td>4174</td>
<td>6154</td>
<td>7617</td>
</tr>
<tr>
<td>40</td>
<td>1504</td>
<td>3994</td>
<td>5031</td>
<td>7417</td>
<td>9181</td>
</tr>
</tbody>
</table>

Table 3. $\mu w$ in cm$^2$s$^{-1}$ for the various positions and wind speeds.
<table>
<thead>
<tr>
<th>$W_{(ms^{-1})}$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>768</td>
<td>64</td>
<td>105</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>768</td>
<td>346</td>
<td>346</td>
<td>346</td>
<td>346</td>
</tr>
<tr>
<td>15</td>
<td>768</td>
<td>977</td>
<td>977</td>
<td>977</td>
<td>977</td>
</tr>
<tr>
<td>20</td>
<td>768</td>
<td>1542</td>
<td>1943</td>
<td>2092</td>
<td>2092</td>
</tr>
<tr>
<td>25</td>
<td>785</td>
<td>2084</td>
<td>2626</td>
<td>3821</td>
<td>3821</td>
</tr>
<tr>
<td>30</td>
<td>1008</td>
<td>2676</td>
<td>3371</td>
<td>4970</td>
<td>6152</td>
</tr>
<tr>
<td>35</td>
<td>1248</td>
<td>3314</td>
<td>4174</td>
<td>6154</td>
<td>7617</td>
</tr>
<tr>
<td>40</td>
<td>1504</td>
<td>3994</td>
<td>5031</td>
<td>7417</td>
<td>9181</td>
</tr>
</tbody>
</table>

Table 4. \( \mu \) in cm$^2$s$^{-1}$ for the various positions and wind speeds, assuming mean spring tidal conditions.
<table>
<thead>
<tr>
<th>$\mathcal{W}_{(\text{ms}^{-1})}$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.52</td>
<td>0.016</td>
<td>0.020</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>10</td>
<td>0.52</td>
<td>0.08</td>
<td>0.08</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>15</td>
<td>0.52</td>
<td>0.26</td>
<td>0.20</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>20</td>
<td>0.52</td>
<td>0.40</td>
<td>0.40</td>
<td>0.30</td>
<td>0.24</td>
</tr>
<tr>
<td>25</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.44</td>
</tr>
<tr>
<td>30</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>35</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>40</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 5. $\mathcal{K}$ in cms$^{-1}$ for the various positions and wind speeds.
<table>
<thead>
<tr>
<th>( N_1 )</th>
<th>( V_s )</th>
<th>( V'_s )</th>
<th>( V_a )</th>
<th>( \xi_a )</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_2 )</td>
<td>( V_s )</td>
<td>7.78</td>
<td>12.85</td>
<td>22.29</td>
<td>36.36</td>
<td>55.10</td>
<td>78.51</td>
<td>106.45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V'_s )</td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_a )</td>
<td>7.35</td>
<td>12.70</td>
<td>22.24</td>
<td>36.33</td>
<td>55.09</td>
<td>78.50</td>
<td>106.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_a )</td>
<td>1.38</td>
<td>0.46</td>
<td>0.13</td>
<td>0.041</td>
<td>0.014</td>
<td>0.005</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N_3 )</td>
<td>( V_s )</td>
<td>7.77</td>
<td>12.98</td>
<td>19.56</td>
<td>31.43</td>
<td>47.29</td>
<td>67.41</td>
<td>91.96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V'_s )</td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_a )</td>
<td>7.34</td>
<td>12.84</td>
<td>19.50</td>
<td>31.40</td>
<td>47.28</td>
<td>67.40</td>
<td>91.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_a )</td>
<td>1.39</td>
<td>0.45</td>
<td>0.19</td>
<td>0.066</td>
<td>0.024</td>
<td>0.010</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N_4 )</td>
<td>( V_s )</td>
<td>7.77</td>
<td>13.08</td>
<td>19.00</td>
<td>25.42</td>
<td>37.19</td>
<td>52.35</td>
<td>71.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V'_s )</td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_a )</td>
<td>7.34</td>
<td>12.94</td>
<td>18.93</td>
<td>25.38</td>
<td>37.17</td>
<td>52.33</td>
<td>71.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_a )</td>
<td>1.39</td>
<td>0.44</td>
<td>0.21</td>
<td>0.117</td>
<td>0.052</td>
<td>0.024</td>
<td>0.012</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N_5 )</td>
<td>( V_s )</td>
<td>7.77</td>
<td>13.06</td>
<td>19.13</td>
<td>25.55</td>
<td>33.06</td>
<td>46.05</td>
<td>62.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V'_s )</td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_a )</td>
<td>7.34</td>
<td>12.92</td>
<td>19.07</td>
<td>25.51</td>
<td>33.04</td>
<td>46.03</td>
<td>62.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_a )</td>
<td>1.39</td>
<td>0.44</td>
<td>0.20</td>
<td>0.116</td>
<td>0.071</td>
<td>0.035</td>
<td>0.018</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Values of \( V_s \), \( V'_s \), \( V_a \) (in \( \text{cms}^{-1} \)) and \( \xi_a \) (in metres) at positions \( N_1 \) to \( N_5 \) for various wind speeds; \( \xi_o = 0.001 \text{m} \).
<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_s$</td>
<td>$V_s'$</td>
<td>$V_a$</td>
<td>$F_a$</td>
<td>$W$ ($m s^{-1}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_1$</td>
<td>6.10</td>
<td>17.23</td>
<td>36.88</td>
<td>66.60</td>
<td>95.42</td>
<td>127.29</td>
<td>161.10</td>
</tr>
<tr>
<td></td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
</tr>
<tr>
<td>$N_2$</td>
<td>7.78</td>
<td>12.85</td>
<td>22.29</td>
<td>36.36</td>
<td>55.10</td>
<td>78.51</td>
<td>106.45</td>
</tr>
<tr>
<td></td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
</tr>
<tr>
<td></td>
<td>7.77</td>
<td>12.98</td>
<td>19.56</td>
<td>31.43</td>
<td>47.29</td>
<td>67.41</td>
<td>91.96</td>
</tr>
<tr>
<td></td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
</tr>
<tr>
<td></td>
<td>7.77</td>
<td>13.08</td>
<td>19.00</td>
<td>25.42</td>
<td>37.19</td>
<td>52.35</td>
<td>71.12</td>
</tr>
<tr>
<td></td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
</tr>
<tr>
<td></td>
<td>7.77</td>
<td>13.06</td>
<td>19.13</td>
<td>25.55</td>
<td>33.06</td>
<td>46.05</td>
<td>62.07</td>
</tr>
<tr>
<td>$N_5$</td>
<td>7.77</td>
<td>13.08</td>
<td>19.00</td>
<td>25.42</td>
<td>37.19</td>
<td>52.35</td>
<td>71.12</td>
</tr>
<tr>
<td></td>
<td>30.00</td>
<td>45.00</td>
<td>60.00</td>
<td>75.00</td>
<td>90.00</td>
<td>105.00</td>
<td>120.00</td>
</tr>
<tr>
<td></td>
<td>13.36</td>
<td>23.34</td>
<td>31.84</td>
<td>45.33</td>
<td>61.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.32</td>
<td>7.14</td>
<td>3.88</td>
<td>1.84</td>
<td>0.932</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Values of $V_s$, $V_s'$, $V_a$ (in $cm s^{-1}$) and $F_a$ (in metres) at positions $N_1$ to $N_5$ for various wind speeds; $F_o = 0.05m$. 
<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.600</td>
<td>0.002</td>
<td>0.002</td>
<td>0.020</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Table 8. $Z_0$ in centimetres at the locations $N_1$ to $N_5$. 
\[ \mathbf{W} \text{ (m s}^{-1} \text{)} \rightarrow \]

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_1)</td>
<td>(V_c)</td>
<td>1.994</td>
<td>5.636</td>
<td>12.060</td>
<td>21.585</td>
<td>31.091</td>
<td>41.947</td>
</tr>
<tr>
<td></td>
<td>(V_c)</td>
<td>2.007</td>
<td>5.645</td>
<td>12.093</td>
<td>21.698</td>
<td>31.280</td>
<td>42.237</td>
</tr>
<tr>
<td></td>
<td>(Z_c)</td>
<td>0.01320</td>
<td>0.02244</td>
<td>0.04141</td>
<td>0.07625</td>
<td>0.08769</td>
<td>0.09995</td>
</tr>
<tr>
<td>(N_2)</td>
<td>(V_c)</td>
<td>0.494</td>
<td>2.391</td>
<td>5.516</td>
<td>10.113</td>
<td>16.263</td>
<td>24.187</td>
</tr>
<tr>
<td></td>
<td>(V_c)</td>
<td>0.497</td>
<td>2.458</td>
<td>5.570</td>
<td>10.475</td>
<td>16.425</td>
<td>24.384</td>
</tr>
<tr>
<td></td>
<td>(Z_c)</td>
<td>0.00005</td>
<td>0.00007</td>
<td>0.00009</td>
<td>0.00012</td>
<td>0.00014</td>
<td>0.00017</td>
</tr>
<tr>
<td>(N_3)</td>
<td>(V_c)</td>
<td>0.216</td>
<td>1.627</td>
<td>4.281</td>
<td>8.032</td>
<td>13.161</td>
<td>19.915</td>
</tr>
<tr>
<td></td>
<td>(V_c)</td>
<td>0.220</td>
<td>1.778</td>
<td>4.539</td>
<td>8.386</td>
<td>13.606</td>
<td>20.146</td>
</tr>
<tr>
<td></td>
<td>(Z_c)</td>
<td>0.00004</td>
<td>0.00007</td>
<td>0.00008</td>
<td>0.00010</td>
<td>0.00012</td>
<td>0.00014</td>
</tr>
<tr>
<td>(N_4)</td>
<td>(V_c)</td>
<td>0.037</td>
<td>0.588</td>
<td>2.317</td>
<td>5.195</td>
<td>8.870</td>
<td>13.781</td>
</tr>
<tr>
<td></td>
<td>(V_c)</td>
<td>0.037</td>
<td>0.596</td>
<td>2.325</td>
<td>5.245</td>
<td>8.934</td>
<td>13.847</td>
</tr>
<tr>
<td></td>
<td>(Z_c)</td>
<td>0.00029</td>
<td>0.00046</td>
<td>0.00061</td>
<td>0.00070</td>
<td>0.00084</td>
<td>0.00100</td>
</tr>
<tr>
<td>(N_5)</td>
<td>(V_c)</td>
<td>0.009</td>
<td>0.265</td>
<td>1.414</td>
<td>3.711</td>
<td>6.988</td>
<td>11.026</td>
</tr>
<tr>
<td></td>
<td>(V_c)</td>
<td>0.009</td>
<td>0.274</td>
<td>1.419</td>
<td>3.720</td>
<td>7.001</td>
<td>11.041</td>
</tr>
<tr>
<td></td>
<td>(Z_c)</td>
<td>0.00024</td>
<td>0.00037</td>
<td>0.00053</td>
<td>0.00064</td>
<td>0.00071</td>
<td>0.00084</td>
</tr>
</tbody>
</table>

Table 9. Values of \(V_c\), \(V_c\) (in \text{cms}^{-1}) and \(Z_c\) (in metres) at positions \(N_1\) to \(N_5\) for various wind speeds.
Figure 1. (a) Vertical water column between sea surface and sea bed and (b) horizontal planform of currents and wind stress.
Figure 2. Positions $N_1, N_2, N_3, N_4, N_5$ marked on the grid of the continental shelf numerical model for tides and storm surges (Flather 1984).
Figure 4(d)

\[ V_s^* \left( u-g \right)^2 \left( v-g \right)^2 \]

Axes: 
- Y-axis: 0 to 40 cm/s
- X-axis: 0 to 60
Figure 6(a) Current profiles at position Na ; (1) denotes the depth-mean, for wind speeds 5-80 ms\(^{-1}\).
Figure 7 (a-d): Current profiles at position Nₕ. @ denotes the depth mean for wind speeds 5 (5-40 m/s).
Figure 8. Diagrammatic representation of the logarithmic boundary layers at (a) the sea surface and (b) the sea bed.