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Competition and Merger in Network Economy

by

Ke Li

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ABSTRACT

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This thesis is concerned about firm's merger and competition behavior in modern economies in which networks are ever-more important and how to optimize merger policy when network externalities present. As a demand-side economics of scale, network externalities bring benefit to consumers through merger and acquisition if the products from different firms are incompatible. Hence, a merger, which is both socially optimal and privately profitable, can exist without considering the supply-side economies of scale. Merger policy should be revised to be able to recognize these "good" mergers and encourage them. Firm's incentive to merge is enlarged by network effect because merged entities can benefit from a larger network, which increases the demand for their product. Moreover, merger and acquisition in network world give the merged entities an advantage in competition over the firms who stand outside the merger. One of the explanations for this advantage is merged entity may inherit indirect network resources, for example complementary products producers, from all merged firms, since the mobile of these resources are costly and slow. Acquiring more firms brings more indirect network resources to merged entity, which makes the products of merged entity more valuable to the consumers. Thus the merged entity can charge a higher price or squeeze more market share. Merged entity can obtain locked-in consumers from all merged firms is another explanation of the advantage. For some information products, such as TV subscription, internet access and mobile phone service, consumers need to sign a contract with the service provider and are locked by these contracts for a fixed period. Merged entity may inherit these locked-in consumers and show a larger initial network to the consumers who are not locked at the beginning of the competition. Social planner should be cautious to the merger in network world because network externalities magnify the power of the merger, which may be utilized by the firms to get dominant position.

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Declaration

I, KE LI declare that the thesis entitled *Competition and Merger in Network Economy* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research, I confirm that:

* this work was done wholly while in candidature for a research degree at this University;

* where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;

* where I have consulted the published work of others, this is always clearly attributed;

* where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

* I have acknowledged all main sources of help;

* where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

* none of this work has been published before submission.

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Introduction and Background

Many of the so-called industrial nations are experiencing a transition to information-based economy, in the sense that ICE (information, communications, and entertainment) comprises an ever-greater share of national economy (Shapiro and Varian 1999). In the "New Economy", the economics of networks takes on greater importance in comparison with traditional economies of scale. Many networks are self-evident: the telephone network, the network of fax machines, the credit-card acceptance network, the Internet etc. The role of networks in the information economy is even larger than it might appear at the beginning, because of the presence of many virtual networks: the network of users of Apple Macintosh computers, the network of owners of compact disk machines, and the network of users of Microsoft software.

It has been recognized that networks can raise problems for competition policy. But as yet, little attention has been paid to how merger policy should be applied to networks. The need for rigorous research on this topic is acute. There have been a number of high-profile mergers in the information sector. A number of mergers have been allowed: Lotus and IBM, Nynex and Bell Atlantic, Microsoft and Hotmail, WebTV, and Vermeer. However, several prominent mergers have been blocked altogether, and been subject to substantial modifications competition authorities.

In 1991, Borland International announced its intention to acquire Ashton-Tate. The firms were two leading suppliers of personal computer based “relational database” programs. This case was an important early test of how mergers in the personal computer software industry would be treated by the anti-trust agencies in the US. The DoJ (US Department of Justice) expressed competition concerns that the merged company would be dominant in the market for relational database software. As a result, Borland agreed to issue FoxPro, a competitor, a license to the dBase code.

In 1994, Adobe announced its intention to acquire Aldus. The firms sold the leading brands of professional illustration software. To prevent the creation of a dominant position in this market, the US Federal Trade Commission required the merged firm to divest the software owned by Aldus to a third firm.

In 1994, Microsoft proposed to acquire Intuit, the owner of the leading personal financial software package. Microsoft's Money product performed many of the same functions. The acquisition was challenged by the DoJ, again on the grounds that the merged firm would be dominant in a particular product market. In response to the DoJ's challenge, Microsoft abandoned the acquisition.

These are just three examples. Detail of these examples and other examples can be found in Shapiro's (2000) work. In each of these cases, the policy-makers have acted either to prevent or to modify the merger. The question is whether these are the correct decisions?

In information economy, the consumer's welfare not only depends on the quality of the products and the price, but also depends on the size of the products' network. Clearly, people value a large networks more than a small network because it is less possible to meet compatibility problem. Merger between two networks may create a larger network, which makes merged entity's products more valuable to consumers without any additional price reduction or quality improvement. If the cost saving of merger is a kind of supply-side economies of scale, this network effect can be similarly defined as demand-side economies of scale. Merger, despite increasing concentration in an industry, may nevertheless be associated with a rise in welfare. Thus, it seems that the social planner should approve more merger applications in network economy. However, we know that demand-side economies of scale create positive feedback: a tendency for the strong to get stronger and the weak to get weaker. Consequently, the merged entity may use their initial advantage to pursue dominant market status, which may finally be harmful to social welfare.

Another thing that needs to raise attention is some network externalities do not directly come from the products. Instead, they come from complementary products market. A larger network will attract more complementary products supplier and increase competition, which will finally reduce the price and benefit the consumers. These indirect network externalities may take effect similar as direct network externalities. However, in most of the time, they are more similar to a kind of resource that has been fixed to a specific product, since the firms in complementary market are costly to switch from supplying one product to another product. The merged entity can inherit these recourses from all of the firms who are involved in the merger and get an advantage in the competition compared with other firms who stand outside the merger. This also supports the idea that the merger in network economy should be more restricted by social planner than in traditional economy.

Many literatures have discussed or emphasized how these characteristics of the network effect the market structure in typical industries, such as finance (Noia 1998), telecommunication (Baranes and Flochel 2003), entertainment (Matteucci 2003), internet (Cremer, Rey and Tirole 1992, Baranes and Cortade 2004) and transportations (Brueckner and Spiller 1992). In this thesis, we are attempting to develop a model without making any specification in a certain industry environment, reveal some general rules about the merger behavior of the firms and provide some propositions for the social planner to consider in a general network world.

This thesis focuses on the following questions:

When is merger privately profitable if there are network externalities present?

Obviously, firms will not undertake mergers that are privately unprofitable. This means that mergers that reach the attention of policy-makers are a sub-set of all possible mergers. Salant et al. (1983) show that, when firms with equal market shares compete in outputs (the industry is a symmetric Cournot oligopoly), with linear demand and constant marginal costs (so that there are no synergies), a merger is profitable only if it involves at least 80% of the firms in the industry. An implication of this result is that, in the absence of synergies, any merger proposal considered by a policy-maker is likely to involve a large increase in industry concentration. This means, firstly that there should be a general policy bias against mergers; and secondly, that this bias can be overturned only if synergies are strong enough. If we detail the acquisition process, we may get some more counterintuitive merger threshold without considering synergies. Kamien and Zang (1990) show that, in an acquisition model with linear demand function, merger equilibrium only exists for an industry with no more than two competitors.

Do these conclusions carry over when network externalities are present? In the first chapter of the thesis, we have found that in the same setting as Salant et al. (1983), small-scale merger can be profitable if the network externalities are sufficiently strong. Hence, a profitable merger need not a sharp increase in concentration. The intuition is simple: merging firms can benefit from having a larger network, which increases the demand for their product. This effect can outweigh the other effects identified by Salant et al., in a way similar to supply-side synergies.

In the second capture, we focus on answering this question under the condition that only indirect network effect presents. Based on the work of Kamien and Zang (1990), we find that merged nash equilibrium is easier to exist than Kamien and Zang's declaration if the firms may inherit indirect network resource. The intuition is, after inheriting indirect network resources from the firms acquired, buyer in the acquisition can afford more payment to the seller, so the merger can occur in a more general market condition. .

When there exist multiple equilibra?

The analysis will be more complicated because of the presence of multiple equilibria. Network externalities naturally give rise to many possible outcomes. These outcomes are driven by different "expectations". If an individual expects, for example, everyone else to buy Firm A's product, then that individual has a strong incentive to buy from Firm A. If everyone thinks in this way, then Firm A becomes dominant, and its competitor (Firm B) suffers. On the other hand, if everyone expects Firm B to be dominant, then this expectation can also be self-fulfilling.

If we don't know the location of the equilibrium after the merger or there exist too many possibilities, it will be difficult for us to carry out further analysis. In this thesis, we assume the consumer's expectation is full filled by firms and firms are aware of this when they make output decisions. This means the consumer's expectation is fully reacted with firms' output and the network utility function can be directly added to the demand function to solve the equilibrium. Since the network utility function can be a curve, multiple equilibria still exist in some cases. However, we find that the multiple equilibra only exist when the network effect is strong enough. The intuition is the traditional reaction function of firms must be bended to a certain level to get more than one cross point. In this thesis, we limit our analysis in the world, in which the network effect is relatively week. In most of the chapters of this thesis, we assume the network utility is a linear function bx, and $b < 0.5$. This assumption eliminate the possibility of multiple equilibria and greatly reduce the difficulty of analysis. We believe most of characteristics of network world are well kept under this assumption, because products with extremely strong network effect are rare. Multiple equilibra and more general utility function can be left as an interesting future work.

When is a merger socially optimal if network externalities present?

The next issue to investigate is whether those mergers which are privately profitable are, or can be, socially optimal. We must therefore determine the effects of merger on consumer surplus. There will be two countervailing effects. The standard effect is that merger increases industry concentration, which generally will be bad for consumer surplus. The new effect, arising from network externalities, is that merger can create larger networks, which, other things equal, is good for consumer surplus.

There are many factors that determine the balance between these two effects. The first one is the strength of network externalities. The second is how concentrated this market is. Degree of compatibility between competing networks also matters. In the extreme case in which compatibility is perfect, merger clearly makes no difference to overall network benefits. The concentration effect then dominates, and (in the absence of supply-side effects) any merger is likely to be socially harmful. In this thesis, we mainly consider the other extreme case ——when all firms' products are entirely incompatible. In this circumstance, merger can surely raise consumer surplus by increasing the size of networks (the main characteristic we want to keep) and the model is simplified without considering degree of compatibility.

Does the merged entity get an advantage in the competition?

As the merged entity may benefit from the reduction of the competitors, all the other firms, who are not involved in the merger, can get benefit as well. Since the merger is costly, firms all prefer to stand outside merger and encourage others to get into it if the reduction of competition is the only effect of merger. However, in network world, there exists a tendency that: the strong (merged entity) to get stronger and the weak (the firms standing outside merger) to get weaker. Thus, there must exist some mechanisms that give the merged entity an advantage in the competition.

The consumers will always consider the size of network when they are planning to purchase. Generally speaking, the merged entity will show a larger existing network than separate ones. This is because some of old consumers are locked by the contract and the merged entity may inherit these locked-in consumers from every firm who are involved in the merger. The new consumers form their expectation of the size of network according to the number of these locked-in consumers, so the merged entity shows an advantage. Chapter three of this thesis gives out a preliminary attempt to model this effect. We assume a duopoly market with a given asymmetric initial locked-in consumers to each firm and show that the firm with more initial locked-in consumers indeed obtains a relative advantage. The merger process hasn't been considered in Chapter three, but our result can be easily applied to analyze a merger case. The only difference is we need to treat the size of the initial locked-in consumers as an exogenous variable when we analyze a merger.

Another origin of this advantage is addressed in Chapter two of this thesis, in which we assume firms may inherit indirect network resource from every component. The more firms the merged entity acquired, the more indirect network resources he may control. The consumers obviously prefer a product with more indirect network externalities (more complementary products supplier), so the merged entity may obtain a relative advantage in the competition. In this chapter, the direct network effect is not considered, but we will get similar result when both the direct and indirect network externalities take effect because these two forces are in the same direction. Moreover, we also reveal that the larger firm is easier to acquire smaller firm than the smaller firm to acquire larger one in an acquisition model. It is because the large firm can afford more acquisition payment than the small one. This may hint some mechanism to explain how the market goes from an asymmetric structure to a monopoly.

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Chapter 1
Horizontal Merger with Network Externalities and Incompatible Products

Abstract

In this study, we analyzed the horizontal merger between firms in oligopoly competition with incompatible products and network externalities. The model is based on Katz and Shapiro's network externalities model (1985), but allow the consumer's expectations to react with the firms' outputs. We reveal that the firms have more incentive to merge if they produce network products. For the social planner's problem, we find that mergers may increase social welfare without the consideration of synergies and we develop some sufficient conditions for the merger to be socially desirable when general form network utility presents. As a special case, we applied our model to a linear network utility world, revealing that private and socially desirable mergers may exist. The sufficient and necessary conditions for the existence of such a "good" merger were also obtained for the merger regulators to consider.

1.1 Introduction

Merger policy has been traditionally recognized as a central concern that horizontal mergers (between firms operating the same product and geographic market) can decrease competition and hence social welfare. To avoid a potential monopoly and great reduction of the competition, mergers are required to be reviewed and authorized by the industry regulation department of the government before they may be carried out. Roughly speaking, only a merger that does not increase the concentration in the industry by very much, or that allows the industry concentration to remain low even after the merger, will be permitted (e.g., the United States Department of Justice (DoJ) 1984 Merger Guidelines, which specified explicit concentration thresholds when determining whether a merger is to be allowed). However, for different market structures, different product characteristics and different times, the consequences of the merger may change the short-term and long-term social welfare greatly (Fridolfsson and Stennek 2000, Buccirossi 2008).

Using a simple rule as a guideline to judge all merger cases is a weakness and may be misleading for the social planner, especially in the "New Economy" (Shapiro 2000). An important reason that the old merger policy should be reconsidered by social planners is that mergers may bring some positive effect to the consumers in network industries. For example, fewer mobile phone service providers suggests that consumers have less of chance to pay the cross net fee. More specifically, the merger between MSN and Yahoo Message suggests you only need to register with one service and can use it to send messages to users in either company.

As a widely accepted rule, large networks offer more value to users than small ones: customers value a popular product or network more than an unpopular one. When a consumer gets more people to join the same network or use the same product, he/she may have a chance to obtain an additional benefit. This creates a particular form of economy of scale often denoted as "network externalities". In the information industry, although horizontal mergers between firms could reduce the competition, consumers may also have a chance to enjoy larger network externalities. It is hard to say whether a merger is good or not without considering the balance between good and bad. We can imagine such a "good" merger: a merger which lets the firms squeeze more profits from the consumers and, at the same time, the consumers are also happy to accept this merger since they receive benefits from network externalities, which may offset their losses from the higher price after the merger. This “good” merger increases social welfare and is welcome by both firms and consumers. The merger policy should consequently be revised to identify these "good" mergers and encourage them.

A “good” merger must firstly be a profitable merger to the firms involved. Salant et al. (1983) revealed that mergers were only privately profitable if they caused a sufficiently large decrease in concentration. This illustrates that mergers in need of attention are only a sub-set of all potential mergers. Our first goal in this study is therefore to determine this sub-set with the effect of network externalities. The literature has revealed that Salant’s condition, where over 80% of the firms must get involved in the merger to make the merger profitable, should be modified if other conditions or limitations are added to the model. Perry and Porter (1985) pointed out that Salant’s model may underestimate the probability of a merger if there is a limit of the output capacities or shortage of capital. Cheung (1992) illustrated that Salant’s threshold may be relaxed to 50% if the demand satisfying the marginal revenue of the industry is decreasing. Fauli-Oller (1997) further investigated Cheung’s work and found that the probability of mergers may depend on the degree of concavity of the demand function.

In Cournot competition, firms’ outputs increase after the merger, so every consumer should receive more or less benefit from the merger because of the network externalities. This benefit allows the firms to charge a higher price to consumers and squeeze out more profits. Thus, the incentive of the firms to merger may increase if the network effect is considered. For different forms of the utility function of network externalities, the demand function may be distorted to a linear function with a smaller slope, or a concave function. The degree of relaxation of Salant’s condition may depend upon how large the network effect is, which is coincident with Fauli-Oller’s (1997) declaration. On the other hand, this result also provides insight that the profitable merger does not need a sharp increase in concentration in the network world, which would normally harm consumer welfare. A more gentle increase in market concentration will be easier to accept for the consumers, who need to balance the losses from the increase of the price and the benefits from the increase in network externalities.

The second step of this study is to provide a further narrowing of the candidate mergers we find in the first step. This means we need to pick up the mergers which may increase social welfare from all the mergers profitable to the firms. Generally speaking, the greater the network effect is, the more benefit the consumers may receive and the larger the possibility that we can find a "good" merger.

It is easy to identify whether the consumer's welfare increases or not if all firms' products are compatible. In a fully compatible case, the network externalities only depend upon the total output from all firms. Thus, the consumer's welfare increases when the merger increases the total output and decreases when the merger decreases the total output. In this thesis, we focus on the situation where the firm's products are completely incompatible. The partial compatibility problem can be a future work and solved under a similar framework.

In the literature, the social welfare of the merger and market concentration has been analyzed extensively (Salant et al. 1983, Ferrell and Shapiro 1990a, 1990b, Gaudet and Salant 1991). Ferrell and Shapiro (1990b) introduced synergies into the competition model and indicated that the merger may be socially desirable if the supply-side economies of scale are strong enough. It is easy to see that the synergy has many similar characteristics to network externalities. With the spirit of Ferrell and Shapiro (1990b)'s work, we can view network externalities as a demand-side economies of scale, and obtain similar conclusions that the merger may be socially desirable if the demand-side economies of scale, or the network effect, is strong enough. The possibility that vertical mergers can raise social welfare in a network world has been recognized in recent literature. Inspired by the work of Katz and Shapiro (1994), Jamison (2002) and Weisman (2005) have conducted work revealing the sufficient conditions for a vertical merger to be socially desirable. However, their analysis was based on the merger between the firms who served multiple markets and their network externalities comes from the internalization of other market's products. In our thesis, we primarily consider the horizontal merger in which all firms only compete in a single market and the network effect only connects with the firms' output in a single product line.

Before we begin the discussion of merger and social welfare, it is crucial that we create a simple and efficient way to model network effect. Thus, in the first part of this chapter, we spend a whole section discussing the modelling of network externalities and the possible equilibrium under our model. One of the most successful models of network externalities to date has been provided by Katz and Shapiro (1985). They developed a static oligopoly Cournot Model of competition with network effects. Most of the other literature (found in the survey paper of Farrell and Klemperer (2004) and the book of Shapiro and Varian (1999)) addresses the characteristics of the network effect based on Katz and Shapiro's work. In their model, they assumed the expectations of consumers were fulfilled and introduced a concave utility function to describe the network utility. Katz and Shapiro predicted that, unlike the traditional Cournot Model, there may exist multiple equilibria for some utility functions because of the distortion of the reaction functions, even though a perfect symmetric assumption to each firm was given. This brings some difficulties to policy makers because of the uncertainty of the ex-post status of the merger activities. Although the authorities may know how the market works at the moment, it is hard for them to predict which equilibrium will be played by the firms in the future.

In our model, we assume the expectated output of the consumers will perfectly change with the change in the firm's real output, rather than assuming that the output of the firms has no effect on the expectations of the consumers. This modification enables us to more precisely simulate the reactions between the firms and consumers. We find that the asymmetric equilibria only existed in a limited number of cases. When the network effect is very strong, there is no equilibrium and when the network effect is relatively weak, only a symmetric equilibrium exists. In this thesis, we restrict our discussion to the case where the network effect is gentle and only a symmetric equilibrium exists, which avoid the complexity of the selection of multiple equilibria. Although our conclusions are compromised on some level, most of the new features that are bought by the network externalities to merger are maintained. The conclusions update our understanding of mergers in the network world. We also provide a list of rules about the reaction functions between the firms and clarify the features of network competition that are not pointed out by Katz and Shapiro's model. This makes a virtual framework for the discussion in the following sections.

In Section 1.2, we will describe the model, attempt to provide new features of the firms' reaction function and derive one of the necessary conditions for the existence of multiple equilibria. In Section 1.3, we will discuss the firm's merger incentives and social welfare in a general utility function context. In Section 1.4, we will introduce a linear utility function and give out sufficient and necessary conditions for the existence of a "good" merger in linear utility circumstances. We will also illustrate a rough way to identify "good" mergers for the social planners under the network environment. Section 1.5 is the conclusion, but also includes future research suggestions.

1.2 Network Externalities with Cournot Competition

1.2.1 The Competition Model with Network Externalities

We investigate an oligopoly market with n firms. These firms produce homogeneous products and chose their outputs to maximize their profits. The products produced by the same firm were compatible, but were not compatible to the products produced by the other firms. This suggests that if a consumer chose a product from one of the firms, he/she may benefit from an increase in the number of consumers selecting the same firm, but the change in the output of other firms will not affect his/her surplus. A consumer can choose either one or zero units of the product from one of the firms. His/her choice depends on the products that can maximize his/her surplus. The surplus that a consumer derives from purchasing a unit of the good depends on the number of the consumers who join the network associated with his/her choice and his/her basic willingness of that product. When the consumers make their decisions, they are not able to see the choices of others, so their purchases are only based on their expectations of the network size. We assume this expectation is identical for every consumer.

The game is played in the following sequence: first, consumers form their expectations about the size of the network of each of the firms. Secondly, the firms play an output competition and make an announcement about their output. When the firms play the competition game, they fully understand that consumers will change their expectations according to their announcement. Thus, we assume that the outputs announced by the firms counted upon the possibility that consumers will change their minds. In the third step, consumers revise their expectations about the size of the network of each firms according to the announcement made by the firms. The firms then commit to their announcement and generate a set of prices for their products. Finally, the consumers make the purchasing decision by comparing their reservation price, which is based on their revised expectations of the network sizes, with the price set by the firms.

The game described here can be compared with Katz and Shapiro's (1985). In Katz and Shapiro's model, the firm's announcement of its planned level of output has no effect on consumers' expectations. Their assumption may reduce the calculations, however, this is unlikely to happen in the real world. The consumers will, more or less, change their expectations of the each firm's network size after they have seen the announcement of the firms. In our model, we assume the consumers fully trust the announcement of the firms since firms will always commit to their announcement. If we use x_i^e to denote the consumers' expectation outputs of one of the firms and use x_i, $i \in \{1, ..., n\}$, to denote the real output of this firm, the assumption can be understood as $\partial x_i^e / \partial x_i = 1$ or $x_i^e \equiv x_i$. Hence, the notation x_i^e will all be writen as x_i in the following thesis.

In our model, we make the assumption that consumers are heterogeneous in their basic willingness to pay for a product without considering the network effect, but homogeneous in their valuation of network externalities. More specifically, we use r to denote each consumer's basic willingness to purchase the simple product and $u(x_i)$ to denote the network externalities that a consumer can obtain when he/she purchases Firm i's product. Based on the definition of the positive network externalities and the characteristics of most information products, we define that the network externality function as having the following characteristics:

$$u(0) = 0; u(x_i) > 0; u'(x_i) > 0; u''(x_i) \leqslant 0$$

Since we assume that all the firms produce incompatible goods, the network size of the product is the output of the firms who produces this product. Without further loss of generality, we assume r is uniformly distributed between minus infinity and 1. The uniform distribution ensures we obtain a linear price function. The assumption that r can go to minus infinity gives us an always opened market which suggests we do not need to consider the corner solution in a covered market.

When a consumer purchases a product from Firm i, he/she will be able to enjoy the product plus the network externalities that the product brings to him and have to pay the price the firm charges. So the consumer's surplus from purchasing a product from Firm i

is $r+ u(x_i) - p_i$. Since the consumers will purchase the product if and only if their surplus is positive, $r+ u(x_i) - p_i$ should be positive for the consumer who makes the purchasing decision. Hence, only the consumers with a type r that is not less than $p_i- u(x_i)$ will enter the market. Obviously the minimum r to make $r+ u(x_i) - p_i$ positive is $p_i - u(x_i)$. Here, we use r^* to denote the consumer who has no difference between purchasing the product or not and only the consumer whose type r is not less than $r^* = p_i- u(x_i)$ enter the market. In addition, $p_i- u(x_i) = p_j- u(x_j)$, $i \neq j \ and \ i, j \in \{1, ..., n\}$ must also be true to keep the consumers are indifferent in purchasing product between all the firms and all the firms have a positive output.

The total output of all the firms is:

$$z = \sum_{i=1}^{n} x_i = 1 - r^* = 1 - p_i + u(x_i) \quad i \in \{1, ..., n\} \tag{1.1}$$

From the equation (1.1), we know that after each firm sets their outputs, they will recieve a price according to its output and the outputs of all other firms, which is defined as:

$$p_i = 1 + u(x_i) - z = 1 + u(x_i) - x_i - \sum_{i \neq j} x_j \quad i, j \in \{1, ..., n\} \tag{1.2}$$

for all i such that $x_i \geqslant 0$.

The profit of Firm i is:

$$\pi_i = p_i x_i = x_i(1 + u(x_i) - z) \quad i \in \{1, ..., n\} \tag{1.3}$$

All the firms choose their outputs to maximize profits. From the first-order condition of the equation (1.3), we can get:

$$1 + u(x_i) + x_i u'(x_i) - 2x_i = \sum_{i \neq j} x_j \quad i, j \in \{1, ..., n\} \tag{1.4}$$

Equation (1.4) is the reaction function of Firm i against the total output of all other firms. If the total output of all other firms $\sum x_j$ is given, we can get the best response of Firm i by solving this equation.

Proposition 1.1 *If the maximum of the profit function of Firm i exists for a given total output of all other firms ($\sum x_j$), the first derivative of the profit function at the maximum is less than 1.*

Corollary 1.1 *If $u'(x_i) \geqslant 1$ for any $x_i \geqslant 0$, the maximum of the profit of the firms does not exist. Firms' outputs are only bounded by their capacity.*

Corollary 1.2 *If x_i^* is the output of the firm i in an equilibrium, there must have $u'(x_i^*) < 1$.*

If we look at equation (1.2), it is clear that the price function is slightly different to the standard Cournot model. In a standard Cournot competition, the price will always decrease with an increase in output because of the increase in competition. However, in this new model, the increase of the output has two effects. On the one side, it increases the competition and causes the price to drop. On the other side, an increase in the output can make goods more competitive because of the increasing network size. Thus, if the benefit from an increase in network size is larger than the loss from the dropping price, the firm will never stop to produce more products. Because $u''(x_i) \leqslant 0$, the increment of the network externalities, when the firm produces one more product, always decreases. Only a small set of network functions satisfy the situation of Corollary 1.1.[1] Because we are only interested in the situation where the equilibrium exists, the utility functions which exhibits $u'(x_i) \geqslant 1$ for any $x_i \geqslant 0$ will not be discussed. This can also be seen as an reinforcement of the definition of the network utility function in our model.[2]

Corollary 1.2 is a clear consequence of Proposition 1.1. According to the definition of an equilibrium, if x_i^* is an equilibrium output, it must maximize the profits of Firm i for a given equilibrium output of all other firms. Thus, we have $u'(x_i^*) < 1$. This corollary limits

1 There does exist such a network function in real world, such as $u(x_i) = bx$, $b \geqslant 1$.

2 In some papers, condition $\lim u'(x) = 0$ as $x \to \infty$ is added in the definition of the utility function. This condition will eliminate the situation in Corollary 1.1. However, the linear utility function is also ruled out by this defintion from all the possible utility functions. Since the linear utiltity function is the main topic we will discuss in Section 4 of the paper, we do not introduce this condition in our definition.

the equilibrium output of the game in certain areas and is helpful when we need to know some propositions of the equilibrium output, but cannot exactly solve the equilibrium.

Proposition 1.2 *The reaction function of Firm i against the total output of all the other firms decreases monotonously.*

This proposition provides us with a clear picture about the monotonicity of the equation (1.4), which indicates that the relationship between the outputs of the Firm i and the total output of all the other firms is always strategical substitution. Moreover, from the monotonicity of the function, we know that only unique best response for Firm i can maximize its profits for a given total output of all other firms. In Katz and Shapiro's model, they indicated that the firms may have more than one best response for a given action of all the other firms after accounting for the network externalities. They explained this declaration using the reaction, stating that it will be a curve rather than a straight line if the network externality function is non-linear. However, from Proposition 1.1, we know that, in our model, no matter what shapes of the network utility functions are, the situation indicated by Katz and Shapiro is not likely to occur.

We can draw the equation (1.4) in the following figure:

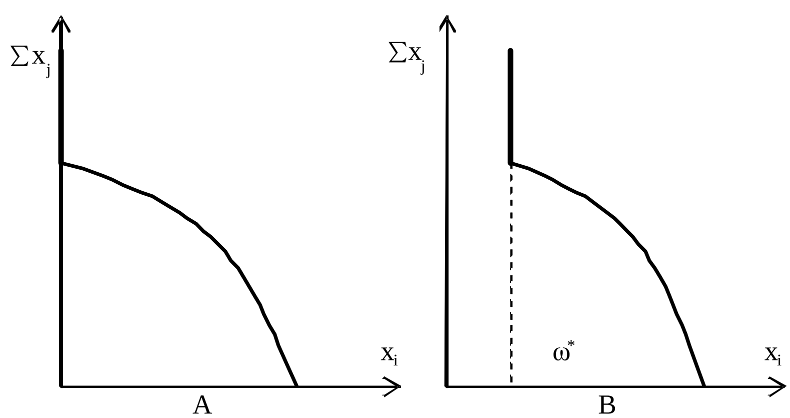

Figure-1.1: Firm i's reaction function

As can be seen, although $\sum x_j$ can be any number larger than 0, the Firm i's best response is bounded. From $u''(x_i) \leqslant 0$, we know $u'(x)$ decreases with an increase in x. If

$u'(0) < 1$, we have $u'(x_i) < 1$ for any $x_i \geqslant 0$. Thus, x_i can be chosen from any positive number without violation of Proposition 1.1. The Figure-1.1-A illustrates this situation. In Figure-1.1-A, when the total output of the other firms is larger than a certain number, Firm i will always set its output at 0. When $\sum x_j$ chooses 0, Firm i will choose its output as the market is a monopoly. If $u'(0) \geqslant 1$, x_i cannot be 0 or any number very near to 0 because of Proposition 1.1. However, a ω ($\omega > 0$) can always be found and $u'(x_i) < 1$ for all $x_i \geqslant \omega$. We define the minimum ω is ω^*. Firm i will always at least produce ω^* if it faces utility functions with $u'(0) \geqslant 1$. The Figure-1.1-B illustrates this situation. When total output chosen by all of the other firms is greater than a certain number, Firm i will always set its output as ω^*. When the total output of all other firms is 0, Firm i sets its output as the market is a monopoly.

From the figure we know that, for some products of which $u'(x_i)$ is very large when x_i is very small, every firm can benefit from setting a positive output regardless of the intensity of the competition in the market. No firm can drive others out of the market by simply increasing its output. However, for some products, of which the utility of network is relatively small, the competition in the market may drive a firm out of the market or force it not to produce when other firms set a large output level..

1.2.1 Symmetric and Asymmetric Equilibrium

In a traditional Cournot competition, there only exists a unique symmetric equilibrium in which all the firms choose equal outputs. The reason behind this is that the reaction function in the traditional Cournot model is a linear line and that two linear lines only have one crossing point. After we add the network externalities into the Cournot model, we can see from Figure-1.1 that the reaction function could be a curve and two curves may cross more than once in First Quadrant. This is illustrated in Figure-1.2:

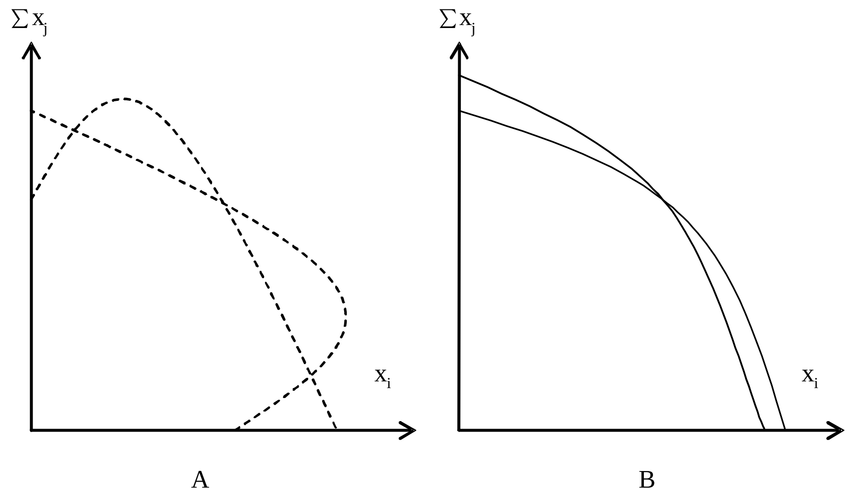

Figure-1.2: Reaction function of Firm i and Firm j

Figure-1.2 illustrates that whether the asymmetric equilibrium exists mainly depends upon the shape of the reaction function, while the shape of the function is determined by the form of the network utility function.

Proposition 1.3 *The necessary condition for the existence of an asymmetric equilibrium is $u'(0) \geqslant 1/2$.*

According to Corollary 1.1, if the effect of network externalities is strong and $u'(x)$ is greater than 1 for any $x \geqslant 0$, the equilibrium does not exist since the firms will continue to increase their output. If the network effect is medium: $u'(0) \geqslant 1/2$ and $u'(x_i) < 1$, for some x_i, multiple equilibria may exist, which is illustrated in Figure-1.2-A. If the network externalities are relatively small: $u'(x_i) < u'(0) < 1/2$, the model only has a unique symmetric equilibrium. This is because the radian of reaction function is not large enough to create an asymmetric equilibrium, which is illustrated in Figure-1.2-B. The characteristics of asymmetric equilibrium are very difficult to determine especially when we don't know the precise form of the network utility function. However, the symmetric equilibrium always exists according to the fixed point theorem and is relatively easy to calculate. The following sections will mainly consider the propositions in a symmetric equilibrium.

1.3 Merger with Network Externalities

1.3.1 Firm's Merger Incentive

We consider a market with n firms, among which $m+1$ firms intend to merge ($0 < m \leqslant n-1$). When $m = 0$, no merger occurs. When $m = n-1$, all the firms merge into one firm and the market becomes a monopoly. We define the percentage of the firms who choose to merge as $\alpha \equiv \frac{m+1}{n}$. From the previous discussion, we know multiple equilibria may exist because of the distortion of the reaction functions by non-linear network externalities. Thus, the output after merger is uncertain since the market could reach any one of the equilibria. However, In this study, we focus on the the situation in which the network effect is limited and the asymmetric equilibrium does not exist.[3] We assume that firms know, before and after a merger, they will always be located in a symmetric equilibrium.

Since all the firms are symmetric, $x_1 = x_2 = ... = x_{n-1} = x_n$. By combining this condition with equation (1.4), we can obtain:

$$1 + u(x_i) + x_i u'(x_i) - 2x_i = (n-1)x_i \quad i \in \{1, ..., n\} \tag{1.5}$$

By solving equation (1.5), we can get the equilibrium output of firms prior to the merger and we use x^* to denote the solution of equation (1.5).

Proposition 1.4 *For any network utility function in our definition, there exists and only exists one symmetric equilibrium.*

The intuition behind this proposition is very simple, especially when we refer to Figure-1.1. According Figure-1.1, the left side of equation (1.5) is monotonically decreasing and we can see the right side of equation (1.5) is a straight line which goes up from origin. These two functions must cross, but cannot cross more than once in their definition

[3] The allocation of the asymmetric equilibrium is difficult to determine when we don't know the exact form of the utility function. A more general discussion about the asymmetric equilibrium would therefore be a good future work and extension of this thesis.

area. This proposition illustrates that, for any given network utility function and the number of firms, n, firms can always set their output as x^* to reach the symmetric equilibrium and that this symmetric equilibrium is unique.

After the merger, only $n - m$ firms remain in the market. From the assumption, we know that they are still located in the symmetric equilibrium. We can use the same method to solve the equilibrium after the merger. By equation (1.5), we can get:

$$1 + u(y_i) + y_i u'(y_i) - 2y_i = (n - m - 1)y_i \quad i \in \{1, ..., n - m\} \tag{1.6}$$

We define the solution of equation (1.6) as y^*. y^* is the symmetric equilibrium output of the firms after the merger. From Proposition 1.4 we know that y^*, which always exists, is bigger than 0 and unique.

Proposition 1.5 *For any given network utility function in our definition, the symmetric equilibrium output of the firms prior to the merger are less than the symmetric equilibrium output of the firms after the merger.*

Proposition 1.5 is equivalent to the conclusion that the solution of equation (1.6), y^*, is always larger than the solution of equation (1.5), x^*. It is easy to find that the left side of equation (1.5) and (1.6) have the same shape and the right side of the equations are both straight lines. Because $m > 0$, the right side of equation (1.6) is always below the right side of equation (1.5). Since the reaction function is monotonically decreasing, it always crosses the line $(n - 1)x_i$ earlier than $(n - m - 1)y_i$. Thus, y^* is always greater than x^*. In a Cournot competition, firms will increase their output to receive more profits if the number of the competitors decreases. When the intensity of the competition is reduced with a decrease in the number of firms in the market, firms have less burden to control their output to keep the price. Although, in our model, network externalities distort the price function in traditional Cournot model, this rule never changes and will hold for any network utility function.

We denote the profits of firms prior to the merger as $\pi(n)$ and the profits of firms after the merger as $\pi(n - m)$. Salant (1983) has pointed out that one of the sufficient conditions for the firms to join the merger is they can obtain extra profits from the merger action. This sufficient condition can be shown in the following equation:

$$\pi(n-m) > (m+1)\pi(n) \tag{1.7}$$

The left side of the equation is the equilibrium profits of the merged entity and the right side is the total profits of the firms who intented to merge prior to merger action taking place. This equation suggests that the merger will occur only when the merged entity can receive more profits than the total profits they earned as an individual prior to the merger. Since we know that $\pi(n-m)$ and $\pi(n)$ are all the profits in equilibrium, we can rewrite equation (1.7) as:

$$y^*p(y^*) > n\alpha x^*p(x^*) \tag{1.8}$$

Here, $p(y^*)$ and $p(x^*)$ are the equilibrium prices in the market before and after the merger.

From equation (1.2), (1.5) and (1.6), we can get:

$$p(x^*) = 1 + u(x^*) - nx^* = x^*(1 - u'(x^*)) \tag{1.9}$$

$$p(y^*) = 1 + u(y^*) - (n + 1 - n\alpha)y^* = y^*(1 - u'(y^*)) \tag{1.10}$$

Since $y^* > x^*$ and $1 - u'(y^*) > 1 - u'(x^*)$, we have $p(y^*) > p(x^*)$.

Proposition 1.6 *For any given network utility function in our definition, the symmetric equilibrium price in the market before the merger is always less than the equilibrium price after the merger.*

Adding network externalities into the demand function will not change the fact that merger reduces the competition between firms and firms can set a higher equilibrium price after merger.

Substituting (1.9) and (1.10) into equation (1.8), we can obtain:

$$\sqrt{n\alpha} < \frac{y^*\sqrt{1 - u'(y^*)}}{x^*\sqrt{1 - u'(x^*)}} \tag{1.11}$$

equation (1.5) and (1.6) can be rewriten as:

$$x^* = \frac{1+u(x^*)+x^*u'(x^*)}{n+1} \quad \text{and} \quad y^* = \frac{1+u(y^*)+y^*u'(y^*)}{n-n\alpha+2} \tag{1.12}$$

By substituting (1.12) into (1.11), we can obtain:

$$\sqrt{\alpha}(n-n\alpha+2) < \frac{n+1}{\sqrt{n}} \frac{(1+u(y^*)+y^*u'(y^*))\sqrt{1-u'(y^*)}}{(1+u(x^*)+x^*u'(x^*))\sqrt{1-u'(x^*)}} \tag{1.13}$$

We now assume

$$\Omega \equiv \frac{(1+u(y^*)+y^*u'(y^*))\sqrt{1-u'(y^*)}}{(1+u(x^*)+x^*u'(x^*))\sqrt{1-u'(x^*)}}, t(\alpha) = \sqrt{\alpha}(n-n\alpha+2)$$

Here, Ω changes with the change of the intensity of network effect. If there is no network externality, $u(x) = 0$ and $u'(x) = 0$. Then, $\Omega = 1$ and equation (1.13) can be rewritten as:

$$t(\alpha) < \frac{n+1}{\sqrt{n}} \tag{1.14}$$

equation (1.14) is a sufficient condition for firms to merge without network externalities.

To understand equation (1.13) and (1.14), we can draw the function $t(\alpha)$ in the following figure:

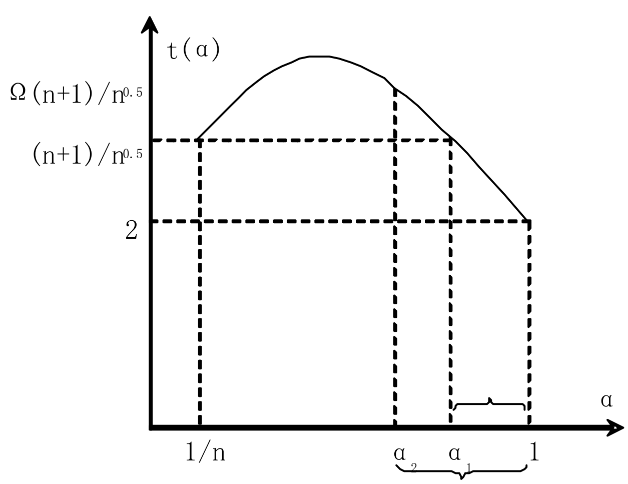

Figure-1.3: Function $t(\alpha)$

From the definition of α, we know that $\alpha \in (\frac{1}{n}, 1]$. When $\alpha = 1$, $t(\alpha) = 2$. When $\alpha = \frac{1}{n}$, $t(\alpha) = \frac{n+1}{\sqrt{n}}$. From Figure-1.3, we can see that the equation (1.14) will only hold when $\alpha \in (\alpha_1, 1]$. From the definition: $n \geqslant 2$, we can obtain $\alpha_1 > 0.8$. This result coincides with Salant's model (1983). For a given n, m and $u(.)$, we can solve equation (1.5) and (1.6) to find x^* and y^* and substitute them into Ω to create a value which reveals the intensity of the effect of network externalities. If $\Omega > 1$, the line $t(\alpha) = \Omega\frac{n+1}{\sqrt{n}}$ moves upward. Then, we can conclude $\alpha_2 > \alpha_1$ from the illustration of Figure-1.3. Since equation (1.13) holds for any $\alpha \in (\alpha_2, 1]$, an Ω, which is larger than 1, relaxes the condition for mergers to be profitable.

Proposition 1.7 *For any network utility function in our definition, the condition for the mergers to be profitable is always relaxed when we consider the effect of network externalities.*

This proposition illustrates that, for any n, m and $u(.)$, Ω is always larger than 1. This means the firms who produce network goods are always more likely to merge than the firms who produce goods without network externalities. The network externalities not only benefits the consumers but also the firms. This can be seen from the fact that, after the merger, the consumers value the same products more and the firms can charge a higher price if network externalities present. If the firms produce network products, they can obtain two aspects of benefits from the merger action. On one hand, the merger reduces the competition between the firms and push up the price in the market. On the other hand, the merger increases every firm's individual output, hence increasing the goods' network externalities and making them more attractive to the consumers. Because firms who produce network products benefit more from the merger, the requirement for the merger to be privately desirable is reduced.

In Salant's model, if we rule out the monopoly case, we need at least five firms in the market and over 80 percent of the firms to join the merger to enable the merger to be privatly profitable. In our model, this condition can be greatly relaxed. Salant et al (1983) indicates that social planner should be cautious to any merger proposal since any privately desirable merger will greatly increase the concentration of the industry. However, in our model, if the firms produce network products, the merger may occur with a relatively small change in the market structure. Moreover, the consumers have as much potential to benefit from the merger behaviour as the firms. This will be discussed in the next section.

1.3.1 Social Welfare

In a general definition, social welfare can be measured as the sum of the total profits of the firms and the total surplus of the consumers. The surplus of a consumer, who is type r, is $r + u(x_i) - p_i$. According to the equation (1.2), $p_i = 1 + u(x_i) - z$, so we can rewrite the surplus as $r + z - 1$. As we know from the previous discussion, only the consumers whose type $r > p_i - bx_i = 1 - z$ enter the market. The total surplus of the consumers can be calculated as:

$$S = \int_{1-z}^{1} (\rho + z - 1)d\rho = \frac{z^2}{2} \tag{1.15}$$

Equation (1.15) indicates that the consumer's total welfare is the function of firms' total output. And the firms' total output is determined by the number of the consumers who entered the market. If the total output increases or more consumers join the market, the consumer's welfare will increase. If the total output decreases or less consumers are willing to pay for the product, the consumer's welfare will decrease. In the equilibrium, we denote the total surplus of the consumers before the merger as $S(n)$ and the total surplus of the consumers after the merger as $S(n - m)$. Combining equation (1.5), (1.6) and (1.15), we can obtain:

$$S(n) = \frac{(nx^*)^2}{2} = \frac{(1 - x^* + u(x^*) + x^*u'(x^*))^2}{2} \tag{1.16}$$

$$S(n-m) = \frac{((n-m)y^*)^2}{2} = \frac{(1 - y^* + u(y^*) + y^*u'(y^*))^2}{2} \tag{1.17}$$

If we know the form of the utility function, we can solve the function (1.5) and (1.6) to obtain x^* and y^*. And then, we may substitute them into the above equations and compare $S(n)$ with $S(n - m)$ to get the effect of the merger to social welfare. If we don't know the details of the utility function, we can also obtain the effect of the merger to social welfare when the utility function satisfies some specific conditions.

Proposition 1.8 *If $-1+2u'(x)+xu''(x) > 0$ for any $x \in [x^*, y^*]$, the total output of the firms and the consumer's welfare increase after the merger. If $-1+2u'(x)+xu''(x) < 0$ for any $x \in [x^*, y^*]$, the total output of the firms and the consumer's welfare decreases after the merger.*

If $-1+2u'(x)+xu''(x) > 0$ for any $x \in [x^*, y^*]$, we have a relatively large $u'(x)$ and small $u''(x)$ in the interval $[x^*, y^*]$. According to the economic interpretation of the first and second derivatives of the network utility function, the first part of this proposition illustrates that, no matter how small the network externality, the merger will always increase total output and consumer's welfare as long as the network externality increases very quickly and this increasing trend is persistent. The second part of the proposition illustrates that the merger will decrease consumer's welfare if the network utility function is relatively flat and the trend of increasing drops very quickly with the increase of x, regardless of the absolute value of the network externalities.

Corollary 1.3 *If $u'(x^*) < \frac{1}{2}$, the consumer's welfare decreases after the merger.*

Corollary 1.4 *If $u'(0) < \frac{1}{2}$, the consumer's welfare decrease after the merger.*

Corollary 1.3 tells us that if we want to increase the consumer's welfare, we must have a relatively small equilibrium output prior to the merger to make $u'(x^*) \geqslant \frac{1}{2}$. Since we know that the equilibrium output decreases with the increase in the number of the firms in the market, this corollary may also indicate that the smaller the number of the firms in the industry, the less likely that the merger increases consumer's welfare and total output. Corollary 1.4 is a more strict, but simple condition for us to identify which network utility function will decrease the total output and consumer's welfare.

From (1.3), (1.5), (1.6), (1.9) and (1.10), we can obtain the total profits of firms in the equilibrium before and after the merger as:

$$\Sigma\pi_{\text{ex-ante}} = nx^*p(x^*) = (1 - x^* + u(x^*) + x^*u'(x^*))x^*(1 - u'(x^*)) \tag{1.18}$$

$$\Sigma\pi_{\text{ex-post}} = (n - m)y^*p(y^*) = (1 - y^* + u(y^*) + y^*u'(y^*))y^*(1 - u'(y^*)) \tag{1.19}$$

From Proposition 1.6, we know that $p(y^*) > p(x^*)$, so the total profits of the firms increase after merger if $(n - m)y^* > nx^*$. $(n - m)y^* > nx^*$ indicates that the total output increases after the merger or the first part of the Proposition 1.8 holds.

It is also easy to know that the firms who stand outside merger will benefit more from the merger action than the firms who join the merger. Thus, we can also get the conclusion that if a merger is privately profitable (make equation (1.13) hold), this merger must increase the total profits of all the firms.

More generally, we have the following proposition:

Proposition 1.9 *For any network utility function in our definition, the total profits of the firms always increase after the merger.*

This proposition indicates that, no matter how many firms join the merger or what the form of the network utility function is, the total profits of the firms always increase with the decrease of the number of the firms in the market. A merger can only exist (privately profitable) when enough firms to join the merger. However, in a social planner's position, any mergers will definitely be profitable for firms as a whole. This proposition is true in both the traditional Cournot competition and the competition with network externalities.

Now we stand at the position of social planner to calculate the effect of the merger to total social welfare. We denote the total social welfare prior to the merger as $W(n)$ and the total social welfare after the merger as $W(n - m)$. Combining equation (1.16), (1.17), (1.18) and (1.19), we can obtain that:

$$W(n) = S(n) + \Sigma\pi_{\text{ex-ante}} = \frac{(1 + u(x^*))^2 - (x^*(1 - u'(x^*)))^2}{2} \tag{1.20}$$

$$W(n-m) = S(n-m) + \Sigma\pi_{\text{ex-post}} = \frac{(1+u(y^*))^2 - (y^*(1-u'(y^*)))^2}{2} \tag{1.21}$$

When we have enough information about the market, for example we know m, n and the form of the network utility function, the standard way for the social planner to determine whether the merger increases the total social welfare or not is: firstly solving the equation (1.5) and (1.6) to obtain x^* and y^*, then substituting them into (1.20) and (1.21) and finally comparing $W(n)$ with $W(n-m)$. If we cannot obtain the full information about the market and products, we can still identify the merger which increase the social welfare with the following proposition:

Proposition 1.10 *If $W'(x) > 0$ for any x in our definition area, the total social welfare increases after the merger. If $W'(x) < 0$ for any x in our definition area, the total social welfare decreases after the merger.*

Here, $W(x)$ is a continuous and differentiable function defined as:

$$W(x) = \frac{(1+u(x))^2 - (x(1-u'(x)))^2}{2} \tag{1.22}$$

and we can get:

$$W'(x) = (1+u(x))u'(x) - x(1-u'(x))(1-u'(x)-xu''(x)) \tag{1.23}$$

Since $y^* > x^*$, $W(n-m)$ is larger than $W(n)$ when $W'(x) > 0$ and is less than $W(n)$ when $W'(x) < 0$. This indicates that, if the social planner finds the utility function satisfies $W'(x) > 0$ for any x, they should be glad to boost the merger activities since any merger can increase the social welfare. One of the typical cases for this situation is $u(x) = bx$ with $b > \frac{1}{2}$.

If we do not consider the synergies, the social welfare will never increase after the merger in a world without network externalties. This can get proved by deleting $u(.)$ from the equation (1.23). If there is no network effect, $W'(x) = -x$, which is negative for any x in definition area.

Corollary 1.5 *If* $-1 + 2u'(x) + xu''(x) > 0$ *for any* $x \in [x^*, y^*]$*, the total social welfare increases after the merger.*

Corollary 1.6 *If* $-1 + 2u'(x) + xu''(x) > 1 - \frac{1}{1-u'(x)}$ *for any* $x \in [x^*, y^*]$*, the total social welfare increases after the merger.*

The form and meaning of equation (1.23) is difficult to understand. However, Corollary 1.5 gives us a stricter but more explainable condition. From Proposition 1.9, we know that the total profits of the firms increase after the merger. Because, in our definition, social welfare equals to the sum of the total profits of the firms and the consumer's total surplus, social welfare will definitely increases after the merger if the merger increases the consumer's welfare. This suggests that the social planner should pay attention to the consumer's welfare first. If they can make sure the merger benefits the consumers, they can make the conclusion that the merger is a "good" merger without further investigation.

Another interesting question is whether there exists any merger which decreases the consumer's welfare, but increase the total social welfare? Corollary 1.6 provides us a more relaxed condition than Corollary 1.5. Since we know that $1 > 1 - u'(x) > 0$ for any $x \in [x^*, y^*]$, $1 - \frac{1}{1-u'(x)}$ is always a negative number. Thus, Corollary 1.6 indicates that there may exist some network utility function which makes the consumer's welfare decrease, but social welfare increase after the merger.

1.4 Merger with Linear Network Utility Function

1.4.1 The Existence of a "Good" Merger

Although we developed some general rules for the social planner to determine whether the merger will increase social welfare or not in the previous sections, these propositions are all sufficient conditions. If we want to obtain the sufficient and necessary conditions, we need to solve equation (1.5) and (1.6) and get the absolute value of x^* and y^*, which will enable us to give a precise comparison about the social welfare before and after the merger.

To make x^* and y^* solvable, we introduce a linear network utility function $u(x) = bx$ instead of the general utility function. Another advantage of the linear utility function is it is very easy for us to measure the intensity of the network externality. For a general form of the network utility function, it is hard to compare the intensity of the network effect between different products, since the relationship may change with the change of the network size. If we use a linear utility function, the second derivative of the function is a constant number b, which can always be seen as the indicator of the intensity of the network effect and will not change with the size of the network. In the following context, we may use b to indicate a more precise relationship between the intensity of the network effect with the merger behaviour of the firms.

We define $u(x) = bx$, $0 < b < 1$. Since $u'(x) = b < 1$ for any x, our assumption does not violate Proposition 1.1. By substituting the linear utility function into equation (1.3) and (1.4), we obtain:

$$\pi_i = p_i x_i = x_i(1 + bx_i - z) \quad i \in \{1, ..., n\}$$

$$1 + 2bx_i - 2x_i = \sum_{i \neq j} x_j \qquad i \in \{1, ..., n\} \tag{1.24}$$

Since we only consider the symmetric equilibrium, we have $x_1 = x_2 = ... = x_{n-1} = x_n$. By solving equation (1.24), we can obtain the unique symmetric equilibrium output of the firms as:

$$x^* = x_i = \frac{1}{n+1-2b}$$

By substituting x^* into equation (1.9), we can get the equilibrium price in the market as:

$$p(x^*) = \frac{1-b}{n+1-2b}$$

Thus, each firm's profit is:

$$\pi(n) = xp = \frac{1-b}{(n+1-2b)^2}$$

After the merger, the total number of the firms in the market is $n-m$. Since the procedure to solve the equilibrium after the merger is identical to the procedure to solve the equilibrium prior to the merger, we only need to change n with $n-m$ to get y^*, $p(x^*)$ and $\pi(n-m)$. If we substitute these results into equation (1.7), we can obtain the condition for the merger to be privately profitable as:

$$\pi(n-m) - (m+1)\pi(n) = -\frac{m(1-b)(n\alpha - A)(n\alpha - B)}{(n-m+1-2b)^2(n+1-2b)^2} > 0 \tag{1.25}$$

Here we use A and B to denote the following equations:

$$A = \frac{2n+3-4b-\sqrt{4n+5-8b}}{2}, B = \frac{2n+3-4b+\sqrt{4n+5-8b}}{2}$$

Proposition 1.11 *The sufficient and necessary condition for the merger to be privately profitable is the proportion of the firms joining the merger is larger than $\frac{A}{n}$.*

This proposition indicates that if the merger is privately profitable, the proportion of the firms joining the merger must be large enough. If we go back to Figure-1.3, we will find that $\frac{A}{n}$ is equal to α_2 in a linear network world. If there is no network effect ($b = 0$), $\frac{A}{n} = \frac{2n+3-\sqrt{4n+5}}{2n}$, which coincides with the result in Salant et al. (1983) 's paper. We can also tell that $\partial A/\partial b < 0$ for any $n \geqslant 2$, which suggests A goes smaller with an increase of b. Since b is the intensity of network effect and $\alpha_2 = \frac{A}{n}$, we may conclude that the condition for the merger to be privately desirable will be relaxed additionally if these is a stronger network effect. Hence, firms may be willing to merge with just a little increase of the industry concentration in a market with sufficient strong network effect.

By substituting x^*, y^* and $u(x) = bx$ into equation (1.20) and (1.21), we can get:

$$W(n-m) - W(n) = \frac{(4b^2 - 2bn - 4b + 1)(n\alpha - C)}{2(n - n\alpha + 2 - 2b)^2(n + 1 - 2b)^2} > 0$$

Here, for ease of notation, we use C to denote the the following equation:

$$C = \frac{2n - 14b - 10bn + 20b^2 - 8b^3 - 2bn^2 + 8b^2n + 3}{4b^2 - 2bn - 4b + 1}$$

Proposition 1.12 *The sufficient and necessary condition for the existence of a merger, which increases social welfare, is* $4b^2 - 2bn - 4b + 1 < 0$ *and the proportion of the firms joining the merger is less than* $\frac{C}{n}$.

This proposition provides an easy way for the social planner to judge whether the potential merger increases social welfare if the network utility function is linear. First, the social planner need to know how strong the network effect (the value of b) is and how concentrated the industry is (the value of n). If the network effect is very weak, the merger, which increases social welfare, can only exsits in a relatively less concentrated market. If the industry is highly concentrated, a relatively large network effect is a must to ensure these exsits a socially desirable merger. If n and b make $4b^2 - 2bn - 4b + 1 \geqslant 0$, social planner should block all the merger application. It is because, in this market condition, any merger will definitely decrease social welfare, no matter how many firms join the merger. If $4b^2 - 2bn - 4b + 1 < 0$, only the merger with proportion α less than $\frac{C}{n}$ is a "good" merger. The social planner needs to compare the proportion of the firms intending to merge with $\frac{C}{n}$ to determine whether the merger should be approved.

Proposition 1.13 *There exists a merger which is privately profitable and socially desirable if and only if* $n > \frac{(2b-1)^2(2b+1)}{4b^2}$.

From Proposition 1.11, we know that if a merger is privately profitable, the proportion of the firms joining the merger should be greater than $\frac{A}{n}$. The firms always want the market goes to more concentrated, since they may charge higher price to the consumers. Proposition 1.12 tells us that if the social planner wants the merger to increases social welfare, the proportion of the firms joining the merger should be limited to below a specific number. This is because if the proportion of the firms joining the merger is larger than $\frac{C}{n}$, all the benefit from the merger (additional network externalities) will offset by the harm from reduction of the competition in the market. Thus, a socially desirable merger must be a merger with gently increasing of market concentration. Proposition 1.13 provides us with a sufficient and necessary condition to determine whether there exists an intersection between Proposition 1.11 and Proposition 1.12. Only the merger which is located in this intersection is a "good" merger which can be realized. To obtain more intuition into Proposition 1.13, we can draw the condition in Proposition 1.13 in the following figure:

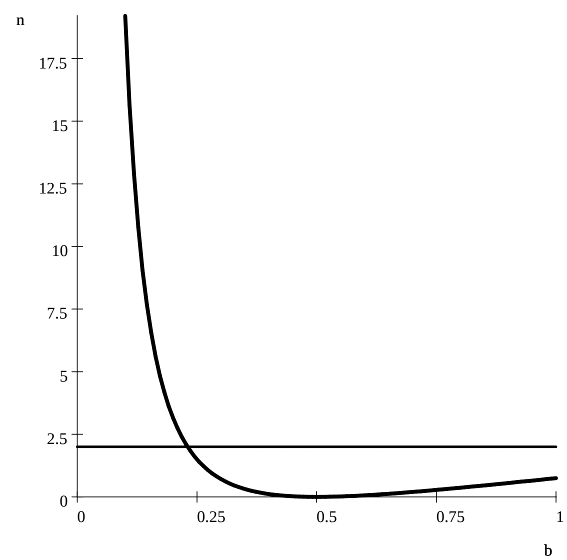

Figure-1.4: $n > \frac{(2b-1)^2(2b+1)}{4b^2}$

The inequality $n > \frac{(2b-1)^2(2b+1)}{4b^2}$ is illustrated in Figure-1.4, where we can see: when b is relatively small, a large n is needed to keep the inequation existing and when b is relatively large, the requirement for n to make the inequality hold is not very strict. Proposition 1.13 indicates that the existence of the intersection between Proposition 1.11 and Proposition 1.12 depends upon the concentration of the industry and the characteristics of the products. In an over-concentrated industry with a relatively weak network effect, there will be less possibility to have a "good" merger. But if the number of the firms in the industry is relatively large and the network effect of the products is strong, there will be a better chance of having a privately profitable and socially optimal merger, or a "good" merger.

Corollary 1.7 *If $b > 0.23$, any potential merger has the possibility to benefit both the firms and society if a suitable proportion of the firms join the merger.*

This Corollary is also illustrated in Figure-1.4. The horizontal line in Figure-1.4 is $n = 2$. Since we know that $n \geqslant 2$, from the figure, we can conclude that, when b is larger than a certain number, any n in our definition will make $n > \frac{(2b-1)^2(2b+1)}{4b^2}$. This provides us a more straight condition: when b is larger than 0.23, there will always exist a "good merger". The only thing that social planner needs to do to realize this "good" merger is to control the number of the firms joining the merger.

1.4.1 Chooseing a Suitable Number of Firms to Join the Merger

According to the definition, $m + 1$ denotes the number of firms who join the merger. From the previous discussions, we know that, if social planner knows that a "good" merger exists in the industry, the next work is to control the proportion, or the number, of the firms joining the merger to make sure that the "good" merger is realized.

Proposition 1.14 *With the condition that $n > \frac{(2b-1)^2(2b+1)}{4b^2}$, if $C \geqslant n$, any m larger than $\frac{2n+1-4b-\sqrt{4n+5-8b}}{2}$ is a suitable m which makes the merger both privately profitable and socially optimal, and if $C < n$, any m which is between $\frac{2n+1-4b-\sqrt{4n+5-8b}}{2}$ and $\frac{(n+1-2b)(4b^2-2bn-6b+2)}{4b^2-2bn-4b+1}$ is a suitable m which can make the merger both privately profitable and socially optimal.*

Corollary 1.8 *If $b > 0.26$, m is only bounded by privately profitable restriction. Any m which is larger than $\frac{2n+1-4b-\sqrt{4n+5-8b}}{2}$ is a suitable m which can make the merger both privately profitable and socially optimal.*

Corollary 1.9 *If $b < 0.19$, m is bounded on two sides. Any m which is between $\frac{2n+1-4b-\sqrt{4n+5-8b}}{2}$ and $\frac{(n+1-2b)(4b^2-2bn-6b+2)}{4b^2-2bn-4b+1}$ is a suitable m which can make the merger both privately profitable and socially optimal.*

If the social planner knows how concentrated the industry is and the intensity of network effect of the products, Proposition 1.14 provides us a criterion to evaluate whether the potential merger is good merger or not. Since it is difficult to understand the intuition behind the relationship between C and n, we can only get very little information from this proposition. However, Corollary 1.8 and 1.9 are more intuitionistic. They illustrate that when the network externalities are very large ($b > 0.26$), the social planner doesn't need to set any restrictions to the merger, since all of the mergers are socially optimal. Therefore, social planner should encourage the firms to merge and boost a dominator for the industry. However, when the network externalities are relatively small ($b < 0.19$), the number of firms who join the merger must be chosen carefully. If m is too large, which means too many firms join the merger, the social welfare will be harmed; if m is very small, the merger is not attractive to the firms anymore.

1.5 Conclusions and Future Work

The network effect distorts the reaction function of the firms in Cournot competition, but this distortion does not change some of the basic characteristics of the reaction function, such as monotonicity. The distortion provides a possibility of the existence of multiple equilibria, which requires a relatively strong network effect. However, extremely large network effect results in a situation in which no equilibrium exists. This is because firms will never stop increasing their outputs if the benefit from network effect can always offset the price losses from producing an additional product. In this study, we limit our discussion to an industry with a relatively week network effect, so the multiple equilibria are ruled out. Although the conclusions in this chapter are only valid for a section of all the network utility functions, it still provides us with some information of how the network effect changes the firm's behaviour in the competition and merger choice and gives us some hints for the study of a more general result.

If we only consider the symmetric equilibrium, the network effect will make firms more zealous in merger activities, compared with the conclusion of Salant's model. Consequently, a merger with only a relatively small proportion of all the firms getting involved, which is not possible in Salant's model, may occur in our model. Merger between the firms, who produce network products, can bring some level of benefit to consumers through network externalities, so there exists a merger which increases social welfare without the consideration of supply-side economics of scale.

A linear network utility function will greatly reduce the calculations and bring some convenience to the denoting of the intensity of the network effect. With the help of the linear network utility function, we find that, in some mergers, the profit of the firms and the social welfare are not always contradictive. If the network externality is very strong or the market is not highly concentrated yet, it is possible to exist a "good" merger which is both privately profitable and socially optimal. Moreover, if the network externality is large enough, the social planner doesn't need to set any restrictions to the merger behaviour since all the mergers which are privately profitable will be socially optimal as well. However, if the network externality is relatively weak, the social planner should set some restrictions to the number of the firms joining the merger in order to make the "good" merger be realized. We recommend that a more general discussion about the situation in which the multiple equilibria exist be conducted in the future.

1.6 Appendix

A1.1 Proof of Proposition 1.1

Assume x_i maximize the profit of firm i for a given total output of all other firms. From the first derivative of equation (1.2), we can get:

$$\frac{\partial p_i}{\partial x_i} = u'(x_i) - 1 \qquad (x_i \geqslant 0)$$

If $u'(x_i) \geqslant 1$, we have $\partial p_i/\partial x_i \geqslant 0$. Assume x_i' is slightly larger than x_i, then we have $p_i' = 1 + u(x_i') - x_i' - \sum x_j \geqslant p_i$. Thus, $\pi_i' = p_i' x_i' > p_i x_i = \pi_i$ and x_i cannot maximize the profit of firm i, which contradicts with our assumption. Hence, we may conclude that if x_i maximize the profit of firm i for a given total output of all other firms, there must have $u'(x_i) < 1$.

A1.2 Proof of Proposition 1.2

Equation (1.4) is the reaction function of firm i against the total output of all other firms. We assume

$$g(x_i) = \sum_{i \neq j} x_j = 1 + u(x_i) + x_i u'(x_i) - 2x_i \tag{1.26}$$

$$\Rightarrow g'(x_i) = u'(x_i) + u'(x_i) + xu''(x_i) - 2 = 2(u'(x_i) - 1) + x_i u''(x_i) \tag{1.27}$$

Here x_i is always the best response for a given $\sum x_j$. From Proposition 1.1, we have $u'(x_i) - 1 < 0$. By the definition, we have $u''(x_i) < 0$ and $x_i > 0$, so it is easy to know $g'(x_i) < 0$. Hence, we may conclude that the reaction function $g(x)$ is monotonically decreasing.

A1.3 Proof of Proposition 1.3

Before we begin the proof, we need to prove the following lemma first.

Lemma 1.1 *For any two functions, if the slope of one of the functions is always larger than other's, these two functions have no more than one crossing point.*

Proof. Assume functions $m(x)$ and $n(x)$ have two crossing point $x1$, $x2$ ($x1 \neq x2$) and $q(x) = m(x) - n(x)$. Since $m(x1) = n(x1)$ and $m(x2) = n(x2)$, $q(x1) = 0$ and $q(x2) = 0$. $q'(x) = m'(x) - n'(x)$. If $m'(x)$ is always bigger than $n'(x)$, $q'(x) > 0$ for any given x. This means $q(x)$ is monotonic increasing and there cannot exist two different value: $x1$ and $x2$ which let $q(x1)$ and $q(x2)$ equal to zero at the same time. So the function $m(x)$ and $n(x)$ cannot have more than one crossing point. ■

Proof of Proposition 1.3:

From the equation (1.27)

$$g'(x_i) = 2(u'(x_i) - 1) + x_i u''(x_i)$$

From the definition $x_i \geqslant 0$ and $u''(x) < 0$, if $u'(0) < 1/2$, $u'(x_i) \leqslant u'(0) < 0.5$. Thus, we have $g'(x_i) < 2(0.5 - 1) + 0 = -1$. Firm i's competitor's reaction is just the inverse of firm i's reaction function against to 45 degree line. If the slope of firm i's reaction function is less than -1 for any x_i, his competitor's reaction function will always bigger than -1. From Lemma 1.1, we know that firm i and his competitor's reaction function will have no more than one crossing point. This means there does not exist more than one equilibrium. Since the symmetric equilibrium is always exist according to the fixed point theorem. If there exist asymmetric equilibrium, we must have $u'(0) \geqslant 1/2$.

A1.4 Proof of Proposition 1.4

What we need to do is to prove that equation (1.5) only has a unique solution. Assume there exist two solution, x_1 and x_2 for equation (1.5) and $x_1 > x_2 \geqslant 0$. we have:

$$g(x_1) = 1 + u(x_1) + x_1 u'(x_1) - 2x_1 = (n-1)x_1 \tag{1.28}$$

$$g(x_2) = 1 + u(x_2) + x_2 u'(x_2) - 2x_2 = (n-1)x_2 \tag{1.29}$$

From the proof of Proposition 1.2, we know that $g(x)$ is a monotonically decreasing function, so $g(x_1) < g(x_2)$. Since $n \geqslant 2$, $(n-1)x_1 > (n-1)x_2$. This makes (1.28) contradict with (1.29). Thus, we may conclude that there is no more than one solution for equation (1.5). We can also rewrite equation (1.5) as:

$$1 + u(x_i) + x_i u'(x_i) - nx_i = x_i \quad i \in \{1, ..., n\} \tag{1.30}$$

Since the left part of (1.30) is continuous and differentiable function, equation (1.5) have more than one solution according to the fixed point theorem. Thus equation (1.5) have and only have one solution and the market exist and only exist one symmetric equilibrium.

A1.5 Proof of Proposition 1.5

Proving Proposition 1.5 equals to prove the solution of equation (1.6), y^*, is bigger than the solution of equation (1.5), x^*. We may define a function:

$$h(x) = 1 + u(x) + xu'(x) - (n+1)x$$

By the definition of the network utility function, we get $h(0) = 1 + u(0) + 0u'(0) - (n+1)0 = 1$. As we know that y^* is the solution of equation (1.6), we have

$$h(y^*) = 1 + u(y^*) + y^* u'(y^*) - (n+1)y^* = -my^* < 0$$

Since $h(x)$ is a continuous function, $h(0) > 0$ and $h(y^*) < 0$, there must exist a $\epsilon \in (0, y^*)$ which makes $h(\epsilon) = 0$.

$$h(\epsilon) = 0 \Longleftrightarrow 1 + u(\epsilon) + \epsilon u'(\epsilon) - (n+1)\epsilon = 0 \tag{1.31}$$

Obviously, equation (1.31) has a same form as equation (1.5), From Proposition 1.4, we know that equation (1.31) has and only has unique solution $\epsilon = x^*$. Since $\epsilon \in (0, y^*)$, we must have $x^* \in (0, y^*)$ or we can say $x^* < y^*$.

A1.6 Proof of Proposition 1.7

We need to prove the following lemma first:

Lemma 1.2 *For any $x \in [x^*, y^*]$ and utility function in our definition,* $1 + u(x) + xu'(x) - 2x \geqslant 0$.

Proof. From (1.26) and (1.27), we know that $g(x) = 1 + u(x) + xu'(x) - 2x$ and $g'(x) = 2(u'(x) - 1) + xu''(x)$. From Corollary 1.2, we know $u'(x^*) < 1$. Since $u''(x) \leqslant 0$, for any $x \geqslant x^*$, we have $u'(x) < 1$, which indicates for any $x \in [x^*, y^*]$, $u'(x) < 1$. So $g'(x) < 0$ for any $x \in [x^*, y^*]$. The minimum of $g(x)$ is $g(y^*)$ if x is chosen from $[x^*, y^*]$. Since y^* is the solution of equation (1.6), we have:

$$g(y^*) = 1 + u(y^*) + y^*u'(y^*) - 2y^* = (n - m - 1)y^* \geqslant 0$$

$g(y^*) = 0$ if and only if $m = n - 1$ which means the merger creates monopoly in the market. So for any $x \in [x^*, y^*]$, $g(x)$ is non-negative. ■

Proof of Proposition 1.7:

We may define a function:

$$k(x) = (1 + u(x) + xu'(x))\sqrt{1 - u'(x)} \qquad x \in [x^*, y^*]$$

As we know from Proof of the Lemma 1.2, $u'(x) < 1$ for any $x \in [x^*, y^*]$, $\sqrt{1 - u'(x)}$ always has a real value for $x \in [x^*, y^*]$. From the definition of the network utility function ($u(x) > 0$, $x > 0$ and $u'(x) > 0$), we know $k(x) > 0$ for any $x \in [x^*, y^*]$ and $k(x)$ is a continuous and differentiable function. The first derivative of function $k(x)$ is:

$$k'(x) = \frac{4u'(x)(1 - u'(x)) - (g(x) + 2xu'(x))u''(x)}{2\sqrt{1 - u'(x)}}$$

From Lemma 1.2, we know that $g(x) \geqslant 0$ for any $x \in [x^*, y^*]$ and we also know that $u(x) > 0$, $x > 0$, $1 - u'(x) > 0$ and $u''(x) < 0$. So it is easy to see that $k'(x) > 0$ for any $x \in [x^*, y^*]$. We can then obtain:

$$\begin{aligned} k(y^*) \quad > \quad & k(x^*) \Longrightarrow \frac{k(y^*)}{k(x^*)} > 1 \\ \Longleftrightarrow \quad & \frac{(1 + u(y^*) + y^*u'(y^*))\sqrt{1 - u'(y^*)}}{(1 + u(x^*) + x^*u'(x^*))\sqrt{1 - u'(x^*)}} = \Omega > 1 \end{aligned}$$

for any network utility function in our definition and any $x \in [x^*, y^*]$.

A1.7 Proof of Proposition 1.8

We can define the total output in the equilibrium as a continuous and differentiable function:

$$\begin{aligned} l(x) &= 1 - x + u(x) + xu'(x) \Longrightarrow \\ l'(x) &= -1 + 2u'(x) + xu''(x) \end{aligned}$$

If $-1 + 2u'(x) + xu''(x) > 0$ for any $x \in [x^*, y^*]$, we have $l'(x) > 0$ and $l(y^*) > l(x^*)$. So we can get:

$$l(y^*) = (n - m)y^* > l(x^*) = nx^*$$

A1.8 Proof of Corollary 1.3 and 1.4

By the definition of the network utility function: $u''(x) < 0$ and

$$-1 + 2u'(x) + xu''(x) = xu''(x) - 2(\frac{1}{2} - u'(x))$$

we can indicate that $-1 + 2u'(x) + xu''(x) < 0$ if $u'(x) < \frac{1}{2}$. We also know that $u'(x) \leqslant u'(x^*) < u'(0)$ for any $x \in [x^*, y^*]$. So if we have $u'(x^*) < \frac{1}{2}$ or $u'(0) < \frac{1}{2}$, we may conclude that $u'(x) < \frac{1}{2}$ for any $x \in [x^*, y^*]$. According to Proposition 1.8, these two conditions are also the sufficient condition for merger to decrease the consumer's welfare and total output.

A1.9 Proof of Proposition 1.9

We define a continuous and differentiable function:

$$\begin{aligned} v(x) &= (1 - x + u(x) + xu'(x))x(1 - u'(x)) \\ v'(x) &= (g(x) + xu'(x))(1 - u'(x) - xu''(x)) + (1 - u'(x))xu'(x) \end{aligned}$$

From Lemma 1.2, Corollary 1.2 and the definition of the network utility function, we know that $g(x) > 0$, $1 - u'(x) > 0$, $u''(x) < 0$ and $x > 0$ for any $x \in [x^*, y^*]$. So $v'(x) > 0$ for any $x \in [x^*, y^*]$. We can then make the conclusion that:

$$v(x^*) = \Sigma\pi_{\text{ex-ante}} < v(y^*) = \Sigma\pi_{\text{ex-post}}$$

for any network utility function and equilibrium output.

A1.10 Proof of Corollary 1.6

The condition $W'(x) > 0$ can be rewrite as:

$$g(x)u'(x) + x\{1 - [1 - u'(x)][2 - 2u'(x) - xu''(x)]\} > 0 \tag{1.32}$$

From Lemma 1.2, we know that $g(x) > 0$ for any $x \in [x^*, y^*]$. So if we want the equation (1.32) holds for any $x \in [x^*, y^*]$, we only need:

$$1 - [1 - u'(x)][2 - 2u'(x) - xu''(x)] > 0$$

$$\Longleftrightarrow -1 + 2u'(x) + xu''(x) > 1 - \frac{1}{1 - u'(x)}$$

holds for any $x \in [x^*, y^*]$.

A1.11 Proof of Proposition 1.11

The proof of this proposition is equivalent to prove that the sufficient and necessary condition for $\pi(n-m) - (m+1)\pi(n)$ to be larger than 0 is $n\alpha - A > 0$. Since we know from the definition that $m > 0$, $1 - b > 0$, if we can prove that $n\alpha - B$ is negative, we can then get the conclusion that $\pi(n-m) - (m+1)\pi(n)$ is a positive number if and only if $n\alpha - A > 0$. From the definition of B:

$$\begin{aligned} \frac{B}{n} &= \frac{2n + 3 - 4b + \sqrt{4n + 5 - 8b}}{2n} \\ &= 1 + \frac{3 - 4b + \sqrt{4n + 5 - 8b}}{2n} > 1 + \frac{3 - 4 + \sqrt{8 + 5 - 8}}{4} > 1 \end{aligned}$$

So $B > n$ for any $0 < b < 1$ and $n \geqslant 2$. Since $\alpha < 1$, we can say $n\alpha - B < 0$. From the definition of A:

$$\frac{A}{n} = \frac{(\sqrt{4n + 5 - 8b} - 1)^2}{4n} < \frac{(\sqrt{4n + 5} - 1)^2}{4n}$$

As we know that $n \geqslant 2$, we can construct the following inequation:

$$
\begin{aligned}
0 &> 6-2\sqrt{13} \geqslant 6-2\sqrt{4n+5} \\
&\iff 4n+5-2\sqrt{4n+5}+1 < 4n \iff \frac{(\sqrt{4n+5}-1)^2}{4n} < 1
\end{aligned}
$$

This shows that $\frac{A}{n} < 1$ for any $0 < b < 1$ and $n \geqslant 2$. Thus, we can make sure there always exists a $\alpha \in (\frac{1}{n}, 1)$ which let $n\alpha - A > 0$.

A1.12 Proof of Proposition 1.12

If $4b^2 - 2bn - 4b + 1 > 0$, we can solve this inequality and obtain:

$$
b < \frac{n+2-\sqrt{(n+2)^2-4}}{4} \text{ or } b > \frac{n+2+\sqrt{(n+2)^2-4}}{4}
$$

Obviously, $b \ngtr \frac{n+2+\sqrt{(n+2)^2-4}}{4}$,since $n \geqslant 2$ and $0 < b < 1$. If $b < \frac{n+2-\sqrt{(n+2)^2-4}}{4}$, we can get $b < 0.134$ since $n \geqslant 2$. By rewriting $4b^2 - 2bn - 4b + 1 > 0$, we can obtain:

$$
\begin{aligned}
2bn &< 4b^2 - 4b + 1 = (2b-1)^2 < 1 \\
&\Longrightarrow 8b^3 + 6nb + 14b < 5 < n + 3 + (20+4n)b^2 \\
&\Longrightarrow -8b^3 + (20+4n)b^2 + (-6n-14)b + n + 3 > 0 \\
&\Longrightarrow 2n - 14b - 10bn + 20b^2 - 8b^3 - 2bn^2 + 8b^2n + 3 \\
&> n(4b^2 - 2bn - 4b + 1) \qquad (1.33)
\end{aligned}
$$

Since we know $4b^2 - 2bn - 4b + 1 > 0$, from (1.33), we can get:

$$
\frac{2n - 14b - 10bn + 20b^2 - 8b^3 - 2bn^2 + 8b^2n + 3}{4b^2 - 2bn - 4b + 1} > n \Longrightarrow C > n
$$

Since $4b^2 - 2bn - 4b + 1 > 0$, the condition to make $W(n-m) - W(n) > 0$ is $n\alpha - C > 0 \iff \alpha > \frac{C}{n} > 1$. But by the definition, we know $\alpha \leqslant 1$, so we can conclude that there does not exist a merger which can increase the social welfare if $4b^2 - 2bn - 4b + 1 > 0$.

If $4b^2 - 2bn - 4b + 1 = 0$, we can obtain:

$$
\begin{aligned}
& 2n - 14b - 10bn + 20b^2 - 8b^3 - 2bn^2 + 8b^2n + 3 \\
= & -2b(4b^2 - 2bn - 4b + 1) + (n+3)(4b^2 - 2bn - 4b + 1) + n = n > 0
\end{aligned}
$$

So $W(n-m) - W(n) < 0$. There does not exist a merger which can increase the social welfare either.

If $4b^2 - 2bn - 4b + 1 < 0$, the sufficient and necessary condition for a merge to increase social welfare can be written as: $\alpha < \frac{C}{n}$.

A1.13 Proof of Proposition 1.13

From Proposition 1.11, we know that if we want the merger to be privately profitable, we need $\alpha > \frac{A}{n}$. From Proposition 1.12, we know that if we want the merger to be social desirable, we need $4b^2 - 2bn - 4b + 1 < 0$ and $\alpha < \frac{C}{n}$. By Combining these two conditions together, we can obtain the sufficient and necessary condition that there exists a merge which can benefit not only the firms but also the whole society is: (i) $4b^2 - 2bn - 4b + 1 < 0$, (ii) $A < C$. If an industry system can satisfy these two condition, there must exist an α, which is between $\frac{A}{n}$ and $\frac{C}{n}$, can satisfy Proposition 1.11 and 1.12 at the same time. From (i), we can get: $n > \frac{(2b-1)^2}{2b}$. From (ii), we can get:

$$\frac{2n + 3 - 4b - \sqrt{4n + 5 - 8b}}{2} < \frac{2n - 14b - 10bn + 20b^2 - 8b^3 - 2bn^2 + 8b^2n + 3}{4b^2 - 2bn - 4b + 1} \tag{1.34}$$

Since we have condition (i), (1.34) can be rewriten as:

$$\sqrt{4n - 8b + 5} > \frac{2n}{-(4b^2 - 2bn - 4b + 1)} - 3 \tag{1.35}$$

If $\frac{2n}{-(4b^2-2bn-4b+1)} - 3 \leqslant 0$, we have $A < C$.

if $\frac{2n}{-(4b^2-2bn-4b+1)} - 3 > 0$, (1.35) can be written as:

$$4n - 8b + 5 > \left(\frac{2n}{-(4b^2 - 2bn - 4b + 1)} - 3\right)^2$$
$$\iff n > \frac{8b^3 - 4b^2 - 2b + 1}{4b^2} = \frac{(2b-1)^2(2b+1)}{4b^2}$$

Since $n > \frac{(2b-1)^2}{2b} + \frac{(2b-1)^2}{4b^2} > \frac{(2b-1)^2}{2b}$, the sufficient and necessary condition of $(i) + (ii)$ can be simplified as: $n > \frac{(2b-1)^2(2b+1)}{4b^2}$.

A1.14 Proof of Proposition 1.14

If $C \geqslant n$, the restriction $\frac{A}{n} < \alpha < \frac{C}{n}$ can be rewritten as $\frac{A}{n} < \alpha \leqslant 1$. This means m isn't bounded by the social welfare restriction.

$$\begin{aligned} \frac{A}{n} &< \alpha \leqslant 1 \Leftrightarrow \frac{A}{n} < \frac{m+1}{n} \leqslant 1 \Leftrightarrow A-1 < m \leqslant n-1 \\ &\Leftrightarrow \frac{2n+1-4b-\sqrt{4n+5-8b}}{2} < m \leqslant n-1 \end{aligned}$$

If $C < n$, m is bounded by both privately profitable restriction and social optimal restriction.

$$\begin{aligned} \frac{A}{n} &< \alpha < \frac{C}{n} \Leftrightarrow \frac{A}{n} < \frac{m+1}{n} < \frac{C}{n} \Leftrightarrow A-1 < m < C-1 \\ &\Leftrightarrow \frac{2n+1-\sqrt{4n+5}}{2} < m < \frac{(n+1-2b)(4b^2-2bn-6b+2)}{4b^2-2bn-4b+1} \end{aligned}$$

A1.15 Proof of Corollary 1.8 and 1.9:

$$C-n = \frac{(4b^2-6b+1)n-(2b-1)^2(2b-3)}{4b^2-2bn-4b+1}$$

:

We know $4b^2-2bn-4b+1<0$, $(2b-1)^2(2b-3)<0$ and $n>0$. Thus,

if $4b^2-6b+1 \geqslant 0$, we have $C-n<0$. From $4b^2-6b+1 \geqslant 0$, we can get $b \leqslant \frac{3-\sqrt{5}}{4} \approx 0.19$.

If $4b^2-6b+1<0$, we can conclude that, when $n < \frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$, $C-n<0$ and when $n \geqslant \frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$, $C \geqslant n$. We may draw the figure of $\frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$ as:

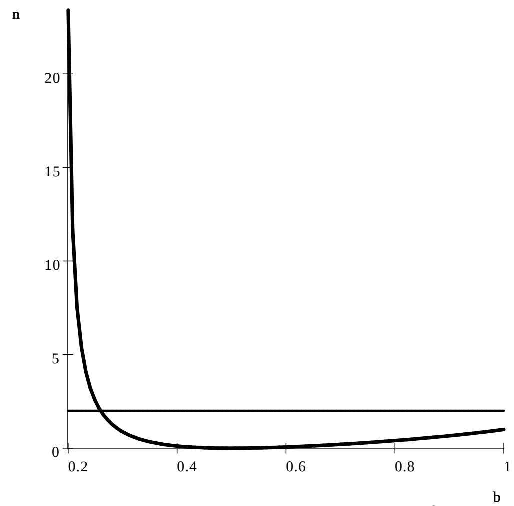

Figure-1.5: $\frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$

From Figure-1.5, we can see $\frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b} < 2$, when $b > 0.26$. So If $b > 0.26$, $C > n$. We also know, if $b < 0.19$, $C < n$. However, if $0.19 < b < 0.26$, the analysis will be a little more complex. In this situation: if $n < \frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$, we have $C < n$. which means m is bounded by both side. If $n \geqslant \frac{(2b-1)^2(2b-3)}{(2b-1)^2-2b}$, we have $C \geqslant n$, which means m isn't bounded by the social welfare restriction.

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Chapter 2
Merger through Acquisition with Inheritable Indirect Network Externalities

Abstract

We investigate firms' acquisition behaviours if they may inherit indirect network externalities from the firms they acquired. For a given symmetric initial market structure, we provide the sufficient and necessary conditions for the existence of an equilibrium in which some firms are acquired by the others and reveal that these conditions are relaxed when the indirect network is inheritable. For an asymmetric previous market structure, we find that larger firms acquiring smaller firms occurs more easily than smaller firms acquiring larger ones. Inheritable indirect network externalities can provide an incentive for the firms to merge and also help the merged entities maintain their advantage position.

2.1 Introduction

In network economy, consumers may benefit from an increase in the number of consumers who use the same or compatible products. This network externality, which significantly affects the behaviours of both consumers and firms, can be explained by two different origins. The first is a direct physical effect of the number of purchasers on the quality of products. A good example of the direct network effect is the mobile phone network —— It is easy to understand that a mobile phone network with more users will be more valuable to its consumers if different networks as a whole are incompatible, or if customers need to pay a significant mount of money to connect to other networks. Study of the complementary products market has given rise to another reason for network externalitites. The idea is: if more consumers choose to use a specific product, there will be more firms that join this product's complementary market. Competition among these downstream firms will lower the price and increase the variety of complementary products, hence increase the utility of the consumers. These indirect network externalities are found in many IT (information technology) products, such as platform/software, and can also exist in many traditional industries, such as automobile/authorized repair agencies.

Indirect network externalities are a potentially important factor that influences consumers when they are choosing products. Generally speaking, the more indirect network externalities he/she may obtain as a result of purchasing the product, the higher price he/she would be happy to pay. However, it is very difficult for consumers to predict the future size of the networks, since it is nearly impossible for everyone have full information about others' choices when making their decisions. In most instances, the consumers evaluate the size of a network in the current period according to the market size of each firm in the last period, which is a more accessible piece of public data and can be seen by everyone. Previous market size determines how many complementary goods are produced and how many complementary goods developers and suppliers, who are slow or costly to switch, are already there. It is therefore resonable that consumers prefer products from firms with larger previous market sizes and are willing to pay more for a product that has established market status and reputation in a previous time. This consumer's behaviour can be explained by the fact that indirect network externalities are inheritable, which means that an increase in the number of consumers who use a product in a previous time will increase the utility of the consumers who choose the same or compatible product in the current period. In this chapter, we assume that firms can only inherit the last period's indirect network externalities and that earlier market structure does not affect the current period.

Inheritable indirect network externality has some similar characteristics with installed base. However, this special installed base comes from the complementary product market instead of the product's market itself. Moreover, installed base does not always bring about network externalities, which is crucial in our model. Goodwill for reputable brand names can also bring about similar effect for consumers, but goodwill is not always size determined and can vary among different consumers.

In the current period, firms can do nothing to influence their previous market size, which means the sizes of the indirect network externalities they may inherit are determined by history. However, if a firm acquires another firm, it may inherit indirect network externalities from the acquired firm by making his product compatible with all acquired firms' complementary goods. This provides a possibility for firms to change the sizes of the indirect network externalities involved in their current period products and can be an important motivation for merger and acquisition activities. A good example of this can occur in the video game console/game software market. One can imagine that a merger between Sony (PlayStation) and Microsoft (XBox) would allow creation of a new product (PS-XBox) that is more competitive in the video game marketplace than Nintendo's Wii, or other separate brands, since PS-XBox consumers can enjoy all of the games that once could only be played on either the PlayStation or XBox consoles. Another advantage of the new game console is that there will exist more game developers for the new PS-Xbox game console, compared with the case in which two firms haven't merged. The reason for this is that those game software developers for PlayStation and XBox, individually, in previous time may prefer to continue to develop software for the new PS-XBox game station, because the switch is slow and costly. When a consumer considers purchasing a game console in current period, he/she may evaluate how many software and software developers already exist in the market and form their willingness to pay for PS-XBox. Clearly, consumers will prefer to pay more for a new, combined game console than previous separate consoles, because this new game console provides more indirect network externalities and increases their utilities to purchase this product. The relatively advantage in the product is more significant if the new syndicate acquires more firms, but on the other hand, payment for the acquisition may limit the firms' benefit from the merger. Thus, the market structures (how many firms in the market and their market size) along with the intensity of the inherited indirect network effect determine the firm's acquisition strategy and the final location of the market equilibrium.

In this chapter, we study firm's acquisition behaviours when they are faceing a given previous market structure and a prospect of inheriting indirect network externalities from the firms they acquired. Our model is based on Kamien and Zang's (1990) acquisition model. However, the main purpose of Kamien and Zang's work is to show that mergers are unlikely to happen in Cournot competitions if there is no other benefit for the merger except an increase of market concentration. Kamien and Zang derive their conclusion only from a necessary condition for the merger. In their model, the sufficient condition is not important and is not discussed since mergers may only occur in very limited cases, given their assumption. If inheritability is considered, the necessary condition for the existence of an equilibrium, in which some firms are merged, will be greatly relaxed and enumeration is impossible. Thus, in our model, the sufficient and necessary condition is critically important for the antitrust social planner because it provides a more accurate indicator to when the merged Nash equilibrium can exist. Another interesting topic we investigate is whether inheritability can motivate firms to merge and how much inheritability affects the concentration of the market. In a traditional Cournot competition, the firms that have acquired other firms have no advantage in the competition compared with the firms that are not involved with the merger. This is because firms only compete with their output and the quality of their products is indifferent. In our model, the inherited indirect network externalities provide certain advantages to the firm that acquires others firms since consumers will likely pay more for the products with more inherited network externalities. Firms that acquire others may use this advantaged to squeeze more profits from their consumers and increase their output. Moreover, firms that successfully set up their output advantage in the current period might secure an advantage position in the next round of mergers and acquisitions. This can explain why it is easier for a larger firm to acquire smaller firm than the other way around and why the markets always tend to be more asymmetric without other exogenous forces, although the merged Nash equilibrium that will be played is random.

In the literature, horizontal merger is always thought to be a phenomenon that requires study and regulation, since antitrust social planners believe that such a merger has great potential to reduce competition and social welfare. However, Salant et al (1983) points out that mergers are much easier said than done. They find that, in the Cournot competition model, mergers may only occur when they include more than 80 percent of the firms in the industry. This is because the merged entities must be able to generate more profit than the sum of the separate pieces did before the merger. Given further discussion of the process of a merger through acquisition, Kamien and Zang (1990) indicate that no merger can happen in an industry with more than seven firms if the demand function is concave. This can be explained by the fact that each merged entity desires to make at least what it could, in terms of profitability, by unilaterally abandoning the merger. If a linear demand function is employed in Kamien and Zang's model, merger through acquisition only exists for an industry with no more than two competitors. Merger under the Bertrand model, with differentiated products, is studied by Denrckere and Davidson (1984, 1985), but acquisition in a Bertrand competition is also limited because the value of the fringe firm may increases when the industry becomes more concentrated (Kosenok, 2005). Stigler (1950) points out another consideration that may reduce the chances of the merger, which is that firms that stay outside of the merger can benefit more than those firms that are involved. Although Inderst and Wey (2004) suggest that this insiders' dilemma can be solved similar to a public goods problem, it is natural for us to believe that other motivations to explain a firm's enthusiasm to merge are likely.

As to the motivation of mergers from the supply side, Perry and Porter (1985) use an alternative cost function to show that the mergers may create cost efficiencies, which makes merger easier to exist than in Salant's estimation. Farrel and Shapiro (1990) give a further investigation and indicate that the merger may have a positive impact on social welfare if the synergy of the merger is considered. From the demand side, Cheung (1992) shows that Salant's threshold may be relaxed to 50% if the demand satisfies the marginal revenue of the industry is decreasing. Fauli-Oller (1997) finds that the profitability of mergers may depend on the degree of concavity of the demand function. In our model, the demand function may varies with the merger result, while firms can change the demand function by choosing different acquisition strategies. However the demand function is linear and fixed after the merger is finished. Huck, Konrad and Muller (2004) provide additional reasons for mergers, including internal organization of the firm, the time structure of decision making, the information aspect of competition, etc. However, size depended inheritable resource from the acquired firms, which can be an important reason for the merger, is not investigated in extant literature while neither is the sufficient and necessary conditions for the existence of a merged Nash equilibrium discussed in the literature.

The direct network effect is firstly modeled by Katz and Shapiro (1985). They also suggest, in another paper, that "hardware/software system can be seen as vertical network which has similar properties as direct network" (Katz and Shapiro, 1994). Empirical analysis shows that this indirect network effect may be important in video game (Clements and Ohashi, 2005) or personal digital assistant market (Nair et al, 2004). Economides and White (1998) suggest that the indirect network may be seen as a two-way network and there are other works (Church and Gandal, 1992; Chon and Shy, 2002) in which the authors try to model this indirect network effect in different markets. More recently, Church et al (2008) review development of the indirect network theory and provide the condition for the existence of the adoption externalities in indirect network industries. Our thesis's indirect network theory is based on these works. We assume that the indirect network effect is inheritable and offer further discussion on its effects to mergers and acquisitions.

In Section 2.2, we discuss the sufficient and necessary conditions for the existence of a Nash equilibrium, in which some firms are merged. The model in Section 2.2 is based on a symmetric initial market and a linear network utility function. In Section 2.3, we investigate whether the larger firms have an advantage in an asymmetric market and try to discuss whether inheriting indirect network externalities can be an incentive for the firms' acquisition behaviors. The possibility of market structure changes under our assumption and its implication to social planners are also discussed in this section. The last section of this chapter provides our conclusion and offers future possible work relative to this chapter.

2.2 The Symmetric Model

2.2.1 The Model

Assume there are n ($n \geqslant 2$) symmetric firms in an oligopoly market and that these firms produce homogeneous products and choose their output to maximize profits. The merger and acquisition between these firms is processed in two stages: in the first stage, every firm simultaneously selects a bid vector that includes the bids for all the other firms and the reservation price for himself. Then, the firms merge according to specific rules and the chosen bid vector. In this step, some firms take the bid offer and leave the market. After observing the acquisition result, consumers form their valuations for each survived firm's products or services. Here, we assume the more firms a firm acquires, the more indirect network externalities this firm may provide to consumers. Consequently, consumers will value this firm's products or services more. In the second stage of the merger and acquisition game, survived firms compete with their output to maximize profits according to the consumer's valuations. The producing cost is not considered in this model. The entire merger process is one-off, thus, the firms are myopic and do not consider the next round bid. In the following paper, we denote the firms that accept the bid offer and leave the market as the *sellers*. And we denote the firms that acquire at least one firm as the *buyers*.

The bid vector of firm i, in the first step, can be written as: $B_i = (b_i^1, b_i^2, b_i^3, ..., b_i^n)$. Here, we denote firm i's bid for firm j as b_i^j and firm i's reservation price (bid for itself) as b_i^i. Since all the firms are symmetric, firms always prefer operating themselves to acquiring others. Thus, firm i's bid for itself is always larger than its bid for others and we have $b_i^i > b_i^j$ $(j \neq i)$.[4] After all the firms select his bid vector, each firm receives an offer vector from all other firms and itself as $B_i' = (b_1^i, b_2^i, b_3^i, ..., b_n^i)$. Firms merge according to the following rules: if there exists a $b_q^i \in B_i'$ $(q \neq i)$ for which $b_q^i \geqslant b_j^i$ $(j = 1..n)$, we call firm q a potential buyer of firm i. Here, we assume that when firm i receives an offer

4 This assumption can successfully avoid the acquisition dilemma, which is the situation that Firm A may acquire Firm B, Firm B may acquire Firm C and Firm C may acquire Firm A. Proof can be found in the Appendix.

equal to its reservation price, the firm will prefer to accept the offer and leave the market to avoid the uncertainty of the competition. If there does not exist such b_q^i (firm i's reservation price is larger than any offer), firm i will not be acquired by any firm and will remain until the next stage. If there is more than one potential buyer (these potential buyers provide the same offer to firm i), the potential buyer with the highest rank will win the bid (firm i's rank is i, which is an artificial, exogenous variable). We also assume that firms will first consider selling themselves. If the firm cannot find a buyer, it then begins to consider acquiring others firms. After the market is restructured by the acquisition procedure, we assume there are only m $(m \leqslant n)$ firms left in the market and each of these has acquired $k_l - 1$ $(n \geqslant k_l \geqslant 1)$ firms (here $l = 1..m$). By the definition of k_l, we can obtain:

$$\sum_{l=1}^{m} k_l = n$$

As is known, the indirect network externalities that consumers can obtain from the buyer are determined by the total initial market size of all the firms that the buyer acquires. We denote the market size of firm i at the beginning of the merger as x_i^0. Since the firms are symmetric, there is: $x_1^0 = x_2^0 = ... = x_n^0 = x^0$. We also simply assume, before the merger and acquisition process, that these n firms compete in a standard Cournot model with the following price function:

$$p = 1 - \sum_{i=1}^{n} x_i$$

Thus, the output of each firm, in the equilibrium, is:

$$x^0 = \frac{1}{n+1}$$

The indirect network externalities consumers obtain from buying one of the buyer's products is:

$$u(\sum x_i^0) = u(k_l x^0) = u(\frac{k_l}{n+1})$$

If firm l, a buyer, acquires more firms in the first stage of the game, its products or services will carry more indirect network externalities and be more valuable to consumers. Similar to the network externality utility function, this utility function should be characterized as: $u(0) = 0, u(x) > 0, u'(x) > 0$ and $u''(x) \leqslant 0$. For convenience of calculation, we may assume the utility function of indirect network externalities to be linear as: $u(x) = bx$

$(0 < b < 1)$. Here, b is the measure of the intensity of the indirect network externality. We also assume the price of firm l's product is determined by the function:

$$p_l = 1 + u(k_l x^0) - \sum_{i=1}^{m} x_i = 1 + \frac{bk_l}{n+1} - \sum_{i=1}^{m} x_i$$

Thus, firm l's profit is:

$$\pi_l = p_l x_l = (1 + \frac{bk_l}{n+1} - \sum_{i=1}^{m} x_i) x_l$$

From the first order condition, we can obtain the optimized output of firm l, x_l^M, is the solution of the function:

$$1 + \frac{bk_l}{n+1} - \sum_{i=1}^{m} x_i - x_l = 0 \tag{2.1}$$

Since these are m firms left after the merger, there exists m solution functions in the same form as equation (2.1). If we add these together, we obtain:

$$m + \frac{b\sum_{l=1}^{m} k_l}{n+1} - (m+1)\sum_{i=1}^{m} x_i = 0$$

$$\Longrightarrow \sum_{i=1}^{m} x_i = \frac{m}{m+1} + \frac{nb}{(m+1)(n+1)} = z \tag{2.2}$$

Here, z is the total output after the merger. Since $b < 1$, $\frac{nb}{n+1} < 1$ and $z < 1$. We may easily get $\partial z / \partial m > 0$ from equation (2.2). This means that total output will decrease with the decrease of number of the firms that survive in the last stage, which is coincident with the standard Cournot competition. We may also find that $\partial z / \partial b > 0$. This means that the stronger the indirect network externalities, the more consumers will join the market. These extra consumers lured by the indirect network externalities to join the market is: $\frac{nb}{(n+1)(m+1)}$.

By substituting z into the optimized output solution function (2.1), we obtain the equilibrium output of firm l after the merger as:

$$x_l^M = \frac{1}{m+1} + \frac{b}{n+1}(k_l - \frac{n}{m+1}) \tag{2.3}$$

while the equilibrium profit of firm l is:

$$\pi_l^M = p_l x_l^M = (\frac{1}{m+1} + \frac{b}{n+1}(k_l - \frac{n}{m+1}))^2 \tag{2.4}$$

If firm l seizes more firms from other buyers, k_l will increase and m does not change. From equation (2.3) and (2.4), we can conclude that firm l will have more output and profits. Acquiring one more firm will extend the firm's output by $\frac{b}{n+1}$. From function (2.2), we know that the total output, z, does not change if n and m are fixed. Thus, if firm l increases its output by $\frac{b}{n+1}$, there must exist a buyer who loses the same amount of output. This is different from Kamien and Zang's model (1990) or our model in perper-1. In these models, after the merger, firms compete in a symmetric status. The final market structure is symmetric and only determined by how many firms are left after the merger (here is m). However, in this model, buyers that acquire more firms have larger market share in the equilibrium. This can provide a clue for us to analyze the incentive of the firms to raise their bids. It is clear that $\partial x_l^M / \partial m < 0$. Thus, if the firm l increases k_l by reducing m, which means the market becomes more concentrated, not only x_l^M will increase, but also all the other firms that remain in the market will benefit as well.

2.2.2 Existence of Merged Nash Equilibrium (MNE)

For convenience to denote in this chapter, we make the following definition:

Definition 2.1 *If the bid game reaches an equilibrium (no firm chooses to change its bid vector given all the other firms' bid vectors) and $m < n$ in the equilibrium, we call this equilibrium a merged Nash equilibrium (MNE).*

In MNE, there at least exists one firm who has acquired some other firm in the first stage of the game. We assume firm l is a firm that has acquired others. Obviously, firm l can choose to give up the acquisition by setting its bid for all the other firms at 0. If it does, the $k_l - 1$ firms, that once may sell themselves to firm l, must get involved in the second stage of the competition or consider selling themselves to other potential buyers. In this effect, the total firms left in the second stage will increase to m' $(m + k_l - 1 \geqslant m' \geqslant m)$. Following the procedure of calculating π_l^M, we can obtain the profits of firm l when it chooses to give up the acquisition in the first stage as:

$$\pi_l^{NM} = (1 + \frac{b}{n+1} - \frac{m' + \frac{nb}{n+1}}{m'+1})^2 \tag{2.5}$$

Since $\partial \pi_l^{NM} / \partial m' < 0$, π_l^{NM} reaches the minimum when $m' = m + k_l - 1$. By substituting $m + k_l - 1$ into equation (2.5), we may obtain:

$$\pi_l^{NM} \min = (\frac{1}{m+k_l} + \frac{b}{n+1}(1 - \frac{n}{m+k_l}))^2$$

This indicates that firm l can always guarantee $\pi_l^{NM} \min$ profit by giving up the acquisition.

Any firm may also make itself un-acquirable by setting its reservation price as infinity. If a firm that decides to sell itself to firm l changes its mind and chooses to keep itself un-acquirable, it may increase the number of the firms that remain to second stage to $m+1$ and get equilibrium profit π^D. π^D can be calculated in the same manner as above:

$$\pi^D = (\frac{1}{m+2} + \frac{b}{n+1}(1 - \frac{n}{m+2}))^2 \tag{2.6}$$

If firm l wants to acquire a firm, it must pay at least π^D to the seller. Otherwise, the seller will choose to remain un-acquirable. Thus, the minimum payment for the firm l to buy $k_l - 1$ firms is $(k_i - 1)\pi^D$.

Proposition 2.1 *For a given n and b, the necessary condition for the existence of a MNE is: there exists a m and a k_l which makes:*

$$\pi_l^M - \pi_l^{NM} \min \geqslant (k_l - 1)\pi^D \tag{2.7}$$

Here, $n \geqslant 2, n > m \geqslant 1, n \geqslant k_l \geqslant 2$ *and* $b \in (0, 1)$.

In equation (2.7), $\pi_l^M - \pi_l^{NM} \min$ is the profit of firm l in acquisition and is also the maximum amount that firm l is willing to pay. We call this amount *the budget* of firm l. This budget must be greater than the minimum payment at which the $k_l - 1$ firms are willing to merge. Otherwise, either firm l will give up on the acquisition or sellers will deviate from the merger.

Proposition 2.2 *The necessary condition for the existence of a MNE is relaxed by the increased intensity of the indirect network externalities.*

From equation (2.6), we know that the increase of b will decrease the incentive of the sellers to deviate in most of the cases[5]. The greater the indirect network externalities, the less buyers must pay for the acquisition. Let's assume firm l is the largest buyer, which means $k_l \geqslant k_i$ $(i = 1...m)$. From equation (2.3), we know firm l's profit after merger increase with the increase of b. The change of b has a different effect to $\pi_l^{NM} \min$ when the number of buyers in the MNE is different. In the case that there is more than one buyer in the MNE, $\partial x_l^{NM} \min / \partial b < 0$ and the buyers will get less from giving up the acquisition. Thus, the firm l is likely to pay more for the acquisition. If there is only one buyer in the MNE, we will have $\partial x_l^{NM} \min / \partial b > 0$, which means buyers will obtain more from giving up the acquisition. However, from the left side of the equation (2.7), we have $\partial(\pi_l^M - \pi_l^{NM} \min)/\partial b > 0$. This indicates that, for firm l, with the increase of b, the increase of profits from the merger is greater than the increase of the profits from giving up the merger. In both of these cases, the total budget that firm l is willing to pay for sellers increases with the increase of b. Combining the effect to the seller and the buyer, the increase of the intensity of indirect network externalities causes firm l's budget to increase and makes sellers are happy to accept a lower offer. We can conclude that the necessary condition for the existence of an MNE will relax.

In Kamien and Zang's model (1990), a MNE only exists for a relatively small n. Especially when the price function is linear, a MNE only exists for $n = 2$. This is because the profit from the merger is very low and the incentive for the sellers to deviate is relatively large if we do not consider any other benefit of the acquisition. However, in the real world, mergers happen more frequently than Kamien and Zang's (1990) declaration, which may be explained partly by Proposition 2.2. If we consider indirect network externalities, firms will be rewarded more for their acquisition behaviors. With the increase of the intensity of

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[5] When and only when $m = n - 1$, the deviation profit of the sellers increases with the increase of b. This special case is discussed in a separate section in the following part of this paper. Proposition 2.2 will not be violated in this special case.

the indirect network, there are many possible MNEs which may lead to symmetric, asymmetric equilibrium or monopoly cases. Kamien and Zang (1990) do not discuss sufficient conditions for the existence of a MNE since a possible merger is very limited and can be solved by enumeration. However, if we consider indirect network externalities, the set of possible MNEs is relatively large, so the sufficient conditions can be important and may provide additional profiles of MNEs.

2.2.1 Merger with a Single Buyer

Similar to MNE, we may give the following definition for convenience to denote in the paper:

Definition 2.2 *If the bid game reaches an equilibrium with $m < n$, and this equilibrium contains only one buyer, we call this equilibrium single buyer merged Nash equilibrium (SBMNE). If this MNE contains more than one buyer, we call the equilibrium multiple buyer merged Nash equilibrium (MBMNE).*

If only one buyer exists, the number of firms that are involved in the second step competition must equal $n+1-k$. Here, we may ignore the subscript of k since only one k exists in a SBMNE. In a general MNE model, π_l^M and π^D are both functions of m and k_l for a given n and b. In this section, we use $\pi^M(k)$ and $\pi^D(k)$ to denote them because they are determined solely on k in SBMNE. If we substitute $m = n+1-k$ into π_l^{NM} min, we can obtain:

$$\pi_l^{NM} \min = (\frac{1}{n+1} + \frac{b}{(n+1)^2})^2 = C \tag{2.8}$$

This indicates that $\pi_l^{NM} \min$ is a constant number for a given n and b. We define this constant number as C.

A Candidate SBMNE and the Sufficient Condition for its Existence

Proposition 2.3 *The sufficient condition for the existence of a SBMNE is that there exists a $k \in [2, n]$, which allows the inequality:*

$$\pi^M(k) - C \geqslant (k-1)\pi^D(k) \tag{2.9}$$

hold for a given $n \geqslant 2$ and $b \in (0, 1)$.

If there exists one or more than one $k \in [2, n]$ that allows the inequality (2.9) hold for a given $n \geqslant 2$ and $b \in (0, 1)$, we may define k^* as the smallest k that makes $\pi^M(k) - (k - 1)\pi^D(k) \geqslant C$ and

$$\epsilon = \frac{\pi^M(k^*) - C}{k^* - 1} - \pi^D(k^*) \tag{2.10}$$

It is obviously that ϵ is always greater than 0. We call the firms that haven't been acquired by the buyer in the SBMNE as non-sellers and the buyer's profit after paying out the bids to all sellers as its net profit.

We can construct such a strategy set of a candidate equilibrium:

{buyer's strategy: (bid for himself: ∞, bid for all the sellers: $\pi^D(k^*) + \epsilon$, bid for all the non-sellers: 0);

sellers' strategy: (bid for himself: $\pi^D(k^*) + \epsilon$, bid for the buyer: 0, bid for all the other sellers: 0, bid for all the non-sellers: 0);

non-seller's strategy: (bid for himself: ∞, bid for the buyer: 0, bid for all the other sellers: 0, bid for all the other non-seller: 0)}

If we prove that the above candidate strategy set is an equilibrium, we can conclude that there always exists a SBMNE since there is only one buyer in this equilibrium.

Analysis of the Candidate Equilibrium

(i) For the buyer's strategy, given all others' strategies

Clearly, there is no incentive for the buyer to decrease or increase its bid for itself since the bids for the buyer from the sellers and the non-sellers are 0. And the buyer has no incentive to increase its bid for the seller because this will only increase its cost of the acquisition and decrease its profit. From inequality (2.9), we also know that the buyer has no incentive to decrease its bid for all the sellers simultaneously. If the buyer does so, it will become a non-buyer. Being a non-buyer is never the best strategy since inequality (2.9) provides potential profitability for the buyer. The buyer also has no incentive to decrease its bid for part of the sellers. This can be proved in the following lemma.

Lemma 2.1 *Given the strategies of sellers and non-sellers as the candidate strategy set, the buyer's net profit will always be less than C if the buyer chooses to only acquire fewer than k^* sellers.*

We may assume that the buyer decreases the number of the firms it acquires to k' ($k' < k^*$). Since k^* is the smallest k that makes the net profit larger than C, the buyer will obtain less net profit than C if it chooses to only acquire k' and pay them $\pi^D(k')$. Moreover, the actual payments for each seller are $\pi^D(k^*) + \epsilon$, which is larger than $\pi^D(k')$. The actual net profit the buyer can get will be even less if he chooses to acquire just k' firms. This means that the buyer has no incentive to decrease its bid for part of the sellers. The buyer also has no incentive to increase or decrease its bid for the non-sellers since the nonseller's bids for themselves are ∞. Considering above discussion together, we can conclude that the buyer has no incentive to change its strategy when all the others' strategies are given as the candidate strategy set.

(ii) *For the non-sellers, given all others' strategies*

Obviously, the non-sellers have no incentive to increase their bids for the buyer since the buyer's bid for itself is ∞. Any non-sller has no incentive to increase its bids for other non-sellers, since all of the non-sellers' bids for themselves are ∞. The non-sellers will not decrease their bids for themselves because all of the others' bids for the non-sellers are 0. Any non-seller has no incentive to increase its bids for one, some, or all of the sellers. This is shown in the following lemma.

Lemma 2.2 *Given the strategies of the buyer, all sellers and all other non-sellers as the candidate strategy set, any non-seller will have less profit if it chooses to pay more than $\pi^D(k^*) + \epsilon$ to acquire one, some, or all of the sellers.*

The non-seller may also lure some sellers by raising its bid over $\pi^D(k^*)+\epsilon$. However, the total number of firms in the last step of the competition will not change and the number of sellers that the non-seller can acquire is limited, so the non-seller cannot benefit from additional reduction in competition intensity. If the non-seller acquires some sellers, it may benefit from an increase of its products' indirect network externalities. However, this is very limited and will be offset by what it needs to pay for the acquisition. The intuition behind the Lemma 2.2 is the insider's dilemma, which means it is always better to stay outside the merger than get into it. Thus, starting a bidding war is not a smart strategy when the buyer has given a non-profitable high bid to sellers. Adding Lemma 2.2 to our previous discussion, we can conclude that an outsider has no incentive to change its strategy when all of the others' strategies do not change.

(iii) *For sellers, given all others' strategies*

Clearly, sellers have no incentive to increase their bids for the buyers or the non-sellers since their bids for themselves are ∞. Sellers have no incentive to decrease their bids for themselves either, since this will only reduce their profits, while they also have no incentive to increase their bids for themselves to be a non-seller, since $\pi^D(k^*)+\epsilon > \pi^D(k^*)$. However, the seller may increase its bid for itself and set a bid larger than $\pi^D(k^*) + \epsilon$ for some or all the other sellers. This makes the seller become a second buyer and allow it to snatch some of the firms from the buyer. The following lemma eliminates the idea that this strategy is better for sellers than our candidate strategy set.

Lemma 2.3 *Given the buyer's strategies, the non-sellers and all of the other sellers as the candidate strategy set, the seller will obtain less net profit than $\pi^D(k^*) + \epsilon$ if it chooses to stay un-acquirable and acquires one, some or all of the rest of the sellers by offering more than $\pi^D(k^*) + \epsilon$.*

$\pi^D(k^*) + \epsilon$ is really a decent pay for the seller, which is even larger than the buyer's net profit C. If one of the sellers becomes a second buyer, the total number of firms in the last step of the competition will increase by 1. This means that the second buyer must face a more intensive competition. If the second buyer stands at the same position as the first buyer, it cannot generate a net profit more than C since the number of firms it may acquire is less than k^* and k^* is the smallest k that makes the net profit larger than C. Thus, if the second buyer must face a even harder competition, what it may generate is even less and obviously less than the payment from the buyer, $\pi^D(k^*) + \epsilon$. According to Lemma 2.3 and the discussion above, we can say that sellers have no incentive to change their strategies given the strategies of buyer and non-sellers as the candidate strategy set.

From (i), (ii) and (iii), we show that the candidate equilibrium is an equilibrium. Thus, if we have a $k \in [2, n]$ that lets (2.9) hold, we can find out k^* by checking all of the numbers less than k, and there always exists an equilibrium same as the candidate equilibrium —– Proposition 2.3 is proved.

If we combine Proposition 2.3 and Proposition 2.1, the following corollary is obtained:

Corollary 2.1 *For a given $n \geqslant 2$ and $b \in (0, 1)$, the sufficient and necessary condition for the existence of a SBMNE is there exists a $k \in [2, n]$, which makes $\pi^M(k) - C \geqslant (k-1)\pi^D(k)$.*

Two Special Cases of SBMNE

One of the special cases of SBMNE is only two firms are involved in the merger. In this case, $m = n - 1$ and $k = 2$. Substituting m and k into equation (2.9), we may easily obtain the following proposition:

Proposition 2.4 *For any given $n \geqslant 3$, if*

$$b \geqslant \frac{((\sqrt{2}-1)n^2 - (\sqrt{2}+1))(n+1)}{(n^2+2n-1)n} \tag{2.11}$$

, there exists a SBMNE in which the buyer only acquires one firm.

Since

$$\lim_{n\to\infty}\frac{((\sqrt{2}-1)n^2-(\sqrt{2}+1))(n+1)}{(n^2+2n-1)n}=\sqrt{2}-1$$

, any b that is greater than $\sqrt{2}-1$ will automatically make (2.11) hold. We can then get the following corollary.

Corollary 2.2 *For any given $n \geqslant 3$ and $b \geqslant \sqrt{2}-1$, there always exists a SBMNE in which the buyer only acquires one firm.*

From Corollary 2.2, we may also indicate that there always exists a MNE when b is not less than $\sqrt{2}-1$. This Corollary will be helpful when we discuss the existence of a general MNE since we only need to consider the situation when b is less than a certain threshold.

Another very important special case is the one in which the market becomes a monopoly after the acquisition. In this case, $m=1$ and $k=n$. Similar to Proposition 2.4, we can obtain the following proposition:

Proposition 2.5 *For any given $n \geqslant 3$, if*

$$b \geqslant \frac{(4n^3+5n^2+7n+24-6(n+1)\sqrt{4n^3-15n+16})(n+1)}{(4n^3-25n^2-35n+12)n} \tag{2.12}$$

, there exists an MNE in which the market becomes a monopoly.

Here, the left side of the inequality (2.12) is divergent when n goes to infinity. In order to understand the relationship between these two special cases, we can draw (2.11) and (2.12) in the following figure:

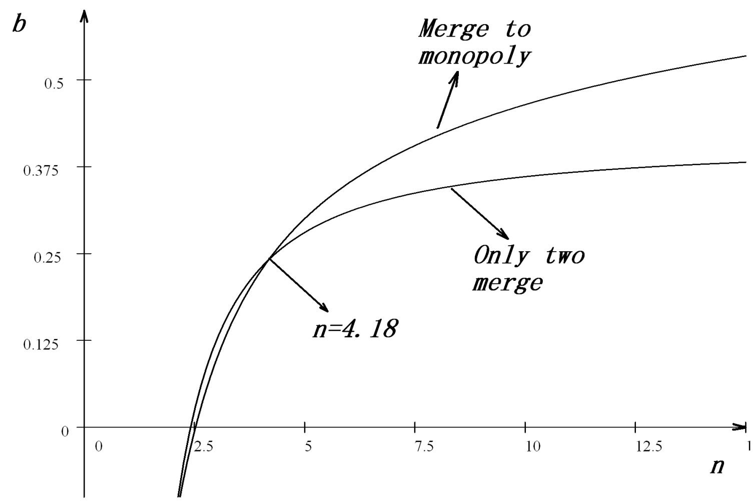

Figure-2.1: Two firms merger vs Merger to monopoly

From Figure-2.1, we see (2.11) and (2.12) are monotonic concave functions. With the increase of n, we need a larger b to satisfy both inequalities. This means we need a larger indirect network effect for the equilibrium to exist with the increasing of the number of the firms in the market. We may also see the line drawn by condition (2.11) is above the line drawn by condition (2.12) for $n = 3$ and 4. For $n \geqslant 5$, (2.12) always locates above (2.11). This tells us that it is easier for a firm to buy all the other firms (to pursue monopoly) than just buy only one firm when n is relatively small. However, when n becomes larger, the cost of the monopoly strategy increases rapidly and a larger indirect network effect is necessary in order for the monopoly to remain profitable. For a relatively large n, a SBMNE with the merger of just two firms is easier to realize. For the antitrust social planner, when there are only a few firms in the market and competition is weak, a merger leading to monopoly will be more likely to happen. However, when the number of firms in the market is large and highly competitive, the merger is more likely to happen between limited firms.

More generally, the sufficient and necessary condition of the existence of a SBMNE can be seen as a quadratic function of b, given n and k. By solving this function, we may get $b = f(n, k)$. Plotting $f(n, k)$ with the condition $n > k \geqslant 2$, we obtain the following figure:

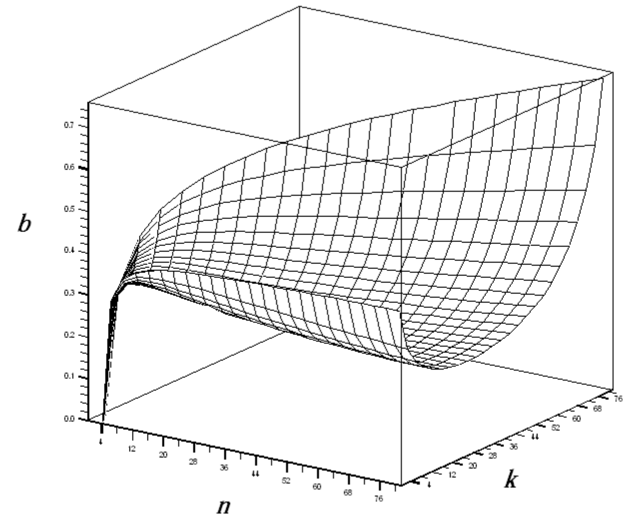

Figure-2.2: $b = f(n,k)$

Figure-2.2 describes the relationship between b, n and k under the condition of (2.9). Any point located in the space above the curved surface causes the SBMNE exist. An interesting proposition here is: if the initial number of firms in the market is given and this number is larger than 4, the SBMNE that can exist by the smallest b will not be any one of the special cases we have discussed above. This threshold equilibrium happens when the buyer chooses a k^* that is located between 2 and n. This can be explained by a double-side effect of the acquisition: increasing the value of buyer's products because of indirect network externalities and increasing the payment for sellers. From Figure-2.2, we can also guess that k^* will become smaller with the increase of n and finally converge at a value when n goes to infinity. This will be left to future work.

2.2.1 Merger with Multiple Buyers

When $n = 2$ and 3, there is no MBMNE. Thus, we only need to consider the situation that $n \geqslant 4$. The simplest multiple buyer merge equilibrium is the one in which there are two buyers and each has acquired one seller. This is the only possible MBMNE when $n = 4$.

Proposition 2.6 *When $n = 4$, the sufficient and necessary condition for the existence of an MBMNE is $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$.*

By substituting $n = 4, m = 2$ and $k = 2$ into (2.7), we can easily check the necessity of Proposition 2.6. For sufficiency, we may construct a candidate equilibrium:

{The first buyer's strategy: (bid for himself: ∞, bid for Seller A: $(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}$, bid for Seller B and the second buyer: 0);

The second buyer's strategy: (bid for himself: ∞, bid for Seller B: $(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}$, bid for Seller A and the first buyer: 0);

Seller A's strategy: (bid for himself: $(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}$, bid for the two buyers: 0, bid for Seller B: 0);

Seller B's strategy: (bid for himself: $(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}$, bid for the two buyers: 0, bid for Seller A: 0)}

We can prove that, for any $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$, this candidate equilibrium is an MBMNE. This provides us the sufficiency for Proposition 2.6.

When $n = 4$, three possible SBMNEs exist: $(k = 2, m = 3)$, $(k = 3, m = 2)$ and $(k = 4, m = 1)$. Substituting these three SBMNEs into inequality (2.9), we can obtain the smallest b that may allow these two SBMNEs exist is $\frac{75\sqrt{2}-85}{92}$, $\frac{75\sqrt{93}-580}{616}$ and $\frac{150\sqrt{3711}-8260}{4127}$, respectively. These three threshold b are all greater than $\frac{\sqrt{2}}{4} - \frac{1}{3}$, so an MBMNE is easier to exist than an SBMNE.

Proposition 2.7 *For any $n > 9$, if an MBMNE exists, an SBMNE must exist at the same time.*

We define a function:

$$f(m) = (k-1)\pi^D + \pi_l^{NM} \min - \pi_l^M$$

For any $n > 9$ and $b \geqslant \sqrt{2} - 1$, we have $\partial f(m)/\partial m < 0$[6] if we assume n, k and b are fixed. According to Proposition 2.1, if an MBMNE exists, we must have an m and k that

[6] The proof is in Appendix

makes the inequality (2.7) hold. Let's define $m^{\sim}$ and $k^{\sim}$ as such a pair that can fulfill these conditions. Since it is an MBMNE, we must have $m^{\sim} + k^{\sim} - 1 < n$. So there exists a $m^* > m^{\sim}$ which makes $m^* + k^{\sim} - 1 = n$. Because $\partial f(m)/\partial m < 0$, $f(m^*) < f(m^{\sim}) < 0$ for a given n, $k^{\sim}$ and b. By substituting $m^* = n+1-k^{\sim}$ into $f(m^*) < 0$, we may find that the $k^{\sim}$ can make the inequality (2.9) in Proposition 2.3 hold. Thus, an SBMNE must exist with only one buyer that has acquired $k^{\sim} - 1$ firms. Proposition 2.7 shows us that if an MBMNE exists, the equilibrium, which is constructed by leaving one buyer and separating all the other mergers, will also exist if $n > 9$.

Combining Proposition 2.6 and 2.7, we may indicate that, when n is relatively small, an MBMNE may exist with a smaller b than an SBMNE. However, when n becomes larger than a certain number, the requirement for the existence of an SBMNE is weaker than the requirement for the existence of an MBMNE. If the indirect network effect is not strong enough to make an SBMNE exist, an MBMNE will not exist either. Therefore, in a market with sufficient competition, the social planners only need to focus on the existence of an SBMNE. If they find that the market is not ready for an SBMNE, an MBMNE will also be impossible. This can be concluded in the following corollary:

Corollary 2.3 *For $n > 9$, the sufficient and necessary condition for the existence of a MNE is there exists a $k \in [2, n]$ which makes the inequality (2.9) hold.*

For $n = 5, 6, 7, 8, 9$, we may test one by one to see whether they satisfy the Corollary 2.3 using a procedure that is very similar to the proof for Proposition 2.6.

2.3 Acquisition with Asymmetric Market Size

2.3.1 Asymmetric Acquisition and the Advantage of the Larger Firm

Assume there are n firms at the beginning of the merger and that these firms have different market sizes in the previous period. Without losing generality, we can assume their initial market sizes are $x_1^0 \geqslant x_2^0 \geqslant ... \geqslant x_n^0$. We also assume that consumers are uniformly distributed between 0 and 1 according to their willingness to buy the product. Then, we have $\sum_{i=1}^{n} x_i^0 < 1$. Suppose there are two firms A and B, which both want to buy r firms and $x_A^0 > x_B^0$. For ease of denotation, we name these r firms: $s1, s2...sr$. Similar to the model above, we still have a linear indirect network utility function: $u(x) = bx$.

Proposition 2.8 *If a firm with larger initial market size cannot benefit from acquiring a set of firms, all other firms with smaller initial market sizes cannot benefit from acquiring this set of firms either.*

First, we consider the situation of Firm $B \notin (s1, s2...sr)$ when Firm A is the buyer and Firm $A \notin (s1, s2...sr)$ when Firm B is the buyer. Following the procedure we use with the symmetric model, we can get the necessary condition for firm A to acquire these r firms is:

$$\begin{aligned} P_A &= (1 + b(\sum_{i=1}^{r} x_{si}^0 + x_A^0) - Z(m))^2 - (1 + bx_A^0 - Z(m+r))^2 \\ &\geqslant \sum_{i=1}^{r} (1 + bx_{si}^0 - Z(m+1))^2 = \Pi^D \end{aligned} \tag{2.13}$$

and the necessary condition for firm B to acquire these r firms is:

$$\begin{aligned} P_B &= (1 + b(\sum_{i=1}^{r} x_{si}^0 + x_B^0) - Z(m))^2 - (1 + bx_B^0 - Z(m+r))^2 \\ &\geqslant \sum_{i=1}^{r} (1 + bx_{si}^0 - Z(m+1))^2 = \Pi^D \end{aligned} \tag{2.14}$$

Here, $Z(x)$ is a function of total market output and x is the number of firms in the competition. From equation (2.2), we know that total market output after the merger only depends on m and will not change with the change of buyers and sellers. So z can be written as a function of the number of firms in the competition. P_A and P_B is the maximum value that Firm A and Firm B are willing to pay for the acquisition. Π^D is what both firms need to pay to prevent deviation of sellers.

$$P_A - P_B = 2b(x_A^0 - x_B^0)(b\sum_{i=1}^{r} x_{si}^0 + Z(m+r) - Z(m)) \tag{2.15}$$

Since $\partial Z(x)/\partial x > 0$, we can obtain $Z(m+r) > Z(m)$. So $P_A > P_B$. This means that the larger firm is easier to allow the necessary condition hold and Proposition 2.8 is proved.

If the target acquisition firms are identical, the total deviation payment for these firms is the same for all buyers. Thus, the right sides of the inequality (2.13) and (2.14) are both Π^D. However, the firm with a larger initial market size has the ability to pay more for the acquisition than the firm with smaller initial market size. This means that the larger firm may have an advantage in the acquisition compared with the smaller firm. Because they benefit more, larger firms are willing to pay more. Thus, the equilibrium is more likely to occur with a larger firm as the buyer. From (2.15), we find that $P_A - P_B$ increases with the increase of $x_A^0 - x_B^0$, b and r. This indicates that the advantage of the larger firm in the acquisition will be more significant if this firm is leading more in market size at the beginning of the merger; the indirect network effect is stronger and the size of the acquisition is larger.

The other case is Firm $B \in (s1, s2...sr)$ when Firm A is the buyer and Firm $A \in (s1, s2...sr)$ when Firm B is the buyer. The necessary condition for the Firm A and B to acquire these r firms will be different from (2.13) and (2.14). The necessary condition for Firm A is:

$$P_A \geqslant \sum_{i=1...r}^{si \neq B} (1 + bx_{si}^0 - Z(m+1))^2 + (1 + bx_B^0 - Z(m+1))^2 \tag{2.16}$$

While the necessary condition for Firm B is:

$$P_B \geqslant \sum_{i=1...r}^{si \neq A} (1 + bx_{si}^0 - Z(m+1))^2 + (1 + bx_A^0 - Z(m+1))^2 \tag{2.17}$$

From (2.16) minus (2.17), we can infer that firm A's advantage in the acquisition is:

$$2b(x_A^0 - x_B^0)(Z(m+r) - Z(m+1)) \geqslant 0 \tag{2.18}$$

We can indicate that in this case, Proposition 2.8 also holds. Similar to equation (2.15), the advantage of the larger firm in the acquisition increases with the increase of $x_A^0 - x_B^0$, b and r. However, when $r = 1$, this advantage disappears simply because the left side of (2.18) goes to 0. This tells us that, for a merger in which only Firm A and B are involved, the larger firm, A, has no advantage in the acquisition compared with the smaller one, B. In this special case, Firm A and B have an identical chance as the buyer. The advantage of larger initial market size can only be shown in a merger that occurs with the introduction of more firms.

In the case that firms merge to form a monopoly, we may obtain the following corollary:

Corollary 2.4 *If the firm with largest previous market size can not afford an acquisition, other firms can not afford this acquisition either.*

2.3.1 The Incentive of the Acquisition and Dynamic Market Structure

The incentive of the firm to be a buyer in an MNE is relatively weak and firms are all want to be a non-seller if there is only one period. We can find this by comparing the benefit of the buyer, the sellers and the non-sellers in the candidate equilibrium in Section 2.2.3. In this equilibrium, the benefit of the non-seller can be calculated by using the profit of the non-seller after the merger, minus the profit if the merger doesn't occur, which is:

$$MB_{non-seller} = (\frac{1}{n+2-k} + \frac{b(2-k)}{(n+1)(n+2-k)})^2 - C$$

From $MB_{non-seller} \geqslant 0$ for any $k \geqslant 2$,[7] we know that non-sellers always benefit from the mergers no matter how many firms are acquired by the buyer because they can obtain more profits by the decreasing number of the competitors, which can be seen from $\frac{1}{n+2-k} -$

[7] It is because

$$(\frac{1}{n+2-k} + \frac{b(2-k)}{(n+1)(n+2-k)}) - C = \frac{(k-1)(b-1)}{(n+1)(n+2-k)} + \frac{b(k-1)}{(n+1)^2(n+2-k)}$$

$\frac{1}{n+1} > 0$. Although non-sellers will suffer some loss from relatively small indirect network externalities ($\frac{b(2-k)}{(n+1)(n+2-k)} < 0$), this loss never exceeds what these firms gain from the reduction of the competition. The benefits of sellers can be calculated using the payment from the buyer, minus the profit if the merger doesn't occur, which is:

$$MB_{seller} = (\pi^D(k^*) + \epsilon) - C$$

From $MB_{seller} \geqslant 0$ for any $k \geqslant 2$,[8] we know the seller can always benefit from the merger as well and the reason is the same as that for the non-seller. The difference is, the non-seller may obtain more benefit than the seller, since $MB_{non-seller} - MB_{seller} > 0$ for any $k \geqslant 2$. From (2.10), which is the expression of ϵ, we may conclude that the buyer's benefit is 0 in our candidate equilibrium. There may exist some other equilibrium by which the buyer's benefit is not 0, via bargaining the offer with the sellers. However, the buyer's bargaining power is limited, since it needs to deter sellers or nonsellers from becoming second buyers by offering a very high bid.

If the firms that survive the current round of merger face another round of the merger, the buyer may have additional bargaining power for the acquisition in the next period than it has in the current period. An important reason for this is that the market would become asymmetric and lead by the buyer. The buyer, whose product has more indirect network externalities, creats more output in the last round of the merger and will become a larger firm compared with the non-sellers at the beginning of the next round of the merger game. From Proposition 2.8, we know that larger firms have some advantage and that this advantage is affected by the firm's initial market size. Thus, we might face a situation in which, in the second round acquisition game just after an SBMNE, only the buyer in the previous round of the game can find a k that will satisfy the condition (2.13) or (2.16), and all other non-sellers cannot satisfy (2.14) or (2.17). In this situation, only one firm, the buyer in previous period, has the chance to acquire others and no others firms have the ability to

[8] This is because

$$\left(\frac{1}{n+3-k} + \frac{b(3-k)}{(n+1)(n+3-k)}\right) - C = \frac{nb(k-2)}{(n+1)^2(n+3-k)}$$

host a merger. In this situation, the buyer does not need to offer a significant price premium to sellers in the second round of the game to deter a potential bidding war. Moreover, the buyer also generates more profit by providing products with higher indirect network externalities to consumers. Combined with the profit of these two periods, the buyers will obtain compensation for what they lost at the first period. Thus, the incentive of the firms to become a buyer in the first period increases.

If there is only one period, the maximum number of firms that the buyer may acquire is easy to predict by the sufficient and necessary condition (2.9). The buyer's ambition is constrained by the number of firms (n) and the intensity of indirect network externalities (b). The buyer cannot acquire as many firms as it expects (for example, merge to monopoly) when n is too large and b is too small, which has been illustrated in the Figure-2.2. However, if there is more than one round of the merger game, social planners should notice that the limits of concentration of the market in the first period can be exceeded via a two step acquisition. Sometimes, if the buyer acquires too many firms in the first period, the merger stops in the second period because the buyer and the non-seller in the second period are both unable to satisfy the necessary conditions. However, limited acquisition in the first period may open the possibility for additional acquisitions in the future and lead to a more concentrated market. Thus, an equilibrium in which that the buyer acquires a very large number of firms in the first period may not always lead to the most concentrated market structure. All of the possibilities above will be shown in the following example.

For example, we assume $n = 100, b = 0.21$. There are 30 possible SBMNE since any $k \in [20, 49]$ may satisfy the inequality (2.9). If equilibrium in the first period is located at $k = 49$, the market goes to the most possible concentration and a MNE cannot be found in the future period acquisition game. This is because, in the next period, the total number of firms in the market will be 52. For the buyer, we cannot find a $r \in [1, 51]$ which may satisfy (2.13) or (2.16). From Proposition 2.8, we know that all other firms cannot be the buyer as well in this situation. Thus, there will be no merger in the future rounds of the game. However, if equilibrium in the first period is located at $k = 30$, we have $n = 71$ after the first period. In the next period, only the buyer in last round of the game may host a merger. This is because inequality (2.14) and (2.17) may not hold for any $n = 71, b = 0.21$ and $r \in [1, 70]$. However, the inequality (2.13) may hold for $r \in [9, 27]$. If $r = 27$, the number of firms left in the market after the second round of the merger is 44, which is less than 52. We may conclude that two-step merger may lead to a more concentrated market structure. In the case that $k = 30$, if there is no discount for time, the total profit of the non-seller in two periods is around 0.00049. Although the buyer will get 0 profit in the first period, it may get around 0.00036 profit after payments for sellers in the second period if equilibrium is located at $r = 20$. If there is no merger in the second period, the buyer will obtain around 0.00070, which is significantly greater than the total profit of the non-seller of approximately 0.00036.

From this example, we see that a merger with a great increase of market concentration may not always be the worst thing for antitrust social planners since it may prevent future mergers. In addition, a merger with only limited firms involved should not be treated lightly since it may lead to a merger with more participating firms.

2.4 Conclusions and Future Work

Inheritable indirect network externalities can be a very important factor that require consideration when we analyze firm acquisition behaviors. The possibility of inheriting market-size-determined network resources may encourage firms to acquire other firms. Hence, the merged Nash equilibrium can more easily exist than the situation in which there is no other benefit for the merger except a reduction of competition. We also find that the single buyer merged Nash equilibrium can more easily exist than a multi-buyer merged Nash equilibrium when the number of firms in the market is relatively large. If social planners find out that a single buyer merger is not possible given current market conditions, they may also rule out multi-buyer mergers. With the sufficient and necessary condition for the merged Nash equilibrium that we indicate in this paper, social planners may more accurately predict whether a merger will create a concern and may determine how to regulate it properly if so. However, the model is based on a linear indirect network externality function and the sufficient and necessary condition in a more general utility function would be valuable future work since it can be helpful for social planners to use to solve more general cases. After calculating the buyer's, seller's and non-seller's benefits from the merger, we find that buyer does not obtain as much benefit from the merger as the seller and the non-seller if the model only has one period. This coincides with what we see in the stock market, wherein the buyer's stock price decreases while the seller's stock price increases after the merger announcement. However, the acquisition provides the buyer an advantage in the competition with the outsiders and creates more output since the acquisition allows the buyer's products to become more valuable to the consumers due to an increase of indirect network externalities. Although buyers needs to pay for obtaining this advantage and may only share a very small part of the benefit from the concentration of the market compared with the sellers and non-sellers in current period, this advantage may bring about significant profits to the buyer in future competition. Moreover, in some special cases, only larger firms can acquire smaller firms, while smaller firms cannot acquire larger ones. Even in the case that all firms have an equal chance to acquire others, larger firms can afford higher payments than smaller firms. Thus, those firms with larger market sizes always find it easier to win bidding wars. This gives the merged entities a better chance to boost their leading market positions and the market may move towards a more concentrated structure endogenously. In this paper, we only provide some examples and use a static model to show the possibility that merged entities uses their market size advantages to capture future addtional revenue. If we seek to model the whole process and detail the firm's strategy and behavior when firms are forward looking, a dynamic model would be more accurate and necessary. The incentive for the firms to sell themselves and leave the market, such as facing better opportunity in other markets, could also be added into the model to more accurately simulate the real world.

2.5 Appendix

A2.1 Proof of Footnote 4

If Firm A can acquire Firm B, we have $b_A^B \geqslant b_B^B$. If Firm B can acquire Firm C, we have $b_B^C \geqslant b_C^C$. According to our assumption $b_i^i > b_i^j$ $(j \neq i)$, we have $b_A^A > b_A^B$, $b_B^B > b_B^C$ and $b_C^C > b_C^A$. Thus, we may indicate that $b_A^A > b_C^A$. This means Firm C's bid for Firm A is always less than Firm A's reservation price and Firm C is not able to acquire firm A. The merger dilemma does not exist.

A2.2 Proof of Proposition 2.1

Assume an MNE exists and there does not exist a pair of m and k which let inequality (2.7) hold. In the MNE, the buyer has to pay $(k-1)\pi^D$ to keep the sellers from deviating. However, the maximum profits the buyer can obtain from the acquisition process is $\pi_l^M - \pi_l^{NM}\min$ which is always less than what it has to pay to keep the merger. Hence, the buyer always has incentive to deviate from the equilibrium by setting itself as a non-buyer. This is contradict to our assumption. Thus, we can say there does not exist any MNE if there does not exist a pair of m and k which let inequality (2.7) hold or there exists a pair of m and k which let inequality (2.7) hold is the necessary condition for the existence of an MNE for a given n and b.

A3: The proof of Proposition 2.2

If there exists an MNE, we have $m \leqslant n-1$. If $m = n-1$, only 2 firms get merged in the first step of the game and we can rewrite the equation (2.7) as equation (2.11). It is easy to find that larger b makes the inequality (2.11) easier to hold when n is fixed.

If $m \leqslant n-2$, we have:

$$1 - \frac{n}{m+2} < 0 \Longrightarrow \frac{\partial\sqrt{\pi^D}}{\partial b} < 0 \Longrightarrow \frac{\partial \pi^D}{\partial b} < 0$$

So π^D decreases with an increase of b.

As we assume $k_l \geqslant k_i$ $(i = 1...m)$, we can obtain:

$$k_l - \frac{n}{m+1} > 0 \Longrightarrow \frac{\partial \pi_l^M}{\partial b} > 0 \tag{2.19}$$

We also know $m + k_l - 1 \leqslant n$. If $m + k_l - 1 < n$, we can get:

$$m + k_l \leqslant n \Longrightarrow 1 - \frac{n}{m+k_l} < 0 \Longrightarrow \frac{\partial \pi_l^{NM}}{\partial b} < 0 \tag{2.20}$$

By adding (2.19) and (2.20) together, we can obtain:

$$\frac{\partial \pi_l^M}{\partial b} - \frac{\partial \pi_l^{NM}}{\partial b} > 0 \Longleftrightarrow \frac{\partial(\pi_l^M - \pi_l^{NM})}{\partial b} > 0$$

If $m + k_l - 1 = n$, the left side of equation (2.7) can be written as:

$$\begin{aligned}
& \pi_l^M - \pi_l^{NM} \min \qquad (2.21) \\
= \ & \left(\frac{1}{m+1} + \frac{b}{n+1}(k_l - \frac{n}{m+1})\right)^2 - \left(\frac{1}{m+k_l} + \frac{b}{n+1}(1 - \frac{n}{m+k_l})\right)^2 \\
= \ & \left(\frac{1}{m+1} - \frac{1}{m+k_l} + \frac{b}{n+1}(k_l - 1 + \frac{n}{m+k_l} - \frac{n}{m+1})\right) \\
& \left(\frac{1}{m+1} + \frac{1}{m+k_l} + \frac{b}{n+1}(k_l + 1 - \frac{n}{m+k_l} - \frac{n}{m+1})\right)
\end{aligned}$$

In equation (2.21), we have:

$$\begin{aligned}
k_l + 1 - \frac{n}{m+k_l} - \frac{n}{m+1} &= k_l + 1 - \frac{n}{n+1} - \frac{n}{m+1} \\
&= k_l + \frac{1}{n+1} - \frac{n}{n+3-k} > 0
\end{aligned} \tag{2.22}$$

$$\begin{aligned}
k_l - 1 + \frac{n}{m+k_l} - \frac{n}{m+1} &= k_l - 1 + \frac{n}{n+1} - \frac{n}{m+1} \\
&= k_l - \frac{1}{n+1} - \frac{n}{n+3-k} > 0
\end{aligned} \tag{2.23}$$

Combining (2.22) and (2.23), we can indicate $\partial(\pi_l^M - \pi_l^{NM})/\partial b > 0$ if $m + k_l - 1 = n$. Thus, the left side of the equation (2.7) increases with an increase of b.

Comparing the change of the left side of the equation (2.7) with the change of π^D, we can conclude that, for the largest buyer, the necessary condition for the existence of an MNE will get relaxed by a larger b.

A2.4 Proof of Lemma 2.1

If the buyer partly decreases its bid for the sellers, total firms acquired by the buyer will be reduced to $k' - 1$. Obviously $k' < k^*$. Given that the strategies of the sellers and non-sellers are fixed, the profits of the buyer from the acquisition with the new strategy is $\pi^M(k') - (k'-1)(\pi^D(k^*) + \epsilon)$. According to the definition of k^*, we have

$$\pi^M(k^*) - (k^*-1)\pi^D(k^*) \geqslant C > \pi^M(k') - (k'-1)\pi^D(k')$$

By substituting $m = n + 1 - k$ into π^D, we may obtain:

$$\pi^D(k) = (\frac{1}{n-k+3}(1 - \frac{nb}{n+1}) + \frac{b}{n+1})$$

Since

$$\frac{\partial \pi^D(k)}{\partial k} > 0 \Longrightarrow \pi^D(k^*) > \pi^D(k')$$

$$\begin{aligned} \Longrightarrow \quad & \pi^M(k') - (k'-1)(\pi^D(k^*) + \epsilon) = \pi^M(k') - (k'-1)\pi^D(k^*) - (k'-1)\epsilon \\ < \quad & \pi^M(k') - (k'-1)\pi^D(k^*) < \pi^M(k') - (k'-1)\pi^D(k') < C \end{aligned}$$

A2.5 Proof of Lemma 2.2

If one of the non-sellers sets its bid for some of the sellers higher than $\pi^D(k^*) + \epsilon$, this non-seller may snatch some firms from the buyer. We assume this non-seller snatches $k'' - 1$ firms from the buyer and its bid for these sellers is $\pi^D(k^*) + \epsilon'$. Here $2 \leqslant k'' \leqslant k^*$ and $\epsilon' > \epsilon$. Although this non-seller snatched some firms from the buyer, the total number of firms in the second stage competition hasn't changed and this number is:

$$Q = \frac{n + 1 - k^* + \frac{nb}{n+1}}{n + 1 - k^* + 1}$$

From the first-order condition of the profit function of this non-seller, we can obtain its output, when the strategy we have defined is chosen, is:

$$x^*_{NS} = 1 + \frac{bk''}{n+1} - Q$$

And its profit after the acquisition is:

$$\pi^*_{NS} = ((1 - \frac{nb}{n+1})\frac{1}{n+2-k^*} + \frac{bk''}{n+1})^2$$

Given k^*, we can indicate that the net profit of the non-seller from snatching k'' firms from the buyer is a quadratic function of k'':

$$P(k'') = \pi^*_{NS} - (k''-1)(\pi^D(k^*) + \epsilon')$$

It is clear that:

$$\begin{aligned} P(k^*) &= \pi^M(k^*) - (k^*-1)(\pi^D(k^*) + \epsilon') \\ &< \pi^M(k^*) - (k^*-1)(\pi^D(k^*) + \epsilon) = C \end{aligned}$$

and

$$\begin{aligned} P(1) &= ((1 - \frac{nb}{n+1})\frac{1}{n+2-k^*} + \frac{b}{n+1})^2 \\ &> ((1 - \frac{nb}{n+1})\frac{1}{n+2-1} + \frac{b}{n+1})^2 = C \end{aligned}$$

Since the quadratic function $P(k'')$ is convex and $P(1) > P(k^*)$, we may conclude that $P(k'') < P(1)$ for any $2 \leqslant k'' \leqslant k^*$. Since $P(1)$ is the profit of this non-seller acquiring nobody, we may say acquiring nobody is always a better strategy than acquiring no more than $k^* - 1$ firms.

A2.6 Proof of Lemma 2.3

If one of the sellers becomes a second buyer, the total number of firms in the last step competition will increase 1 to

$$Q' = \frac{n+2-k^* + \frac{nb}{n+1}}{n+3-k^*} > Q$$

Assume this seller acquires $k''' - 1$ firms and its bid to these sellers is $\pi^D(k^*) + \epsilon'$. Here $2 \leqslant k''' \leqslant k^*$ and $\epsilon' > \epsilon$. Following the identical procedure as the proof of Lemma 2.2, we can obtain the benefit for the seller being a second buyer is:

$$\begin{aligned}
P_s &= (1+\frac{bk'''}{n+1}-Q')^2-(k'''-1)(\pi^D(k^*)+\epsilon') \\
&< (1+\frac{bk'''}{n+1}-Q)^2-(k'''-1)(\pi^D(k^*)) \\
&< \pi^M(k''')-(k'''-1)\pi^D(k''')<C \qquad (2.24)
\end{aligned}$$

We also know:

$$\begin{aligned}
\pi^D(k^*)+\epsilon &= ((1-\frac{nb}{n+1})\frac{1}{n+3-k^*}+\frac{nb}{n+1})^2+\epsilon \\
&> \pi^D(2)+\epsilon = C+\epsilon > C \qquad (2.25)
\end{aligned}$$

Combining (2.24) and (2.25), we can get:

$$\pi^D(k^*)+\epsilon > C > P_s$$

This means the seller will always obtain less than $\pi^D(k^*)+\epsilon$ profit if it tries to be a second buyer.

A2.7 Proof of Proposition 2.4

In the second stage of the game, if only one buyer exists and this buyer only acquires one firm, we have $m=n-1$ and $k=2$ by the definition of m and k. Hence, $\pi^M(k)$, $\pi^{NM}(k)$ and $\pi^D(k)$ can be rewritten as:

$$\pi^M(2)=(\frac{1}{n}+\frac{b}{n+1})^2$$

$$\pi^{NM}(2)=\pi^D(2)=(\frac{1}{n+1}+\frac{b}{(n+1)^2})^2$$

The inequality (2.9) can be written as:

$$(\frac{1}{n}+\frac{b}{n+1})^2 \geqslant 2(\frac{1}{n+1}+\frac{b}{(n+1)^2})^2$$

By solving this inequality for b, we can obtain (2.11). This means that, for any $n \geqslant 3$, if b satisfies (2.11), there exists a $k=2$ which makes inequality (2.9) hold. According to Proposition 2.2, we always have an SBMNE if the inequality (2.9) holds. We can then conclude that there exists an SBMNE in which the buyer only acquires one firm ($k=2$), if b satisfies (2.11).

A2.8 Proof of Proposition 2.5

If the market finally becomes a monopoly, the only possibility is one buyer acquired all the other firms. Thus, this MNE must be a SBMNE with $m = 1$ and $k = n$ according to our definitions. By substituting $k = n$ into $\pi^M(k)$, $\pi^{NM}(k)$ and $\pi^D(k)$, we can obtain:

$$\pi^M(k) = \frac{1}{4}(1 + \frac{nb}{n+1})^2$$
$$\pi^{NM}(k) = (\frac{1}{n+1} + \frac{b}{(n+1)^2})^2$$
$$\pi^D(k) = \frac{1}{9}(1 + \frac{(3-n)b}{n+1})^2$$

and the inequality (2.9) can be written as:

$$\frac{1}{4}(1 + \frac{nb}{n+1})^2 - (\frac{1}{n+1} + \frac{b}{(n+1)^2})^2 \geqslant (n-1)\frac{1}{9}(1 + \frac{(3-n)b}{n+1})^2$$

By solving the above inequality, we can get (2.12). This means, for any $n \geqslant 3$, if we have a b which satisfies (2.12) , the inequality (2.9) will hold when $k = n$. From Proposition 2.3, we know that an SBMNE must exist in which all the firms merge to become a monopoly.

A2.9 Proof of the sufficiency of Proposition 2.6

We only need to prove the candidate equilibrium is an MBMNE when $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$.

(i) For the first buyer, fixed the strategy of all other three firms

When $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$, we have inequality (2.7). Thus, the first buyer has no incentive to decrease its bid for Seller A. Obviously the first buyer also has no incentive to increase its bid for Seller A because this will decrease its net profit. The first buyer has no incentive to decrease or increase its bid for the second buyer since the second seller's bid for itself is ∞. The first buyer may increase its bid for Seller B in order to snatch Seller B from the second buyer. If the first buyer chooses to do so, he needs to pay Seller B at least $\frac{\sqrt{2}}{4} - \frac{1}{3}$ and its net profit would be:

$$P \leqslant (\frac{1}{3} + \frac{b}{5}(3 - \frac{4}{3}))^2 - 2(\frac{1}{5} + \frac{b}{6}(1 - \frac{4}{5}))^2 \leqslant \frac{1}{4^2} \text{ for } b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$$

We know the net profit, that the first buyer may get if he chooses the strategy as our candidate equilibrium, is $\frac{1}{4^2}$, so the first buyer will not choose to snatch Seller B from the second buyer. Obviously the first buyer has no incentive to change its bid for itself since all the others only bid 0 for him. In a conclusion, the first buyer has no incentive to change its strategy when other players' strategies are fixed as the candidate equilibrium.

(ii) For the Seller A, fixed the strategy of all other three firms

If Seller A chooses to deviate, he may get $\frac{1}{4^2}$ profit. When $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$, we have

$$(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16} \geqslant \frac{1}{4^2}$$

Thus, Seller A has no incentive to simply increase its bid for itself and deviate from the acquisition. He also has no incentive to decrease his bid for himself because this will only decrease his profit. Seller A has no incentive to increase his bid for the two buyers since the buyers' bids for themselves are ∞. He may choose to increase his bid for Seller B to a number larger than $(\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}$ and increase his bid for himself to ∞ at the same time. This would make himself the only buyer. In this case, Seller A's net profit is:

$$P_A \leqslant (\frac{1}{4} + \frac{1}{5}b)^2 - ((\frac{2}{15}b + \frac{1}{3})^2 - \frac{1}{16}) < \frac{1}{4^2}$$

Thus, Seller A will not choose this strategy. In a conclusion, seller A has no incentive to change its strategy when other players' strategies are fixed as the candidate equilibrium.

The analysis of the second buyer and seller B is similar to (i) and (ii), since the equilibrium is symmetric and the two buyers and two sellers are in an identical position. We may then indicate that the candidate equilibrium is an MBMNE when $b \geqslant \frac{\sqrt{2}}{4} - \frac{1}{3}$, and the sufficiency of Proposition 2.6 is proved.

A2.10 Proof of Proposition 2.7

We define a function:

$$f(m) = (k-1)\pi^D + \pi_l^{NM} \min - \pi_l^M \Longrightarrow$$

$$\begin{aligned} -\frac{\partial f(m)}{\partial m} &= (1 - \frac{bn}{n+1})(\frac{k-1}{(m+2)^3} + \frac{1}{(m+k)^3} - \frac{1}{(m+1)^3}) \\ &+ \frac{b}{n+1}(\frac{k-1}{(m+2)^2} + \frac{1}{(m+k)^2} - \frac{k}{(m+1)^2}) \end{aligned}$$

We also define:

$$\alpha = \frac{k-1}{(m+2)^3} + \frac{1}{(m+k)^3} - \frac{1}{(m+1)^3}$$
$$\beta = \frac{k-1}{(m+2)^2} + \frac{1}{(m+k)^2} - \frac{k}{(m+1)^2}$$

It is easy to test that $\alpha > 0$ and $\beta < 0$ when $n \geqslant 5$,. From Corollary 2.2, we also know that there always exists an SBMNE if $b \geqslant \sqrt{2}-1$. So, here, we only need to consider the situation in which $b < \sqrt{2}-1$. Combining all conditions above with $n \geqslant m+k-1$, we can obtain:

$$\begin{aligned} -\frac{\partial f(m)}{\partial m}(n+1) &\geqslant (n+1-bn)\alpha + b\beta \\ &> (n+1-(\sqrt{2}-1)n)\alpha + (\sqrt{2}-1)\beta \\ &\geqslant ((2-\sqrt{2})(m+k-1)+1)\alpha + (\sqrt{2}-1)\beta \end{aligned} \tag{2.26}$$

We plot the right side of (2.26) in the following figure:

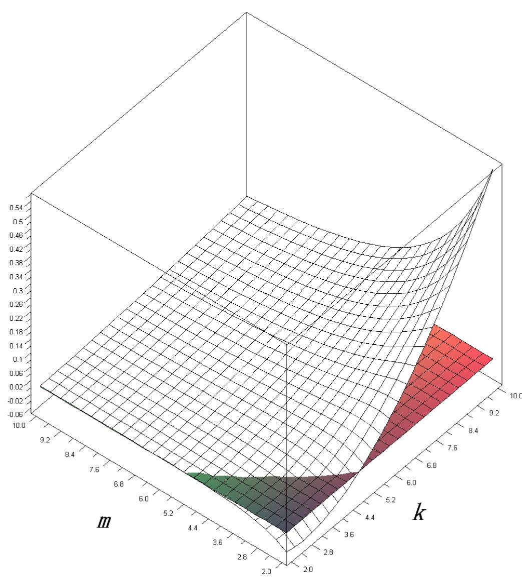

Figure-2.3

From Figure-2.3, we can see that only 11 pair of m and k will make the right side of (2.26) be negative. They are $(m = 1, k = 3, 4, 5, 6, 7)$, $(m = 2, k = 2, 3, 4)$, $(m = 3, k = 2, 3)$ and $(m = 4, k = 2)$. By the definition of m and k, we know $n < m * k$. Hence, when

$n > 9$, none of the above 11 pairs of m and k exists. Thus, we may conclude, when $n > 9$,

$$-\frac{\partial f(m)}{\partial m}(n+1) > 0 \Longrightarrow \frac{\partial f(m)}{\partial m} < 0$$

A2.11 Some calculations in the example of Section 2.3.2

When $n = 100$ and $b = 0.21$, we may rewrite the inequality (2.9) as:

$$(\frac{0.792}{102-k} + 0.00208k)^2 - 0.000098 - (k-1)(\frac{0.792}{102-k} + 0.00208k) \geqslant 0 \qquad (2.27)$$

By solving (2.27), we may get $49.5 \geqslant k \geqslant 20.96$. Since k is a natural number, $k \in [20, 49]$.

If $k = 30$, $m = 71$ in the first period and buyer's market size is:

$$x_A^0 = 1 + \frac{bk}{n+1} - \frac{m}{m+1} - \frac{nb}{(m+1)(n+1)} = 0.07338$$

The non-seller's market size is:

$$x_B^0 = 1 + \frac{k}{n+1} - \frac{m}{m+1} - \frac{nb}{(m+1)(n+1)} = 0.01308$$

From the definition of z, (2.2), we can also obtain:

$$\begin{aligned} Z(m) &= \frac{71-r}{72-r} + \frac{71b}{72(72-r)} \\ Z(m+1) &= \frac{72-r}{73-r} + \frac{71b}{72(73-r)} \\ Z(m+r) &= 0.98899 \end{aligned}$$

Thus, the inequality (2.13) can be written as:

$$\begin{aligned} P_A &= (1 + b(rx_B^0 + x_A^0) - Z(m))^2 - (1 + bx_A^0 - Z(m+r))^2 \\ &\geqslant r(1 + bx_B^0 - Z(m+1)) \end{aligned}$$

, which may be solved and get $r \in [9, 27]$. The inequality (2.14) can be written as:

$$\begin{aligned} P_B &= (1 + b(rx_B^0 + x_B^0) - Z(m))^2 - (1 + bx_B^0 - Z(m+r))^2 \\ &\geqslant r(1 + bx_B^0 - Z(m+1)) \end{aligned}$$

, which has no solution when $r > 0$.

The case when $k = 49$ may be solved in a very similar way.

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Chapter 3
Locked-in by Contract, Competition and Network Externalities

Abstract

This paper investigates a single period Cournot competition model in duopoly market with some of the consumers are locked by the contract. The number of the consumers who have been locked by each firm is exogenous. We reveal that multiple equilibria may exist, while the firms and social planner always have conflicting incentives in the selection of the equilibrium. In an extended discussion, we add network externalities and asymmetric initial market structure into our model. We show that the firm with more initial locked-in consumers have an advantage in the competition if there exists a network effect. This advantage will be extended when the intensity of the network effect increases. Hence, obtaining more locked-in consumers could be an important incentive for the firms to merge in network world.

3.1 Introduction

In the current information economy, many service providers, such as Telecommunications companies and Internet service providers (ISP), are asking their consumers to sign a contract with monthly payment, which may last one or several years. During the contract period, the consumer must pay the service with an ex-ante specific price, no matter whether he/she consumes it or not. While this distribution method may not reduce the competition between firms (Farrell and Shapiro 1989), it is still very popular in the service industry. Two aspects have been defined as potential reasons for this popularity: one is that firms may use contracts to maintain their market share, deter potential entrants and reduce the uncertainty of future profits; the second is that consumers will normally overestimate their future consumption and purchase more than they really need (Vigna and Malmendier 2004).

Many service goods, such as TV subscription, internet access and mobile phone communications, are homogenous and the utility that the consumers can obtain from these goods will not increase simply by repeated purchase. Moreover, firms in these industries may not successfully lure the consumers who have signed a contract with other firms by cutting their prices. For example, if one of the consumers has already purchased one year of unlimited internet access from an ISP, he/she will not obtain any more utility from purchasing another internet access and obviously he/she will not consider purchasing more internet access from another ISPs during that year, no matter how inexpensive it is.

Since consumers who are locked-in to their service providers with a contract may be less likely to modify their choice or purchase more, firms are actually competing in the part of the market which constructed by two kinds of potential customers: those with a strong willingness to buy, who can afford a relatively high price and have just finished a contract; and those with a weak willingness to purchase, who still stand outside the market in a previous rounds competition and will not purchase the service unless the price is low enough. Since some of the consumers with a high willingness to purchase are locked-in by the contract, the chance for the firms to meet a consumer who has very high willingness to buy is relatively lower than the chance that the firms will meet a low willingness to purchase consumer. In other words, the density of the consumers who have a high willingness to buy and just finished a contract is lower than the density of the consumers who has low willingness to buy and are outside the market. This difference affects the price sensitivity of the total output.. To illustrate this further, we may think such an example: when the price of the mobile phone is over 1000 USD, only 1 additional consumer will purchase it if the firms reduce the price by 1 USD. However, because of the difference in density, when the price is lower than 100 USD, 100 new consumers will join the market if the firms reduce the price by 1 USD. Here, the price of the mobile phone is more sensitive to the total output when the price is high and when the total output is low. Firms may face a kinked demand functions with different slope in different output level.

Compared with traditional linear demand function Cournot competition, the kinked demand function may result in multiple equilibria. Firms may reach an equilibrium at steeper part of the demand function, which means they only deal with high willingness consumers and choose a relatively low output level. They may also reach an equilibrium at flatter part of the demand function, which indicates they compete in an expanding market. The existence of these equilibria and their location both depends on the initial market structure. Our main focus is to locate the equilibrium outputs and reveal the relationship between these equilibria and the initial market structure. Moreover, we will also investigate the social welfare and firms' incentives in equilibrium selection and discuss its implications to the social planner.

Some economic phenomenons as studied in the literature, have very similar characteristics as a locked-in effect of contract, for example, consumer's loyalty. The locked-in consumers can be defined as 100 percent loyal to the firms with which they signed the contract. Rosenthal (1980), Deneckere et al (1992), and Fisher and Wilson (1995) have studied a single period model where part of the consumers only purchase from specific firms. The difference between loyalty and locked-in by contract is that the firm's strategy will affect loyal consumers' behaviours and surplus but will not affect the consumers who are locked by the contract. Although the loyal consumers will not purchase from other firms, they can choose to stand outside the market. If they choose to join the market, they need to pay the price in the current period. However, for the consumers who have signed a contract with a specific firm, they must pay and only need to pay the goods at the previously fixed price. This means firms completely do not need to consider locked-in consumers when they design their competition strategy. Some other similar cases are developed by Varian (1980) and Padilla (1992), who investigated a model with part of the consumers who cannot choose freely because they can only observe some specific firms' price.

Ferrell and Klemperer (2006) point out that the previous cases (loyalty and imperfect price information) can all be consolidated and analyzed or interpreted as a single period model with switching cost. This concept was first discussed by Weizsacker (1984) and well developed by Klemperer. In fact, the contract in our model can also be partly explained by switching cost. Since the consumers locked-in by the contract will not choose other firms' product, we can say these consumers have an infinite switching cost with the choice of service providers. However, switching cost model cannot fully characterize the locked-in contract. This is because all the consumers in the switching cost model (similar to the loyalty model) must face current market prices. In our model, the locked-in customer only needs to pay an ex-ante specified price. Another development of our work compared with the literature about switching costs is that we use an open market instead of a covered market. In Klemperer's earlier work (1987a, 1988, 1989), he uses a two period model to investigate the effect of switching cost to the firms' behaviour. Since he assumes the firms follow Cournot or Bertrand competition in the first period, the market always shrink in the second period compared with the total output of the first period. Thus, he does not need to consider the possibility that some new consumers, who never purchase from any firms in the first period, may enter the market in the second period. Following this structure, most of the literatures about the switching cost (Klemperer 1987b, 1995, Beggs and Klemperer1992) choose to examine a more conveniently covered market or a linear city model. In this chapter, we consider a single period competition with exogenous initial market structure, which brings the potential possibility that the market may expand in the equilibrium (new consumers entering the market) and multiple equilibria may exist in some circumstances.

The development of an open market model may illustrate some new characteristics and help to solve the market equilibrium for an arbitrary initial market condition.

Network externalities are another important feature of the information economy. In a simple model without network effects, only the total number of the locked-in consumers in the initial setting may influence the equilibrium, while the component of these locked customers is irrelevant. However, if we consider the network externalities, the initial market structure will be every important in determination of the equilibrium. The firm with more locked-in consumers will inherently have an advantage in the competition, since their product is more attractive to the consumers if all other conditions are equivalent. As an extension to the basic model, in the second part of this chapter, we provide a further study of a locked-in model with network effect and asymmetric initial market structure. The modelling of network externalities is based on the work of Katz and Shapiro (1985)[9].

Different to the basic model, the asymmetric initial market structure may result in an asymmetric equilibrium. The firms with more locked-in consumers are able to charge a higher price and take a larger percentage of the market share. In some extreme cases, they may even deter other firms from entering the market. The size of the effect of initial advantage is closely connected with the intensity of the network externalities. Strong network externalities may enhance the effect of initial advantage, but weak network externalities will make this advantage insignificant. Moreover, pure strategy equilibrium will not always exist in the extended model. The number and location of the equilibrium vary with the change of the initial market structure and the intensity of the network externalities. We will solve all the equilibria and provide the sufficient and necessary conditions for their existence. The locked-in model with network externalities could be important when study the firms' merger behaviour in network economy. If the locked-in consumers from all merged firms can be inherited by the new entity, obtaining more locked-in consumers could be an incentive for the firms to merge. This is because these locked-in consumers may bring a relative advantage for the new entity in competition through the network effect.

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9 More literatures about the network externalities can be found in the survey paper by Farrell and Klemperer (2006).

In Section 3.2, we will describe the basic model, solve for the equilibrium in the duopoly and oligopoly market and give out a discussion about the social welfare in different equilibria. In Section 3.3, we add the network effect and asymmetric initial market structure to our basic duopoly model, solve the equilibrium with different asymmetrical levels and intensities of the network effect. We will also provide the sufficient and necessary conditions for the existence of all the pure strategy equilibrium in extended model. Section 3.4 is the conclusion and also recommends future work.

3.2 The Model with Locked-in

Assume there are only 2 firms, A and B, in the market. They produce homogeneous products and compete with their outputs. Consumers are heterogeneous in their basic willingness to pay for the product. We denote their basic willingness to pay for the product as r. r varies across the consumers and is assumed to be uniformly distributed between minus infinity and 1 with a density of one. The uniform distribution assumption allows us to obtain a linear demand function for the products. This means that if we nominate a person, who would like to pay the highest price to purchase the product, he/she has a willingness that equals 1. And we assume people who dislike the product may have a willingness that equals minus infinity. Our model is an opened market, so we do not need to discuss corner solutions. These assumptions will make it convenient for us when we discuss the model with network externalities in the second part of the chapter. We also assume the consumers can only purchase one product from one of the firms or stands outside the market. Obviously if both firms want to have a positive output, they must set their prices to be equal, because the products are homogeneous to the consumers. Thus, there is only one price in the market and we denote it as p $(1 > p > 0)$. When a consumer purchases the product from one of the firms, the surplus he can obtain is $r - p$. We know that the consumer will buy the product only if he/she can obtain a positive surplus from the purchase. Only those consumers with their willingness (r) larger than p enter the market. Given the uniform distribution, the market size can be written as z $(z = 1 - p)$. Since $p > 0$, the market size, or the total output of the firms, z, is strictly less than 1.

3.2.1 Locked-in and Equilibrium in Duopoly Market

[10]Now we assume the number of the consumers who are already in the market before the firms begin the competition is x_0. As we know that if a consumer chooses to purchase the product, any consumer with a willingness larger than him will also choose to purchase the

[10] We only investigate pure strategy nash equilibrium in this chapter.. There may exist mixed strategy nash equilibrium, but these strategies are not discussed here. However, they may be included in furture work.

product, it is easy to see that these insiders in previous round competition are uniformly distributed between $1 - x_0$ and 1. We also assume that half of the insiders in previous time are locked-in with their service provider because of the purchasing contracts they signed. This means half of the consumers who joined the market in previous time can neither change their mind to choose another firm's product nor quit the market. Hence, when the firms choose their outputs, they do not need to consider these locked-in consumers, because their decisions will not change the behaviour of these customers. To make things easy, we also assume that these consumers, who are locked in the market, are also uniformly distributed. The density of the people, who are located and unlocked in $[1 - x_0, 1]$, is half of the density of the people located from $1 - x_0$ to minus infinity. The allocation of the consumers and their willingness can be illustrated in the following figure:

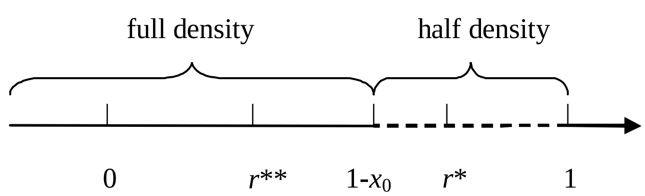

Figure-3.1: Consumer's willingness

From Figure-3.1 we can see that the consumers' density in dashed area, $[1 - x_0, 1]$, is just half of the density in solid line area, $(-\infty, 1 - x_0]$.

We use x_A and x_B to denote the output of Firm A and B in current round competition respectively. Here, x_A and x_B do not include the outputs for the consumers who are already locked in the market. The firms choose the output to maximize their profits and price p is determined by the market. There exists a person who has no difference between purchasing the product or not under price p. We denote this consumer's willingness as r^* if it is located in $[1 - x_0, 1]$. From Figure-3.1, we know r^* is located in the dashed half density area. According to our assumption, the people located in the right of the axes have more incentive to get into the market than the ones located at relatively left. Thus, only the consumers on the right side of r^* purchase the product. In this situation, we have:

$$x_A + x_B = \frac{1 - r^*}{2} \leqslant \frac{x_0}{2} \tag{3.1}$$

and the market size $z = \frac{x_0}{2} + \frac{1-r^*}{2}$, which is the sum of the consumers who voluntarily joined the market and the consumers who are locked by the contract. Since $z = \frac{x_0}{2} + \frac{1-r^*}{2} \leqslant x_0$, we know that some of the consumers who purchased produce previously choose to stand outside the market in current round competition. In other words, the market shrinks compared with the previous market size.

If the consumer, who has no difference between purchasing the product or not, is located on the left side of $1 - x_0$. we call his/her willingness r^{**}. From Figure-3.1, we can see that r^{**} is located in the full density area. For the same reason as above, we have :

$$x_A + x_B = \frac{x_0}{2} + [(1 - x_0) - r^{**}] > \frac{x_0}{2} \tag{3.2}$$

and the market size $z = \frac{x_0}{2} + x_A + x_B = 1 - r^{**}$. In equation (3.2), $\frac{x_0}{2}$ represents the consumers in the half density area, $(1 - x_0) - r^{**}$ is the number of the consumers who are located in $[r^{**}, 1 - x_0]$ with full density. By the definition, $r^{**} < 1 - x_0$, so we know $z = 1 - r^{**} > x_0$. This means, besides all the consumers who purchased the product in previous time, some new consumers with a smaller r are enticed to enter the market. We can also say that the market expanded compared with the previous market size.

The consumers who have been locked-in by their previous choices must buy the product according to contract price they signed with the firms in previous round competition. These previous prices have no effect on the firm's decision right now, and all the prices we discuss in this chapter are the prices that firms set for the consumers who can freely choose (the current market price). The consumers who haven't been locked-in will enter the market only if their willingness r is no less than p . Thus, the willingness of the consumer, who have no difference between purchasing and not purchasing, must be p. If $x_A + x_B \leqslant \frac{x_0}{2}$, we denote the current market price as p^* and we have $r^* = p^*$. By rearranging the equation (3.1), we can get:

$$p^* = 1 - 2(x_A + x_B) \tag{3.3}$$

If $x_A + x_B \geqslant \frac{x_0}{2}$, we denote the price as p^{**} and we have $r^{**} = p^{**}$. By rearranging the equation (3.2), we will obtain:

$$p^{**} = 1 - \frac{x_0}{2} - (x_A + x_B) \tag{3.4}$$

Proposition 3.1 *Firm A and B's outputs are always less than $1 - \frac{x_0}{2}$ when they have a positive profit.*

Since the price has a negative relationship with the total output, Firm A and B will always curb their output to maintain a positive price. Proposition 3.1 reveals the upper limit of the firm's output. This proposition can easily be illustrated in Figure-3.1. From Figure-3.1, we can see that if any firm sets its output larger than $1 - \frac{x_0}{2}$, some consumers located on the left side of 0 enter the market. This situation has been ruled out, because the consumers located on the left side of 0 will not enter the market unless the price is negative.

Different to the traditional model with a linear single slop demand function, here, the slop of demand function will change at different output levels. In order to provide more intuition about the relationship between the firm's output and the market price, we combine the equation (3.3) and (3.4) and draw the price function as the output of Firm A for a given output of Firm B in the following figure:

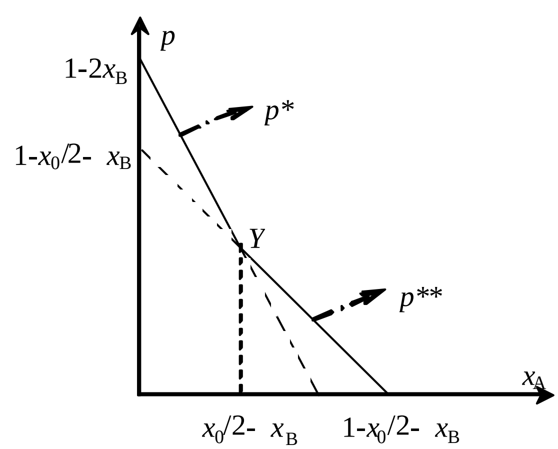

Figure-3.2: Firm A's price function given x_B.

In Figure-3.2, we can see the market price will be 0 when x_A is larger than $1-\frac{x_0}{2}-x_B$, so x_A is bounded by $[0, 1-\frac{x_0}{2}-x_B)$. From Proposition 3.1, we know $1-\frac{x_0}{2}-x_B > 0$. If $x_A \leqslant \frac{x_0}{2}-x_B$, the market price will obey the function (3.3); if $x_A \geqslant \frac{x_0}{2}-x_B$, the price function in the market is function (3.4). Function (3.3) and (3.4) joint at the kink point $Y(\frac{x_0}{2}-x_B, 1-x_0)$. Obviously, with the change of the value of x_0 and x_B, kink point Y will move upwards or downwards and the shape of the price function p^* and p^{**} will also change. In an extreme case ($\frac{x_0}{2} < x_B$), the kink point Y will go to negative side. Hence, p^* part of the price function does not exist and the price is solely determined by p^{**}. The intuition behind this situation is that the market size will surely expand if Firm B chooses an output bigger than $\frac{x_0}{2}$. Here, since x_0 is less than 1, kink point $Y(\frac{x_0}{2}-x_B, 1-x_0)$ will never locate below the x_A axis. Thus, p^{**} part of the price function always exists. This is because Firm A can always choose an output to make the market expand if Firm B chooses a relatively small output.

Figure-3.2 also reveals that the main difference between our model and the traditional Cournot model is that the price function in our model is constructed by two straight lines with different slopes, but in the traditional Cournot comptition, the price function has a unique slope. From equation (3.3) and (3.4), we know that when $x_i \in (\frac{x_0}{2}-x_{-i}, 1-\frac{x_0}{2}-x_{-i})$, $\partial p^{**}/\partial x_i = 1$; when $x_i \in (0, \frac{x_0}{2}-x_{-i}]$, $\partial p^{*}/\partial x_i = 2$. This means that if the output increases by Δx in the locked-in area, the demand (price) of the product will reduce $2\Delta x$. However, in the area that all the consumers can freely join the market, the price will only decrease Δx with an increase of Δx in output. For convenience to denote, we call the consumers who have purchased the product at previous round competition the old consumers and the consumers who are outside the market in previous time the new consumers. Combining Figure-3.1 and Figure-3.2, we may conclude that the price is more sensitive to output if the total output has not reached the level in which some new consumers join the market. In addition, the firms have less incentive to increase their output when their output are relatively low. This is because the price will decrease very quickly when they increase by a relatively small amount of output. However, if the output has reached a threshold, the price will be less sensitive to the output and the firms have more incentive to increase their output. This demand structure provides the possibility of the existence of multiple equilibria.

Firm A will choose an output, x_A, from $[0, 1 - \frac{x_0}{2} - x_B)$ to maximize its profit π_A. If x_A is chosen from $(0, \frac{x_0}{2} - x_B]$, we denote the profit of Firm A as π_A^* and

$$\pi_A^* = x_A p^* = x_A(1 - 2(x_A + x_B)) \tag{3.5}$$

From the first order condition, we can conclude that when

$$x_A = x_A^* = \frac{1}{4} - \frac{x_B}{2} \tag{3.6}$$

, profit π_A^* reaches its maximum:

$$\pi_A^* \max = 2(\frac{1}{4} - \frac{x_B}{2})^2 \tag{3.7}$$

Here, we need to be cautious that we don't know whether $\pi_A^* \max$ can be reached or not. This is because we don't know whether x_A^* is located in $(0, \frac{x_0}{2} - x_B]$ or not without knowing the value of x_B.

In a similar way, if x_A is chosen from $[\frac{x_0}{2} - x_B, 1 - \frac{x_0}{2} - x_B)$, we denote the profit of Firm A as π_A^{**} and

$$\pi_A^{**} = x_A p^{**} = x_A[1 - \frac{x_0}{2} - (x_A + x_B)] \tag{3.8}$$

From the first order condition of equation (3.8), we can obtain that when

$$x_A = x_A^{**} = \frac{1}{2} - \frac{x_0}{4} - \frac{x_B}{2} \tag{3.9}$$

, profit π_A^{**} reaches its maximum:

$$\pi_A^{**} \max = (\frac{1}{2} - \frac{x_0}{4} - \frac{x_B}{2})^2 \tag{3.10}$$

$\pi_A^{**} \max$ can only be reached when $x_A^{**} \in [\frac{x_0}{2} - x_B, 1 - \frac{x_0}{2} - x_B)$.

As we have illustrated in Figure-3.2, when $x_B \geqslant \frac{x_0}{2}$, the market will expand no matter what output Firm A chooses, while Firm A knows only p^{**} part of the price function exists. In this situation, Firm A will always choose x_A^{**} as its best respondence to Firm B's output. With the condition $x_B \geqslant \frac{x_0}{2}$ and Proposition 3.1, we can get $\frac{x_0}{2} - x_B \leqslant x_A^{**} < 1 - \frac{x_0}{2} - x_B$. Hence, $\pi_A^{**} \max$ can always be reached.

If $x_B < \frac{x_0}{2}$, for a given x_B, whether the market expands or shrinks will depend on Firm A's behaviour. So Firm A will choose its strategy by comparing the maximum profit it can obtain when market expands with the maximum profit it can obtain when market shrinks. Now we draw π_A^* and π_A^{**} according to the different locations of the x_A^{**} and x_A^* in the following figure:

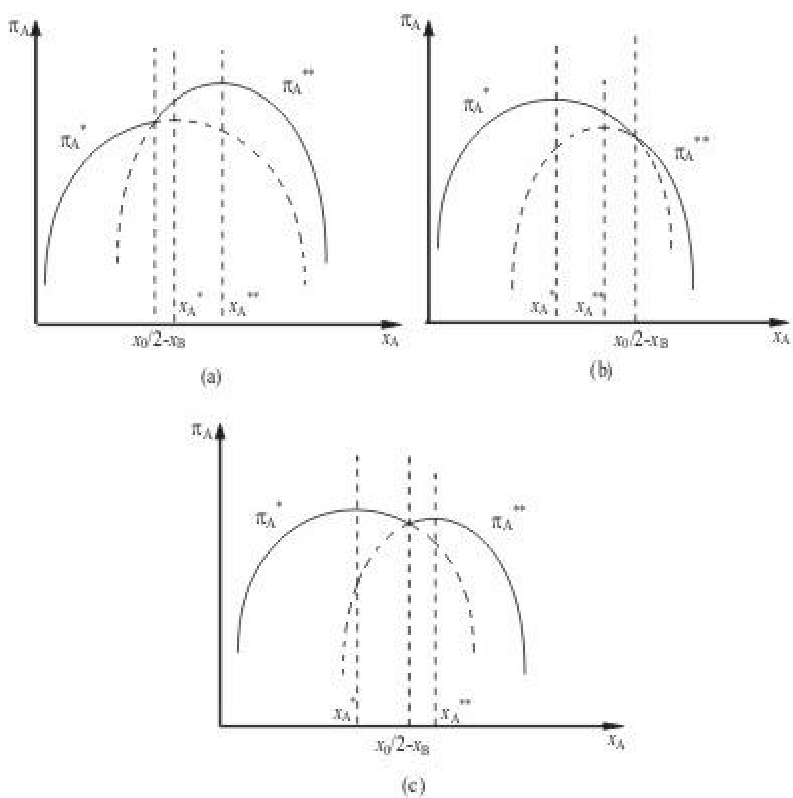

Figure-3.3: The profit function of Firm A given Firm B's output.

From equation (3.6), (3.9) and $x_0 < 1$, we have $x_A^{**} - x_A^* = \frac{1-x_0}{4} > 0$, so x_A^{**} is always on the left side of x_A^* in Figure-3.3. From Proposition 3.1 and $x_B < \frac{x_0}{2} < \frac{1}{2}$, we have $1 - \frac{x_0}{2} - x_B > x_A^{**} > x_A^* > 0$. This means x_A^{**} and x_A^* are not bounded by our definition.

If $x_A^{**} > x_A^* \geqslant \frac{x_0}{2} - x_B$, we get Figure-3.3-a. It is easy to see $\pi_A^* \max$ cannot be reached in Figure-3.3-a and π_A^* is maximized when $x_A = \frac{x_0}{2} - x_B$. However, $\pi_A^{**} \max$ can be reached and we have $\pi_A^*(x_A = \frac{x_0}{2} - x_B) = \pi_A^{**}(x_A = \frac{x_0}{2} - x_B) < \pi_A^{**} \max$. From equation (3.6) and $x_A^* \geqslant \frac{x_0}{2} - x_B$, we obtain that the condition to enter the situation in

Figure-3.3-a is $x_B \geqslant x_0 - \frac{1}{2}$. Thus, Figure-3.3-a tells us that, for a given $x_B \in [x_0 - \frac{1}{2}, \frac{x_0}{2})$, Firm A will choose x_A^{**} as its best response and maximize its profit at $\pi_A^{**}\max$. As we have discussed previously, when $x_B \geqslant \frac{x_0}{2}$, Firm A will always choose x_A^{**} as its output. Combining this with the situation in Figure-3.3-a, we can show, for a given $x_B \in [x_0 - \frac{1}{2}, 1 - \frac{x_0}{2})$, Firm A's best choice is x_A^{**}.

If $\frac{x_0}{2} - x_B \geqslant x_A^{**} > x_A^{*}$, we obtain Figure-3.3-b. In this situation, $\pi_A^{**}\max$ cannot be reached. π_A^{**} is maximized when $x_A = \frac{x_0}{2} - x_B$. But $\pi_A^{*}\max$ can be reached and we have $\pi_A^{**}(x_A = \frac{x_0}{2} - x_B) = \pi_A^{*}(x_A = \frac{x_0}{2} - x_B) < \pi_A^{*}\max$. From equation (3.9) and $\frac{x_0}{2} - x_B \geqslant x_A^{**}$, we obtain that the condition to enter the situation in Figure-3.3-b is $x_B \leqslant \frac{3}{2}x_0 - 1$. Figure-3.3-b tells us, for a given $x_B \in [0, \frac{3x_0}{2} - 1]$, Firm A will choose x_A^{*} as its best output, which maximizes its profit as $\pi_A^{*}\max$.

If $x_A^{**} > \frac{x_0}{2} - x_B > x_A^{*}$, we obtain Figure-3.3-c. Here, $\pi_A^{*}\max$ and $\pi_A^{**}\max$ can both be reached, so whether Firm A choose x_A^{*} or x_A^{**} as its best output depends on which maximum profit is larger. From (3.7) and (3.10):

$$\pi_A^{**}\max \geqslant \pi_A^{*}\max \Longrightarrow x_B \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} \tag{3.11}$$

Inequality (3.11) tells us, if $\pi_A^{*}\max$ and $\pi_A^{**}\max$ can all be reached, Firm A's best response is x_A^{**} when $x_B > \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$; Firm A's best response is x_A^{*} when $x_B < \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$; and Firm A's best response can be either x_A^{**} or x_A^{*} when $x_B = \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, since both of which yield the same maximum profit.

If we put all three situations together, it is easy for us to obtain the following proposition.

Proposition 3.2 *For a given $x_{-i} \in [0, 1 - \frac{x_0}{2})$, when $x_{-i} \leqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, Firm i's best response is $\frac{1}{4} - \frac{x_{-i}}{2}$ and when $x_{-i} \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, Firm i's best response is $\frac{1}{2} - \frac{x_0}{4} - \frac{x_{-i}}{2}$. ($i = A$ or B)*

The intuition behind Proposition 3.2 is: when one of the competitors sets his output very large (larger than $\frac{x_0}{2}$), the market will surely expand. Then, its opponent has to follow the expanding strategy (x_i^{**}) and choose a relatively large output to maximize its profit.

However, when one of the competitors sets its output at a level, which is not large enough to surely make the market expand, its opponent will have two choices: one is to set an output to make the market expand; the other is to let the market shrink. Which one is the best choice depends upon how large the first competitor sets its output. If the first competitor's output is relatively small, its opponent will also set a relatively small output to make the market shrink. If the first competitor chooses an aggressive strategy to make a relatively large output, making the market expand will be a better choice for its opponent.

If both of the firms follow the expanding market strategy, we will have:

$$x_A = \frac{1}{2} - \frac{x_0}{4} - \frac{x_B}{2} \tag{3.12}$$

$$x_B = \frac{1}{2} - \frac{x_0}{4} - \frac{x_A}{2} \tag{3.13}$$

By solving equations (3.12) and (3.13), we can get $x_A = x_B = \frac{1}{3} - \frac{x_0}{6}$. The equilibrium occurs when both the firms set their outputs equal to $\frac{1}{3} - \frac{x_0}{6}$.

If both of the firms choose the shrinking market strategy, we will have:

$$x_A = \frac{1}{4} - \frac{x_B}{2} \tag{3.14}$$

$$x_B = \frac{1}{4} - \frac{x_A}{2} \tag{3.15}$$

By solving equations (3.14) and (3.15), we can obtain $x_A = x_B = \frac{1}{6}$. This means the equilibrium occurs if both of the firms set their output equal to $\frac{1}{6}$.

If one of the firms follows the expanding market strategy and the other one chooses the shrinking market strategy, there will be no equilibrium. This can be illustrated by the fact that there is no solution if we substitute (3.12) to (3.15) or substitute (3.13) to (3.14). The reason behind this is straightforward. After the firms determine their outputs, the market must either expand or shrink. If the market expands, the firm who follows the shrinking strategy can simply change to choose expanding strategy to increase its profit; if the market shrinks, the firm who chooses expanding strategy will change it mind in order to obtain more profits. By the definition of equilibrium, there does not exist an equilibrium in which two firms choose different strategies.

Proposition 3.3 *For a given $x_0 \in (0, \frac{5-2\sqrt{2}}{3})$, the market will expand and there exists a unique symmetric equilibrium in which both of the firms choose their output as $\frac{1}{3} - \frac{x_0}{6}$. If $x_0 \in (5-3\sqrt{2}, 1)$, the market will shrink and there exists unique symmetric equilibrium in which both of the firms set their output at $\frac{1}{6}$. If $x_0 \in [\frac{5-2\sqrt{2}}{3}, 5-3\sqrt{2}]$, there exists two symmetric equilibria: $(x_A = x_B = \frac{1}{3} - \frac{x_0}{6})$ and $(x_A = x_B = \frac{1}{6})$.*

To illustrate Proposition 3.3, we can draw the reaction function of Firm A and B in the following figure:

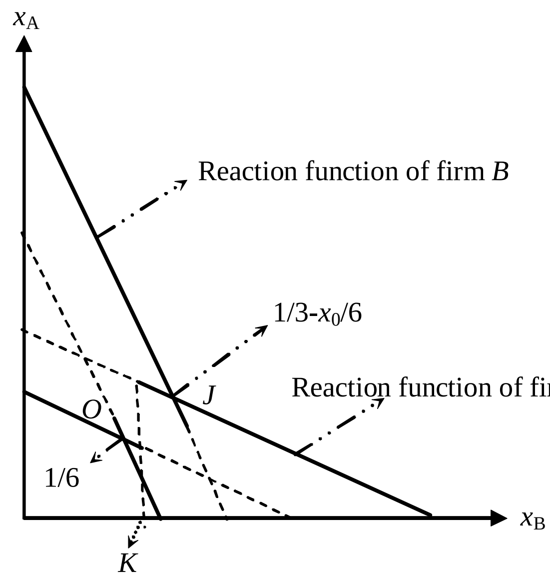

Figure-3.4: Reaction function and equilibria.

In Figure-3.4, the solid lines are the reaction functions of Firm A and B. We can see the reaction functions of both firms are constructed by two parallel lines. When x_B is small, Firm A follows the lower one and when x_B becomes large, Firm A jumps to follow the higher one from point K. From Proposition 3.2, we know $K = \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$. Here, O and J are the crossing points of the two reaction functions and they are also equilibrium points. Since K is determined by x_0, point K will move along the x_B axis when the value of x_0 changes. When x_0 is relatively small, K will move to the left side of point O $(K < \frac{1}{6})$. In this situation, the lower part of the reaction function of Firm A and B will be too short to cross. Thus, the two reaction functions only cross at one point, J. This means only unique equilibrium exists and both of the firms follow the expanding strategy.

When x_0 becames larger, K will move to the right side of point J ($K > \frac{1}{3} - \frac{x_0}{6}$). In this situation, the higher part of the two reaction functions will be too short to cross. The reaction function of Firm A and B will only cross at point O. This means the market only has a unique equilibrium and both of the firms follow the shrinking strategy.

For some x_0, K will locate between O and J ($\frac{1}{3} - \frac{x_0}{6} \geqslant K \geqslant \frac{1}{6}$). In this situation, the reaction function of Firm A and B will cross at two point and multiple equilibria exist.

Proposition 3.3 tell us that the initial market status (how many consumers are locked-in) directly determines the location of the equilibrium. If only a few top-end consumers are locked, firms will prefer to choose to explore new markets and entice new consumers to join the market. However, if most of the consumers with positive willingness to pay have already been locked, firms prefer to choose the shrinking strategy. This is because consumers in the new market have a low willingness to buy and firms must reduce the price to a sufficiently low level to attract them. If the price is greatly reduced, firms will lose more profits from those consumers with high willingness. This explains that why firms prefer to reduce their outputs and increase the price to squeeze profits from those high willingness customers rather than explore new market.

3.2.1 Social Welfare in Equilibrium

We define the social welfare as the sum of the consumers' surplus plus the profits of the firms. Here, we do not consider the welfare of the consumers who are locked-in the market by previous contracts, since the behaviour of the firms in the current period has no effect to their welfare and will not change the profits squeezed from them. For a type r consumer, his/her surplus equals $r - p$ if he/she chooses to purchase the product and his/her surplus is zero if he/she stands outside the market. The total surplus of the consumers is the integration over all the new consumers with their willingness from 1 to r^* in a shrinking market or from 1 to r^{**} in an expanding market.

For the equilibrium in the expanding market, we define the number of the consumers in the market, except the consumers who are locked-in, as z^{**}. From Proposition 3.3, we have:

$$z^{**} = 2x_i^{**} = 2(\frac{1}{3} - \frac{x_0}{6}) = \frac{2}{3} - \frac{1}{3}x_0 \tag{3.16}$$

The number of the consumers in the full density area is:

$$z^{**} - \frac{x_0}{2} = \frac{2}{3} - \frac{5x_0}{6} \tag{3.17}$$

In equation (3.16), z^{**} decreases with an increase of x_0. However, in the expanding market, the number of the consumers in half density area is $\frac{x_0}{2}$ which increases with an increase of x_0. This indicates that the number of new consumers in the full density area (outside locked-in area) will decrease very fast with an increase of x_0. This is illustrated in equation (3.17) in which the coefficient of x_0 is $-\frac{5}{6}$. We can substitute the expanding market outputs of the firms into equation (3.4) and obtain:

$$r^{**} = p^{**} = \frac{1}{3} - \frac{1}{6}x_0 \tag{3.18}$$

We denote the consumers' surplus in the expanding market as S^{**}. By integrating the consumers' surplus in the half density area and full density area, we can obtain:

$$\begin{aligned} S^{**} &= \frac{1}{2}\int_{x_0}^{1}[r - (\frac{1}{3} - \frac{1}{6}x_0)]dr + \int_{\frac{1}{3}-\frac{1}{6}x_0}^{x_0}[r - (\frac{1}{3} - \frac{1}{6}x_0)]dr \\ &= \frac{5}{36} - \frac{5}{36}x_0 + \frac{25}{72}x_0^2 \end{aligned} \tag{3.19}$$

We use Π^{**} to denote the firms' total profits in the expanding market. By multiplying (3.17) and (3.18), we get:

$$\Pi^{**} = z^{**}p^{**} = (\frac{2}{3} - \frac{1}{3}x_0)(\frac{1}{3} - \frac{1}{6}x_0) = \frac{2}{9} - \frac{2}{9}x_0 + \frac{1}{18}x_0^2 \tag{3.20}$$

Adding (3.19) and (3.20) together, we can obtain the total social welfare:

$$W^{**} = S^{**} + \Pi^{**} = \frac{13}{36} - \frac{13}{36}x_0 + \frac{29}{72}x_0^2 \tag{3.21}$$

For the equilibrium in the shrinking market, only the consumers in the locked-in area (half density area) join the market. The number of the new consumers in this situation are easy to calculate by adding the shrinking equilibrium outputs of Firm A and firm B together. If we use z^* to denote the total outputs in shrinking market, $z^* = 2x_i^* = \frac{1}{3}$. From equation (3.3), we can obtain $r^* = p^* = \frac{1}{3}$. Thus, the sum of the consumers' surplus is:

$$S^* = \frac{1}{2}\int_{\frac{1}{3}}^{1}(r - \frac{1}{3})dr = \frac{1}{9}$$

The firms' total profit is $\Pi^* = z^*p^* = \frac{1}{9}$ and the social welfare is $W^* = S^* + \Pi^* = \frac{2}{9}$.

Proposition 3.4 *In the expanding equilibrium, consumer's surplus and the total social welfare increases with an increase of the initial locked-in consumers, but the total profit of the firms decreases with an increase of the initial locked-in consumers. In a shrinking equilibrium, consumer's surplus, total social welfare and the total profits of the firms are all constant.*

In expanding equilibrium, from (3.17) and (3.18), the total output and the market price will both decrease with an increase of the initial locked-in consumers, so the total profit of all the firms must have a negative relationship with x_0. However, from Proposition 3.4, we know that the social welfare has a positive relationship with x_0. This is because the consumers' surplus increases faster than the decrease in the firm's profit, so, in aggregate, the total welfare of the expanding equilibrium increases with an increase in x_0. In the shrinking equilibrium, only the consumers in the half density area will join into the market, so the firms actually compete in a traditional Cournot model with half density demand. From Proposition 3.3, we know the firms will always choose their outputs as $\frac{1}{6}$, so the location of the equilibrium and the total welfare will not change with the change of the initial setting.

Proposition 3.5 *If multiple equilibria exist, the social welfare in the expanding equilibrium is always larger than the social welfare in the shrinking equilibrium. However, the firms obtain fewer profits in the expanding equilibrium than the shrinking equilibrium.*

Proposition 3.5 tells us the firms' incentives always contradicts with the social planner's incentive if there exists two equilibria. The firms always want to go to the equilibrium with the smaller output and make the market shrink. However, the social planner prefers that the firms compete at the larger output equilibrium (expanding equilibrium). The implication for the social planner is: if x_0 results in two possible equilibria and there are no regulations for the competition, the final equilibrium may locate at the one with less social welfare. Sometimes, firms will even set up self-regulations or collude in the competition to make sure they reach the shrinking equilibrium since this equilibrium benefits both of them. If the social planners want to maximize the social welfare, they should set up some mechanism to urge the firms to produce more and reach the equilibrium which makes the market expand.

3.2.1 Locked-in Competition in Oligopoly Market

The duopoly model can be easily extended to an oligopoly market. The competition analysis is very similar to the duopoly case. The only difference is that firms consider their opponent as the total output of all other firms when they choose their competition strategy. Assuming there are n firms in the market, we define these firms are Firm i and their outputs as x_i $(i = 1, 2..n)$. Other notations have the same meaning as previous section. Following a similar procedure, we can obtain Firm i's reaction function to the total output of all other firms in a shrinking market as:

$$x_i^* = \frac{1}{4} - \frac{1}{2}\sum_{i \neq j} x_j \quad i, j \in \{1, ..., n\} \tag{3.22}$$

and its reaction function in an expanding market as:

$$x_i^{**} = \frac{1}{2} - \frac{x_0}{4} - \frac{1}{2}\sum_{i \neq j} x_j \quad i, j \in \{1, ..., n\} \tag{3.23}$$

Comparing equation (3.22) and (3.23) with equation (3.6) and (3.9), we can see the only difference between these equations is that $\sum x_j$ substitutes x_B, since every firm makes its decision by considering the aggregate output of all other competitors. According to Proposition 3.2 and the analysis of Figure-3.3, we can declare that, in an oligopoly market, when $\sum x_j \leqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, Firm i's best response is equation (3.22) and when $\sum x_j \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, firm i's best response is equation (3.23) $(j \neq i \; and \; i, j \in \{1, ..., n\})$.

Since the market will either expand or shrink at the end of the period, the firms who choose to follow the shrinking strategy (x_i^*) will always want to change their mind if the market finally expands and the firms who choose to follow the expanding strategy (x_i^{**}) will change their minds if they find the market finally shrinks. By the definition of equilibrium, all the firms must choose the same strategy in equilibrium, so they will either all choose (3.22) or all choose (3.23).

If all the firms choose shrinking strategy, by solving the equation (3.22)[11], we can obtain the symmetric shrinking equilibrium output as $\frac{1}{2(n+1)}$. If all the firms choose the expanding strategy, by solving the equation (3.23), we can get the symmetric expanding equilibrium output as $\frac{1}{n+1} - \frac{x_0}{2(n+1)}$. Following the proof of Proposition 3.3, we can obtain the following proposition:

Proposition 3.6 *When $x_0 \geqslant 1 - \frac{2\sqrt{2}-2}{n+1}$, there always exists a shrinking equilibrium in which all firms symmetrically set their output as $\frac{1}{2(n+1)}$. When $x_0 \leqslant 1 - \frac{2}{(\sqrt{2}+2)n+\sqrt{2}}$, there always exists an expanding equilibrium in which all firms symmetrically set their outputs as $\frac{1}{n+1} - \frac{x_0}{2(n+1)}$.*

For convenience to denote, we define:

$$\theta_1 = 1 - \frac{2\sqrt{2}-2}{n+1} \tag{3.24}$$

$$\theta_2 = 1 - \frac{2}{(\sqrt{2}+2)n+\sqrt{2}} \tag{3.25}$$

then:

$$\theta_2 - \theta_1 = \frac{2(\sqrt{2}-1)(n-1)}{(n+1)(2n+\sqrt{2}n+\sqrt{2})} \tag{3.26}$$

Since $n > 1$, $\theta_2 - \theta_1 > 0$, and θ_2 is always in the left side of θ_1. From equation (3.24) and (3.25), it is easy to find that both θ_1 and θ_2 are located between 0 and 1 for any n in our definition. In order to get a more understanding about Proposition 3.6, we draw θ_1 and θ_2 in the following figure:

[11] Since all the firms are symmetric at the equilibrium, equation (3.22) can be rewritten as $x = 1/4 - 1/2(n-1)x$. Thus, we can obtain the equilibrium output by solving this equation. A similar method can be used to solve the expanding equilibrium with equation (3.23).

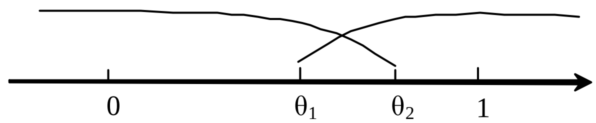

Figure-3.5: Location of θ_1 and θ_2

According to Proposition 3.6 and Figure-3.5, there exists a unique equilibrium when x_0 is located between 0 and θ_1 or between θ_2 and 1. When x_0 is relatively small $(x_0 < \theta_1)$, the firms will all follow an expanding strategy and reach a unique expanding equilibrium. When x_0 is relatively large $(x_0 < \theta_2)$, the firms will all choose a shrinking strategy and the unique equilibrium is the shrinking equilibrium. When x_0 is located between θ_1 and θ_2, two equilibria exist. In this situation, we cannot determine which equilibrium will actually be played without more information. If we set $n = 2$, these conclusions coincide with Proposition 3.3.

From equation (3.24), (3.25) and (3.26), we know $\lim(\theta_1) = \lim(\theta_2) = 1$ and $\lim(\theta_2 - \theta_1) = 0$ if $n \longrightarrow \infty$. This means θ_1 and θ_2 both move towards 1 when the number of the firms in the market increases. However, with the increase of n, θ_1 moves more quickly than θ_2, so the gap between θ_1 and θ_2 decreases. When n is very large (perfect competition market), both θ_1 and θ_2 will be very close to 1 and the gap between them will be very small. The implication here is, with the increase of n, the chance of existing shrinking equilibrium decreases. This is because the intensity of competition increases with an increase in the number of the firms in the market. Thus, the chance for the firms to squeeze more profits from consumers by using the method of cutting outputs will decrease when n becomes larger. The firms will be more willing to choose an expanding strategy in market with relatively large n. When n goes to infinity, the market is under perfect competition. Thus, there is no chance for the firms to reach a shrinking equilibrium and the market output will be fixed at 1.

3.3 The Model with Locked-in and Network Externalities

In the previous model, the firms are perfectly symmetric. They produce homogeneous products and face an identical price function. We also have proven that only symmetric equilibrium exists in this situation. However, the firms may compete based on asymmetric initial market conditions. In the model we discussed previously, the firm's market shares at previous round competition will not affect the competition in the current period. However, this is only correct in a world without network externalities. If the firms produce network products, their previous market status may benefit or harm their competition in the current period through the locked-in effect. The firm with a larger market share in previous time will have more locked-in consumers in the current period and these locked-in consumers will guarantee a larger network size. Hence, the firm, which has larger market size previously, is more attractive to the consumers who can freely choose the product in current period. In other words, the firm with larger output in the previous period will have an inherent advantage. In this section, we make a modification to our model in Section 3.2 by adding the network externalities and an asymmetric initial market structure. We want to find how much the asymmetric market status will affect the equilibrium output in a network world.

We consider a duopoly market with Firm A and B competing with their outputs. If the products have a network effect, the final surplus, which a consumer can obtain after he/she chooses to join the market, will not only depend on the price of the product but also on how many consumers make the same choice. We assume the two firms produce completely incompatible products and the network externalities a consumer can obtain are $u(X_i)$[12] when he/she purchases Firm i's product ($i = A$ or B). Here, X_i is the sum of the number of the consumers who choose Firm i's product in current period (x_i) and the number of the consumers who are locked-in by Firm i. Now, we assume firm A and B have x_A^0 and x_B^0 consumers in the previous time ($x_A^0 > 0, x_B^0 > 0$) and Firm A has a competitive advantage ($x_A^0 \geqslant x_B^0$). Similar to previous model, we assume half of x_i^0 are locked-in with

[12] According to the generally accepted characteristics of the network utility function, we assume $u(x) > 0$, $u'(x) > 0$, $u''(x) \leqslant 0$.

their previous service provider. These locked-in consumers are uniformly distributed. If the consumer chooses Firm A's product, the network externalities he/she can obtain from the product is $u(\frac{x_A^0}{2} + x_A)$[13]. The surplus that a consumer with willingness r can obtain when he/she chooses Firm A's product, is $r + u(\frac{x_A^0}{2} + x_A) - p_A$. Here, if both of the firms have a positive output, there must have:

$$u(\frac{x_A^0}{2} + x_A) - p_A = u(\frac{x_B^0}{2} + x_B) - p_B \tag{3.27}$$

This is because the surplus a consumer can obtain from purchasing Firm A or Firm B's product must have no difference, otherwise all the consumers will choose to buy just from one firm. From equation (3.27), we can conclude that the firms may have different prices if they provide different network externalities. The larger network the firm can provide, the higher price he can charge to the consumers. If the firm has a relatively small network, it has to reduce the price to attract the consumers to choose its product.

Since only the consumer whose surplus is bigger than zero will enter the market, we have:

$$r + u(\frac{x_i^0}{2} + x_i) - p_i \geqslant 0 \Longleftrightarrow r \geqslant p_i - u(\frac{x_i^0}{2} + x_i)$$

For the consumer, who have no difference of whether to join the market or not, we have:

$$r = p_i - u(\frac{x_i^0}{2} + x_i) \tag{3.28}$$

Here, the consumer with a negative willingness can also join the market when $p_i < u(x_i^0/2 + x_i)$. Unlike the model without network externalities, the market size is not bounded by 1 anymore.

3.3.1 Linear Network Externalities and the Behaviour of the Firms

We assume, for both products, these is a linear network utility function: $u(x) = bx$ ($0 < b < b\max$)[14]. We also assume that $x_A^0 + x_B^0$ is bound by 1 ($x_A^0 + x_B^0 < 1$). Similar to the

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13 Actually, the consumer's choice depends on the expectation of the output of the firms, since they cannot know the exact output of the firms prior to making their decision. In this model, we assume the expected output of the firms perfectly changes with the real output of the firms and the expected output can always be reached. As a result, the firm's output and the consumer's expectation have no difference.

14 In fact, b can be any positive number. However, we limite our discussion to a relatively small network

previous model, we can obtain the price function of Firm A with a given output of firm B, x_A^0 and x_B^0. If the consumer, who has no difference of whether to join the market or not, has a willingness, r, that locates in $(1-(x_A^0+x_B^0),1)$, the market will shrink. In this situation, we denote this critical consumer's willingness as r^*. According to Figure-3.1 and equation (3.28), we now have:

$$\begin{aligned} x_A + x_B &= \frac{1-r^*}{2} = \frac{1-[p_A^* - b(x_A + \frac{x_A^0}{2})]}{2} \Longrightarrow \\ p_A^* &= 1 + \frac{bx_A^0}{2} - (2-b)x_A - 2x_B \end{aligned} \tag{3.29}$$

If the consumer, who has no difference of whether to join the market or not, has a willingness, r, that locate in $(-\infty, 1-(x_A^0+x_B^0))$, the market will expand. We then denote his/her willingness as r^{**}. From Figure-3.1 and equation (3.28), we have:

$$\begin{aligned} x_A + x_B &= \frac{x_A^0 + x_B^0}{2} + [1-(x_A^0+x_B^0) - r^{**}] \Longrightarrow \\ p_A^{**} &= 1 + \frac{bx_A^0}{2} - \frac{x_A^0+x_B^0}{2} - (1-b)x_A - x_B \end{aligned} \tag{3.30}$$

If the critical willingness $r = 1-(x_A^0+x_B^0)$, the market will maintain its previous size and equation (3.29) and (3.30) will be identical.

We temporarily treat x_B, x_A^0 and x_B^0 as exogenous variables. The price of Firm A's product is determined by p_A^* when $x_A \leqslant \frac{x_A^0+x_B^0}{2} - x_B$ and by p_A^{**} when $x_A \geqslant \frac{x_A^0+x_B^0}{2} - x_B$. To obtain a clearer picture of the price function of Firm A, We will illustrate it in the following figure:

externality by setting b is strictly less than $b\max$. ($b\max \approx 0.457$). $b\max$ is largest b which allows the point K in Figure-3.7 less than 1. This assumption maintains a majority characteristics of the network effect and greatly reduces the cases we need to discuss.

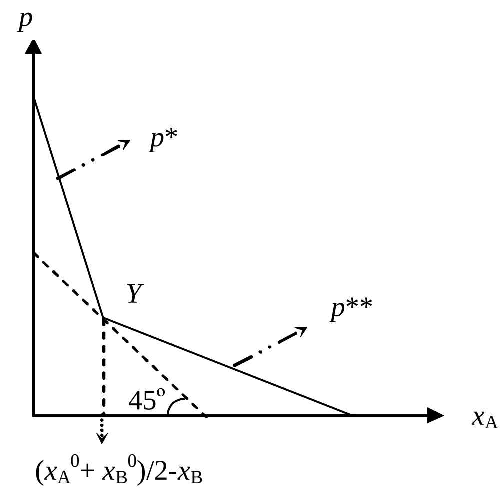

Figure-3.6: Price function of Firm A given x_B, x_A^0 and x_B^0

From Figure-3.6, the slope of the price function of Firm A in a shrinking market is $\partial p_A^*/\partial x_A = -2+b$. Since $0 < b < \frac{1}{2}$, $\partial p_A^*/\partial x_A$ is located in $(-2,-1)$. In the expanding market, the slope of firm A's the price function is $\partial p_A^{**}/\partial x_A = -1+b > -1$. Comparing Figure-3.6 with Figure-3.2, we can find the price function p_A^* and p_A^{**} are both flater in the network world and Figure-3.6 can be seen as a graph constructed by pulling every point of the price function in Figure-3.2 to the left side. The force to pull the price functions can be explained by the network externalities. This is because, for a given price, more consumers are willing to join the market if the firms produce network products and consumers' surplus is larger in the network model than in the model without network externalities. For a concave network utility function, network externalities may force the price function to be concave. But, with a linear network utility assumption, the price functions will maintain linear.

In Figure-3.6, Y is the joint point of two price functions. At point Y, firms keep the market size unchanged and the price in the market is:

$$p_A^* = p_A^{**} = 1 - (x_A^0 + x_B^0) + \frac{bx_A^0}{2} + b[\frac{x_A^0 + x_B^0}{2} - x_B] \tag{3.31}$$

When $\frac{x_A^0+x_B^0}{2} - x_B \leqslant 0$, point Y goes to the negative side of the x_A axis and the price function is dominated by p_A^{**}. This means that Firm B has chosen a relatively large output, hence the market will expand no matter what Firm A's strategy is. In this situation, Firm A

has to follow the expanding strategy. From equation (3.31) and $x_A^0 + x_B^0 < 1$, we know that the price is always positive at point Y when $\frac{x_A^0+x_B^0}{2} - x_B > 0$. This means, when p_A^* exists, point Y will never go downward to the negative side of the p_A axis and the price function will never be dominated by p_A^*.

To demonstrate with simple notation, we set:

$$1 + \frac{bx_A^0}{2} \equiv \sigma_A,\, 1 + \frac{bx_B^0}{2} \equiv \sigma_B \text{ and } \frac{x_A^0 + x_B^0}{2} \equiv \epsilon \tag{3.32}$$

By our definition, σ_A and σ_B are larger than 1 and $\epsilon < \frac{1}{2}$.

If $x_B \geqslant \epsilon$ and the price function is dominated by p_A^{**}, both of the firms will follow the expanding strategy and the market will reach the expanding equilibrium. Now, we consider the circumstances that $x_B < \epsilon$ and Firm A faces a kinked price function. In this situation, if $x_A \leqslant \epsilon - x_B$, the market price is determined by p_A^* and the profit of the Firm A is:

$$\pi_A^* = x_A p_A^* = x_A[\sigma_A - (2-b)x_A - 2x_B] \tag{3.33}$$

From the first order condition, π_A^* is maximized when:

$$x_A = x_A^* = \frac{\sigma_A - 2x_B}{2(2-b)} \tag{3.34}$$

and

$$\pi_A^* \max = \frac{(\epsilon - 2x_B)^2}{4(2-b)} \tag{3.35}$$

If $x_A \geqslant \epsilon - x_B$, the market price is determined by p_A^{**} and the profit of the Firm A is:

$$\pi_A^{**} = x_A p_A^{**} = x_A[\sigma_A - \epsilon - (1-b)x_A - x_B] \tag{3.36}$$

From the first order condition, π_A^{**} is maximized when:

$$x_A = x_A^{**} = \frac{\sigma_A - \epsilon - x_B}{2(1-b)} \tag{3.37}$$

and

$$\pi_A^{**} \max = \frac{(\sigma_A - \epsilon - x_B)^2}{4(1-b)} \tag{3.38}$$

Here, we must be cautious that $\pi_A^{**} \max$ and $\pi_A^* \max$ can only be reached when x_A^* and x_A^{**} are located in their definition area.

Proposition 3.7 *When $\sigma_i \geqslant (2-b+\sqrt{(2-b)(1-b)})\epsilon$, Firm i will always choose function x_i^{**} as its best response for a given output of Firm $-i$. When $\sigma_i < (2-b+\sqrt{(2-b)(1-b)})\epsilon$, if $x_{-i} \leqslant \alpha_i$, Firm i will choose x_i^* as its best response; and if $x_{-i} \geqslant \alpha_i$, Firm i will choose x_i^{**} as its best response. ($\alpha_i = \epsilon + \frac{2\epsilon-\sigma_i}{\sqrt{(1-b)(2-b)}-b}$, $i = A$ or B)*

In order to get a more clear illustration to Proposition 3.6, we draw $x_A^0 = x_B^0$, $x_A^0 + x_B^0 = 1$ and $\sigma_i = (2-b+\sqrt{(2-b)(1-b)})\epsilon$ ($i = A$ and B) in the following figure:

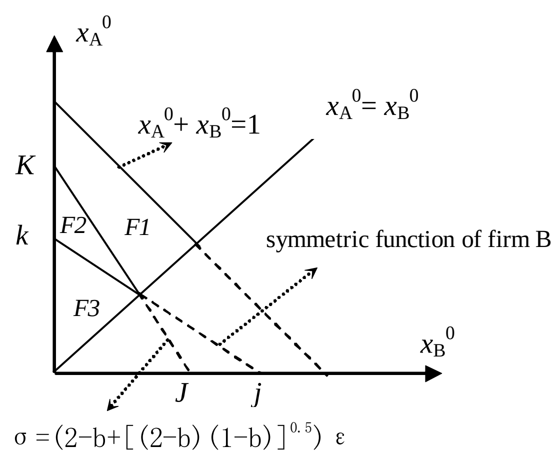

Figure-3.7: The allocation of x_A^0 and x_B^0

In Figure-3.7, the line KJ is the function $\sigma_A = (2-b+\sqrt{(2-b)(1-b)})\epsilon$ and the line kj is the function $\sigma_B = (2-b+\sqrt{(2-b)(1-b)})\epsilon$. By assuming $x_B^0 = 0$, we can solve the function KJ and get:

$$x_A^0(K) = \frac{2}{2(1-b)+\sqrt{(2-b)(1-b)}} \tag{3.39}$$

In a similar way, by assuming $x_B^0 = 0$, we can solve function kj and get:

$$x_A^0(k) = \frac{2}{2-b+\sqrt{(2-b)(1-b)}} \tag{3.40}$$

Comparing (3.39) and (3.40), we find that K is always located above k. When $x_A^0 = x_B^0$, σ_A equals σ_B, so line KJ and kj cross at a point located on line $x_A^0 = x_B^0$. By our definition that $x_A^0 \geqslant x_B^0$ and $x_A^0 + x_B^0 < 1$, x_A^0 and x_B^0 can only be chosen from the triangular area constructed by $x_A^0 = x_B^0$, $x_A^0 + x_B^0 = 1$ and x_A^0 axis. This triangular area is divided by

KJ and kj into three parts. We define these parts as the $F1$, $F2$ and $F3$ area according to Figure-3.7.

From Proposition 3.7, we know Firm A will always choose the expanding strategy if x_A^0 and x_B^0 are located below line KJ and Firm B will always choose the expanding strategy if x_A^0 and x_B^0 are located below line kj. Thus, in $F3$, both firms will choose a reaction function which makes the market expand, because x_A^0 and x_B^0 are very small in this area. When x_A^0 and x_B^0 are located above line KJ, Firm A will choose x_A^* as its best response if $x_B \leqslant \alpha_A$ and choose x_A^{**} as its best response if $x_B \geqslant \alpha_A$. Similarly, when x_A^0 and x_B^0 are located above line kj, Firm B will choose x_B^* as its best response if $x_A \leqslant \alpha_B$ and choose x_B^{**} as its best response if $x_A \geqslant \alpha_B$. Since all the points in the $F1$ area are located above KJ and kj, Firm A and B will all choose a piecewise function as their reaction function when x_A^0 and x_B^0 belongs to $F1$. For any points belonging to the $F2$ area, Firm B will choose a piecewise function as its reaction function and Firm A will always follow an expanding strategy.

The network externalities will affect the structure of Figure-3.7 and the size of $F1$, $F2$ and $F3$. For any b in our definition area, we have $\partial x_A^0(K)/\partial b > 0$ and $\partial x_A^0(k)/\partial b > 0$. From (3.39) and (3.40), we can find that $x_A^0(K) = x_A^0(k) = 2 - \sqrt{2}$ when $b = 0$. This tells us that if b or the intensity of network effect decrease, point K goes down and point k goes down as well. Hence, $F1$ area will increase. However, the speed for K to go down is much quicker than that for point k. When $b = 0$, k will be caught up by K and the $F2$ area will disappear. This coincides with the model without network externalities in previous sections: KJ and kj become the same line and both have a slope equal to 1. When b increases, points K , k , J and j will all go up or go right, the $F1$ area will become smaller and the $F3$ area will become larger. This is because the increasing in the network externalities causes Firm A and B more prefer to use the expanding strategy and they might to choose the expanding strategy even if the initial market locked-in size is relatively large.

3.3.1 Equilibrium Analysis

Any pair of x_A^0 and x_B^0 in our definition must be located in the $F1$, $F2$ or $F3$ area, so we separate our discussion of the equilibrium into three cases. There is a proposition which can be applied to all three cases:

Proposition 3.8 *Firm A's expanding reaction function (x_A^{**}) will never cross with Firm B's shrinking reaction function (x_B^{*}); and Firm A's shrinking reaction function (x_A^{**}) will never cross with Firm B's expanding reaction function (x_B^{**}) in each reaction function's definition area.*

Proposition 3.8 indicates that there does not exist an equilibrium in which Firm A chooses the expanding strategy but Firm B chooses the shrinking strategy or Firm A chooses the shrinking strategy but Firm B chooses the expanding strategy. The reason behind this proposition is that only one status exists for the final market. Every firm will change its mind if the wrong reaction function according to the final market situation has been chosen. By the definition of the equilibrium, there does not exist an equilibrium in which two firms follow different strategies.

x_A^0 and x_B^0 in $F3$ and $F2$ area

If x_A^0 and x_B^0 are located in the $F3$ area, Firm A and B will both choose expanding strategy. Equation (3.37) is the reaction function of Firm A in the expanding market. Similarly, we can obtain the reaction function of Firm B in the expanding market as:

$$x_B^{**} = \frac{\sigma_B - \epsilon - x_A}{2(1-b)} \tag{3.41}$$

We draw (3.37) and (3.41) in the following figure:

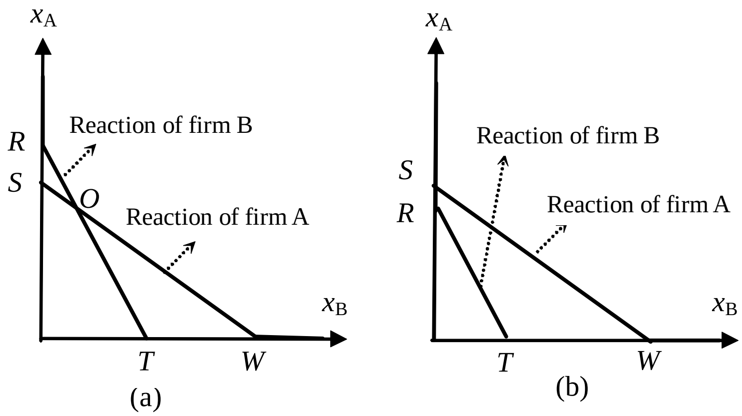

Figure-3.8: The reaction functions when x_A^0 , x_B^0 are located in $F3$

Since $x_A^0 \geqslant x_B^0$, we have $\sigma_A \geqslant \sigma_B$ and $W \geqslant R > T$. If $R > S$, we will get Figure-3.8-a and the equilibrium will occur at point O. Combining (3.37) and (3.41), we can solve this equilibrium:

$$x_A = \frac{2(1-b)\sigma_A - \sigma_B - (1-2b)\epsilon}{4(1-b)^2 - 1}, x_B = \frac{2(1-b)\sigma_B - \sigma_A - (1-2b)\epsilon}{4(1-b)^2 - 1} \tag{3.42}$$

If $R \leqslant S$, we will get Figure-3.8-b and the equilibrium will occur at point S. By solving equation (3.37) with $x_B = 0$, we can obtain the equilibrium:

$$x_A = \frac{\sigma_A - \epsilon}{2(1-b)}, x_B = 0 \tag{3.43}$$

If x_A^0 and x_B^0 are located in the $F2$ area, Firm A will always follow the expanding strategy, but Firm B will consider a piecewise reaction function. This case is very similar to the situation in which x_A^0 and x_B^0 are located in the $F3$ area. We can make a small revision to Figure-3.8 to obtain the graph about the reaction functions in this case:

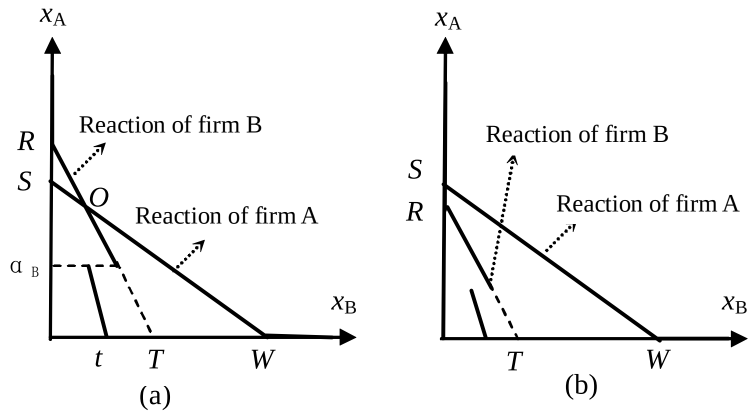

Figure-3.9: The reaction functions when x_A^0 , x_B^0 are located in $F2$

In Figure-3.9, Firm B's reaction function is constructed by two parts: x_B^* and x_B^{**}. Obviously $x_B^{**}(x_A = \alpha_B) > x_B^*(x_A = \alpha_B)$ and we also know T is always on the left side of t[15]. Hence, x_B^* is always on the left side of the RT line. If $R \leqslant S$, we will obtain Figure-3.9-b and the equilibrium occurs at point S. This equilibrium is (3.43), which is the same as Figure-3.8-b. If $R > S$, we obtain Figure-3.9-a. In this case, whether there exists a pure strategy equilibrium depends on the position of α_B. When α_B is small, the equilibrium is point O and the equilibrium output of the firms is given out by (3.42). When α_B goes large, the reaction function of Firm A (x_A^{**}) may just cross the gap between x_B^{**} and x_B^*. There will be no crossing point between the two firm's reaction functions. Or, we can say there is no pure strategy equilibrium. However, α_B is determined by x_A^0 and x_B^0, which are chosen from $F3$, so α_B can only vary in a limited area. In fact, α_B may not go above point O if x_A^0, x_B^0 and b are chosen from our definition area[16]. This means that the reaction function of Firm A and B will always intersect and there always exists a pure strategy equilibrium, no matter whether x_A^0 and x_B^0 are located in $F2$ or $F3$ area. We can

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15 We can obtain the reaction function of Firm B from (3.41) and (3.44). By setting $x_B = 0$, we can obtain T and t and

$$T - t = \frac{\sigma_B - \epsilon}{2(1-b)} - \frac{\sigma_B}{2(2-b)} = \frac{\sigma_B - 2\epsilon + b\epsilon}{2(1-b)(2-b)}$$

Since $\sigma_B > 2\epsilon, T - t > 0 \Longleftrightarrow T > t$.

16 The proof can be seen at the proof of Proposition 3.9.

combine the equilibrium analysis when x_A^0 and x_B^0 are located in $F3$ and $F2$ together and obtain the following proposition:

Proposition 3.9 *When x_A^0 and x_B^0 are located in the $F2$ and $F3$ area, there always exists a unique pure strategy Nash equilibrium. If $x_A^0 \geqslant \gamma$, the equilibrium is: $(x_A = \frac{\sigma_A - \epsilon}{2(1-b)}, x_B = 0)$ and if $x_A^0 < \gamma$, the equilibrium is: $(x_A = \frac{2(1-b)\sigma_A - \sigma_B - (1-2b)\epsilon}{4(1-b)^2-1}, x_B = \frac{2(1-b)\sigma_B - \sigma_A - (1-2b)\epsilon}{4(1-b)^2-1})$. (Here, $\gamma = \frac{2(1-2b)+(4b-2b^2-1)x_B^0}{1-b}$)*

Proposition 3.10 *If $b \geqslant b^*$, there exists a pair of x_A^0 and x_B^0, which are located in the $F3$ or $F2$ area and leads the market to the equilibrium $(x_A = \frac{\sigma_A - \epsilon}{2(1-b)}, x_B = 0)$. If $b < b^*$, the equilibrium $(x_A = \frac{\sigma_A - \epsilon}{2(1-b)}, x_B = 0)$ does not exist for any given pair of x_A^0 and x_B^0, which are located in $F3$ or $F2$ area. (Here $b^* \approx 0.361$ and is the solution of $12b^3 - 12b^2 + 1 = 0$)*

To obtain more intuition about Proposition 3.9 and 3.10, we can draw $x_A^0 = \gamma$ in Figure-3.7 and get the following figure:

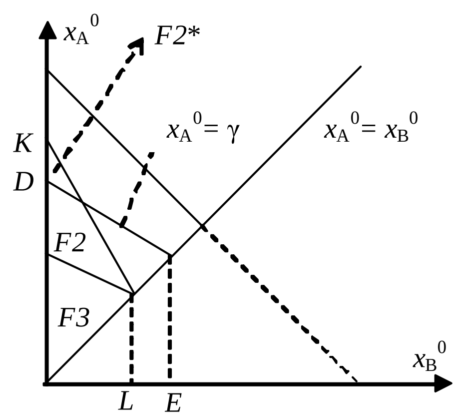


Figure-3.10: $x_A^0 = \gamma$

From Proposition 3.9, we know the market will reach equilibrium (3.43) when x_A^0 and x_B^0 are located above $x_A^0 = \gamma$ and the market will go to equilibrium (3.42) when x_A^0 and x_B^0 are located below $x_A^0 = \gamma$. Since $E > L$ for any b in the definition area[17], there

[17] $E = \frac{2(1-2b)}{2-5b+2b^2}$ and L is given in A3.8. It is easy to test, for any $b \in (0, b\max)$, $E - L > 0$.

exists a pair of x_A^0 and x_B^0 in the $F3$ or $F2$ area, which is located above $x_A^0 = \gamma$ if and only if $D < K$. The location of point D depends on b. When b is relatively small, D is very large and located above K. When b increase, point D will go down. From Proposition 3.10, we know that when $b = b^*$, D equals K. Since $b^* < b\max$, these exists a b, which is in out definition area and larger than b^*. When $b > b^*$, D is located below K and any pair of x_A^0 and x_B^0 located in $F2^*$ area ($F2^* \subset F2$) will lead the market to equilibrium (3.43).

The implication of Proposition 3.9 illustrates that there exists a situation that Firm B cannot obtain any new consumers because Firm A has an extremely large initial number of locked-in consumers. In this case, the difference in initial locked-in consumers is so large that any positive price offered by Firm B will not attract consumers to join its network. Thus, Firm B has to just produce for their previous locked-in consumers. In Figure-3.10, any point located in $F2^*$ satisfies this condition. We can see these points in $F2^*$ all have a large x_A^0 and relatively small x_B^0. However, Proposition 3.10 also tells us that only a relatively large difference in initial number of locked-in consumers is not enough for one firm to deter another. The Firm with an advantage needs network externalities as a catalyst to enable its advantage in the initial market structure to become the advantage in the competition. The firm with a larger number of locked-in consumers can always benefit more from network externalities and obtain certain advantages, while larger network effect will make this advantage more significant. More specifically, if Firm A wants to drive Firm B out of the market, the intensity of network effect must reach a critical level. To the social planner, larger network externalities increases the chance that the market becomes a monopoly by increasing the size of $F2^*$ area.

x_A^0 and x_B^0 are located in the $F1$ area

If x_A^0 and x_B^0 are located in the $F1$ area, both Firm A and Firm B will choose a piecewise function as their reaction function.

Proposition 3.11 *If x_A^0 and x_B^0 are located in $F1$ area, there does not exist an equilibrium in which Firm A have a positive output and Firm B have zero output.*

From the proof of Proposition 3.8, we know that function x_A^* and x_B^* are located below the line $x_A^0 + x_B^0 = \epsilon$ and x_A^{**} and x_B^{**} are located above the line $x_A^0 + x_B^0 = \epsilon$. If Firm A wants to deter Firm B from getting new consumers, x_B^{**} must locate under function x_A^* [18]. However, this is impossible, since x_i^{**} is always located above x_i^*.

According to Proposition 3.8, only two possible equilibria exist. One is located at the intersection of functions x_A^* and x_B^* and we define this equilibrium as a shrinking equilibrium with outputs (x_A^S, x_B^S). The other is located at the intersection of functions x_A^{**} and x_B^{**} and we define this equilibrium as an expanding equilibrium with outputs (x_A^E, x_B^E).

Assume both of the firms choose the shrinking strategy. By solving x_A^* and x_B^*, we can obtain the shrinking equilibrium outputs:

$$x_A^S = \frac{(2-b)\sigma_A - \sigma_B}{2(1-b)(3-b)} \tag{3.44}$$

$$x_B^S = \frac{(2-b)\sigma_B - \sigma_A}{2(1-b)(3-b)} \tag{3.45}$$

The sufficient and necessary condition of the existence of the shrinking equilibrium is that the intersection of the two shrinking strategy functions is located in the definition area. This equals the conditions that $x_A^S \leqslant \alpha_B$ and $x_B^S \leqslant \alpha_A$.

In the same way, by solving x_A^{**} and x_B^{**}, we can obtain the expanding equilibrium outputs as:

$$x_A^E = \frac{2(1-b)\sigma_A - \sigma_B - (1-2b)\epsilon}{4(1-b)^2 - 1} \tag{3.46}$$

$$x_B^E = \frac{2(1-b)\sigma_B - \sigma_A - (1-2b)\epsilon}{4(1-b)^2 - 1} \tag{3.47}$$

The sufficient and necessary condition of the existence of the expanding equilibrium is that the intersection of x_A^{**} and x_B^{**} is located in the definition area, which equals to the condition that $x_A^E \geqslant \alpha_B$ and $x_B^E \geqslant \alpha_A$. Combining these conditions together, we can obtain the following proposition:

[18] The illustration of this can be found in Figure-3.14 in Appendix.

Proposition 3.12 *For a given x_A^0 and x_B^0 located in the $F1$ area, if $x_B^S \leqslant \alpha_A$ and $x_B^E < \alpha_A$, there exists only one shrinking equilibrium (x_A^S, x_B^S); if $x_B^S \leqslant \alpha_A$ and $x_B^E \geqslant \alpha_A$, there exist two equilibrium (x_A^S, x_B^S) and (x_A^E, x_B^E); if $x_B^S > \alpha_A$ and $x_B^E \geqslant \alpha_A$, there exists only one expanding equilibrium (x_A^E, x_B^E); if $x_B^S > \alpha_A$ and $x_B^E < \alpha_A$, no pure strategy Nash equilibrium exists.*

In order to illustrate this proposition clearly, we draw the following figure[19]:

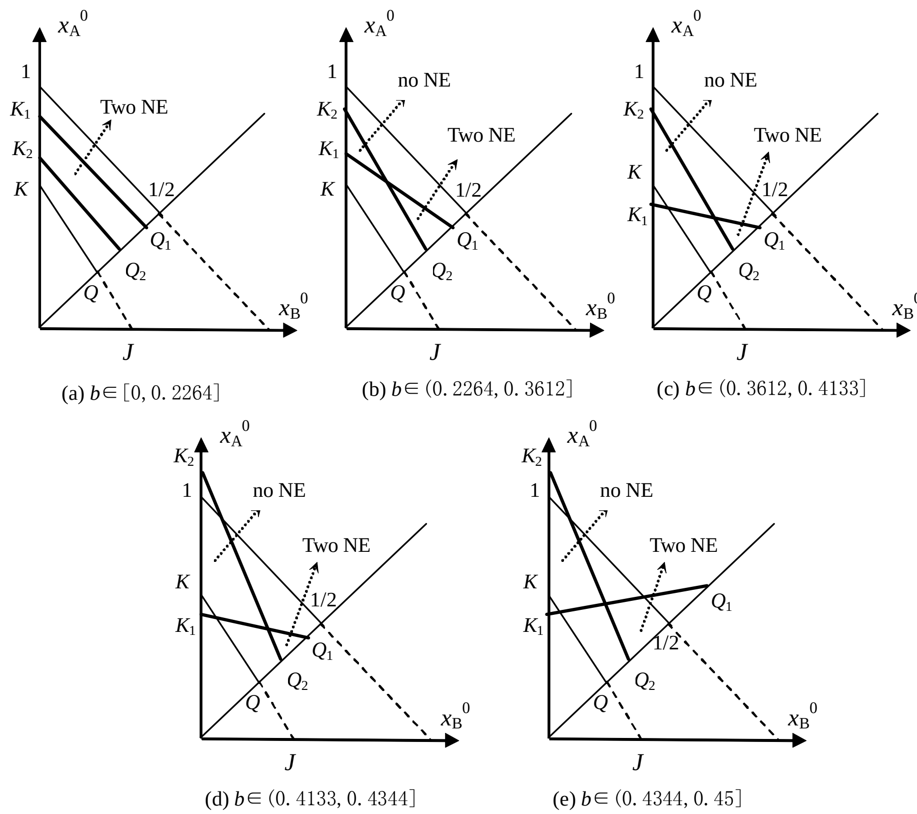


Figure-3.11: The equilibrium analysis when x_A^0 and x_B^0 are located in the $F1$ area.

In Figure-3.11, K_1Q_1 is the function $x_B^E = \alpha_A$ and K_2Q_2 is the function $x_B^S = \alpha_A$. From Proposition 3.12, we know that the expending equilibrium will exist if and only if $(x_A^0,\, x_B^0)$ is located below K_1Q_1 and the shrinking equilibrium will exist if and only if $(x_A^0,\, x_B^0)$ is located above K_2Q_2. When b is very small, we obtain Figure-3.11-a. In this

[19] The figures, which illustrate the relationship of $\{1, K, K_1, K_2\}$ and $\{1, Q, Q_1, Q_2\}$ with the change of b, can be found in Appendix. (NE: pure strategy Nash equilibrium.)

case, K_1Q_1 is always above K_2Q_2. If x_A^0 and x_B^0 are located between K_1Q_1 and K_2Q_2, both of the equilibriums will exist. When b goes larger, K_2, K_1, K move towards 1. Since K_2, K move more quickly than K_1, K_2 will be larger than K_1 when b is larger than 0.2264. If $b \in (0.2264, 0.3612]$, we obtain Figure-3.11-b. Here, for some x_A^0 and x_B^0, there does not exist a pure strategy Nash Equilibrium. When b goes over 0.3612, point K moves over K_1 and we get the Figure-3.11-c. Here, the area in which these is no pure strategy Nash Equilibrium increases and the area where there exists two equilibria shrinks. When b is larger than 0.4133, K_2 goes above 1 and we get the Figure-3.11-d. No matter how b changes, we always have $Q_1 > Q_2 > Q$. When b gets close to $b\max$, $Q_1 > 0.5$ and we obtain Figure-3.11-e. However, Q_2 will always be located between Q and 0.5.

The implication here is: when the the intensity of network effect is very small, the model we discuss is very similar to the model we discuss in Section 3.2. When the network externality is larger than a critical value, it is possible that pure strategy Nash equilibrium does not exist in the market. However, this situation will only happen when the difference of the initial market share is relatively large. If neither firm has a significant advantage in the number of locked-in consumers, the pure strategy equilibrium will always exist. The possibility that the market goes to a situation where no pure strategy Nash equilibrium exists will increase with an increase in the intensity of network effect.

Proposition 3.13 *In equilibrium, the differences of output, price and profit between Firm A and Firm B increase with an increase of $x_A^0 - x_B^0$ or b.*

In all the equilibriums (shrinking or expanding equilibrium), Firm A's advantage in the initial locked-in consumers will enable it produce more than its competitor. Moreover, Firm A can charge a higher price than Firm B since its product brings more network externalities to consumers. Thus, Firm A will obtain more profit than Firm B. This advantage in output, price and profit will increase with the increase of the intensity of network effect, because the network effect is the key connection between the firm's advantage in initial market structure and the advantage in competition.

This locked-in model with network externalties may provide some hints for social planners and merger regulators. In network world, obtaining more initial locked-in consumers could be an important incentive for the firms to merge. If a firm acquired another firm, the buyer can inherit another firm's locked-in consumers. This will bring the merged entity an advantage over other firms who stand outside the merger. Sometimes, firms may even utilize a merger strategy to deter other competitors from getting new consumers. The social planner should be cautious to the merger in network world, that enable the merged entity to obtain a dominant position with the help of inheriting locked-in consumsers and network effect.

3.4 Conclusion and Future Work

In the first part of this chapter, we investigate a duopoly Cournot competition model with half of the consumers in previous round competition are locked by a contract. The locked-in effect made the density of the consumers who can freely choose service providers not be uniformly distributed, hence the demand is distorted into a kinked function which may lead to multiple equilibria. When the number of locked-in consumers are relatively small, firms choose the expanding strategy and reach an expanding equilibrium. When the number of the locked-in consumers are relatively large, firms choose the shrinking strategy and reach a shrinking equilibrium. If the number of the locked-in consumers is in a specific interval, there exist two symmetric equilibria. In multiple equilibria, firms prefer to choose the shrinking equilibrium but the expanding equilibrium always provides a higher social welfare. The model can also be extended to an oligopoly market with a similar analysis procedure. In an oligopoly market, the possibility for the market to reach the shrinking equilibrium decreases with an increase in the number of firms.

In the second part of the paper, we add a network effect into our duopoly model. Comparing with the model in Section 3.2, the main difference is that the previous locked-in consumers may affect firms' competition through network externalities. We illustrate that the firm with the previous market share advantage may have an advantage in current round competition. Sometimes, the previous market leader may even deter its competitor from getting new consumers if the network effect is strong and the initial advantage is significant. Moreover, the pure strategy Nash equilibrium will not always exist in the network world. We found out the sufficient and necessary conditions for the existence of the pure strategy equilibrium and showed that there may only exist mixed strategy Nash Equilibrium when the network effect is strong and the difference of the two firm's previous market size is relatively large. Since more initial locked-in consumers may bring more advantage in the current round competition, obtaining more initial locked-in consumers could be an important incentive for firms to acquire others when the buyer may inherit the initial locked-in consumers from all the firms acquired.

3.5 Appendix

A3.1 Proof of Proposition 3.1

Since $x_0 < 1$, we have $1 - \frac{x_0}{2} > \frac{x_0}{2}$. If any of the firm choose their output equal or larger than $1 - \frac{x_0}{2}$, the price function will determined by p^{**}. From equation (3.4), we know:

$$p^{**} = 1 - \frac{x_0}{2} - x_A - x_B$$

It is easy to see p^{**} is less than 0 if any firm choose an output larger than $1 - \frac{x_0}{2}$. So if the firms want to have a positive profit, they must keep the market price positive, or we can say they must limit their output no more than $1 - \frac{x_0}{2}$.

A3.2 Proof of Proposition 3.2

The proof of this proposition has been shown in the discussion of the three cases in Figure-3.3. Here, the only thing which need to be clarified is $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} \in (\frac{3x_0}{2} - 1, x_0 - \frac{1}{2})$. We can rewrite $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ as:

$$\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} = x_0 - \frac{1}{2} - (\sqrt{2}-1)\frac{(1-x_0)}{2}$$

Since $x_0 < 1$, we have $(\sqrt{2}-1)\frac{(1-x_0)}{2} > 0$ and $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} < x_0 - \frac{1}{2}$. We can also rewrite $\frac{3}{2}x_0 - 1$ in a similar way:

$$\frac{3}{2}x_0 - 1 = x_0 - \frac{1}{2} - (\frac{1-x_0}{2})$$

Since $\sqrt{2} - 1 < 1$, we have:

$$\begin{aligned} x_0 - \frac{1}{2} - (\sqrt{2}-1)\frac{(1-x_0)}{2} \quad &> \quad x_0 - \frac{1}{2} - (\frac{1-x_0}{2}) \\ &\Longleftrightarrow \quad \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} > \frac{3x_0}{2} - 1 \end{aligned}$$

So in our definition, we have $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} \in (\frac{3x_0}{2} - 1, x_0 - \frac{1}{2})$.

A3.3 Proof of Proposition 3.3

From the Proposition 3.2, we know the reaction function of firm A is equation (3.14) for $x_B \leqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ and is equation (3.12) for $x_B \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$. The reaction function of firm B is equation (3.15) for $x_A \leqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ and is (3.13) for $x_A \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$. It is easy to see that x_A and x_B must be both larger than $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ or both less than $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ in equilibrium, since there are no solution by combining equation (3.12) with (3.15) or combining equation (3.13) with (3.14).

For any $x_0 \in (0, \frac{5-2\sqrt{2}}{3})$, we have:

$$\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} < \frac{1}{6}$$

If both x_A and $x_B \leqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} < \frac{1}{6}$, there is no solution by substituting (3.14) into (3.15). However, if we substitute (3.12) into (3.13), we can get $x_A = x_B = \frac{1}{3} - \frac{x_0}{6} > \frac{1}{6} > \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$. Thus, there exists and only exists one equilibrium $x_A = x_B = \frac{1}{3} - \frac{x_0}{6}$. In this equilibrium, both of the firm follow the expanding strategy, so the market will expand.

When $x_0 \in (5 - 3\sqrt{2}, 1)$,

$$\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} > \frac{1}{3} - \frac{x_0}{6}$$

If both x_A and $x_B \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} > \frac{1}{3} - \frac{x_0}{6}$, there is no solution by substituting (3.12) into (3.13). But if we substitute (3.14) into (3.15), we can get $x_A = x_B = \frac{1}{6} < \frac{1}{3} - \frac{x_0}{6} < \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$. Thus, there exists unique equilibrium $x_A = x_B = \frac{1}{6}$. In this equilibrium, both of the firm follow the shrinking strategy, so the market will shrink.

When $x_0 \in [\frac{5-2\sqrt{2}}{3}, 5 - 3\sqrt{2}]$,

$$\frac{1}{3} - \frac{x_0}{6} \geqslant \frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2} \geqslant \frac{1}{6} \tag{3.48}$$

By substituting (3.12) into (3.13), we can get $x_A = x_B = \frac{1}{3} - \frac{x_0}{6}$. And from (3.48) we know both x_A and x_B are larger than $\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$ in the equilibrium solution. Thus, the equilibrium in which the market expands exists. From (3.14) and (3.15), we get $x_A = x_B = \frac{1}{6}$. According to (A-8), we know, in this equilibrium, both x_A and x_B are less than

$\frac{\sqrt{2}+1}{2}x_0 - \frac{\sqrt{2}}{2}$, so the equilibrium in which market shrink exists as well. This proved that two equilibria exist when $x_0 \in [\frac{5-2\sqrt{2}}{3}, 5-3\sqrt{2}]$.

A3.4 Proof of Proposition 3.4

From (3.19), we can obtain

$$\frac{\partial S^{**}}{\partial x_0} = \frac{25}{36}x_0 - \frac{5}{36} > 0 \text{ for any } x_0 \in [\frac{5-2\sqrt{2}}{3}, 1)$$

Since $x_0 > \frac{5-2\sqrt{2}}{3}$ for expanding equilibrium, $\partial S^{**}/\partial x_0 > 0$ for any given x_0 in the expanding equilibrium. This means the total surplus of consumers will increase with a larger initial locked-in area. From (3.20), we can obtain:

$$\frac{\partial \Pi^{**}}{\partial x_0} = \frac{1}{9}x_0 - \frac{2}{9}$$

By our definition ($x_0 < 1$), $\partial \Pi^{**}/\partial x < 0$ for any given x_0. This means the total profit of the firms in the expanding equilibrium will decrease with an increase of x_0. From (3.21), we have

$$\frac{\partial W^{**}}{\partial x_0} = \frac{29}{36}x_0 - \frac{13}{36} > 0 \text{ for any } x_0 \in [\frac{5-2\sqrt{2}}{3}, 1)$$

Since $x_0 \in [\frac{5-2\sqrt{2}}{3}, 1)$ is the condition for the existence of expanding equilibrium, $\partial W^{**}/\partial x_0 > 0$ for any x_0 which may yields the expanding equilibrium exist. Thus, we can conclude that, in expanding equilibrium, we will have a larger social welfare if the number of the locked-in consumers increases.

In the shrinking equilibrium, Consumer's surplus, total social welfare and the total profit of the firms are constant.

A3.5 Proof of Proposition 3.5

From Proposition 3.3, we know, if there exist two equilibrium, we must have $x_0 \in [\frac{5-2\sqrt{2}}{3}, 5-3\sqrt{2}]$. Under this condition, $\partial \Pi^{**}/\partial x_0 < 0$, so Π^{**} is maximized when $x_0 = \frac{5-2\sqrt{2}}{3}$. From equation (3.20), $\Pi^{**}(x_0 = \frac{5-2\sqrt{2}}{3}) < \frac{1}{9} = \Pi^*$, so the profits in expanding market is always less than the profits in shrinking market. We also have $\partial W^{**}/\partial x_0 > 0$ in this interval, so

W^{**} is minimized when $x_0 = \frac{5-2\sqrt{2}}{3}$. From equation (3.21), $W^{**}(x_0 = \frac{5-2\sqrt{2}}{3}) > \frac{2}{9} = W^*$, so the welfare of the expanding equilibrium is always larger than the welfare of the shrinking equilibrium.

A3.6 Proof of Proposition 3.7

Here, wee only pride the proof for the case that $i = A$ and the proof of the case that $i = B$ is similar.

If we want π_A^* max can be reached by Firm A, there must have:

$$x_A^* \leqslant \epsilon - x_B \Longleftrightarrow x_B \leqslant \frac{2(2-b)\epsilon - \sigma_A}{2(1-b)} \equiv M = \epsilon + \frac{2\epsilon - \sigma_A}{2(1-b)} \tag{3.49}$$

If we want π_A^{**} max can be reached by Firm A, there must have:

$$x_A^{**} \geqslant \epsilon - x_B \Longleftrightarrow x_B \geqslant \frac{(3-2b)\epsilon - \sigma_A}{1-2b} \equiv N = \epsilon + \frac{2\epsilon - \sigma_A}{1-2b} \tag{3.50}$$

Here, M and N are defined according to equation (3.49) and (3.50). From $\sigma_A > 1$ and $\epsilon \leqslant \frac{1}{2}$, we can get $2\epsilon - \sigma < 0$. Since $b < \frac{1}{2}$, we have $2(1-b) > 1 > 1-2b > 0$. So it is easy to see $M > N$ for any b, x_A^0 and x_B^0 in our definition.

If π_A^* max and π_A^{**} max can both be reached, the Firm A need to evaluate which strategy will lead to a bigger maximum profit. If π_A^* max $> \pi_A^{**}$ max, Firm A will choose shrinking strategy and if π_A^* max $< \pi_A^{**}$ max, Firm A will follow expanding strategy. By comparing π_A^* max with π_A^{**} max, we can get:

$$\pi_A^* \max > \pi_A^{**} \max \Longleftrightarrow x_B < \epsilon + \frac{2\epsilon - \sigma_A}{\sqrt{(1-b)(2-b)} - b} = \alpha_A \tag{3.51}$$

When all two maximums can be reached, Firm A will choose x_A^* as its best response if $x_B \leqslant \alpha_A$; Firm A will choose x_A^{**} as its best response if $x_B \geqslant \alpha_A$.

By the definition of b and $2 - b > \sqrt{(1-b)(2-b)} > 1 - b$, we have $2(1-b) > \sqrt{(1-b)(2-b)} - b > 1 - 2b > 0$. Comparing (3.51) with 3.49) and (3.50), we may easily obtain $M > \alpha > N$. Moreover, from (3.34) and (3.37), we know the slopes of function x_A^* and x_A^{**} are all larger than -1 and the slope of x_A^* is larger than the slope of x_A^{**}. Now, we can combine the above result together and draw the reaction function of Firm A for a given output of Firm B in the following figure:

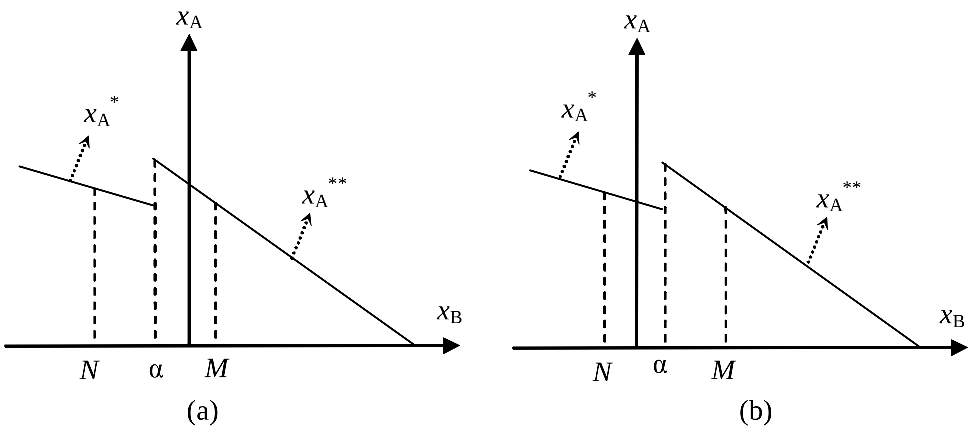


Figure-3.12: The reaction function of Firm A

If $\sigma_A \geqslant (2-b+\sqrt{(2-b)(1-b)})\epsilon$, $\alpha_A \leqslant 0$ and we obtain Figure-3.12-a. In Figure-3.12-a, Firm A will only choose function x_A^{**} as its reaction function for a positive output of Firm B. If $\sigma_A < (2-b+\sqrt{(2-b)(1-b)})\epsilon$, $\alpha_A > 0$ and we get Figure-3.12-b. In Figure-3.12-b, Firm A will choose its reaction function as x_A^* if $x_B \leqslant \alpha_A$ and choose to follow reaction function x_A^{**} if $x_B \geqslant \alpha_A$.

A3.7 Proof of Proposition 3.8

From Proposition 3.7, we know the definition area of reaction function x_A^* is $x_B \leqslant \alpha_A$ and the definition area for reaction function x_B^{**} is $x_A \geqslant \alpha_B$. Assume they cross at point $O(m, n)$, there must have $m \leqslant \alpha_A$ and $n \geqslant \alpha_B$. By substituting m, n into function x_A^*, we may obtain:

$$
\begin{aligned}
n &= \frac{\sigma_A - 2m}{2(2-b)} \\
&\Longrightarrow \quad m + n - \epsilon = \frac{\sigma_A + 2(1-b)m}{2(2-b)} - \epsilon \leqslant \frac{\sigma_A + 2(1-b)\alpha_A}{2(2-b)} - \epsilon \\
&= \frac{(2\epsilon - \sigma_A)[2 - b - \sqrt{(1-b)(2-b)}]}{2(2-b)[\sqrt{(1-b)(2-b)} - b]}
\end{aligned}
$$

Since $2\epsilon - \sigma_A < 0, 2 - b - \sqrt{(1-b)(2-b)} > 0$ and $2(2-b)[\sqrt{(1-b)(2-b)} - b] > 0$, we have $m + n - \epsilon < 0 \Longrightarrow$

$$m + n < \epsilon \tag{3.52}$$

If we substitute m, n into function x_B^{**} :

$$\begin{aligned} m &= \frac{\sigma_B - \epsilon - n}{2(1-b)} \\ &\Longrightarrow \quad m + n - \epsilon = \frac{\sigma_B - \epsilon + (1-2b)n}{2(1-b)} - \epsilon \geqslant \frac{\sigma_B - \epsilon + (1-2b)\alpha_B}{2(1-b)} - \epsilon \\ &= \frac{[\sqrt{(1-b)(2-b)} - (1-b)](\sigma_B - 2\epsilon)}{2(1-b)[\sqrt{(1-b)(2-b)} - b]} > 0 \end{aligned}$$

Thus, we have $m + n > \epsilon$, and this contradict with (3.52). We can make the conclusion that there does not exist such a $O(m, n)$, which is the crossing point of function x_A^* and x_B^{**}, when $m \leqslant \alpha_A$ and $n \geqslant \alpha_B$. For the same reason, x_B^* cannot cross with x_A^{**} in their definition area as well.

In order to illustrate this proof more clearly, we may draw the following figure:

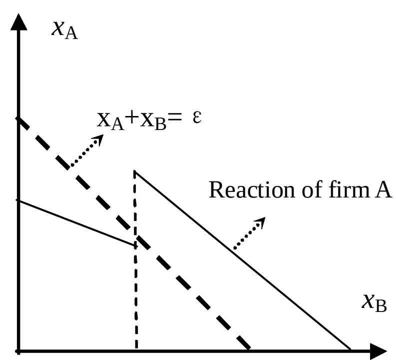


Figure-3.13: The bounded area for x_A^* and x_A^{**}.

This proof shows that x_A^* is always below the dashed line, $x_A + x_B = \epsilon$, and x_A^{**} is always above this dashed line. This dashed line also seperates x_B^* and x_B^{**}. We can see that $x_A + x_B = \epsilon$ divides the plane into two part and x_A^* and x_B^{**} are located in different part, so they will never cross. This figure can also explain why there is no cross point for function x_B^* and x_A^{**}.

A3.8 Proof of Proposition 3.9

From Figure-3.9, we know that we only need to prove α_B line is always below point O, then we can say that there always exists unique equilibrium when x_A^0 and x_B^0 are located in $F2$ area. From (3.42), we can get the x_A value of point O is $\frac{2(1-b)\sigma_A-\sigma_B-(1-2b)\epsilon}{4(1-b)^2-1}$ and we also know $\alpha_B=\epsilon+\frac{2\epsilon-\sigma_B}{\sqrt{(1-b)(2-b)}-b}$ by the definition of α_i. Thus, we may define a new function:

$$y(x_A^0,x_B^0)=\alpha_B-x_A \tag{3.53}$$

Here, we only need to prove that there does not exist a pair of x_A^0 and x_B^0 located in $F2$ and $F3$ area can make $y(x_A^0,x_B^0)>0$. If we draw $y(x_A^0,x_B^0)=0$ in Figure-3.7, we can obtain the following figure:

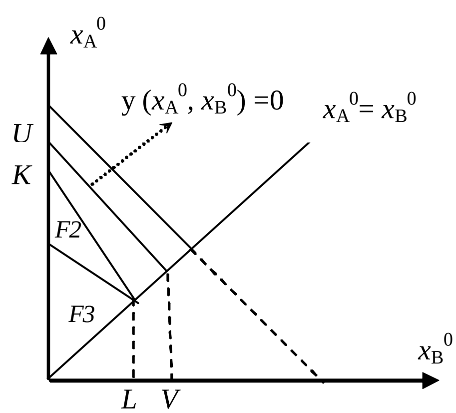


Figure-3.14: $y(x_A^0,x_B^0)=0$

By solving the function $y(x_A^0,0)=0$, we can get:

$$U=\frac{(1-2b)[3(1-b)+\sqrt{(1-b)(2-b)}]}{(2-6b+3b^2)\sqrt{(1-b)(2-b)}+(1-b)(3-7b+3b^2)}$$

Combining (3.39) and the value of U, we can obtain:

$$U-K=\frac{2-11b+17b^2-8b^3+(1-3b+4b^2)\sqrt{(1-b)(2-b)}}{(2(1-b)+\sqrt{(1-b)(2-b)})((2-6b+3b^2)\sqrt{(1-b)(2-b)}+(1-b)(3-7b+3b^2))}$$

When $0 < b < b\max$, $U - K > 0$. This means point U is always located above point K. In a similar way, by solving the function $y(x_B^0, x_B^0) = 0$, we can get :

$$V = \frac{6(1-b) + 2\sqrt{(1-b)(2-b)}}{12 - 17b + 5b^2 - (3-b)\sqrt{(1-b)(2-b)}}$$

If we substitute $x_A^0 = x_B^0$ into $\sigma_A = (2 - b + \sqrt{(2-b)(1-b)})\epsilon$, we can get:

$$L = \frac{1}{2 - \frac{3}{2}b + \sqrt{(1-b)(2-b)}}$$

Then, we can obtain:

$$\begin{aligned} & V - L \\ = \quad & \frac{4(2-3b)(1-b+\sqrt{(1-b)(2-b)})}{(12-17b+5b^2-(3-b)\sqrt{(1-b)(2-b)})(2-\frac{3}{2}b+\sqrt{(1-b)(2-b)})} \end{aligned}$$

When $0 \leqslant b \leqslant 0.45$, $V - L > 0$. So in Figure-3.14, function $y(x_A^0, x_B^0) = 0$ is always located above the function $\sigma_A = (2 - b + \sqrt{(2-b)(1-b)})\epsilon$ in our definition area. This means all the points (x_A^0, x_B^0) in $F2$ and $F3$ area make $y(x_A^0, x_B^0) < 0$ and we may indicate that α_B line is always below point O.

From the analysis of Figure-3.8 and 3.9, we know there always exist unique equilibrium if (x_A^0, x_B^0) is located in $F2$ and $F3$ area and the location of the equilibrium depends on the relation between R and S. We can construct the following inequality function:

$$R \leqslant S \Longleftrightarrow x_A^0 \geqslant \frac{2(1-2b) + (4b - 2b^2 - 1)x_B^0}{1-b} = \gamma$$

So when $x_A^0 \geqslant \gamma$, we have $R \leqslant S$ and the equilibrium is (3.43). When $x_A^0 < \gamma$, we have $R > S$ and the equilibrium is (3.42).

A3.9 Proof of Proposition 3.10

From Proposition 3.9, we know that if the equilibrium (3.43) exists, we must have a pair of x_A^0 and x_B^0 which causes $x_A^0 \geqslant \gamma$. In another words, there must exist some points located in $F2$, $F3$ area and also located above $x_A^0 = \gamma$ line. We know the slope of function KJ is less than -1 and the slope of $x_A^0 = \gamma$ is $\frac{4b-2b^2-1}{1-b} > -1$. Thus, the sufficient and necessary condition for $x_A^0 = \gamma$ line crossing $F2$, $F3$ area is that the intercept of $x_A^0 = \gamma$ is less than K and larger than zero. By solving

$$\frac{2(1-2b)}{1-b} \leqslant \frac{2}{2(1-b)+\sqrt{(1-b)(2-b)}} = K$$

we can obtain that $b^* \approx 0.3612$ and is one of the solution of $12b^3 - 12b^2 + 1 = 0$.

A3.10 Proof of Proposition 3.11

If there exists a equilibrium that firm A have a positive output and firm B have zero output, the two firms' reaction function must be located as the following figure:

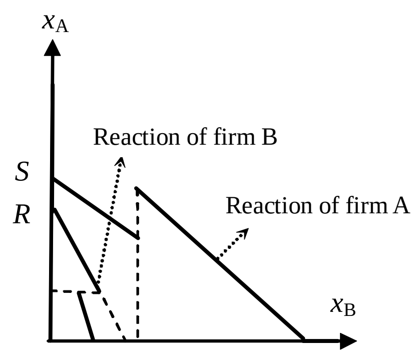


Figure-3.15: Firm A drives Firm B out of the market.

In Figure-3.15, the equilibrium occurs at point S, so we must have $S \geqslant R$. We can get the value of S and R by substituting $x_B = 0$ into reaction functions x_A^* and x_B^{**}.

$$\begin{aligned} S \quad &\geqslant \quad R \Longleftrightarrow x_A^0 \geqslant \frac{6-4b}{4-b} - \frac{(1-b)(2-b)}{4-b} x_B^0 \\ &\Longrightarrow \quad x_A^0 + x_B^0 \geqslant 1 + \frac{2-3b}{4-b} + \frac{2-2b-b^2}{4-b} x_B^0 > 1 \end{aligned}$$

According to our assumption, $x_A^0 + x_B^0 \leqslant 1$, $S \geqslant R$, as which we illustrated in Figure-3.15, will never occur when x_A^0 and x_B^0 are located in the $F1$ area. Or we can say $S < R$ for any give x_A^0 and x_B^0 located in $F1$ area

A3.11 Proof of Proposition 3.12

Lemma 3.1 *For any b in our definition area and any pair of x_A^0 and x_B^0 located in the $F1$ area, $x_A^S \leqslant \alpha_B$ if $x_B^S \leqslant \alpha_A$.*

Proof. From (3.42) and the definition of α_B, we get:

$$\frac{(2-b)\sigma_A - \sigma_B}{2(1-b)(3-b)} \leqslant \alpha_B$$

$$\begin{aligned}
\iff & \frac{(2-b)\sigma_A - \sigma_B}{2(1-b)(3-b)} \leqslant \epsilon + \frac{2\epsilon - \sigma_B}{\sqrt{(1-b)(2-b)} - b} \\
\iff & (\frac{2(1-b)(3-b)}{\sqrt{(1-b)(2-b)} - b} - 3 + b)\sigma_B \\
\leqslant & (2(1-b)(3-b)(1 + \frac{2}{\sqrt{(1-b)(2-b)} - b}) - b(2-b))\epsilon - 2(2-b)
\end{aligned} \tag{3.54}$$

From (3.43) and the definition of α_A, we get:

$$\frac{(2-b)\sigma_B - \sigma_A}{2(1-b)(3-b)} \leqslant \alpha_A$$

$$\begin{aligned}
\iff & \frac{(2-b)\sigma_B - \sigma_A}{2(1-b)(3-b)} \leqslant \epsilon + \frac{2\epsilon - \sigma_A}{\sqrt{(1-b)(2-b)} - b} \\
\iff & (\frac{2(1-b)(3-b)}{\sqrt{(1-b)(2-b)} - b} - 3 + b)\sigma_A \\
\leqslant & (2(1-b)(3-b)(1 + \frac{2}{\sqrt{(1-b)(2-b)} - b}) - b(2-b))\epsilon - 2(2-b)
\end{aligned} \tag{3.55}$$

Since $\frac{2(1-b)(3-b)}{\sqrt{(1-b)(2-b)}-b} - 3 + b > 0$ for any $b \in (0, b\max)$ and $\sigma_A \geqslant \sigma_B$, (3.55) is a stricter condition than (3.54). ■

Lemma 3.2 *For any b in our definition area and any pair of x_A^0 and x_B^0 located in the $F1$ area, $x_A^E \geqslant \alpha_B$ if $x_B^E \geqslant \alpha_A$.*

Proof. From (3.44) and the definition of α_B, we get:

$$\frac{2(1-b)\sigma_A - \sigma_B - (1-2b)\epsilon}{4(1-b)^2 - 1} \geqslant \alpha_B$$

$$\iff \frac{2(1-b)\sigma_A - \sigma_B - (1-2b)\epsilon}{4(1-b)^2-1} \geqslant \epsilon + \frac{2\epsilon - \sigma_B}{\sqrt{(1-b)(2-b)}-b}$$

$$\iff (\frac{4(1-b)^2-1}{\sqrt{(1-b)(2-b)}-b} - 3 + 2b)\sigma_B$$

$$\geqslant (4 - 12b + 6b^2 + \frac{2(4(1-b)^2-1)}{\sqrt{(1-b)(2-b)}-b})\epsilon - 4(1-b) \tag{3.56}$$

From (3.45) and the definition of α_A, we get:

$$\frac{2(1-b)\sigma_B - \sigma_A - (1-2b)\epsilon}{4(1-b)^2-1} \geqslant \alpha_A$$

$$\iff \frac{2(1-b)\sigma_B - \sigma_A - (1-2b)\epsilon}{4(1-b)^2-1} \geqslant \epsilon + \frac{2\epsilon - \sigma_A}{\sqrt{(1-b)(2-b)}-b}$$

$$\iff (\frac{4(1-b)^2-1}{\sqrt{(1-b)(2-b)}-b} - 3 + 2b)\sigma_A$$

$$\geqslant (4 - 12b + 6b^2 + \frac{2(4(1-b)^2-1)}{\sqrt{(1-b)(2-b)}-b})\epsilon - 4(1-b) \tag{3.57}$$

Since $\frac{4(1-b)^2-1}{\sqrt{(1-b)(2-b)}-b} - 3 + 2b < 0$ for any $b \in (0, b\max)$ and $\sigma_A \geqslant \sigma_B$, (3.57) is a stricter condition than (3.56). ■

From Lemma 3.1, the sufficient and necessary condition for the existence of the shrinking equilibrium can be reduced as $x_B^S \leqslant \alpha_A$. From Lemma 3.2, the sufficient and necessary condition for the existence of the expanding equilibrium can be reduced as $x_B^E \geqslant \alpha_A$. Combining these two reduced sufficient and necessary conditions together, we can obtain this proposition.

A 3.12 Proof of Proposition 3.13

From (3.44) and (3.45), we can obtain:

$$x_A^S - x_B^S = \frac{b(x_A^0 - x_B^0)}{4(1-b)}$$

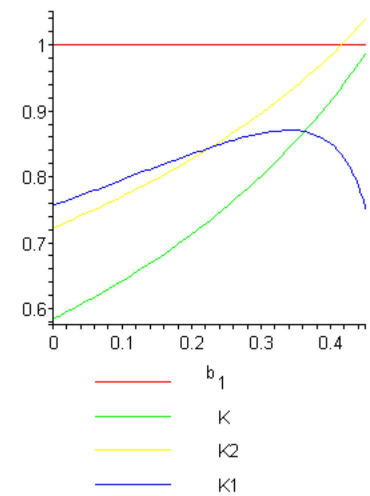
Thus, $\partial(x_A^S - x_B^S)/\partial b > 0$ and $\partial(x_A^S - x_B^S)/\partial(x_A^0 - x_B^0) > 0$.

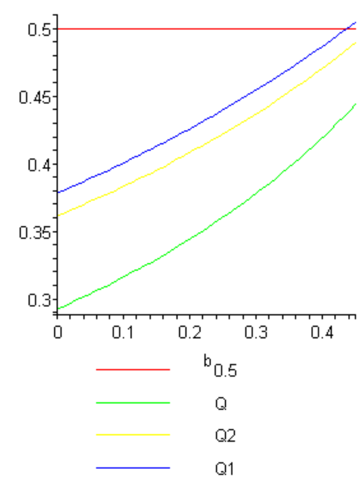
From (3.46) and (3.47), we can obtain:

$$x_A^E - x_B^E = \frac{(2-b)(x_A^0 - x_B^0)}{4(1-b)^2 - 1}$$

Thus, $\partial(x_A^E - x_B^E)/\partial b > 0$ and $\partial(x_A^E - x_B^E)/\partial(x_A^0 - x_B^0) > 0$.

A3.13 Relationship of $\{1, K, K_1, K_2\}$ and $\{1/2, Q, Q_1, Q_2\}$ with the Change of b





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