DESIGN OF A FIBRE-OPTIC ACOUSTIC-SENSOR ARRAY
SENSITIVITY AND NOISE PROPERTIES

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A thesis submitted for the degree of
Doctor of Philosophy

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October 2002
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In this thesis the study of a multiplexed Fibre-Optic Acoustic-Sensor (FOAS) system is presented. Narrowband reflectors (Fibre Bragg Gratings or FBG’s) define sensing sections of 12.5 metres, which then act as Fabry-Perot (FP) cavities. Low-coherence interferometry is used to interrogate the sensors with an accuracy of 50 $\mu$rad/$\sqrt{\text{Hz}}$, in good agreement with the theoretically predicted value. Heterodyne signal processing is used to eliminate low frequency environmental noise. The performance of the sensor is checked with a sinusoidal calibration signal generated by a PZT fibre stretcher. The sensor has a flat frequency response at 10 kHz with a high sensitivity of 50 $\mu$rad/$\sqrt{\text{Hz}}$ and a dynamic range of 80 dB. The use of FBG-based interferometers allows the use of Wavelength Division Multiplexing (WDM) technology allowing us to multiplex large number of sensors in the system.

The sensing system uses Amplified Spontaneous Emission (ASE) sources for illumination purposes. ASE sources are an attractive option for interrogating arrays of FBG sensors. The coherence features of broadband ASE light makes it attractive to be used in sensing applications, since Coherence Multiplexed (CM) systems interrogated with these sources do not suffer from phase induced intensity noise, which is a problem when employing laser sources.

It is well known that ASE sources suffer from excess photon noise, which is the dominant type of noise and hence limits the system’s sensitivity. To get an idea of the impact of this type of noise on the performance of the system, the noise properties have been studied in detail both theoretically and experimentally. Noise spectra are calculated from the autocorrelation function of the output detector current for a thermal-like source.

It is well known that unbalanced interferometers (with delay time $T$) act as filtering elements and produce a noise spectrum with peaks at integer multiples of $1/T$, due to filtered source intensity noise. The noise analysis is used to evaluate the performance of the sensor system, and to calculate the optimum reflectivity of both FBG’s in the FP sensing cavity. Optimum reflectivities for both FBG’s in the FP sensors have been found. Theoretical calculations show that the best phase resolution and visibility is obtained for $R_1 = 40\%$ and $R_2 = 100\%$. This has been verified with experiments. We also established the robustness of the system to FBG spectral drift.

A first demonstration of a FOS interrogation system using a low-coherence ASE source with a Semiconductor Optical Amplifier (SOA) is presented. The SOA is Gain-Saturated and thereby reduces the dominant intensity noise originating from the ASE source, improving the system’s Signal to Noise Ratio (SNR).
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Acknowledgements

I would like to express my gratitude to the following people for all their help and support while completing this research.

First of all my thanks goes out to my supervisor Prof. David Richardson for making it possible for me to study for this degree and for all his help, support and encouragement’s even in difficult times throughout the years. Also a special thanks to Prof. Michael Zervas for reading this work and to Dr. Neil Broderick, Dr. Hyo-Sang Kim and Francisco Javier Madruga Saavedra (Universidad de Cantabria, Spain) for all their help with the experimental and theoretical work and many useful discussions.

To the past and present members of the Richardson group. To Christos Riziotis, Christos Grivas, Periklis Petropoulos, Peh Teh, Zulfadhly Yusoff, Kentaro Furusawa, Carlos Alegría, Amin Abdolvand, Chao-Yi Tai, Yuh-Tat Cho, Romeo Selvas-Aguilar, Joo-Nyung Jang, Li-Bin Fu, Jose Alvarez-Chavez and Oliver Hadeler, the network officer Arthur Longhurst, the technicians Simon and Timothy, the secretaries Eveline Smith and Joyce Aburrow and all the people I mist out. I would like to thank the 'Italian gang’ King Walter Belardi, Fabio Ghiringhelli, Enzo Matera, Alessandro Busacca, Gilberto Brambilla, Costantino Corbari and to Ju-Han Lee and Paulo Jorge dos Santos Almeida, you are all great friends. I also want to thank Morten Ibsen for being a great friend and for manufacturing all the gratings described in this thesis. Keep the V-12 rolling!

I want to thank my parents for always supporting me during my years as a student. I also want to thank family De Paepe for giving me support during my years in England.

There is one person I want to thank the most, a very special thank you goes out to you Anne. Thank you for always being there for me. I could not have done this without you.

Lastly, I am greatly indebted to the ORC for their generous financial support.
Principal abbreviations

(i/p) Optical input power to the SOA.
(o/p) Optical output power from the SOA.
A Ampere.
ASE Amplified Spontaneous Emission source.
C (Fibre-optic directional) Coupler.
CM Coherence Multiplexing.
Cov (Auto)covariance function.
CW Constant Wave
cm Centimetre.
dB Decibel.
dBm Decibel re 1 mW (0 dBm = 1 mW).
DWDM Dense Wavelength Division Multiplexing.
EDFA Erbium Doped Fibre Amplifier.
FBG Fibre Bragg Grating.
FC/PC Fibre Connector / Physical Contact.
FDM Frequency Division Multiplexing.
FOAS Fibre-Optic Acoustic-Sensor.
FOG Fibre-Optic Gyroscope.
FOS Fibre-Optic Sensor.
FP Fabry-Perot.
FS Frequency Shifter.
FSR Free Spectral Range.
FWHM Full Width at Half Maximum.
GaInAsP Gallium Indium Arsenide Phosphide.
GS-SOA Gain-Saturated Semiconductor Optical Amplifier.
Hi-Bi High-Birefringence.
Hz Hertz (1 Hz = 1 s⁻¹).
ITU International Telecommunications Union.
Im Imaginary.
InGaAs Indium Gallium Arsenide.
LED Light Emitting Diode.
m  Metre, or
   milli.
MZI  Mach-Zehnder Interferometer.
N  Newton.
NSD  (electrical) Noise (power) Spectral Density.
OBC  Ocean Bottom Cable.
ORC  Optoelectronics Research Centre.
OSA  Optical Spectrum Analyser.
ppm  Parts Per Million.
Pa  Pascal (1 Pa = 1 N/m²).
PC  Polarisation Controller.
PF  Polarisation Flipping.
PP  Polarisation Preserving.
PM  Polarisation Maintaining.
PZT  Lead Zirconate Titanate (Piezoelectric transducer or fibre stretcher).
re  Relative to.
rad  Radians.
Re  Real.
Res. BW  Resolution Bandwidth.
RF  Radio Frequency.
RIN  Relative Intensity Noise.
s  Second.
SLD  Super Luminescent Diode.
SNR  Signal to Noise Ratio.
SMF  Single-Mode Fibre.
SOA  Semiconductor Optical Amplifier.
SOP  State Of Polarisation.
TDM  Time Division Multiplexing.
TE  Transverse Electric.
TEC  Thermal Electric Cooler.
TF  Tuneable Filter.
TM  Transverse Magnetic.
V  Volt.
W - Watt.
WDM - Wavelength Division Multiplexing.

**List of symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>-</td>
<td>Correction factor for the non-flat frequency response of the receiver.</td>
</tr>
<tr>
<td>A</td>
<td>-</td>
<td>Nested function of the complex field amplitude $F$.</td>
</tr>
<tr>
<td>b</td>
<td>-</td>
<td>Correction factor for the non-flat frequency response of the receiver.</td>
</tr>
<tr>
<td>$B_{tot}$</td>
<td>m</td>
<td>Total optical bandwidth of the source.</td>
</tr>
<tr>
<td>$B$</td>
<td>Hz</td>
<td>Measurement bandwidth.</td>
</tr>
<tr>
<td>$c$</td>
<td>m/s</td>
<td>Speed of light.</td>
</tr>
<tr>
<td>$cc$</td>
<td>-</td>
<td>Complex conjugate of the preceding term.</td>
</tr>
<tr>
<td>dyne</td>
<td>-</td>
<td>Force ($1$ dyne $= 10$ $\mu$N).</td>
</tr>
<tr>
<td>$e$</td>
<td>-</td>
<td>Cartesian unit vector, or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.71828182846$.</td>
</tr>
<tr>
<td>$E$</td>
<td>V/m</td>
<td>Electric field.</td>
</tr>
<tr>
<td>$f$</td>
<td>Hz</td>
<td>Radio frequency.</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Hz</td>
<td>Signal frequency.</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Hz</td>
<td>Modulation frequency.</td>
</tr>
<tr>
<td>$f_h$</td>
<td>Hz</td>
<td>Heterodyne frequency.</td>
</tr>
<tr>
<td>$F$</td>
<td>-</td>
<td>Complex field amplitude.</td>
</tr>
<tr>
<td>$F$</td>
<td>-</td>
<td>Fourier transformation.</td>
</tr>
<tr>
<td>$g$</td>
<td>V/A</td>
<td>Receiver transimpedance gain.</td>
</tr>
<tr>
<td>$H$</td>
<td>dB re $1$ $\mu$Pa/$\sqrt{Hz}$</td>
<td>Reference scale for acoustic pressure.</td>
</tr>
<tr>
<td>$I$</td>
<td>W</td>
<td>Light intensity.</td>
</tr>
<tr>
<td>$I_1$, $I_2$</td>
<td>W</td>
<td>Light intensity in the two arms of an MZI.</td>
</tr>
<tr>
<td>$i$</td>
<td>-</td>
<td>Complex number ($\sqrt{i} = -1$), or</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>Receiver output current.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Unit</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>$J$</td>
<td></td>
<td>Bessel function or electrical signal component.</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>Path-imbalance of an interferometer.</td>
</tr>
<tr>
<td>$l$</td>
<td>m</td>
<td>Interferometer arm length.</td>
</tr>
<tr>
<td>$l_0$</td>
<td>m</td>
<td>Initial interferometer arm length.</td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>m</td>
<td>Change in interferometer arm length.</td>
</tr>
<tr>
<td>$l_c$</td>
<td>m</td>
<td>Coherence length of the light.</td>
</tr>
<tr>
<td>$\ln$</td>
<td></td>
<td>Natural logarithmic (i.e. $\log_e$).</td>
</tr>
<tr>
<td>$\Delta L$</td>
<td>m</td>
<td>Length mismatch of a coherence multiplexed interferometer.</td>
</tr>
<tr>
<td>$M, n, m, p, q$</td>
<td></td>
<td>Integer numbers.</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Pa$^{-1}$</td>
<td>Normalised pressure sensitivity.</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td>Refractive index, or Channel number.</td>
</tr>
<tr>
<td>$N$</td>
<td></td>
<td>Integer number.</td>
</tr>
<tr>
<td>$N$</td>
<td>dBm/Hz</td>
<td>(electrical) Noise(power) Spectral Density.</td>
</tr>
<tr>
<td>$P_0$</td>
<td>W</td>
<td>Source output power.</td>
</tr>
<tr>
<td>$P_1, P_2$</td>
<td>N/m$^2$</td>
<td>Acoustic pressure.</td>
</tr>
<tr>
<td>$r$</td>
<td>%</td>
<td>Field reflection coefficient.</td>
</tr>
<tr>
<td>$R$</td>
<td>%</td>
<td>Intensity reflection coefficient, or Field reflection coefficient, or Receiver output load resistance.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td></td>
<td>Signal spectrum.</td>
</tr>
<tr>
<td>$t$</td>
<td>s</td>
<td>Time.</td>
</tr>
<tr>
<td>$T$</td>
<td>°C</td>
<td>Temperature, or Delay time, or Detection time, or</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>Field transmission coefficient.</td>
</tr>
<tr>
<td>$U$</td>
<td>V</td>
<td>Receiver output voltage.</td>
</tr>
<tr>
<td>$V$</td>
<td>%</td>
<td>Interferometer fringe visibility.</td>
</tr>
<tr>
<td>$V_{pp}$</td>
<td>V</td>
<td>Volt peak-to-peak.</td>
</tr>
</tbody>
</table>
\( x \) - Random variable, or
- Cartesian polarisation axis, or
\( \text{Hz} \) Relative optical frequency \((\nu - \nu_0)\), or
\( \text{dB} \) Loss of optical power.
\( y \) - Cartesian polarisation axis.
\( z \) - Complex number.

**Greek symbols**

\( \alpha \) \( \text{A/W} \) Receiver responsivity.
\( \delta \) - (Dirac) Delta function, or
- Field loss.
\( \omega_m \) \( \text{rad/s}^{-1} \) Angular modulation frequency.
\( (\phi_0)_{\text{min}} \) \( \mu\text{rad}/\sqrt{\text{Hz}} \) Minimum detectable phase (Phase resolution).
\( \phi_m \) \( \text{rad} \) Optical modulation depth.
\( \phi_b \) \( \text{rad} \) Optical phase bias.
\( \phi \) \( \text{rad} \) Optical phase.
\( \sigma \) \( \text{Hz} \) Bandwidth dependent factor, defined as:
\[
2\sigma^2 = \frac{(\Delta \nu_0)^2}{4\ln 2}.
\]
\( \Delta \nu_0 \) \( \text{Hz} \) FWHM optical bandwidth of the source.
\( \nu_0 \) \( \text{Hz} \) Centre frequency of the source light.
\( \gamma \) - Complex degree of coherence.
\( \varepsilon \) \% Mechanical strain.
\( \psi \) - Normalised envelope function of the optical spectrum.
\( \lambda \) \( \text{m} \) Wavelength of the light.
\( \nu \) \( \text{Hz} \) Optical frequency of the source.
\( \tau \) \( \text{s} \) Time delay.
\( \tau_c \) \( \text{s} \) Source coherence time.
\( \Gamma \) - Correlation function.
**Superscripts**

1. Channel 1
2. Channel 2
* Complex conjugate
\textit{eff} Effective reflectivity

**Subscripts**

0 Launched light intensity, or initial.
AC Alternating Current (denotes the signal power).
b (Phase) Bias.
c Centre, or Coherence, or Cavity.
DC Direct Current (denotes the average).
drop Drop-filter.
FP Fabry-Perot.
h Heterodyne.
l Optical intensity.
i (Photo) Receiver output current.
m Modulation.
MZ Mach-Zehnder.
max Maximum.
min Minimum.
out Output.
overall Overall reflectivity.
p Pulse, or Pick-out filter.
ref, r Reference.
rms Root-mean-square.
s Signal.
x, x' Cartesian polarisation axis.
y, y' Cartesian polarisation axis.
Introduction

Acoustic sensors have found their use in many application areas for many years. The oil industry and the military are though perhaps the greatest users of acoustic sensors, employing them for the exploration of oil and gas or for the detection of enemy vessels respectively. Previously electrical sensors were used, but ever since the invention of the optical fibre 20 years ago or so the interest in optical sensors has grown. Optical fibres have several attractive features for sensor applications. The fibres that propagate light from the source to the sensor and back to the receiver have low loss and high bandwidth so they can permit remote operation over greater distance and at lower powers than conventional electrically powered and interrogated sensors. The sensors and the transmission fibres are also entirely immune to electromagnetic interference, which is an important issue for many applications. Other advantages of optical fibre sensors over their electrical counterparts include their lightweight and small size.

Application of Fibre-Optic Acoustic-Sensors

Acoustic sensors or hydrophones are being employed in huge hydrophone arrays (typical length 1 – 10 kilometres) for the exploration of oil and gas at sea. These arrays are towed slightly below the surface of the sea behind a surface ship in conjunction with an air gun, which is used to produce ~ 20 bar acoustic detonations into the water. The acoustic waves generated reflect of different rock formations in the seabed. The acoustic sensors are capable of sensing the minute acoustic waves reflected from these rock formations. The signals are detected on the ship where they then give the operators vital seismic information, which can be used to infer the presence of oil or gas in the seabed. Moreover, the added weight and size advantage of optical hydrophones over their electrical counterparts reduces the towing load of the surface vessel considerably.

Performance of Fibre-Optic Acoustic-Sensors

In order to map as big an area as possible, a massive number of sensors (typically 1000’s) are employed in these towed hydrophone arrays. The performance required from these sensors is exacting. For example, in order to detect the faint acoustic waves reflected back from the rock formations in the seabed in the presence of the massive bang generated at the backend of the surface ship, the sensors need to have a dynamic range of order 100 dB (electrical). They also need to have a linear response over this
range. Moreover, for detection of these faint signals a high receiver sensitivity is required. Most of these type of sensors are based on interferometers and require a typical minimum phase resolution of 10 to 20 μrad/√Hz at frequencies from 10 Hz to 20 kHz. A further critical requirement for these sensing systems is that they must possess low crosstalk levels between adjacent channels in order to allow the sensor channels to be fully distinguished from each other (again these are exacting requirements by anybody's standards, typically –100 dB electrical crosstalk is required).

FOAS arrays based on Fibre Bragg Gratings (FBG's) have been a major research topic in recent years [1]. The use of FBG based interferometers allows the use of Wavelength Division Multiplexing (WDM) technology, which in turn allows us to multiplex large number of sensors in the system. Chapter (1) of this thesis is devoted to highlighting several different FOAS systems and explaining their operating procedures.

*Narrowband Amplified Spontaneous Emission sensor interrogation versus laser interrogation*

Conventionally, FBG based interferometers are interrogated using narrowband (often single frequency) laser sources [2, 3] with associated long coherence lengths and these long coherence lengths can lead to significant problems with respect to phase noise [4, 5]. Whilst these problems can be overcome using for example time gating techniques [6] this can only be done with an increase in system complexity (and cost), and this becomes a severe problem for large channel counts. Figure (1) below shows a FOAS system interrogated with a laser source, employing a time gating technique. To eliminate coherent interference at the output coupler of the Mach-Zehnder Interferometer (MZI), the laser source is optically gated with a modulator or a fast optical switch.
Figure 1: Schematic showing the approach for sensor interrogation with a laser source and a time-gating technique. An optical modulator is used to optically gate the laser source.

With the system drawn in Figure (1) above, a pulse from the laser source is split into two pulses on passing through the MZI. These two pulses are directed to the FP sensing cavity. Four pulses are returned from the sensor, two from the first grating and two from the last grating. Since the sensing cavity is length matched to the MZI, the two information carrying pulses overlap (pulses 1 and 2' overlap and interfere resulting in pulse b containing the phase information $\phi_1$) in the time domain after being returned to the receiver where the phase information is retrieved. With this system, excellent levels of phase sensitivity can be achieved. These systems have in the past shown phase resolutions of 40 $\mu$rad/$\sqrt{\text{Hz}}$ [6].

*Sensing system interrogated with narrowband Amplified Spontaneous Emission source*

The aforementioned interrogation issues, with respect to phase noise and with respect to the time-gating approach, were recognised at an early stage by a local company,
GeoSensor. This company was interested in oil and gas exploration technology, and they developed, in conceptual form, the idea of using spectrally sliced incoherent light to interrogate arrays of FBG interferometers. There are numerous attractive features of this approach:

1. Elimination of phase noise issues (no need to pulse the source).
2. One master source can be sliced into many channels and used to address many sensors thereby greatly reducing system cost.
3. The coherence of the light can be used as a further means of multiplexing and which should further improve crosstalk performance.

Whilst appealing conceptually however there were, at the time GeoSensor approached the ORC, a large number of unresolved issues concerning the viability of the approach. This thesis assesses the technical viability of the approach and establishes what performance specifications can be expected from such a system. A typical FOAS system architecture employing a narrowband Amplified Spontaneous Emission (ASE) source is drawn schematically in Figure (2) below.
**Figure 2:** Schematic showing the basic concept of a Wavelength-Division Multiplexed (WDM) FOAS system with \((n+1)\) channels, interrogated with a multi-wavelength narrowband ASE source.

The source spectrum (with total optical bandwidth \(B_{\text{tot}} = 40\) nm) is shown in the figure above and it comprises of slices of light with each slice (of bandwidth \(\Delta \lambda\)) illuminating a separate sensor channel. The full 40 nm bandwidth of the source can be sliced into multiple channels, each slice having a bandwidth of around 0.05 nm to 1 nm. The multi-wavelength source used with the sensing system shown schematically in Figure (2) above is redrawn in more detail, in Figure (3) below.
Figure 3: Schematic of the multi-wavelength \((n+1)\) channels) narrowband ASE source.

The narrowband ASE source will be presented in full detail, together with experimental work on the narrowband ASE source, in chapter (2).

Issues concerned with narrowband Amplified Spontaneous Emission interrogation

The primary issue concerns the specific noise properties of broadband light. ASE sources, in common with Light Emitting Diodes (LED’s), have thermal-like properties since they emit light as Amplified Spontaneous Emission (ASE) and consequently have an intrinsic intensity noise, termed excess photon noise [7, 8, 9]. A detailed discussion on this with experimental results on source noise will be presented in chapter (2). This intensity noise is inherent to the ASE source and limits the Signal to Noise Ratio (SNR) of the sensing system. Source noise from a broadband source has the characteristic that the SNR does not improve with increasing source power beyond a certain level. Moreover, the noise level is inversely proportional to the optical bandwidth of the ASE source. A narrowband source is preferred in this application since the channel spacing can be reduced and the sensor length matching becomes less stringent (i.e. longer coherence length), however this compromises the SNR. Clearly, a compromise must be made between a narrow bandwidth (i.e. higher coherence – higher noise) or, a broader bandwidth (i.e. lower coherence – lower noise) source. In order to establish the answers to issues such as impact of excess photon noise, optimal source bandwidth, and length matching, we have developed both theoretical and experimental studies on single sensor system to understand the trade-offs that exist.
Source noise reduction using a Semiconductor Optical Amplifier

Several groups have shown that intensity noise can be suppressed with the use of a Gain-Saturated Semiconductor Optical Amplifier (GS-SOA) [10, 11]. In this work we adopt the same principle by using a GS-SOA to reduce the inherent intensity noise originating from the ASE source. Operating an SOA in gain saturation tends to average out the intensity fluctuations input to the SOA, due to the fast gain (carrier) dynamics of the SOA. This work is described in section (2.6) of chapter (2).

Impact of a single Mach-Zehnder Interferometer on the source noise

Chapter (3) of this thesis describes the impact of MZI’s on the source noise. An estimate of source noise on a given system performance is complicated by the filtering effect in multiple-path optical systems, such as interferometers, which can cause redistribution of the optical noise power in the spectral domain [8]. This is also true for Frequency Shifters (FS), which are often employed in such systems for the function of heterodyne detection. To investigate the impact of interferometer systems on the noise power, we first considered a single unbalanced MZI (with path-imbalance \( L_{MZ} \) much longer than the coherence length of the source), and observed the impact of this on the optical noise power. The experimental set-up, to investigate this impact, is drawn schematically in Figure (4) below.

![Figure 4](image)

**Figure 4**: Set-up for experimental investigation of the noise filtering of an interferometer and the influence of SOP within the interferometer. Shown is a single unbalanced MZI in conjunction with the narrowband ASE source, a photo-receiver and an RF spectrum analyser, for noise measurement. A Polarisation Controller (PC) was incorporated in one of the MZI arms to set the SOP in the interferometer.

Our work has indicated that the noise power redistribution depends significantly on the State Of Polarisation (SOP) of the light in each arm of the MZI. A PC was incorporated
in one of the MZI arms to conveniently set the SOP of the light in that arm relative to the SOP in the other arm of the interferometer. In order to understand the effect of the SOP within the interferometer, two extreme cases were considered. The first case is where the polarisation of the light passing through the two arms of the MZI is the same, therefore called the Polarisation Preserving (PP) case. In the second case, the SOP of the light in one arm of the interferometer is changed to be orthogonal to the light in the other arm. The latter, is called the Polarisation Flipping (PF) case. It was shown that noise power showed a periodicity in the frequency domain and the noise power was reduced by as much as 2.5 dB for the PF configuration compared to the PP configuration.

_Dual Mach-Zehnder Fibre-Optic Acoustic-Sensing system_

The impact of a dual MZI based FOAS system on the source noise is presented in the second part of chapter (3). A dual MZI system has been considered by several groups to be a viable FOAS system [12]. In such a system, a second unbalanced MZI (the sensing interferometer) is cascaded after the first one (the compensating interferometer). The two MZI’s are length-matched to within the coherence length of the narrowband ASE source. This is shown schematically in Figure (5) below.

![Figure 5: A dual MZI based FOAS system interrogated with a narrowband ASE source. The sensing MZI incorporates a piezoelectric transducer (PZT).](image)

For heterodyne detection purposes, a FS was incorporated in one of the arms of the first MZI. To allow us to apply a well-defined level of phase modulation, a piezoelectric transducer (made from piezoceramics such as Lead Zirconate Titanate or PZT) was incorporated in the sensing MZI. One PC in each of the two MZI interferometers was included in the system to allow us to set the SOP within the system for either PP or PF implementations. Calculations and experimental observations were made on the effect
of noise redistribution due to the FS [8]. The effect the SOP within this dual MZI system has on noise redistribution was investigated in detail, both theoretically and experimentally. It was found that the noise level for the PF case was 5 dB lower compared to the PP case [8]. We have shown that this system, with an optimised heterodyne frequency, source bandwidth and SOP, is capable of phase resolutions of 62 µrad/√Hz, for a practical length mismatch of around 1 cm [8].

**Grating-based Fibre-Optic Acoustic-Sensing system**

We continue our discussion on source noise redistribution in chapter (4) where a cascaded interferometer system, comprising a MZI and a Fabry-Perot (FP) cavity will be discussed. The FP cavity comprised two, inline FBG’s with similar Bragg wavelengths. The use of FBG based interferometer systems as opposed to MZI based FOAS systems, gives the system significant scope for multiplexing, since it allows us to use WDM to increase the number of sensors in the system. However, there are issues related to the use of FBG based sensing systems, which have to be addressed in order for the system to be proved viable. The first issue, which has to be addressed, is the multiple-reflection nature of the cavity created by the two, inline, FBG’s. The existence of multiple reflections can reduce the sensitivity of the sensing system. Optimum reflection coefficients for both FBG’s comprising the FP sensing cavity have been found using our theoretical model, giving the system the best phase resolution. This was verified with experiments.

**Issues related to the grating-based Fibre-Optic Acoustic-Sensing system**

Chapter (5) deals with other issues related to grating based interferometer systems. A second issue relates to the temperature sensitivity of FBG’s, i.e. their Bragg wavelength shifts with temperature (or strain). Therefore FBG based interferometers are sensitive to temperature fluctuations. Drift of the Bragg wavelength of the FBG’s in the FOAS system could have a significant impact on the system performance. To investigate the influence of the spectral drift of the gratings in the system several experiments were performed on a single-channel system. The influence of drift of the spectral wavelength of the source was investigated. In further experiments, spectral drift of the FBG’s in the sensor cavity was investigated. A third issue relates to the influence of phase response from the use of FBG technology. Experiments were performed to see whether any phase sensitive response exists.
A fourth issue relating to WDM technology has to do with crosstalk. To increase the sensor density in the system, the channel spacing is kept as small as possible. Intra-channel crosstalk imposes a lower limit on the channel spacing. Crosstalk level for different channel spacings were investigated in our system (for a 2-channel sensing system). The issue of crosstalk is directly linked to the issue of source noise, since a Dense WDM (DWDM) system uses narrow bandwidth channels and narrow bandwidth ASE source shows increased excess photon noise level. High channel isolation can be achieved by using channels with well-defined sharp edges to their spectrum, so no spectral overlap exists with adjacent channels. This was achieved in our case by using channel FBG’s that were apodised so as to greatly reduce the side lobes on the FBG’s [13].

Results on the operation of the grating-based FOAS system when interrogated with the narrowband ASE source incorporating the GS-SOA will be presented in chapter (5). Improvement of the system sensitivity will be discussed. This thesis closes with chapter (6) in which all the major conclusions from the complete thesis are summarised.
Chapter One

Overview of Fibre-Optic Acoustic-Sensing

Overview: This chapter describes the technology involved in the field of Fibre-Optic Acoustic-Sensing and defines what the past and current state of the art is. Several FOAS schemes will be highlighted and their operating procedures discussed.

1.1. Applications of Fibre-Optic Acoustic-Sensors

There has been very strong research and development interest in FOAS over the past 25 years or so. Sensing of acoustic pressure fields using optical sensing systems has numerous applications in both industry and military fields, and the development of Fibre-Optic Acoustic-Sensors (FOAS) coincided with the invention of the optical fibre. These days, FOAS are employed in fields such as oil exploration at sea, oil well monitoring down an oil producing well, structural monitoring, enemy vessel detection at sea, medical applications, transportation, and numerous other fields as well.

In the oil industry the use of FOAS or hydrophones in seismic surveying and reservoir monitoring has been growing rapidly in recent years. Hydrophones measure changes in the surrounding pressure field (which are the same as sound waves in the ocean or in air, i.e. variations in pressure). FOAS arrays have a number of applications, on land, in the ocean, and down oil wells. A major military application of acoustic sensors is in antisubmarine warfare, where seismic streamers (on a much smaller scale) are towed behind a submarine, for enemy vessel detection purposes (area surveillance). Seismic arrays can also be mounted on the hull of a submarine.

For commercial ocean-based acoustic systems, the two main applications are seismic streamers and seabed arrays, the latter are known as Ocean Bottom Cables (OBC's). Seismic surveying is a method for creating detailed sub-surface images of
geological structures and defining the characteristics of individual formations. These systems are either rapidly deployable or fixed systems. OBC’s are also used in the military for area surveillance.

1.1.1. Oil well monitoring using Fibre-Optic Acoustic-Sensors

In reservoir characterisation one measures the physical properties of a reservoir (temperature, pressure) that are relevant to hydrocarbon exploration and production [14]. It images the gas/oil or oil/water contacts in a producing formation throughout its productive life by repeating surveys over the field at regular intervals to determine movements in the fluid boundary in the reservoir. The objective of this type of study is to delineate the production characteristics of a reservoir for optimising the recovery of oil and gas reserves from existing fields. These surveys can increase the efficiency (the production plus line charge divided by the total time) of a producing well by as much as 30 % to 70 %.

The size of the borehole at the surface should be large enough that with a series of telescopic pipes the planned drilling depth can be achieved. It often starts out fairly large at the surface, sometimes in excess of 50 cm in diameter. The diameter quickly narrows to as small as 13 cm and is further reduced by the insertion of pipe or casing that prevents unconsolidated rock layers from collapsing and blocking the hole, and prevents the mud (i.e. drilling fluids) from being contaminated by fresh surface water. Mud is used to clean the borehole by removing the cuttings near the drill bit down the bottom of the oil well (but also to cool the drill bit). The mud is pumped down the oil well under a pressure that is higher than that of the pressure of the formation at which the drilling is being done. This over pressure prevents oil or gas from escaping to the surface and also prevents the borehole from collapsing. To extract gas and oil, the engineers reduce the pressure of the mud so that it reaches a value below the formation pressure, and production of oil and gas can start. For this they use production tubing and other hydraulic or electrical devices that control the flow of water, gas, and oil.

The sensors are packaged in a single armoured hose to protect them from corrosive fluids (carbon dioxide, hydrogen sulphide, and oil and gas fluids), but also from the high subsurface temperatures and the static pressure at depth. The sensors are protected so that they are sensitive to very small variations in pressure (0.05 microbars). The diameter of the sensor is small enough to slip it into some tight places
underground in the production plumbing, and can have a several kilometre long lead-in. These sensors detect the transient pressure of a passing seismic wave (i.e. changes in a surrounding pressure field) in the earth down the oil well. The acoustic/seismic signal is then sent back up to the detection part of the system, where it gives vital information on e.g. the location of oil and gas reserves.

1.1.2. Ocean Bottom Cables for seismic surveying

OBC's are seismic arrays laid on to the seabed for the same purpose of picking up acoustic energy reflected from the different rock formations. They are normally laid in a grid that is continuously deployed and recovered. Figure (1-1) below shows a surface vessel deploying OBC's. OBC surveys are slower to carry out, but usually give higher quality data. By placing seismic acquisition cables on the ocean floor, OBC captures 3D seismic data that produces sharper seismic images of the sub-surface. The OBC method allows the recording of seismic data in shallow or obstructed areas where access is difficult (e.g. shallow water, coral reefs, natural subsea formations and man-made objects such as pipelines, platforms and fishing vessels).

Figure 1-1: Schematic showing a surface ship deploying Ocean Bottom Cables for seismic surveying.
1.1.3. Streamers or towed arrays of Fibre-Optic Acoustic-Sensors

Streamers contain many thousands of hydrophones and are several kilometres in length, and are filled with kerosene to provide near natural buoyancy. Streamers are towed behind surface vessels during seismic surveys (part of the oil exploration process). This is drawn schematically in Figure (1-2 a) below.

![Diagram of ocean exploration](image)

(a) (b)

**Figure 1-2**: Schematic (a) and picture (b) of oil exploration at sea using streamers (hydrophone array).

Figure (1-2 b) shows a picture of an actual deployment of a large streamer at sea. Streamers are used as acoustic receivers in conjunction with impulsive sources to build up a picture of the subsurface geology. For the impulsive sources an air gun is used, which is a device that emits acoustic energy by releasing a burst of compressed air into the water, thereby generating an acoustic shock wave. Seismic sources are towed behind the vessel slightly beneath the surface of the water. Acoustic sensing systems need to have a huge dynamic range in order to distinguish the faint acoustic waves reflected of the different rock formations in the seabed, from the massive bang from the air gun behind the towing ship.
1.2. **Types of Fibre-Optic Acoustic-Sensors**

Generally, FOS’s can be divided into two main groups, the phase (interferometric) sensitive detection sensors and the intensity based sensor. With intensity (or amplitude) modulated sensors, the physical perturbation interacts with the fibre (or some transducer attached to the fibre) and directly modulates the intensity of the light in the fibre. This type of fibre sensor is generally simple in construction compared to a phase-modulated sensor. With phase modulated sensors the measure of pressure is attributed to variations in the relative phase of the light propagating in two arms of an interferometer. This work focuses on the latter type of sensor.

1.2.1. **Intensity modulated sensors**

Several type of intensity based sensor were invented to act as hydrophones. Though this thesis focuses on the interferometric type of sensors, a few intensity-based sensors will be presented next.

*Moving grating Fibre-Optic Acoustic-Sensor*

An intensity based Fibre-Optic pressure sensor using moving gratings was considered by Spillman *et al.* [15]. In this sensor a small air gap separates two fibre ends. A pair of gratings is placed in the small air gap, perpendicular to the two fibres. Relative motion between the gratings in the direction perpendicular to the line pattern will modulate the transmitted light. One of the gratings is mounted rigidly on the base plate of the housing while the other grating is attached to a flexible membrane or diaphragm. A schematic of this sensor is shown in Figure (1-3) below.
Light escaping from the fibre tip is collimated by a lens and then partially transmitted through the gratings. The transmitted light is then focused into the second fibre by another lens. Assuming the period of the refractive index modulation of both gratings is, say, 5 μm, the transmitted light intensity will vary periodically with grating movement. The response of this sensor is illustrated in the inset of Figure (1-3). The period of this change in intensity is double the period of the refractive index modulation, thus 10 μm. The sensor can be set to its most sensitive point (quadrature point) by setting the relative grating displacement to half the period of the refractive index modulation (modulo this period), i.e. 2.5 μm, 7.5 μm, 12.5 μm, etc. The sensitivity can be increased by decreasing this grating period, at the cost of dynamic range. A major disadvantage of this kind of sensor is that the light is not confined to the fibre, which may have a significant impact on the long-term stability.
Microbend Fibre-Optic Acoustic-Sensor

The microbend FOAS is shown schematically in Figure (1-4) below and was devised by Fields et al. [16]. This type of sensor utilises induced bending loss in an optical fibre. The transduction element comprises a deforming device such as a pair of toothed or separated plates that introduce small bends in the fibre. Light from a laser is launched into the core of the fibre and passed through the deforming device. Depending on the force exerted on the deforming device light is partly coupled into the cladding of the fibre. After the deforming device the fibre is passed through a mode stripper to remove the light from the cladding of the fibre, so only light propagating in the core of the fibre will reach the photodetector.

![Diagram of microbend FOAS](image)

**Figure 1-4:** Schematic showing the principle of the microbend FOAS. After Fields et al.

Care was taken to remove all possible cladding light before the deforming device by incorporating a mode stripper before the deforming device. The light intensity in the fibre core, reaching the end of the fibre, is monitored. Very small changes in the light intensity of a relative intense beam need to be detected (hence the name brightfield microbend sensor). A so-called darkfield microbend sensor employs the principle that the light intensity coupled into the cladding of the fibre at the mode stripper after the
deforming device is monitored. As with the moving grating fibre-optic sensor, one deformer plate is mounted on a rigid part of the sensor housing and the second plate is attached to a thin diaphragm.

_Evanescent field Fibre-Optic Acoustic-Sensor_

Sheem _et al._ [17] reported on an evanescent type of sensor. It basically consists of a pair of SMF or multi-mode fibres. When the cores of two fibres are nearly adjacent over some distance, light is coupled from one core to the other. In the transduction element itself the cladding is reduced in thickness, or entirely removed, so that the distance between the cores is small enough to permit evanescent field coupling between the two fibres over some small interaction length. Potting the interaction response in a fluid or flexible elastomer, with the same refractive index as the cladding, may enhance coupling. Substantial changes in the light coupled into the pickup fibre are a function of slight variations in the fibre spacing, the interaction length, and the refractive index, which are induced by the particular force field of interest.

Two more sensors, which I will just mention but not explain in detail, since this thesis focuses on interferometric sensors, are the polarisation FOAS, as reported on by Rashleigh [18] and the frustrated total internal reflection FOAS by Spillman _et al._ [19]. In the polarisation FOAS the acoustic perturbation modifies the polarisation characteristics of light in the fibre resulting in an amplitude modulation of the output signal. In the frustrated total internal reflection FOAS acoustic energy is measured by measuring the intensity of the light transmitted from one angle cleaved fibre end to a second one.

_1.2.2. Phase modulated sensors_

The basic transduction mechanism employed in many FOS now being developed is the phase modulation of light propagating through a section of SMF by the action of the energy field that is to be detected. Phase modulation cannot be directly detected due the fact that the light frequency is approximately $10^{14}$ Hz. Photodetectors are unable to respond to such high frequencies. However, the techniques of optical interferometry may be used to detect these phase shifts in lightwaves, by converting the phase modulation to amplitude modulation prior to detection. These techniques allow for extremely high sensitivities. There are basically three different interferometric
configurations currently being employed in FOS. These are the Michelson, the Mach-Zehnder and the Fabry-Perot configurations. There is one important aspect that these interferometric sensors have in common. In each one, the output beam from an optical source is split into two or more portions. After travelling along different paths, these separate beams are then recombined to be sent on to a photosensitive detector.

One of the first demonstrations of optical detection of sound was performed by Bucaro et al. [20] and Cole et al. [21] using large lasers and bulk optical components for beam splitters. Their system is shown schematically in Figure (1-5) below and comprises a bulk optical MZI illuminated with a laser.

![Diagram of bulk optic MZI based FOAS](image)

**Figure 1-5:** Schematic of a bulk optic MZI based FOAS. After Cole et al.

The laser beam is split into two beams, one of which (denoted the detection beam) traverses the interaction region (acoustic waves generated in a tank of water). The other beam is isolated from this region (hence denoted the reference beam). The two beams are then combined and are directed to a photodetector, where the phase information of the detection beam is analysed. Any variation in the relative path lengths between the sensor and the reference arm (referred to as the path-imbalance $L_{MZ}$) will show up as an interference pattern or phase change in the recombined light signal emerging from the
sensor. The reference arm is unaffected (constant phase), and the path length difference of the recombined light signal results in a phase difference that is proportional to the pressure change. The SNR of this sensor in this first experiment was low due to the large ambient acoustic noise in the optical paths between the optical components.

In order to increase the SNR, Cole et al. replaced the detection beam of their MZI set-up with a SMF. The laser light of the detection beam was focused into the core of the SMF via a microscope objective. Employing optical fibres reduced the stray acoustic noise considerably, though laser to fibre coupling noise still played a role. The sensitivity of a FOAS system also depends on the length of sensing fibre that interacts with the field to be measured (the interaction region in Figure (1-5)) and clearly this can be much longer with a fibre interferometer. The application of single-mode fibre-optic interferometers as physical sensors is now an established area of technology. One of the principal advantages of FOAS is their geometric versatility. Sensor elements can be fabricated which are light and flexible. By employing optical fibres as the paths of interferometers, the limitation of sensor length is immediately removed. Path lengths on the order of a kilometre are easily achieved.

The signal seen at the receiver of an interferometric sensor is generally given by [22]:

\[
I = \frac{I_0}{2} \left( 1 + V \cdot \cos(\phi_b) \right)
\]

Equation 1-1

This function is often referred to as the transfer function of an interferometer. Where \( V \) is the fringe visibility, \( I_0 \) is the input intensity to the interferometer and \( \phi_b \) represents the interferometric phase bias (the optical phase difference between the two arms of the interferometer). This is generally given by \( \phi_b = (2\pi\nu_b T) \) with \( T \) being the time difference experienced by the two interfering light beams on passing through the interferometer and \( \nu_b \) is the optical frequency. Ideally, the MZI is biased at its most sensitive point, i.e. at \( \phi_b = (2\pi\nu_b T_{MZ}) \equiv \frac{2\pi}{4} \). This, so called, quadrature biasing results in a modulation centred on the most linear portion of the sinusoidal modulation transfer function. The slope of the transfer function is the highest at the quadrature points (indicated with the symbol Q in Figure (1-6) below). The detected output intensity is
plotted as a function of the interferometer phase bias in Figure (1-6) below.

![Diagram of interferometer fringe pattern](image)

**Figure 1-6:** Illustration of an idealised two beam interferometer fringe pattern or transfer function, i.e. the output intensity as a function of the phase bias.

The quality or sharpness of the fringes, produced by an interferometer, as shown in Figure (1-6) above, can be described quantitatively using the visibility $V$. From the transfer function we can calculate the interferometer fringe visibility $V$ by using the general definition:

$$V = \frac{\langle I(t) \rangle_{\text{max}} - \langle I(t) \rangle_{\text{min}}}{\langle I(t) \rangle_{\text{max}} + \langle I(t) \rangle_{\text{min}}} \cdot \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

**Equation 1-2**

Here $\langle I(t) \rangle_{\text{max}}$ and $\langle I(t) \rangle_{\text{min}}$ are the maximum and minimum output intensity respectively, as indicated in Figure (1-6) above. The last part of Equation (1-2) represents the power imbalance of the interferometer ($I_1$ and $I_2$, are the intensities in the two arms of the
interferometer). The average intensity between the two arms is given by \( \frac{1}{2}(I_1 + I_2) \), and the mean intensity is given by \( \sqrt{I_1 I_2} \). Since \( \frac{1}{2}(I_1 + I_2) > \sqrt{I_1 I_2} \), we can never get maximum visibility when there is a power imbalance. In the rest of this thesis we will assume this power imbalance to be insignificant and will therefore be neglected.

When an acoustic wave is incident on the signal arm of the interferometer, this wave will modulate the optical phase of the light in this arm by \( \phi_s \). The interferometer transfer function now becomes:

\[
I = \frac{I_0}{2} \left( 1 + V \cdot \cos(\phi_s + \phi_i) \right)
\]

Equation 1-3

with \( \phi_s \) the phase modulation caused by the physical field to be measured (e.g. acoustic pressure field). This is illustrated graphically in Figure (1-7) below.

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**Figure 1-7:** Illustration of a periodic phase modulation on the interferometric transfer function due to a signal.
Applying a signal with frequency $f_s$ (in Hz) and a low modulation depth $\phi_s$ (in radians) to the interferometer biased at a quadrature point, causes a larger signal at the output of the system. The out-of-quadrature points are denoted by $M$. Note that when the interferometer is biased at the out-of-quadrature points, the interferometer response will be at double the signal frequency, i.e. at $2f_s$. Recovery of the signal $\phi_s$ can be done by various demodulation schemes. In all interferometric sensing systems, the physical field to be measured is recovered in essentially the same manner, i.e. by measuring the optical output intensity of the system and using for example Equation (1-1). To obtain maximum sensitivity, the system has to stabilised at one of its quadrature points. By employing some kind of stabilisation technique the bias point can be kept stable against environmental effects such as temperature changes that cause a slow change in the relative length difference of the two arms of the interferometer (random low-frequency phase drifts). This can be achieved by incorporating a PZT in one of the arms of the MZI. With the PZT it is possible to physically stretch the fibre in one arm of the interferometer, thereby making it possible to set the phase bias to a fixed point. This way it is also possible, for example, for homodyne detection to be used. Later we will show that a Frequency Shifter (FS) can also serve for this purpose in a technique known as heterodyne detection.

Since generally the length change of optical fibre is relatively small when exposed to acoustic perturbations, thereby inducing a relatively small phase change with acoustic pressure, a means is required to increase this sensitivity. Earlier work into this, by Lagakos et al., involved making the fibre more sensitive by coating it with plastics [28]. On the other hand one wants to minimise the pressure sensitivity of the reference arm of the interferometer. Hughes et al. reported on coating the reference arm with metals or other high bulk modulus materials to desensitise the fibre [29]. Another approach involves winding the fibre of the sensing arm on a hollow air-backed cylinder. The fibre on the reference arm is wound around a solid mandrel that is insensitive to pressure variations. As sound waves or pressure changes occur around the hydrophone, the hollow cylinder changes its diameter, just slightly. The dimensional change in the mandrel can be very small. This diameter change stretches the fibre that’s wrapped around the cylinder and increases the sensor path length, thereby inducing the phase information $\phi_s$ on to the system output. With the sensing fibre wound on a hollow air-backed cylinder, the device is sensitive to small changes in depth. Therefore, devices are usually backfilled with caster oil.
**Homodyne detection**

Stabilisation can be achieved by periodically modulating the path-imbalance of the interferometer. This is done by physically stretching the fibre in one of the arms of the interferometer using for example a piezoelectric transducer (PZT). The phase modulation of, for example, the reference arm, given by $\phi_{\text{ref}} = \phi, \sin(2\pi f_r t)$, causes a modulation of the phase of the output signal of the interferometer. The phase changes due to the signal are then added upon this modulation signal. The interferometer output intensity can now be expressed as:

$$I = \frac{I_0}{2} \left(1 + V \cdot \cos(\phi_0 + \phi, \sin(2\pi f_r t) + \phi_1)\right)$$

Equation 1-4

The second phase term is the extra phase change induced by control of the reference arm of the interferometer (via the PZT) with frequency $f_r$ and amplitude or modulation depth $\phi_1$.

**Heterodyne detection**

A more sensitive detection technique is the heterodyne detection method. This method is different from the homodyne technique in that that the frequencies of the light in the two arms of the interferometer are different. This can be achieved by introducing a FS in the reference arm. The phase of the reference arm now becomes $\phi_{\text{ref}} = (2\pi f_r t)$, and the interferometer output can be written as:

$$I = \frac{I_0}{2} \left(1 + V \cdot \cos(\phi_0 + 2\pi f_r t + \phi_1)\right)$$

Equation 1-5

Moreover, the heterodyne detection scheme produces a much more sensitive interferometer system, compared to the homodyne detection scheme mentioned in the previous section. This is because the modulation occurs in a linear fashion instead of a harmonic fashion used in homodyne detection. A practical ultrasonic hydrophone,
using a heterodyne detection scheme, was considered by Fisher et al. [23]. Christmas et al. reported a similar system suitable for strain sensing [24].

*Phase Generated Carrier (PGC) detection*

Unbalanced interferometers are sensitive to the frequency (or wavelength) of the input light. Therefore, random fluctuations of the frequency of the laser light are converted to intensity noise at the output of a strongly unbalanced interferometer. As was shown by Dandridge et al. this increases linearly with path-imbalance $L$ [4]. The dependence of the phase difference $\phi$ on wavelength shift $\Delta \lambda$ is given by [1]:

$$\Delta \phi = \frac{2\pi n L}{\lambda} \left( \frac{\Delta \lambda}{\lambda} \right) = \frac{2\pi L}{c} \Delta \nu$$

*Equation 1-6*

This property can be used as another kind of recovery technique in order to produce a useful signal. The PGC method relies on the principle that the optical frequency of the input light of the interferometer is periodically modulated [25].

$$\Delta \phi \cdot \sin(2\pi f_m t) = \frac{2\pi L}{c} \Delta \nu \cdot \sin(\omega_m t) = \frac{2\pi n L}{\lambda} \left( \frac{\Delta \lambda}{\lambda} \right) \cdot \sin(2\pi f_m t)$$

*Equation 1-7*

Here $f_m$ is the modulation frequency and $\omega_m$ is the angular modulation frequency. Whilst interferometric sensors offer extreme sensitivity, this range is generally limited, owing to the ambiguity arising when the change in signal path-length imbalance exceeds $2\pi$ radians (see Figure (1-7)). To overcome this problem, fringe-counting techniques can be used.

*Micelison based Fibre-Optic Acoustic-Sensor*

Several authors have reported the use of imbalanced Michelson interferometers in FOAS. A possible Michelson arrangement, after Jackson et al. [26], is shown schematically in Figure (1-8) below.
This is a reflective rather than a transmissive sensor. The reflective fibre end could be a metal coated fibre end, or a Faraday rotator mirror, which then eliminates fading of the fringe visibility caused by random fluctuations of the SOP within the sensor [27]. Using reflective rather than transmissive sensors has the disadvantage that Rayleigh scattering can limit the sensor performance [27].

*Mach-Zehnder based Fibre-Optic Acoustic-Sensor*

FOAS generally employ the MZI arrangement. Light from a source is split by a fibre coupler (C), half of the power of the beam being transmitted by a reference fibre and the other part being transmitted in a sensing fibre. The latter is exposed to the acoustic field while the reference fibre is shielded from the acoustic field. The two beams are recombined using another fibre coupler and allowed to interact coherently on the surface of a photodetector to produce observable interference at the output. This is shown schematically in Figure (1-9) below.
Interrogation of a dual MZI configured FOAS with a pulsed laser source was reported by Brooks et al. [6] and by Lim et al. [12]. Lim’s system comprised a dual MZI configuration, both with a similar path-imbalance. Elimination of phase induced intensity noise generated by random fluctuation of the laser frequency was done by properly gating the laser light. Their system is shown schematically in Figure (1-10) below.
Figure 1-10: Schematic of a dual Mach-Zehnder based FOAS system interrogated with a pulsed laser source and employing heterodyne detection. After Lim et al.

A pulse of light with optical frequency $v_0$ is separated into two pulses in an unbalanced MZI and passed to a second unbalanced MZI. The duration of the pulse $t_p$ is much shorter than the time of flight through the MZI ($T_{MZI}$). A FS employed in one of the arms of the first MZI shifts one pulse in optical frequency to $v_1$. Since the delay time of the second MZI is the same as the time separation between the two pulses entering this interferometer, two of the four pulses exciting from this interferometer will overlap with each other in the time domain (both with frequency $v_0$ and $v_1$) resulting in a single pulse (b). These two overlapping pulses are coherent with each other and will thus produce interference resulting in a single pulse (b) with a heterodyne differential frequency $(v_1 - v_0) = f_h$. This pulse conveys the phase information $\phi_h$ imprinted on the sensing MZI by the acoustic wave. This phase information is then decoded at the output of the system. This is similar to the system shown in Figure (1) in the Introduction of this thesis, except that the second (sensing) MZI is replaced with a FP sensing cavity.

In our system laser phase noise, as mentioned above, is overcome by employing
a low-coherence ASE source. That is, a source is used with a coherence length much shorter than the path-imbalance of the MZI \((L_{MZ})\). The coherence length of this source is very much less than that of a laser. The FOAS configured as a dual MZI is drawn schematically in Figure (1-11 a) below.

![Diagram of dual MZI with ASE source and FOAS configuration](image)

**Figure 1-11:** Schematic of an ASE interrogated dual Mach-Zehnder FOAS (a) and the four possible paths the source light can travel through this system (b).

Interference can only be observed when the optical path differences within the dual MZI are well balanced (i.e. within the coherence length of the source). There are basically four possible paths \((n = 0, 1, 2\) and 3) the light from the source can travel through this dual MZI system. These four paths are shown in Figure (1-11 b). Since the two MZI’s are length matched to each other (i.e. \(L_{MZ1} \approx L_{MZ2}\)), within the coherence length of the source, the light in the paths \(n = 1\) and \(n = 2\) will have travelled mutually the same distance through the system and will be coherent with each other. These two paths will produce interference at the receiver. Any perturbation of the optical phase in either of the two MZI’s will be detected at the output of the system.
**Fabry-Perot based Fibre-Optic Acoustic-Sensor**

The initial work in this area (performed by Henning et al. [30]) utilised partially reflective points in the fibre made by mechanical splices between fibre segments of the sensor array, to form an inline array of low-finesse FP cavities. In this system switching is done with a Bragg cell which also acts as a FS for heterodyning. In later work on FP interferometer based FOAS, the reflection points were formed by small air-gaps in the SMF held together by capillary tubes, as shown in Figure (1-12) below.

![Diagram of Fabry-Perot cavity](image)

**Figure 1-12:** Schematic of a Fabry-Perot type of sensor based on the principle of reflection of light in a small air-gap, between two fibre ends. Note picture is not to scale.

The reflectivities obtained via the mechanical splice were found to be unstable and lossy, thus limiting the usefulness of the technique. One of the most exciting developments in the field of fibre optics has been the introduction of the FBG. These are partial reflectors that are written in the fibre with a laser by the photorefractive effect. The advent of FBG’s, provided a practical means of producing reliable, low insertion loss, in-fibre partial reflectors, as was shown by Henderson et al. [31]. In this
case the FBG’s serve merely as reflectors which define the interferometric paths. In addition to merely acting as full or partial reflectors, the wavelength selective nature of FBG’s allows unique capabilities and configurations to be implemented.

1.3. Multiplexing Fibre-Optic Acoustic-Sensors

The very modest signal bandwidths involved with sensing applications, together with the fact that the SNR only decreases as the square root of the optical power\(^1\) (receiver power within the shot noise limit), point towards the concept of dividing the power among an array of sensors, or multiplexing. Seismic surveys at sea in which the data density is much higher than with conventional surveys in terms of the amount of data recorded from each reflection point and the lateral spacing between lines of sub-surface reflection points offer higher quality 'images'. Also demodulation electronics will become more cost effective on a per-channel basis. Several techniques can be employed to multiplex many sensors in a FOAS system. Three of the most important techniques, knowing Time Domain Multiplexing (TDM), Frequency Division Multiplexing (FDM), Coherence Multiplexing (CM) and Wavelength Division Multiplexing (WDM), will be discussed next. The majority of authors describing multiplexed systems have employed the MZI configuration. However, significant advantages are offered by employing FP systems as will be shown in the next part.

1.3.1. Time-Domain Multiplexing

TDM uses the fact that the signals returned from each sensor are separated in time, as reported by Brooks \textit{et al.} [6]. A time-gated measurement has to be used to de-multiplex the returned signals. This requires the use of a pulsed optical source, appropriate fibre lengths between the sensor elements and high-speed photo detection and switching electronics. A fibre laser is normally run CW at around 1550 nm, but the signal is then pulsed using either acousto-optic or electro-optic modulators to produce the pulses required for time multiplexing. The pulse length \(t_p\) is typically in the region of 100 ns – 1 \(\mu\)s.

For TDM systems, the lasers send out intermittent bursts of light; time-of-flight

\(^1\) When interrogated with laser sources.
is used to separate returns from different sensors. That is, a light packet from the laser travels sequentially to each sensor in the group, and the first signal to return from the group on the gather fibre will be from the 'nearest' sensor with the shortest travel path and so on. Each sensor or channel will have a unique time-slot within a sensor group.

1.3.2. Frequency Division Multiplexing

In this technique each sensor has a specific frequency at which the sensor signal operates, within the detection bandwidth. A heterodyne detection technique is used to recover each specific frequency multiplexed sensor signal. Heterodyne means that the optical frequencies in the interferometer arms are unequal, which is achieved using an Acousto-Optic Modulator (AOM) or Frequency Shifter (FS) producing the heterodyne carrier frequency. For FDM systems, the lasers are on all the time for all sensors; frequency and electronic demodulation separate the channels.

In one of the earliest approaches frequency ramped laser source (chirped frequency source) were used to scan each of the MZI sensors within an array. This approach is illustrated schematically in Figure (1-13) below.
Figure 1-13: Schematic showing an approach for Frequency Division Multiplexed Mach-Zehnder Interferometer sensors.

Unbalanced interferometers are sensitive to the frequency (or wavelength) of the input light (see Equation (1-6)). A beat pattern is generated at the interferometer output with a period that is a function of the path-imbalance of the interferometer, the frequency excursion of the chirp, and the chirp rate. Giving each MZI a different path-imbalance, thus a different beat frequency, distinction can be made between each sensor.

1.3.3. Coherence Multiplexing
This technique makes use of the fact that when an interferometer is interrogated with a low-coherence source (such as an ASE source), then interference can only be observed when the optical path differences within the interferometer system are well balanced (i.e. within the coherence length of the source). In Coherence Multiplexed (CM) systems, the optical light paths which contain the signal, are length matched to within the coherence length of the source illuminating them. Liyama et al. [3] reported on CM
unbalanced Michelson interferometers using an optical loop. A more general CM system is the dual MZI interrogated with a short coherence length source reported by Brooks et al. [2] and Wentworth [32]. A dual MZI system interrogated with a short coherence length source is shown schematically in Figure (1-14) below.

\[ L_{MZ1}, L_{MZ2} \cdots L_{MZn} >> l_c \]

![Schematic showing a set of Coherence Multiplexed MZI's.](image)

**Figure 1-14:** Schematic showing a set of Coherence Multiplexed MZI's.

With the employment of an ASE source there is an issue related to source noise. This will be discussed in the next chapter.

### 1.3.4. Wavelength Division Multiplexing

WDM increases bandwidth by using many wavelength channels. To vastly increase the number of sensors in a single line via WDM, the channel separation is kept to a minimum. This means that the bandwidth of the source also has to be small, which in turn has consequences for the noise of the source output (when interrogated with ASE sources). This will be explained in the next chapter. Most telecommunication systems employ 50 GHz (0.4 nm) or 100 GHz (0.8 nm) channel spacing in the 1540 – 1560 nm
region, according to the International Telecommunication Union (ITU) grid. WDM places requirements on wavelength stability and crosstalk. FBG technology is ideally suited for WDM since each grating has its own specific Bragg wavelength. However, there is an issue associated with WDM systems and that is crosstalk. It is essential to reduce crosstalk levels between adjacent channels in acoustic sensing systems. This will be discussed in the fifth chapter. Apodising gratings is a way of reducing intra-channel crosstalk, by fabricating gratings with sharper reflecting edges.

By combining WDM with TDM it is possible to 'double-up' on common TDM time-slots and use wavelength to separate the return signals from different sensor groups, thereby getting more channels for less cost and fewer fibres. A typical TDM/WDM architecture uses a pair of fibres to carry time/wavelength-multiplexed signals to and from the arrays, and would then branch off individual wavelengths into sub arrays of a number of channels. A sensor system can consist of a combination of multiplexing schemes to even further increase the multiplexing efficiency. In the system shown schematically in Figure (1-15) below, numerous sensors are multiplexed using FM, CM and WDM. Light from an n-channel narrowband ASE source is launched into n MZI's by a (1 x n) coupler, and is subsequently fed into the sensing arrays by a (n x n) coupler (all inputs of this coupler are coupled to all of its outputs). There are n number of unbalanced compensating MZI’s, each with their own specific path-imbalance \( L_{MZ} \).
Figure 1-15: Schematic of a typical FOAS architecture employing FM, CM, and WDM. The FP sensors are addressed by frequency (FM), path length (CM) and wavelength (WDM).

The output of each MZI is coupled to each sensing array that consists of FP-sensors whose path-lengths \( L_{FP} \) are matched to the path-imbalance of the appropriate length matched MZI \( L_{MZ} \). A different modulation frequency \( f_h \) is used for each MZI. Each of the \( n \) receivers receives \( n \) phase modulated signals centred on \( n \) different heterodyne frequencies \( (f_h) \), which can then be demultiplexed electronically. The frequency spectra as seen on the receivers are illustrated in Figure (1-16) below.
The multiplexing efficiency of this system can be even further increased by employing TDM by for example modulating the output of the narrowband ASE source.

Only the most important multiplexing techniques have been mentioned so far. Several other multiplexing techniques exist, for example Polarisation Interleave Multiplexing (PIM), where each channel is orthogonal polarised to the neighbouring channels to reduce crosstalk in telecommunication systems and Intensity and Wavelength Dual coding Multiplexing (IWDM) which has been investigated by Zhang et al. [33].
Chapter Two

Noise properties of Amplified Spontaneous Emission sources

Overview: The development and performance of a narrowband ASE source are described in this chapter. The noise properties of the source are described both theoretically and experimentally. The narrowband ASE source was found to behave as a thermal source, as expected. A dual-channel narrowband ASE source has been developed and used for sensor interrogation. It was also shown that the inherent intensity noise from the ASE source, which limits the SNR of the FOAS system, could be reduced by as much as 20 dB. This was done, by incorporating a GS-SOA into our narrowband ASE source. The intensity noise was suppressed by the fast gain dynamics of the GS-SOA.

2.1. Background study

During the past ten years, tremendous advances have been made in the development of rare-earth-doped fibre fabrication and associated semiconductor pump laser technology. The advances have been driven by the need for efficient, high-performance amplifiers within the telecommunications industry, but have resulted in the parallel development in a range of high-performance fibre laser systems with applications across an extended range of market sectors. Fibre-based ASE sources are one such example offering high optical powers (> 10 dBm) over broad spectral bandwidths (> 40 nm). Such sources have potential telecommunication applications, for example when spectrally sliced they can be used for a range of WDM transmission applications where crosstalk and nonlinearity are a key issue [34]. However, their use is common in the optical sensing
field where they are an established tool for interrogating fibre-based sensors such as Fibre-Optic Gyroscopes (FOG’s) [35] and FBG-based systems [1].

The coherence properties of spectrally filtered, narrowband ASE sources make them attractive for use with interferometric FOAS arrays. The performance of sensor interrogation systems based on CM method and conventional laser sources [2, 3] is limited by phase induced intensity noise [4, 5]. This limitation can be eliminated by the use of appropriate optical gating, and good sensitivity has been achieved [6]. However, imperfect switching, coupled to the inevitable switching speed limitations, can cause problems due to crosstalk between channels and can give rise to signal processing difficulties, alternative approaches are therefore of great interest.

In principle, the use of a broadband (> 20 nm bandwidth) source such as a Super Luminescent Diode (SLD) or a fibre ASE source can enhance the inherent sensitivity of a coherence multiplexed system and provides improved resistance to crosstalk effects. However, this is not a practical solution for an extended fibre-based system (e.g. an acoustic sensing system, where sensor length is required to obtained sufficient acoustic sensitivity), since in order to obtain the required level of visibility length matching within the interferometer needs to be obtained to within the coherence length of the optical source (several tens of μm for a 20 nm bandwidth).

A realistic practical level is generally accepted to be around 1 cm which implies the use of optical bandwidths of ~ 0.1 nm. Such a bandwidth is readily obtained using narrowband ASE sources. For example, using FBG technology it is possible to fabricate filters of almost arbitrary spectral shape (and width) with which to spectrally slice the output from conventional broadband sources, providing a powerful and flexible means of generating light of well-controlled (and defined) coherence properties. It should be apparent that in contrast to the use of broadband ASE sources, narrowband ASE also has the advantage of offering increased scope for sensor multiplexing through the use of WDM technology when used with appropriate narrowband FBG-based sensing interferometers. However, there is a major issue concerned with the use of narrowband ASE sources that needs to be addressed – that of source noise.
2.2. General background on the origin of source noise

In general, the operational performance of any optical system is influenced by various internal and external noise sources. Therefore, a complete analysis of the system’s noise performance involves consideration of all the relevant noise sources. In our particular acoustic sensor system the dominating noise source is the ASE source, and in this chapter we develop a detailed theoretical model of ASE noise in order to predict its dependence on critical source parameters relevant to our system’s operation, such as source bandwidth and optical power. We then go on to demonstrate experimentally that our particular ASE source behaves as expected. With our basic source model validated we are then able to use it with good faith to predict the noise properties of our sensor system.

2.2.1. Statistical properties of thermal light

In general, the complex emission field of optical sources is given by \( E(t) = E_0 e^{2\pi i \nu_0 t + \phi(t)} \). Here \( i \) indicates the complex number \((\sqrt{-1} = -1)\), \( \nu_0 \) is the centre frequency of the emission field, and \( E_0 = |E(t)| \) represents the amplitude of the complex emission field \( E(t) \). Both the phase \( \phi(t) \) and the amplitude \( E_0 \) of \( E(t) \) vary with time \( t \) in a random fashion, and are therefore, statistical in nature [9]. These random quantities can be fully described statistically. When light is produced by thermal sources, fluctuations in amplitude \( E_0 \) arise mainly because the electric field wave \( E(t) \) consists of a large number of Fourier components which are independent of each other, such that their superposition gives rise to a fluctuating field \( E(t) \) which can only be described in statistical terms. For thermal light, the emission field \( E(t) \) is described by a Gaussian random process [9].

In contrast, in a laser (Light Amplification by Stimulated Emission of Radiation) source the various field components may be coupled to some extent. Since light from laser sources is emitted mainly through stimulated emission, the fluctuations in amplitude (or intensity fluctuations) can often be ignored \(^2\). Fluctuations in the output field \( E(t) \) of a laser source is usually dominated by fluctuations in phase [9]. We will not discuss laser noise any further, since this work focuses on thermal sources and not on laser sources.

\(^2\) This is not strictly true, since in laser light the effect of spontaneous emission is never entirely absent.
2.2.2. Correlation functions

The statistical properties of a field are best characterised by using correlation functions. Correlation functions are certain average quantities whose values are determined by the amount of correlation of a time-varying wave at a number of time instants. In general, the \((n+m)th\)-order correlation function of a wave, say \(x(t)\), is defined by [36]:

\[
\Gamma^{(n+m)}_x(t_1, \ldots, t_n, t_{n+1}, \ldots, t_{n+m}) = \left\langle x(t_1) \cdots x(t_n) x^*(t_{n+1}) \cdots x^*(t_{n+m}) \right\rangle_T
\]

Equation 2-1

Where \(t_1, t_2, \ldots, t_{n+m}\) are \((n+m)\) arbitrary time instants, and the angled brackets \(\langle \cdots \rangle_T\) signify the time averaging operation over the detection time \(T\) (where \(T\) is much longer than one cycle of the optical frequency, \(1/\nu_0\)), i.e.

\[
\left\langle x(t_1) x^*(t_2) \right\rangle_T = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t_1) x^*(t_2) dt .
\]

Statistically it means that many measurements of the product \(x(t_1) x^*(t_2)\) are taken over the detection time, and that only the average of these measurements is recorded. In Equation (2-1), the superscript \((n+m)\) refers to the order of the correlation function, while the subscript \(x\) refers to the wave being examined, which might be either the complex output field \(E(t)\), or the instantaneous output intensity \(I_0\). The star \(^*\) indicates the complex conjugate of the complex wave \(E(t)\). Second-order correlation functions \((n+m=2, \text{with } n=m=1)\), which are also known as self-correlation functions, are of special importance in the analysis of many optical systems. From now on, we will omit the superscript 2, and the subscripts \(E\) and \(T\) from the frequently referred to second-order correlation functions of the output electric field \(E(t)\), i.e. instead of \(\Gamma^{(2)}_E(t_1, t_2) = \left\langle E(t_1) E^*(t_2) \right\rangle_T\), we simply write:

\[
\Gamma(t_1, t_2) = \left\langle E(t_1) E^*(t_2) \right\rangle
\]

Equation 2-2

For a random process \(\Gamma\) depends only on the time separation \(\tau = (t_2 - t_1)\) and we can set the time origin \(t_1 = 0\):
\[ \Gamma(\tau) = \langle E(0)E^*(\tau) \rangle \]

Equation 2-3

The second-order correlation function has the dimensions of intensity \([37]\). It is convenient to normalise the correlation function. This function is then referred to as the complex degree of coherence, \(\gamma(\tau)\), and is defined as:

\[ \gamma(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)} = \frac{\Gamma(\tau)}{I_0} \]

Equation 2-4

Coherence can be described as a manifestation of the correlation, which may exist between the fluctuations at two (or more) points in time. The coherence of a source has to do with the ability of two relatively delayed light beams to interfere and form fringes. The amplitude \(E_0\) and phases \(\phi\) of \(E(0)\) and \(E(\tau)\) will somehow fluctuate in time. If these fluctuations are completely independent, then \(\Gamma(\tau) = \langle E(0)E^*(\tau) \rangle\) will go to zero, since \(E(0)\) and \(E(\tau)\) can be either positive or negative with equal likelihood, and their product averages to zero. In that case no correlation exists, and \(\Gamma(\tau) = |\gamma(\tau)| = 0\), i.e. there is no coherence. On the other hand, if the field at the two time-instances \(t_1\) and \(t_2\) were completely correlated to each other, then their relative phase would remain unaltered, and \(\Gamma(\tau) = |\gamma(\tau)| = 1\), i.e. there is maximum coherence. If the two were slightly correlated then we would have \(0 < |\gamma(\tau)| < 1\), i.e. there is partial coherence. The coherence time, \(\tau_c\), of a complex electric field, \(E(t)\), is defined as:

\[ \tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau \]

Equation 2-5

In other words, the coherence time is directly related to the complex degree of coherence (equal to the area under the modulus squared of the complex degree of
coherence). The complex degree of coherence can be measured with a correlator. A correlator is a device that carries out the required time shift $\tau$, determines the products and integrates over the detection time interval. An interferometer is a kind of correlator.

2.2.3. Noise Spectral Density

In almost all, optical systems, the physical quantity ultimately measured, and from which the useful information is derived, is the current of a photo-receiver. Optical source noise measurements are also most usually and conveniently derived from current measurements. According to the Wiener-Khintchine theorem, the noise spectrum (or NSD, which describes the spectral energy distribution of the light) can be obtained from the Fourier transform of the covariance function of the current of a photo-receiver [38]. The autocovariance function, $\text{Cov}_i(\tau)$, of the receiver current is obtained by subtracting the second-order average receiver current from the corresponding second-order correlation function. The covariance function is defined as [37, 38]:

$$\text{Cov}_i(\tau) = \langle i(t)i(t+\tau) \rangle - \langle i(t) \rangle^2$$

Equation 2-6

where $i(t)$ is the receiver current at time $t$, and $\tau$ is the time delay. The electrical noise power spectrum as a function of frequency, can be written as:

$$N(f) = R \int_{-\infty}^{\infty} \text{Cov}_i(\tau) \cdot e^{2\pi if\tau} d\tau$$

Equation 2-7

where $R$ is the output load resistance of the receiver. The current $i(t)$ in the above equations can be substituted by the optical intensity at the receiver $I(t)$ when we consider only the noise component due to the optical source itself, and can thus be written as:
\[ \text{Cov}_r(\tau) = (g \alpha)^2 \text{Cov}_r(\tau) \\
= (g \alpha)^2 \left( \langle f(t) f(t+\tau) \rangle - \langle f(t) \rangle^2 \right) \]

Equation 2-8

where, \( g \) is the transimpedance gain (in V/A) and \( \alpha \) is the responsivity (in A/W) of the receiver. The product \( g \alpha \) represents a conversion factor (in V/W) from the optical intensity incident to the receiver to electrical voltage at the output of the receiver. This factor is commonly called the scale factor of the receiver. For a polarised thermal source \( \text{Cov}_r(\tau) \) can also be expressed as:

\[ \text{Cov}_r(\tau) = \| \Gamma(\tau) \|^2 = \left| \langle E^*(t) E(t+\tau) \rangle \right|^2 \]

Equation 2-9

If the optical spectrum of the source is:

\[ I(\nu) = I_0 \psi(\nu - \nu_0) \]

Equation 2-10

where \( \psi(x) \) is the normalised envelop function of the optical spectrum with a centre frequency of zero, \( \nu \) is the optical frequency and \( \nu_0 \) is the centre frequency of the source, \( \Gamma(\tau) \) can be written as:

\[ \Gamma(\tau) = I_0 e^{-2\pi i \nu_0 \tau} \int_{-\infty}^{\infty} \psi(x) \cdot e^{-2\pi i \nu x} \, dx \]

Equation 2-11

and by the Wiener-Khintchine theorem, the electrical NSD, \( N(f) \), can be written as:

\[ N(f) = (g \alpha)^2 R_0^2 \int_{-\infty}^{\infty} \psi(x - \frac{f}{2}) \psi(x + \frac{f}{2}) \, dx \]

Equation 2-12
From this expression we can see that the noise power at frequency \( f \) is the summation of all pairs of spectral components whose frequency difference is \( f \).

2.3. The autocovariance function of the output intensity of an Amplified Spontaneous Emission source

According to the Equations (2-7) and (2-9), we first have to calculate the correlation function of the output electric field of the ASE source in order to obtain the source NSD. We will assume that the optical spectrum of a thermal source is given by a Gaussian lineshape, i.e. the normalised envelope function of the optical spectrum is given by:

\[
\Psi(\nu - \nu_0) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{\frac{(\nu - \nu_0)^2}{2\sigma^2}}
\]

Equation 2-13

Where: \( \sigma \) = bandwidth dependent factor, defined as: \( 2\sigma^2 = \frac{(\Delta \nu_0)^2}{4 \ln 2} \).

\( \Delta \nu_0 \) = FWHM bandwidth of the source in Hz

The correlation function of its output electric field is given by Equation (2-11):

\[
\Gamma(\tau) = I_0 \cdot e^{-2\pi \nu_0 \tau} \int_{-\infty}^{\infty} \Psi(\nu - \nu_0) \cdot e^{-2\pi i(\nu - \nu_0)\tau} d(\nu - \nu_0)
\]

Equation 2-14

Thus:

\[
\Gamma(\tau) = I_0 \cdot e^{-2\pi \nu_0 \tau} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma^2}} e^{\frac{(\nu - \nu_0)^2}{2\sigma^2}} \cdot e^{-2\pi i(\nu - \nu_0)\tau} d(\nu - \nu_0)
\]

Equation 2-15
This can be rewritten as:

\[
\Gamma(\tau) = \frac{I_0}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{\frac{(\nu - \nu_0)^2}{2\sigma^2}} e^{-2\pi i \nu \tau} d(\nu - \nu_0)
\]

Equation 2-16

And the exponent can now be written as follows:

\[
\Gamma(\tau) = \frac{I_0}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{x^2 - 2xy - 2\pi i \nu \tau} d(\nu - \nu_0)
\]

Equation 2-17

Where: \( x^2 = \frac{(\nu - \nu_0)^2}{2\sigma^2} \)

\[ 2xy = 2\pi i \nu \tau (\nu - \nu_0) \]

Let \( \beta = (x - y) \) then, after some algebraic manipulation we obtain the correlation function. For a thermal source the second-order correlation function of its output electric field is now given by:

\[
\Gamma(\tau) = I_0 e^{-2\pi^2 \sigma^2 \tau^2} e^{-2\pi i \nu_0 \tau}
\]

Equation 2-18

The covariance function of the detected output intensity is given by Equation (2-9):

\[
\text{Cov}_I(\tau) = |\Gamma(\tau)|^2
\]

Hence for the intensity covariance function we find:
\[ \text{Cov} \left( \tau \right) = I_0^2 e^{-4\pi^2 \sigma^2 \tau^2} \]

Equation 2-19

As seen from Equation (2-19), the intensity covariance function is a real function. This property is expected since light intensity is a real quantity, and so is its autocovariance function. Now that we have found the expression for the covariance function of the output intensity of the ASE source, we are able to calculate the NSD of the detected output intensity of the ASE source.

2.4. The Noise Spectral Density of the detected output intensity of an Amplified Spontaneous Emission source

The (electrical) NSD at frequency \( f \) is given by Equation (2-7):

\[ N(f) = (g \alpha)^2 R \int_{-\infty}^{\infty} \text{Cov} \left( \tau \right) \cdot e^{2\sigma f \tau} d\tau \]

Now we can calculate the NSD given by this expression after substitution of the covariance function given by Equation (2-19):

\[ N_{\text{ASE}}(f) = (g \alpha)^2 R I_0^2 e^{-4\pi^2 \sigma^2 \tau^2} \cdot e^{2\sigma f \tau} d\tau \]

Equation 2-20

When we replace \( 2\sigma^2 = \frac{(\Delta V_0)^2}{4 \ln 2} \), we find:

\[ N_{\text{ASE}}(f) = (g \alpha)^2 R I_0^2 \frac{2 \ln 2}{\pi} \frac{1}{\Delta V_0} \cdot e^{-\frac{2 \ln 2}{(\Delta V_0)^2} f^2} \]

Equation 2-21
Note that the noise spectrum of a Gaussian spectrum also has a Gaussian shape with a FWHM bandwidth of $\sqrt{2}\Delta v_0$, caused by the beating between the various frequency components making up the source bandwidth [7]. From Equation (2-21) above, there are two points that highlight the unusual characteristics of thermal light. Firstly, it is clear that the NSD of the thermal source, $N_{\text{ASE}}(f)$, scales with the square of the optical intensity $I_0$. Secondly, and more significantly from the perspective of this study, is that the NSD scales with the inverse source bandwidth $\Delta v_0$. The compromise we have to make between multiplexing capability and length mismatch is thus optical noise $N(f)$. To verify whether our narrowband ASE source behaves as a thermal source as expressed with Equation (2-21), experiments on source NSD were performed which will be presented in section (2.5) of this chapter.

Example

In order to gain an appreciation of the magnitude of the electrical noise powers relevant to our experiments, we will calculate the NSD for the narrowband ASE source (FWHM bandwidth used in our experiments $\Delta v_0 = 6.2$ GHz). The NSD will be calculated at a frequency $f = 110$ MHz and optical power $P_0 = -23$ dBm ($\approx 5.0$ µW) by using the following constants.

\[
g = 40 \cdot 10^3 \text{ V/A} \\
\alpha = 0.95 \text{ A/W} \\
R = 50 \Omega
\]

These parameters relate to the high gain transimpedance receiver used in our experiments (New Focus, InGaAs PIN diode, model 1811).
\[ N_{\text{ASE}}(f) = (g\alpha)^2 R I_0^2 \sqrt{\frac{2 \ln 2}{\pi}} \frac{1}{\Delta \nu_0} e^{-\frac{2 \ln 2}{(\Delta \nu_0)^2} f^2} \]

\[ \approx 194 \frac{pW}{Hz} \]

\[ \approx -97.1 \frac{dBm}{Hz} \]

If the detection bandwidth \( B \) is much narrower than the optical bandwidth \( \Delta \nu_0 \), the NSD in the receiver bandwidth \( (B = 125 \text{ MHz}) \) can be approximated as [7]:

\[ N_B = \int_{f-\frac{B}{2}}^{f+\frac{B}{2}} N_{\text{ASE}}(f) df \approx (g\alpha)^2 R I_0^2 \sqrt{\frac{8 \ln 2}{\pi}} \frac{B}{\Delta \nu_0} \]

\[ \approx \left(40 \cdot 10^3 \cdot 0.95\right)^2 \cdot 50 \cdot \left(5.0 \cdot 10^{-6}\right)^2 \sqrt{\frac{8 \ln 2}{\pi}} \frac{125 \cdot 10^6}{6.2 \cdot 10^9} \]

\[ \approx 50 \text{ mW} \]

Equation 2-22

That is, the noise power is linearly proportional to the detection bandwidth \( B \). This result is obtained by expanding the exponent in Equation (2-21) as a Taylor series, i.e. set \( e^{-x} = \left[1 - x + \frac{x^2}{2} - \cdots \right] \).

2.5. Noise measurements on the narrowband Amplified Spontaneous Emission source

2.5.1. Description of the narrowband Amplified Spontaneous Emission source

In this section we describe the construction of our narrowband ASE sources and describe our initial experiments to confirm that the source behaves as a true thermal source. The narrowband ASE source used in our initial experiments is drawn
schematically in Figure (2-1) below.

![Diagram of ASE source](image)

**Figure 2-1:** Schematic of the narrowband ASE source employed in this work for FOAS interrogation. The pick-out filter grating (denoted by FBG$_{p1}$) was strained to suitably shift its Bragg wavelength $\lambda_1$.

The narrowband ASE source comprises a broadband ASE source, which is basically an Erbium Doped Fibre Amplifier (EDFA) without input, a Tuneable Filter (TF) and an EDFA to increase the power spectral density around the centre wavelength of the FBG ($\lambda_1$) used to define the shape of the ASE spectrum (therefore referred to as a pick-out filter). The ASE source emits broadband light (typically 40 nm bandwidth). The light emitted by the ASE source comprises of photons that are naturally randomly polarised. The broadband spectrum is then filtered by a TF with a 160 GHz (1.3 nm) bandwidth around the centre wavelength of the pick-out filter ($\lambda_1 = 1550$ nm). The optical spectrum after the TF has an extinction ratio between peak and background transmission of $\approx 26$ dB and an insertion loss at peak centre of $\approx 3$ dB. The average power after the TF is $\approx -23$ dBm. The resulting spectrum after EDFA$_1$ is shown in Figure (2-2) below.
Figure 2-2: Measured optical spectrum of the ASE source after EDFA1 in Figure (2-1) on page 50. The FWHM bandwidth of this spectrum is 160 GHz (bandwidth of the TF) and the extinction ratio between the peak and ASE background is ~ 26 dB. The resolution bandwidth of the optical spectrum analyser was 0.05 nm.

The bandwidth remains 160 GHz and the extinction ratio 26 dB. The amplified filtered ASE is then passed on to the pick-out filter used to provide the final spectral shaping. We had a number of different FBG pick-out filters with different bandwidths and lineshapes that we could use within the source to define different output spectra, and with which we could validate the theory of the previous sections of this chapter. Details of these pick-out filters are shown in Table (2-1) below.
Table 2-1: Pick-out filter specifications, listing the bandwidth (FWHM) and centre wavelength of the different FBG’s used as pick-out filters in the experimental narrowband ASE source.

<table>
<thead>
<tr>
<th>Grating</th>
<th>Centre wavelength [nm]</th>
<th>FWHM [pm]</th>
<th>FWHM [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBG1</td>
<td>1559.47</td>
<td>18</td>
<td>2.2</td>
</tr>
<tr>
<td>FBG2</td>
<td>1545.8</td>
<td>45</td>
<td>5.6</td>
</tr>
<tr>
<td>FBG3</td>
<td>1529.5</td>
<td>60</td>
<td>7.7</td>
</tr>
<tr>
<td>FBG4</td>
<td>1529.3</td>
<td>81</td>
<td>10.4</td>
</tr>
<tr>
<td>FBG5</td>
<td>1529.3</td>
<td>115</td>
<td>14.7</td>
</tr>
<tr>
<td>FBGₚ₁</td>
<td>1549.3</td>
<td>50</td>
<td>6.2</td>
</tr>
<tr>
<td>FBGₚ₂</td>
<td>1549.5</td>
<td>50</td>
<td>6.2</td>
</tr>
</tbody>
</table>

For the majority of our later experiments we used pick-out filter FBGₚ₁ which had a bandwidth of around 6 GHz and in these initial discussions we describe the source characteristics with this grating in place. This pick-out filter finally determines the source bandwidth (FWHM ≈ 6.2 GHz) and spectral profile. The pick-out filter was spliced on to port 2 of a 3-port circulator (make FOCI, 70 dB extinction port 1 to port 3), using a conventional fusion splicer. The other end of the pick-out filter was terminated (angle cleaved and soaked in index matching gel) to eliminate any optical feedback from this end. The peak reflectivity of the pick-out filter was chosen to be as high as possible (≈ 90 %), to reflect as much optical power as possible. The spectrum of the FBG pick-out filter used in this work to slice the broadband spectrum of the ASE source, is shown in Figure (2-3 a) below.
Figure 2-3: (a) Measured reflection spectrum of FBG_{p1} used as a pick-out filter in the narrowband ASE source shown in Figure (2-1). The spectral shape of this FBG, at its top, closely resembles a Gaussian function. This filter was written using an apodising technique, to suppress the sidelobes. The FWHM bandwidth of the grating spectrum is 6.2 GHz. Notice the extinction to the small sidelobes is around 35 dB and this drops around 40 dB further away from the centre reflection peak. This spectrum was taken by scanning a narrowline laser source across the grating. (b) Illustration of the refractive index profile along the length of the apodised FBG (Blackman profile). Position ‘0’ indicates the centre of the FBG, and the positions −1 and +1 indicate both ends of the FBG.

FBG_{p1} was written in a 5 cm long piece of photosensitive fibre. The spectral shape of the FBG is at its top, to a close approximation, a Gaussian shape. The FBG’s used in our experiments have been designed to give very low side bands to avoid inter channel crosstalk. The side lobes appear in the spectrum of an FBG due to end effects. These sidelobes were suppressed during manufacturing using a writing technique known as apodisation (from Greek ‘to take away’). Apodising reduces the end effects by reducing the amplitude of the refractive index modulation towards both ends of the FBG (i.e. impedance matching of the FBG to the fibre). Maximum amplitude of the index modulation is in the centre of the FBG, and minimum modulation at both ends. This is illustrated in Figure (2-3 b) above. Shown in this figure is the envelope of the variation of the refractive index, n, of the fibre core along the length of the apodised FBG (i.e. the refractive index modulation) in arbitrary units. The apodisation profile shown is a
Blackman profile (a profile which is close to a Gaussian) and was used for the FBG’s used in this work. The centre of the FBG is indicated with ‘Position along FBG length’ = 0, and either end of the FBG with -1 and +1.

In Figure (2-3 a) above, the extinction to the small sidelobes shown on this FBG pick-out filter can be seen to be 35 dB, dropping 40 dB further away from the main reflection peak. This particular FBG spectrum, shown in Figure (2-3 a) above, had a FWHM bandwidth of 6.2 GHz. Its centre wavelength could be tuned by physically straining the pick-out filter, to ensure that the reflection spectra of the pick-out filters and those of the FBG’s defining the FP cavities of the sensors (see Figure (2) in the Introduction) spectrally overlap. The relationship between the normalised Bragg wavelength shift $\lambda_B$, to applied strain $\varepsilon$, for a constant temperature $T$, is given by [1]:

$$\frac{1}{\lambda_B} \left| \frac{d\lambda_B}{d\varepsilon} \right|_T \approx 0.78 \cdot 10^{-6} \text{ } \mu \varepsilon^{-1}$$

Equation 2-23

Thus, when there is no temperature change of the FBG, its reflection spectrum will shift by about 1.2 pm to a higher wavelength for every micro-strain applied at a wavelength of 1550 nm. This is a linear and fully reversible effect, i.e. if the applied strain is released, the Bragg wavelength will shift to lower wavelengths by the same amount. The pick-out filters were strained by gluing them on to a small translation stage. With the pick-out filter mounted on this stage it was possible to suitably change the centre wavelength of the source output spectrum by simply straining the FBG (tune to higher wavelength) or to release the strain (tune to lower wavelength). In other experiments we filtered the ASE source spectrum with pick-out filter FBG1. The reflection spectrum of FBG1 is shown in Figure (2-4) below.
Figure 2-4: The normalised reflectivity profile of pick-out filter FBG1.

The source spectrum after being filtered by the pick-out filter is sent through a final amplifier. The input power to the final amplifier is $\approx -8$ dBm and the output power after EDFA$_2$ is $\approx +11.5$ dBm. Finally, the unpolarised light is polarised by a polariser, with a resulting net insertion loss of $\approx 3$ dB. To conveniently set the SOP of the output light a 3-paddle Polarisation Controller (PC) was incorporated, which had an insertion loss of around 0.7 dB. From Figure (2-5) below it is seen that the final spectrum has an extinction ratio of 42 dB, and 37 dB to the small lobe on the long wavelength part of the main reflection peak. This small lobe is a feature of the reflection spectrum of the pick-out filter FBG$_{p1}$ itself (see Figure (2-3 a)).
Figure 2-5: Measured optical output spectrum of the narrowband ASE source shown in Figure (2-1) on page 50. Resolution bandwidth of the optical spectrum analyser was 0.02 nm. Note the two small side lobes on either side of the main peak, which can be seen in the reflection spectrum of the pick-out filter itself (Figure (2-3 a)). The FWHM bandwidth of this spectrum is 6.2 GHz (bandwidth of the FBG) and the dynamic range is 37 to 42 dB.

The importance of the source bandwidth to the system performance was already mentioned in the first section. The source bandwidth used in our experiments was $\Delta \nu_0 = 6.2$ GHz (i.e. $\Delta \lambda_0 = 0.05$ nm), a bandwidth chosen for maximum system performance for a practical sensor length mismatch of around 1 cm. This bandwidth was obtained from our calculations and will be discussed in detail in the next two chapters of this thesis. The coherence length of the source output light is $l_c = \left(\frac{c}{n}\right) \sqrt{\Delta \nu_0} \approx 3.3 \ cm$, where $c$ denotes the speed of light in vacuum and $n$ is the refractive index of the fibre core (typically $n = 1.46$). A maximum source output power of about $P_0 = +8 \ dBm$ was obtained.
2.5.2. Principle and set-up for noise measurements

Figure (2-6) below shows a schematic of the set-up used to measure the source NSD and to confirm that it obeyed the expected performance of a thermal source. The output of the source is directed via a variable attenuator (not shown in the figure) to a DC coupled receiver, the output of which is fed directly into an RF spectrum analyser.

![Schematic diagram of noise measurement set-up](image)

**Figure 2-6:** Schematic of the experimental set-up for the measurement of the optical source noise (NSD). The FWHM bandwidth of FBG$_{p1}$ (unchirped) was 6.2 GHz. The receiver is a 125 MHz low-noise photo-receiver. Noise Spectral Density (NSD) was measured with an RF spectrum analyser at a frequency of 110 MHz and a 1 Hz bandwidth.

NSD measurements were taken with a DC coupled 125 MHz low-noise photo-receiver (make New Focus, InGaAs PIN diode, model 1811), with a transimpedance gain $g = 40 \cdot 10^3$ V/A, a responsivity $\alpha = 0.95$ A/W, and a saturation power of $-12.7$ dBm. This receiver had an experimental noise floor of $-119.5$ dBm·Hz$^{-1}$. NSD was measured versus the received optical power (controlled by the variable attenuator). The variable attenuator was a JDS Fitel model no. VA 5505-506. Each EDFA was operated at its maximum pump power, far above its threshold value, to ensure stable operation. Source NSD was measured at the heterodyne frequency of 110 MHz on an RF spectrum analyser in a 1 Hz resolution bandwidth. The RF spectrum analyser (make Marconi 100 Hz – 400 MHz, model no. 2382/2380), had an experimental electronic noise floor
at 110 MHz of -157 dBm·Hz⁻¹, and had a special function with which it was possible to select a 1 Hz resolution bandwidth. Selecting this function on the instrument meant that the noise power read from the instrument was automatically corrected for filter shape to a 1 Hz bandwidth [39]. An averaging function was used on the spectrum analyser for the measurement of the NSD.

Relative Intensity Noise (RIN) measurements, which are NSD measurements that are normalised to the average noise power, were also taken. The \( RIN \) at frequency \( f \) in dB·Hz⁻¹ is generally given by:

\[
RIN(f) = \left( \frac{\left\langle \Delta \hat{i}(f)^2 \right\rangle_B}{i_{DC}^2 B} \right)
\]

Equation 2-24

Where: \( \left\langle \Delta \hat{i}(f)^2 \right\rangle_B \) = mean square current (or intensity) fluctuation spectral density at frequency \( f \) over bandwidth \( B \)

\( i_{DC} \) = average receiver output current

\( B \) = measurement bandwidth

A \( RIN \) of \( 10^{-6} \) Hz⁻¹ (−60 dB·Hz⁻¹) corresponds to random variations of the intensity of one part in a million, measured over a time interval of 1 second. To show explicitly how to convert the measured NSD to \( RIN \) I give an example calculation.

**Example**

For the narrowband ASE source with a 6.2 GHz bandwidth we measure a NSD of \( N(f) = -84.4 \) dBm·Hz⁻¹ at a (heterodyne) frequency of about \( f = 110 \) MHz and at a received optical power of \( P = -14.0 \) dBm. We measure a DC voltage of 1.2 Volts at the output of our 125 MHz receiver on an oscilloscope with a 50 Ω impedance. We first have to convert measured noise power in logarithmic unit, dBm, to the linear unit, Watt.
\[ N(f) = 10^{-84.4/10} \approx 3.63 \cdot 10^{-9} \frac{mW}{Hz} \]
\[ \equiv 3.63 \frac{pW}{Hz} \]

**Equation 2-25**

With Ohm’s law we can calculate the mean square current fluctuation at the receiver output from the NSD, \( N \), and the impedance \( R \):

\[
N(f) = \left( \frac{\Delta i(f)}{\bar{R}} \right)^2 \cdot R \quad \Rightarrow \quad \left( \frac{\Delta i(f)}{\bar{R}} \right)^2 = \frac{N(f)}{R}
\]
\[= \sqrt{\frac{3.63 \cdot 10^{-12}}{50}}
\]
\[\approx 269 \, nA\]

**Equation 2-26**

With Ohm’s law we also know the average DC output current of the receiver from its output voltage \( U \) and the impedance \( R \):

\[
U_{DC} = i_{DC} \cdot R \quad \Rightarrow \quad i_{DC} = \frac{U_{DC}}{R} = \frac{1.2}{50} = 24 \, mA
\]

The narrowband ASE source \( RIN \) at frequency \( f \) can now be readily obtained with Equation (2-24). Measurement of the NSD was done in a 1 Hz bandwidth on the RF spectrum analyser, hence \( B = 1 \) Hz, we get:

\[
RIN(f) = \frac{\left( \frac{\Delta i(f)}{\bar{R}} \right)^2}{B} = \frac{(269 \cdot 10^{-9})^2}{(24 \cdot 10^{-3})^2} \approx 126 \cdot 10^{-12} \, Hz^{-1}
\]
\[\approx -99.0 \, \frac{dB}{Hz} \quad (\text{at } f = 110 \, MHz \wedge -14.0 dBm)\]

- 59 -
The results of the source noise measurements of the narrowband ASE source will be presented in the next two sub-sections, with $RIN$ measurements presented in subsection (2.5.8) of this chapter.

2.5.3. Source Noise Spectral Density as a function of source bandwidth

In certain experiments on source NSD described later we wished to vary systematically the source bandwidth. In certain instances this was most readily achieved by changing the FBG for other pick-out filters of different bandwidths. Five narrowband FBG's were used as pick-out filters, each of different spectral bandwidths (FWHM) in the range of $2 - 15$ GHz, see Table (2-1).

Initially, the NSD at three different frequencies (1, 10, 100 MHz) was measured as a function of received optical power for pick-out filter FBG3 (FWHM bandwidth = 7.7 GHz). The experimental results (total NSD) are summarised in Figure (2-7) below. Also plotted in this figure are the experimentally obtained receiver noise floor, the theoretical source NSD, and the receiver shot noise (in this instance the detector used was an Analog Modules Inc., model no. 712A-8-B, DC coupled, low noise InGaAs receiver, 200 MHz bandwidth, power to current conversion factor of 360 A/W).
Figure 2-7: Measured and predicted source NSD versus optical power at three different frequencies (□ = 1 MHz, ○ = 10 MHz, △ = 100 MHz). The solid and dashed lines are theoretical plots (except for the detector noise floor, which was obtained experimentally). For these measurements, the narrowband ASE source incorporated pick-out filter FBG3 (7.7 GHz bandwidth).

From Figure (2-7) above we can see that the NSD is flat across the measured frequency range (□ = 1 MHz, ○ = 10 MHz, △ = 100 MHz). This is due to the relatively broad bandwidth of the source (7.7 GHz FWHM) compared to the detection bandwidth (200 MHz). From these results the $RIN$ of the source was calculated to be $RIN_{7.7\,\text{GHz}} = -97.8$ dB/Hz, using Equation (2-24). At low optical power the NSD is independent of the received power, since the source noise is lower than the receiver noise, which is dominant at received powers below around −35 dBm. At received powers above −35 dBm, the source noise becomes higher than the receiver noise. Increasing the received optical power shows a linear increase in NSD (on a logarithmic scale). This conforms to our expectation (see Equation (2-21)) that the NSD of our source scales linearly with its optical output power (in logarithmic units dBm), i.e. the square of its output intensity $I_0$ (in linear units Watt). This can be seen from the fact that the slope of the plot above is $\theta = 2$ (for received optical powers higher than −35 dBm).
dBm). Therefore the SNR of the system can not be improved above a certain optical power. Not only the signal power $S$ increases linearly with an increase in optical power, but also the noise power $N$ increases linearly with an increase in optical power (i.e. the SNR stays constant). These results are in agreement with results obtained earlier on the NSD of a narrowband ASE source [8].

![Graph showing spectral power vs. source linewidth (FWHM) for different optical powers.](image)

*Figure 2-8:* Measured and predicted source NSD versus source bandwidth for two different received optical powers. Measurements were taken at 10 MHz.

Figure (2-8) above shows the measured (indicated with the symbols ■ and ●) and the calculated NSD (indicated with the solid — and dashed ---- lines) at 10 MHz as a function of source bandwidth for optical powers of –20 (denoted by ■) and –23 dBm (denoted by ●). Again the results confirm the noise dependency on inverse source bandwidth, predicted in section (2.4) (Equation (2-21)).

As is apparent from Equation (2-12), the detailed shape of the narrowband ASE source NSD is directly related to the optical lineshape. To confirm this to be the case, the NSD of the narrowband ASE source was measured with a fast receiver (Tektronix, 6 GHz bandwidth, scale factor 44 V/W), on a fast RF spectrum analyser (Tektronix...
The narrowband ASE source optical lineshape was defined by pick-out filter FBG1 (see Figure (2-4) on page 55) in this instance. Since the source noise decreases at high frequencies, a further EDFA was included after the polariser to boost the source noise above the receiver noise floor in the high frequency regime. The ASE input power to the additional EDFA was -13 dBm, and the output power was set to be 0 dBm. The normalised reflectivity profile of pick-out filter FBG1 is shown in Figure (2-4), and the both measured and theoretical NSD, calculated using FBG1 reflectivity profile in Equation (2-12), are shown in Figure (2-9), the agreement is seen to be extremely good.

![Graph showing spectral power versus frequency](image)

**Figure 2-9:** Calculated and measured source Noise Spectral Density, when filtered with pick-out filter FBG1.

The NSD spectrum shown in Figure (2-9) above has a Gaussian shape with a FWHM bandwidth of \( \sqrt{2\Delta \nu_0} \), as was predicted earlier with Equation (2-21) in section (2.4).
2.5.4. The chirped narrowband Amplified Spontaneous Emission source

In other instances it was more convenient to chirp the FBG pick-out filter so as to broaden the bandwidth rather than replace the pick-out filter itself. Chirping means that, instead of a uniform refractive index profile along the length of the FBG, a non-uniform profile is induced along the length, i.e. by changing the FBG’s pitch along its length. Chirping of the FBG causes the reflection spectrum of the FBG and therefore the source output spectrum to broaden. Chirping can be most conveniently induced, by applying a temperature gradient along the length of the FBG. The pick-out filter to be chirped (FBG\textsubscript{p1} in Table (2-1) with FWHM bandwidth 6.2 GHz, and optical reflection spectrum shown in Figure (2-3 a)) was placed inside a small metal tube (inner diameter \(\approx 2\) mm). This tube was heated at one end and, at the same time, cooled at the other end. With the chirping rig it was possible to chirp the bandwidth to around three times its original bandwidth, i.e. a bandwidth of around 21 GHz FWHM. For that amount of chirp applied, the temperature gradient applied to the metal tube containing the FBG was approximately 20 °C/cm (i.e. one end being \(\sim 33^\circ\) C and the other end being \(\sim 130^\circ\) C over a total FBG length of about 5 cm). Since the FBG was suspended inside this tube, the heating of the FBG was caused by radiation, instead of conduction, from the tube to the FBG. Consequently the temperature gradient applied to the FBG was thus substantially lower than 20 °C/cm (probably around 2 °C/cm, since a 21 GHz chirp was observed), however this was still adequate for our purposes. Below are two optical spectra taken from the output of the source, for two degrees of chirp applied to the pick-out filter.
Figure 2-10: Measured optical output spectra of the narrowband ASE source using FBG, shown in Figure (2-3 a). The dashed spectrum (----) is the output spectrum when the pick-out filter was not chirped (6.2 GHz bandwidth) and the solid graph (-----) is the source output spectrum with the pick-out filter chirped (bandwidth 21 GHz FWHM). Resolution bandwidth of the spectrum analyser was 0.02 nm.

Figure (2-11) below shows NSD measurements as a function of source bandwidth, which were taken with the chirped narrowband ASE source. Also plotted in the graph is the theoretical line for the source NSD as a function of the source bandwidth, as calculated with Equation (2-21). The measurements were taken for a constant received power of −14 dBm. NSD was measured at a frequency of 110 MHz.
Figure 2-11: Theoretical (—) and experimental (■) source Noise Spectral Density (NSD) versus source bandwidth at a constant received optical power of -14.0 dBm for the chirped narrowband ASE source. Measurements were taken at a frequency of 110 MHz.

This plot again confirms the inverse dependence of noise power on source bandwidth (as predicted in section (2.4) with Equation (2-21)). These results agree well with the results obtained earlier in the previous section. These results indicate that our model, based on a Gaussian random process, can be used with confidence for the prediction of optical source noise [8].

2.5.5. The dual-channel narrowband Amplified Spontaneous Emission source

As mentioned earlier, the use of FBG technology is a very flexible way of selecting a specific spectral shape and bandwidth by using FBG’s as pick-out filters. This technology provides significant scope for multiplexing. Using multiple FBG’s, the broadband spectrum emitted by the ASE source can be sliced into several lines, as can
be seen in Figure (2-12) below.

**Figure 2-12:** Schematic of the dual-channel narrowband ASE source used in the 2-channel experiments. Its output spectrum is shown in the figure below. The symbols are explained in the text. Each FBG is strained individually.

The output spectrum of the source is shown in Figure (2-13) below. The two distinct reflection peaks from the pick-out filters FBG_{p1} and FBG_{p2} in the source at \( \lambda_1 = 1550.0 \) nm and \( \lambda_2 = 1550.8 \) nm respectively can clearly be seen. The separation between the two reflection peaks is 100 GHz (0.8 nm), according to the International Telecommunications Union (ITU) grid.
Figure 2-13: Measured optical output spectrum of the narrowband ASE source with two channels, as drawn schematically in Figure (2-12). The FWHM bandwidth of the two peaks is 6.2 GHz. The channel spacing is 100 GHz, but could be changed accordingly. Resolution bandwidth of the spectrum analyser was 0.02 nm.

Each pick-out filter (FBG_{p1} and FBG_{p2}) was mounted on a separate stage, thereby making it possible to tune their centre wavelengths individually. Therefore it was possible to change the channel spacing for crosstalk experiments. The extinction ratio is seen to be 35 to 40 dB. The FWHM bandwidth of the two peaks in the spectrum above is 6.2 GHz. This 2-channel source was used in the WDM experiments, which will be discussed in chapter five.

2.5.6. The narrowband Amplified Spontaneous Emission source incorporating a Semiconductor Optical Amplifier

It was already mentioned that excess photon noise from the ASE source, is the major limiting factor of the sensing system. A way of reducing this type of noise is to incorporate a GS-SOA into the source configuration to act as a ‘noise eater’ [10, 11].
Compared to EDFA’s, SOA’s have a short carrier lifetime [40] and this means that the non-linear effect of gain-saturation can be used to suppress noise up to fairly high frequency (around a few GHz).

The source with the SOA (manufactured by Alcatel, GaInAsP, optical bandwidth 30 nm FWHM, peak gain between 1530 and 1550 nm, TE/TM differential gain < 1.5 dB) that we incorporated is drawn schematically below.

![Schematic of the narrowband ASE source incorporating a Semiconductor Optical Amplifier (SOA) incorporated, to suppress the inherent intensity noise from the ASE light by operating the SOA in gain-saturation. The pick-out filter (FBG_p1) is the one shown previously in Figure (2-3) with a 6.2 GHz bandwidth. The optical input power to the SOA is denoted by (i/p) and the optical output power by (o/p).](image)

Figure 2-14: Schematic of the narrowband ASE source incorporating a Semiconductor Optical Amplifier (SOA) incorporated, to suppress the inherent intensity noise from the ASE light by operating the SOA in gain-saturation. The pick-out filter (FBG_p1) is the one shown previously in Figure (2-3) with a 6.2 GHz bandwidth. The optical input power to the SOA is denoted by (i/p) and the optical output power by (o/p).

The output from the FBG is amplified (by EDFA_2) then polarised and subsequently fed into the SOA. The optical input power (i/p) to the SOA is controllable with variable attenuator 1. The SOP input to the SOA is controlled with PC_1. The source output power P_0 is controlled with variable attenuator 2.

### 2.5.7. Optimum operating point for the Semiconductor Optical Amplifier

First an optimum operating point for the SOA had to be found. This operating point has to be in the saturation regime of the SOA for the intensity fluctuations to be reduced. This is illustrated in Figure (2-15) below. This figure shows a plot of the (o/p) of the SOA as a function of its (i/p), for a constant drive current (i.e. a gain characteristic). Clearly, at low (i/p), modulations in the (i/p) (represented by (a)) cause a larger fluctuation in the SOA’s (o/p) (represented by (d)). On the other hand, at high (i/p),
substantially lower fluctuations in the SOA’s \((o/p)\) are observed (represented by (b)) for similar fluctuations in the \((i/p)\) (represented by (c)). This phenomenon results from gain-saturation dynamics which itself are dependent on the SOA drive current. Therefore, measurements were first taken of the \((o/p)\), of the SOA versus its \((i/p)\), for three different drive currents.

\[\text{Figure 2-15: Graphical illustration of noise reduction in the SOA due to the SOA's gain saturation. Shown is a gain characteristic of the SOA (i.e. } (o/p) \text{ versus } (i/p) \text{ for a constant drive current). Intensity noise input to the SOA is represented by (a) and (c), and intensity noise at the output of the SOA is represented by (d) and (b) respectively.}\]

The operating point in the SOA’s gain-saturation regime was determined as follows. Using the source configuration shown in Figure (2-14), calibration plots were measured for three different drive currents for the SOA. First the SOA was operated far below saturation, by reducing the drive current. Then the paddles on the PC at the input of the SOA (PC1 in Figure (2-14)) were adjusted and subsequently the \((o/p)\) of the SOA was observed, which was then maximised (i.e. the gain from the SOA was maximised). After that, the SOA’s drive current was set to the required value (i.e. 50 mA, 100 mA or 150 mA), and the \((i/p)\) to the SOA was set with variable attenuator 1 (in the range of \(-10 \text{ dBm to } +7 \text{ dBm})\). Then the \((o/p)\) from the SOA was measured for 5 different
values of \((i/p)\) and this set of measurements was repeated for two other drive currents. Shown below is the experimentally obtained calibration plot for the SOA. It shows the three gain characteristic of the SOA (i.e. for the three different drive currents).

![Operating point](image)

**Figure 2-16:** The experimentally obtained gain characteristic of the SOA, for three different drive currents (\(\blacktriangle = 50\ mA, \bullet = 100\ mA, \text{ and } \blacktriangle = 150\ mA\)). The solid lines are smoothed curves fitted to each of the three experimental data sets.

The solid lines in the plot are smoothed curves fitted to each of the three experimental data sets. From the graph above, we can see that when driving the SOA at 150 mA (denoted by \(\blacktriangle\)), the \((o/p)\) does not change much with a change in \((i/p)\) in the range of \(-5\) to \(+5\ dBm\) (referred to as gain saturation). At \(-5\ dBm\ (i/p)\), fluctuations in \((i/p)\) of say 5 dB gives only a 1 dB change in \((o/p)\). This effect is enhanced, by increasing the drive current of the SOA.
2.5.8. Source Noise measurements

Having found the operating point for the SOA (i.e. \((i/p) = -5.0 \text{ dBm and SOA drive current} = 200 \text{ mA})\), the source \(\text{RIN}\) was measured versus received optical power at this fixed operating point. First, the \((i/p)\) to the SOA was set to \(-5.0 \text{ dBm}\), by adjusting the variable attenuator 1 (mini VOAT from JDS Uniphase, model 4700C LBR) at the input of the SOA. Then, the SOP of the light to the input of the SOA was set on the principal axis of the SOA (i.e. the Transverse Electric, TE or the Transverse Magnetic, TM mode of the SOA). This was done by maximising the \((o/p)\) by rotating the paddles on \(\text{PC}_1\), while driving the SOA at low current (i.e. below saturation). When the SOA is not operated in saturation, there is a difference in \((o/p)\) of around 0.5 dB when launching the light either on the TE or the TM axis of the device. When the gain of the SOA is saturated, this difference in \((o/p)\) is negligible. Next, the SOA was driven at 200 mA (to ensure gain-saturation) and subsequently \(\text{RIN}\) was measured as a function of received optical power by adjusting the combined source output power \(P_0\) with variable attenuator 2 (make JDS Fitel, model no. VA 5505-506).

For \(\text{RIN}\) measurements for the narrowband ASE source without the SOA, the source, as depicted in Figure (2-14) was configured as follows. The SOA, variable attenuator 2, and \(\text{PC}_2\) were altogether removed from the source configuration shown in Figure (2-14). The output power of the source (denoted by \(P_0\)) was adjusted with variable attenuator 1, and subsequently \(\text{RIN}\) measurements of the narrowband ASE source were taken as a function of received optical power. Figure (2-17) below shows the source \(\text{RIN}\) as a function of received optical power. Data is plotted for both, the narrowband ASE source with the SOA incorporated (indicated with \(\square\)), and without the SOA incorporated (indicated with \(\bullet\) and \(\triangle\)).
Figure 2-17: Calculated and measured source $RIN$ versus received optical power, for the narrowband ASE source with a 6.2 GHz bandwidth (● and ---), the narrowband ASE source which incorporates a GS-SOA with a 9 GHz bandwidth (□), and for the chirped narrowband ASE source (bandwidth 9 GHz FWHM) without the SOA (△ and ····). Measurements were taken at a frequency of 110 MHz.

If we first consider the NSD measurements taken for the unchirped narrowband ASE source with a 6.2 GHz bandwidth (denoted by ●), we can clearly see that at optical powers below −35 dBm, the receiver noise dominates the total NSD. For the narrowband ASE source with the SOA incorporated (denoted by □), this figure is increased to an optical power of −20 dBm. For the narrowband ASE source with the SOA, at optical powers above −20 dBm the source noise becomes the dominant noise factor. Clearly, the $RIN$ of the narrowband ASE source can be reduced by as much as 20 dB at received optical powers above −20 dBm when incorporating a GS-SOA. Also plotted in the graph is the theoretically predicted $RIN$ (Equation (2-21)), indicated with the solid and dashed lines. The dark blue line (---) is the theoretical $RIN$ for the unchirped narrowband ASE source without the SOA (6.2 GHz bandwidth), and the dashed red line (····) represents the theoretical $RIN$ for the chirped narrowband ASE
source without the SOA (9 GHz bandwidth). There are no free fit parameters used in these theoretical NSD and RIN predictions. Agreement between the corresponding experimental points is seen to be good. With the model presented in the previous chapter it is not possible to predict the NSD for the narrowband ASE source with the GS-SOA incorporated.

Because of the inherent high non-linearity of the SOA, there is an issue related to line broadening, i.e. the optical output spectrum of the SOA is broadened in wavelength relative to the optical input spectrum. Since line broadening is a second factor in reducing the intensity noise (besides a reduced coherence), the RIN measurements from the narrowband ASE source with the SOA incorporated (denoted by □) were compared with RIN measurements obtained from the chirped narrowband ASE source without the SOA incorporated (denoted by Δ). The narrowband ASE source spectrum was chirped to a similar bandwidth as the output spectrum of the narrowband ASE source with the SOA incorporated, i.e. 9 GHz (0.08 nm FWHM). A 20 dB reduction in RIN, when incorporating the SOA in the source can clearly be seen (as indicated in Figure (2-17)). Also, comparing the RIN for the unchirped narrowband ASE source with 6.2 GHz bandwidth (experimental denoted by ● and theoretical indicated with —) with the RIN for the chirped narrowband ASE source with a 9 GHz bandwidth (experimental denoted by Δ and theoretical indicated with ---), reduction of around 2 dB in RIN is observed (see Figure (2-17)). This is expected, since broadening of the bandwidth of the narrowband ASE source reduces the NSD, see Equation (3-21). Also, a 2 dB decrease in RIN was expected when chirping the narrowband ASE source from a 6.2 GHz to 9 GHz bandwidth. Since NSD of our narrowband ASE source scales with the inverse bandwidth, if you increase the ASE bandwidth by a factor of 9 GHz / 6.2 GHz, then the source RIN decreases by the same factor, i.e.:

\[
RIN_{ASE\ 9\text{GHz}} = RIN_{ASE\ 6.2\text{GHz}} - \frac{9\text{ GHz}}{6.2\text{ GHz}} \\
= -99.0 - 1.5 \\
\approx -101\ \frac{\text{dB}}{\text{Hz}} \quad (\text{at } f = 110\ \text{MHz} \land P \approx -14.0\ \text{dBm})
\]

Comparing the values \(RIN_{ASE\ 9\text{GHz}} = -99.0\ \text{dB/Hz}\) and the \(RIN_{ASE\ 6.2\text{GHz}} = -101\ \text{dB/Hz}\) with the plots (—) and (---) in Figure (2-17), the 2 dB reduction in RIN can be seen. The broadened output spectrum of the narrowband ASE source with the GS-SOA
incorporated is shown in Figure (2-18) below.

**Figure 2-18:** Measured optical output spectrum of the narrowband ASE source with the GS-SOA incorporated (see Figure (2-14)). The source spectrum (defined by FBG$_{p1}$ shown in Figure (2-3 a)) is broadened by the non-linearity of the SOA. The ($i/p$) to the SOA was $-5$ dBm and the SOA drive current was 200 mA. The FWHM bandwidth of this spectrum is 9 GHz. The resolution bandwidth of the optical spectrum analyser was 0.02 nm.

This graph shows that the spectrum of the ASE is broadened by the SOA, from a FWHM bandwidth of about 6.2 GHz (as defined by pick-out filter FBG$_{p1}$ shown in Figure (2-3 a)) to around 9 GHz. The ($i/p$) to the SOA was $-5.0$ dBm, and the SOA drive current was 200 mA. Also, notice that the spectral output of the SOA does not resemble a Gaussian spectral shape anymore like at the input of the SOA. This is due to the non-linearity of the SOA. The results presented in this section indicate that the inherent intensity noise originating from our narrowband ASE source can be suppressed by as much as 20 dB by incorporating a GS-SOA into the narrowband ASE source to act as a ‘noise eater’.
2.6. Conclusions and Discussion

A spectrally sliced narrowband ASE source was developed and tested. The narrowband ASE source used for FOAS interrogation has been characterised and found to obey the expected properties of a thermal source. A model has been written with which it is possible to predict the noise properties of our narrowband ASE source. This model, based on a Gaussian random process, agrees well with our experimental results. Experiments were performed whereby the $RIN$ of the narrowband ASE source was measured as a function of the optical bandwidth. These measurements confirmed the inverse relationship of the source noise on the source bandwidth. Again, this was predicted by our model, and agreed well with the experiments. The intensity squared dependence of the source NSD found experimentally, was also predicted by our model. It was also shown that the inherent intensity noise from the ASE source, which limits the SNR of the FOAS system, could be reduced by as much as 20 dB. This was done, by incorporating a GS-SOA into our narrowband ASE source. The intensity noise was suppressed by the fast gain dynamics of the GS-SOA and could not be explained by the associated 50 % spectral broadening observed in these experiments alone.
Chapter Three

Noise redistribution in Mach-Zehnder Interferometer systems interrogated with Amplified Spontaneous Emission sources

Overview: The previous chapter discussed the noise properties of ASE sources. In this part of the thesis we go on to consider how the inherent source noise impacts the ultimate resolution of FOAS systems. Issues like noise redistribution associated with interferometers (well known as the 'filtering effect' in interferometers) and the effect of the State Of Polarisation (SOP) within the system on the redistribution of the noise power will be addressed. The performance of several different FOAS systems based on the Mach-Zehnder configuration, and interrogated with ASE sources, will be discussed both theoretically and experimentally.

3.1. Background

It is known that fibre ASE sources exhibit thermal source like noise properties [7], characterised in terms of excess photon noise [9, 41] as was fully described in the previous chapter. ASE sources exhibit significant intensity noise relative to conventional lasers and this noise power increases in proportion to the reciprocal bandwidth of the source. Indeed, in most optical systems employing ASE sources, the system performance is ultimately limited by the optical source noise. Clearly then, it is important to establish the impact of source noise on the use of narrowband ASE for the interrogation of interferometers. It is well established that the statistical properties of thermal light can be explained based on a Gaussian random process model [36]. However, an estimate of the true impact of source noise on a given system performance
is complicated by the filtering effect in multiple-path optical systems, such as interferometers [42, 43], which can cause redistribution of the optical noise power in the spectral domain. Note that this is also true of components such as polarisation modulators and Frequency Shifters (FS) frequently incorporated within such systems for functions such as polarisation control, and heterodyne signal processing. Consequently, in order to appreciate the full impact of the higher levels of source noise, and to assess the true potential of the approach, we need to consider in detail the full interferometer system. In earlier works [42, 44], a suitable mathematical formulation applicable to the use of a generalised low-coherence source was developed and applied to a few simple systems. In this thesis, I have focused my study on the use of an erbium fibre-based, narrowband ASE source for the interrogation of both MZI based and FP based interferometers, which are leading candidates for CM systems [2, 12, 32]. I generalise the earlier theoretical approach to include the effects of the SOP in the optical paths and to cover the inclusion of a FS (as required for heterodyne-based demodulation schemes). We then compare the results of our detailed noise analysis with our experimental data.

3.2. The Mach-Zehnder Interferometer

Estimating the noise level for an interferometer is not straightforward. The filtering effect due to the multiple optical paths significantly modifies the noise distribution and leads to periodic structure in the frequency domain. Note also, that this noise distribution is a function of the SOP within the system. We demonstrate the above two features in the following sections, in which we evaluate, both analytically and experimentally, the noise properties of several interferometer systems. In our calculations we assume for simplicity that the coupling ratio of the couplers (C) used within the interferometer is 50:50.

The drawing below shows a schematic of a MZI with a delay time $T_{MZ}$. Light with intensity $I(t) = |E(t)|^2$, entering the MZI, is split into two paths of equal power, at the input coupler (C) of the interferometer. The two light paths reunite at the output coupler of the MZI, where one of the light paths has experienced a time delay $T_{MZ}$.
Figure 3-1: A schematic of a MZI with a path-imbalance or time delay $T_{MZ}$. The directional couplers are indicated by C and are 3-dB or 50:50 couplers. The two possible paths through the interferometer (path $n = 0$ and $n = 1$) are shown for the PP case (b) and for the PF case (c), where $x$ denotes the input SOP and $x'$ and $y'$ denote the output states.

In order to understand the effect of the SOP of the light within the interferometer, we considered the two extreme cases, shown in Figure (3-1 b and c). In the first case, indicated by Figure (3-1 b), the polarisation of the light passing through the two paths of the MZI is the same and hence we refer to this case as the Polarisation Preserving (PP) configuration. In the second case (Figure (3-1 c)), the SOP of the light in one arm of the interferometer is changed to be orthogonal to the light in the other arm (denoted the Polarisation Flipping (PF) case).
3.2.1. The covariance function of the output of a Mach-Zehnder Interferometer

*Polarisation Preserving case*

The output field $E_{\text{out}}(t)$ of a MZI with polarisation maintaining paths (PP case in Figure (3-1 b)), and a time delay, $T_{MZ}$, can be expressed as:

$$E_{\text{out}}(t) = \frac{1}{2} \left( E(t) + E(t - T_{MZ}) \right)$$

Equation 3-1

Note, that for convenience, the relative $\pi/2$ phase shift which occurs between the two arms of the coupler is not included in Equation (3-1) since it is irrelevant to the overall result of the analysis. The correlation function of the output electric field, $\Gamma_{\text{out}}(\tau)$, is given by [37]:

$$\Gamma_{\text{out}}(\tau) = \langle E_{\text{out}}^*(t) E_{\text{out}}(t + \tau) \rangle$$

Equation 3-2

After substitution of Equation (3-1) we obtain four terms, each of which represents an electric field correlation function, given by:

$$\begin{align*}
\langle E^*(t) \cdot E(t + \tau) \rangle &= \Gamma(\tau) \\
\langle E^*(t) \cdot E(t + \tau - T_{MZ}) \rangle &= \Gamma(\tau - T_{MZ}) \\
\langle E^*(t - T_{MZ}) \cdot E(t + \tau) \rangle &= \Gamma(\tau + T_{MZ}) \\
\langle E^*(t - T_{MZ}) \cdot E(t + \tau - T_{MZ}) \rangle &= \Gamma(\tau)
\end{align*}$$

Equation 3-3

After substitution of these four terms back into Equation (3-2) we can write the electric field correlation function of the MZI output in the following form:
\[ \Gamma_{out}(\tau) = \frac{1}{2^2} \left( 2\Gamma(\tau) + \Gamma(\tau - T_{MZ}) + \Gamma(\tau + T_{MZ}) \right) \]

Equation 3-4

According to Equation (2-9) in previous chapter, the covariance function of the output intensity, \(\text{Cov}_I(\tau)\), can be written in terms of the correlation function of the output intensity. After substitution of Equation (3-4) into the covariance function, and after multiplying out the modulus squared term, we obtain for the covariance function, given by Equation (2-9):

\[
\text{Cov}_I(\tau) = \frac{1}{2^4} \left( 4\Gamma^2(\tau) + \Gamma^2(\tau - T_{MZ}) + \Gamma^2(\tau + T_{MZ}) + 2\Gamma(\tau)\Gamma(\tau - T_{MZ}) + \Gamma(\tau + T_{MZ}) + cc \right)
\[
\quad + \Gamma(\tau - T_{MZ})\Gamma(\tau + T_{MZ}) + cc
\]

Equation 3-5

Here \(cc\) denotes complex conjugate of the preceding term. According to Equation (2-7) in the second chapter, the NSD can be obtained by Fourier transforming the covariance function given by Equation (3-5). In the case the SOP is set to be the PF case, the covariance function is modified, as we will show next.

**Polarisation Flipping case**

Consider now the second case for which both polarisation orientations are orthogonal (Figure 3-1 c)). We need to consider separately the noise interference effects within the two polarisation eigenstates of the system. In this case incoherent summing of the two orthogonal polarisation states takes place [45] and the output electric field can be written in the following form:

\[ E_{out}(t) = e_x E_x(t) + e_y E_y(t) \]

Equation 3-6

here \(e_x\) and \(e_y\) are Cartesian unit vectors for the two orthogonal polarisation states.
With $E_x(t)$ aligned with $E(t)$ and $E_y(t)$ aligned with $E(t - T_{MZ})$, as shown in Figure (3-1 a and c), we get for the electric field at the output of the MZI:

$$E_{out}(t) = \frac{1}{2} \left( e_x E(t) + e_y E(t - T_{MZ}) \right)$$

Equation 3-7

Noting that for polarised thermal like sources, $E(t)$ can be assumed to obey circularly Gaussian statistics so that fourth-order moments can be factored into the appropriate combinations of second-order moments [36], i.e.:

$$\langle E(t_1) E^*(t_2) E(t_3) E^*(t_4) \rangle = \langle E(t_1) E^*(t_2) \rangle \langle E(t_3) E^*(t_4) \rangle$$
$$+ \langle E(t_1) E^*(t_4) \rangle \langle E(t_3) E^*(t_2) \rangle$$

Equation 3-8

Using this expression, we can write the covariance function, given by Equation (2-8) in the previous chapter, in the following form:

$$\text{Cov}_1(\tau) = \left| \langle E_x^*(t) E_x(t + \tau) \rangle \right|^2 + \left| \langle E_y^*(t) E_y(t + \tau) \rangle \right|^2 + 2 \left| \langle E_x^*(t) E_y(t + \tau) \rangle \right|^2$$

Equation 3-9

The covariance function for the PF case becomes:

$$\text{Cov}_1(\tau) = \frac{1}{2^4} \left( 4|\Gamma(\tau)|^2 + |\Gamma(\tau - T_{MZ})|^2 + |\Gamma(\tau + T_{MZ})|^2 \right)$$

Equation 3-10

3.2.2. Noise Spectral Density of the output of a Mach-Zehnder Interferometer

Polarisation Preserving case

The NSD at a frequency $f$ is given by Equation (2-7) in the previous chapter. After
substitution of the electric field correlation function, Equation (2-18), into the expression for the covariance function given by Equation (3-5), we find for the NSD:

\[
N_{\text{MZ}}(f) = (g\alpha)^2 \frac{I_0^2}{2^4} \left( \int_{-\infty}^{\infty} 4e^{-4\pi^2\sigma^2\tau^2} \cdot e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} \cdot e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} \cdot e^{2\pi if\tau} d\tau \right)
\]

\[
+ \int_{-\infty}^{\infty} 2e^{-2\pi^2\sigma^2\tau^2} e^{-2\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{-2\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} 2e^{-2\pi^2\sigma^2\tau^2} e^{-2\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau
\]

\[
= (g\alpha)^2 \frac{I_0^2}{2^4} \left( \int_{-\infty}^{\infty} 4e^{-4\pi^2\sigma^2\tau^2} \cdot e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau + \int_{-\infty}^{\infty} e^{-4\pi^2\sigma^2(\tau+\tau_{\text{MZ}})} e^{2\pi if\tau} d\tau \right)
\]

Equation 3-11

Where \( \sigma \) is the bandwidth dependent factor given by Equation (2-13) in the second chapter. We can solve each Fourier integral within the brackets individually. Solving the integrals, and taking the constant \( \frac{1}{\sqrt{4\pi\sigma^2}} \) and the exponential function \( e^{-\frac{f^2}{4\sigma^2}} \) out of the brackets, the NSD can be written as follows:

\[
N_{\text{MZ}}(f) = (g\alpha)^2 \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{\sigma^2}} \left( 4 + 2\cos(2\pi f T_{\text{MZ}}) + 8e^{-\pi^2\sigma^2 T_{\text{MZ}}^2} \cdot \cos(2\pi f T_{\text{MZ}}) \cdot \cos(2\pi f T_{\text{MZ}}) + 2e^{-4\pi^2\sigma^2 T_{\text{MZ}}^2} \cdot \cos(4\pi f T_{\text{MZ}}) \right)
\]

Equation 3-12

From Equation (3-12) two things are immediately apparent. First, the noise power exhibits a periodic structure in the spectral domain (see Figure (3-2)), with a period, \( f_{\text{MZ}} \) defined by the time delay \( T_{\text{MZ}} \) (or the path-imbalance \( L_{\text{MZ}} \)) of the MZI. This period is often referred to as the Free Spectral Range (FSR), i.e.:
\[ FSR = f_{MZ} = \frac{1}{T_{MZ}} = \left( \frac{n}{c} \right) \cdot I_{MZ} \]

Equation 3-13

Here \( c \) is the speed of light in vacuum and \( n \) is the refractive index of the optical fibre core (typically \( n = 1.46 \)). This effect is well known and is referred to as the 'filtering effect' of an interferometer [43]. Second, the noise spectrum is highly dependent on the phase difference between the two arms of the interferometer (assuming the coherent regime i.e. delay time of the interferometer \( T_{MZ} \), is much shorter than the coherence time, \( \tau_c \), of the source), i.e. the phase bias, \( \phi_b = (2\pi v_0 T_{MZ}) \). For an unbalanced MZI, with a path-imbalance of 25 metres, the FSR is around 8 MHz. In the incoherent regime (i.e. delay time of the interferometer \( T_{MZ} \), is much longer than the coherence time, \( \tau_c \), of the source) Equation (3-12) reduces to:

\[ N_{MZ}(f) \approx (g \alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 4 + 2\cos(2\pi f T_{MZ}) \right) \]

Equation 3-14

The NSD of the MZI output (given by Equation (3-12)) is plotted as a function of frequency, in Figure (3-2) below (for a path-imbalance of around 25 metres). A source bandwidth of 6.2 GHz was used in the calculation.

**Polarisation Flipping case**

In the case of two mutually perpendicular polarisation states the covariance function is given by Equation (3-10). After substitution of the electric field correlation function, Equation (2-18) in the previous chapter, into the expression for the covariance function given by Equation (3-10), we find for the NSD:

\[ N_{MZ}(f) = (g \alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 2 + 2\cos(2\pi f T_{MZ}) - 2e^{-4\pi^2 f^2 \tau_{MW}^2} \right) \]

Equation 3-15
Clearly there is still periodicity in the noise spectrum for two orthogonally polarised light beams within the strongly unbalanced MZI. Comparing Equations (3-14) and (3-15) it is clear that only the offset of the noise spectrum has been affected by the SOP within the MZI. The two noise spectra for the PP and the PF configuration, given by Equations (3-12) and (3-15) respectively, are plotted in Figure (3-2) below. Superposed on the graph is the calculated NSD from the narrowband ASE source only, i.e. when the receiver directly faces the source (Equation (2-21) from the previous chapter). It is straightforward to see that Equation (3-15) reduces to Equation (2-21) when setting $T_{MZ} = 0$.

![Graph](image)

**Figure 3-2:** Calculated Noise Spectral Density at the output of the unbalanced MZI for two settings of the SOP (--- = PP, ---- = PF). Superposed is also the NSD when the source directly faces the receiver (--- = source). The MZI path-imbalance was 25 metres. The source bandwidth was 6.2 GHz and received power was -23 dBm throughout.

In the calculations, the source bandwidth was 6.2 GHz and received optical power was set to -23 dBm. From Figure (3-2) shown above it can be seen that the PF setting (----)
gives lower noise power (~ 2.5 dB) compared to the PP case (—), over the full
detection bandwidth. Remember, that as shown in Figure (2-7) in the previous chapter,
the noise power of the source itself at the -23 dBm power level is approximately -
105.6 dBm/Hz and is independent of frequency in the 0 – 20 MHz frequency range
(this is indicated with — in Figure (3-2) above). The periodic structure is due to the
filtering effect of the MZI, with the period equal to the inverse of the elapsed time for
light to propagate through the 25 metre long fibre. The source bandwidth does not
influence the overall shape of the periodic structure in the spectral domain, it only
affects the offset of this periodic structure (see Equation (3-14)). The NSD reaches a
maximum when \( \cos(2\pi v_0 T_{MZ}) = 1 \) in Equation (3-12), i.e. the point of maximum
output intensity (see Equation (1-1) and Equation (3-19)). We can also see that, even
for a well-balanced interferometer (i.e. \( T_{MZ} \approx 0 \)) there is still excess photon noise left
(note that there is no periodic structure in the spectral domain in this instance).
Equation (3-12) now reduces to:

\[
N_{MZ}(f) \approx (g\alpha)^2 R \frac{I_0^2}{2} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot 16
\]

Equation 3-16

The NSD is inversely proportional to the optical bandwidth. This agrees well with the
previous results on the inverse relationship between NSD and source bandwidth (see
the second chapter). It should be noted this noise power spectrum is also dependent on
the SOP within the MZI.

3.2.3. Fringe visibility of a Mach-Zehnder Interferometer

For the PP case, the time averaged output intensity of an optical system is given by:

\[
\langle I(t) \rangle = \langle |E_{out}(t)|^2 \rangle = \langle E_{out}(t)E_{out}^*(t) \rangle
\]

Equation 3-17

The random fluctuations of \( E(t) \) and thus also of the average output intensity, \( \langle I(t) \rangle \),
will manifest themselves as additional noise at the detector output current. Substitution
of Equation (3-1) for $E_{\text{out}}(t)$ for the MZI, and replacing all four second-order terms with the corresponding correlation function (Equations (3-3)), we get:

$$\langle I(t) \rangle = \frac{1}{2^2} \left( 2\Gamma(0) + \Gamma(\tau) + \Gamma^*(\tau) \right)$$

Equation 3-18

After substitution of the electric field correlation function (Equation (2-18) in the second chapter), and recognising that two complex exponential terms represent a cosine function, we find for the average output intensity of the MZI:

$$\langle I(t) \rangle = \frac{I_0}{2^2} \left( 2 + 2e^{-2\pi^2\sigma^2 T_{\text{MZ}}} \cos(2\pi v_0 T_{\text{MZ}}) \right)$$

Equation 3-19

This interferometer transfer function represents a raised cosine function, as shown in Figure (1-6) in the first chapter. Equation (3-19) can be written as follows, according to Equation (1-1) in the first chapter:

$$\langle I(t) \rangle = \frac{I_0}{2^2} \left( 2 + 2V \cdot \cos(2\pi v_0 T_{\text{MZ}}) \right)$$

Equation 3-20

Comparing expressions (3-19) and (3-20), we recognise that the visibility can be expressed as [46]:

$$V = e^{-2\pi^2\sigma^2 T_{\text{MZ}}} \equiv \left| \int_{-\infty}^{\infty} \psi(x) e^{-2\pi\sigma T_{\text{MZ}}} dx \right|$$

Equation 3-21

From the expression above it is clear that the visibility is a function of the source bandwidth, $\Delta \nu_b$, via $\sigma$ defined by Equation (2-13) in the second chapter and also a
function of the path-imbalance of the MZI, $L_{MZ}$. The visibility, given by Equation (3-21), is plotted as a function of the interferometer path-imbalance for four different source bandwidths (see Figure (3-3) below). It can be seen that as the bandwidth of the source decreases (for a fixed $L_{MZ}$), there is a gain in coherence and, henceforth, the visibility increases.

**Figure 3-3:** Predicted fringe visibility of a MZI as a function of the path-imbalance, for four different source bandwidths ($\Delta v_0 = 1$ GHz, 6.2 GHz, 10 GHz and 20 GHz).

Figure (3-3) above shows that for a path-imbalance of 10 mm and a 6.2 GHz source bandwidth (indicated with $-$), a maximum 75 % visibility is obtained. Broadening the bandwidth of the source to 10 GHz (indicated with $\ldots$), reduces the visibility by as much as 30 % to a maximum value of around 50 %. Broadening the bandwidth even further to, say, 20 GHz (indicated with $-$), reduces the interferometer fringe contrast to almost zero. Clearly, an optimum source bandwidth for a practical path-imbalance (around 1 cm) must exist.
3.2.4. Phase resolution of a Mach-Zehnder Interferometer

To obtain the ultimate (phase) resolution of an interferometer, we have to calculate its SNR. The noise spectrum has been calculated in the previous sections. To calculate its signal spectrum we have to introduce the periodic phase modulation (i.e. a signal), \( \phi_s \sin(2\pi f_i t) \) due to the sinusoidal modulation of the length, of one of the interferometer arms. This is shown schematically in Figure (3-4) below, where the transfer function is plotted together with an input signal and consequent interferometer response (i.e. output signal).

![Diagram showing phase resolution of a Mach-Zehnder Interferometer](image)

**Figure 3-4**: Illustration of the output of a balanced MZI as a function of the phase difference. The quadrature settings are labelled Q and the out-of-quadrature settings are labelled M (for maximum or minimum output). A periodic sinusoidal signal is applied to the interferometer with modulation depth (signal amplitude) \( \phi_s \) and frequency \( f_i \). The interferometer is biased at its quadrature point (\( \phi_b = 2\pi/4 = \pi/2 \)).

Note that for a quadrature-biased interferometer (points Q in Figure (3-4) above) the noise spectrum, given by Equation (3-12) yields:
\[ N_{MZ}(f) \approx (g \alpha)^2 R \frac{I_0^2}{4^4} \frac{1}{\sqrt{4\pi \sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 4 + 2 \cos(2\pi f T_{MZ}) - 2e^{-4\pi^2\sigma^2 T_{MZ}} \right) \]

Equation 3-22

The time averaged output intensity, given by Equation (3-20) can now be written in the following form:

\[ \langle I(t) \rangle = \frac{I_0}{2^2} \left( 2 + 2V \cdot \cos \left( 2\pi v_0 T_{MZ} + \phi_s \sin(2\pi f_s t) \right) \right) \]

Equation 3-23

Using the trigonometric identity \( \cos(\alpha \pm \beta) = \cos(\alpha) \cdot \cos(\beta) \mp \sin(\alpha) \cdot \sin(\beta) \), we can write this as:

\[ \langle I(t) \rangle = \frac{I_0}{2^2} \left( 2 + 2V \cdot \left( \cos(2\pi v_0 T_{MZ}) \cdot \cos(\phi_s \sin(2\pi f_s t)) - \sin(2\pi v_0 T_{MZ}) \cdot \sin(\phi_s \sin(2\pi f_s t)) \right) \right) \]

Equation 3-24

When the interferometer is quadrature-biased, then \( \phi_s = (2\pi v_0 T_{MZ}) \approx \frac{2\pi}{4} \approx \frac{\pi}{2} \), so \( \sin(2\pi v_0 T_{MZ}) \approx 1 \) and \( \cos(2\pi v_0 T_{MZ}) \approx 0 \). Also, when we assume the signal applied to be small, i.e. \( \cos(\phi_s \sin(2\pi f_s t)) \approx 1 \) and \( \sin(\phi_s \sin(2\pi f_s t)) \approx \phi_s \sin(2\pi f_s t) \) we obtain:

\[ \langle I(t) \rangle \approx \frac{I_0}{2^2} \left( 2 + 2V \cdot \phi_s \sin(2\pi f_s t) \right) \]

Equation 3-25

The signal spectrum, \( S(f) \), of the detected average output intensity can be obtained by Fourier transforming the average output intensity, given by Equation (3-25), i.e. \( \mathcal{F}[\langle I(t) \rangle] \). The Fourier transform of this continuous function contains both phase and amplitude information and we are only interested in the latter. The amplitude signal
spectrum of the average output intensity, $|S(f)|$ is readily obtained by taking the modulus square of the Fourier transformation, i.e.:

$$|S(f)| = \left| \mathcal{F}\left[ I(t) \right] \right|^2$$

Equation 3-26

Since the Fourier transform of a complex exponential function $e^{ix}$ is a Delta function, we get for the Fourier transform of $\sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$,

$$\mathcal{F}[\sin(x)] = \frac{1}{2i}(i\delta(x) + i\delta(-x)),$$

also the Fourier transform of a constant is a Delta function, i.e. $\mathcal{F}[2] = 2\delta(x)$. For the (electrical) amplitude signal spectrum of the detected average output intensity given by Equation (3-25) we obtain:

$$|S(f)| = (g\alpha)^2 R \frac{I_0^2}{2^4} \left| 2\delta(f) + 2V \cdot \frac{\phi_i}{2} \left( i\delta(f + f_s) - i\delta(f - f_s) \right) \right|^2$$

Equation 3-27

Multiplying out the terms that contain a Delta function, with a shifted version of the Delta function cancel from Equation (3-27), since per definition $\delta(f_1) \cdot \delta(f_2) = 0$. Hence, we are only left with terms where the Delta function is multiplied with an identical Delta function, i.e. $\delta(f) \cdot \delta(f) \neq 0$.

$$|S(f)| = (g\alpha)^2 R \frac{I_0^2}{2^4} \left( 4\delta(f) + 4V^2 \cdot \frac{\phi_i^2}{2^2} \left( \delta(f + f_s) + \delta(f - f_s) \right) \right)$$

Equation 3-28

Since we can only measure positive frequencies, we can take the negative frequency components together with the identical positive ones:
\[ |S(f \pm f_s)| = (g\alpha)^2 R \frac{I_0^2}{2^4} \left[ 4\delta(f) + 4V^2 \cdot \frac{\phi_s^2}{2^2} \delta(f \pm f_s) \right] \]

Equation 3-29

An illustration of this (amplitude) signal spectrum can be seen in Figure (3-5) below. The fundamental or DC component of the spectrum, represented by the Delta function \(\delta(f)\) in Equation (3-29) is situated at zero frequency, with the two other Delta functions \(\delta(f \pm f_s)\) centred around the fundamental, separated by the signal frequency \(f_s\).

![Figure 3-5: Illustration of the amplitude signal frequency spectrum as seen at the receiver output of a single balanced MZI. Note that only positive frequencies will be displayed on the RF spectrum analyser.](image)

Note that negative frequencies are purely mathematical and can not be displayed on the RF spectrum analyser. Therefore, strictly speaking, only the right hand side of Figure (3-5) above with the signal \(\delta(f + f_s)\) will be shown on the RF spectrum analyser.

The sensitivity limit of the interferometer is defined by the minimum detectable phase change, i.e. \((\phi_s)_{\text{min}}\). When we equate this signal spectrum, given by Equation (3-29) to the noise spectrum at the signal frequency of interest \((f \pm f_s)\), i.e. \(|S(f \pm f_s)| = |N(f \pm f_s)|\), or \(\text{SNR} = 1\), we can calculate the system's phase resolution (i.e. the minimum detectable phase shift \((\phi_s)_{\text{min}}\)):
\[ |N(f \pm f_s)| = (g \alpha)^2 R \frac{I_0^2}{2^2} \left( 4 \delta(f) + 4 V^2 \cdot \frac{\phi_s^2}{2} \delta(f \pm f_s) \right) \]

Equation 3-30

The Delta functions drop, since, in this instance, we do not look at one specific frequency (we integrate over a certain frequency range and \( \delta = 1 \)). Solving for the phase resolution we find:

\[ (\phi_s)_\text{min}^2 = \frac{2^4 \left| N(f \pm f_s) \right|}{I_0^2 \left( g \alpha \right)^2 R} \frac{4}{V^2} \]

Equation 3-31

Where the total noise power spectral density of the detected output intensity of the system \( |N(f \pm f_s)| \) is given previously by Equation (3-12) for the PP case and Equation (3-15) for the PF configuration. Substitution of the NSD given by Equation (3-12) and the visibility \( V \) given by Equation (3-21) we obtain:

\[ (\phi_s)_\text{min}^2 = \frac{1}{4 \pi \sigma^2} \cdot e^{\frac{f^2}{4 \sigma^2}} \cdot e^{4 \pi^2 \sigma^2 \tau_{MZ}^2} \left( 4 + 2 \text{Cos}(2 \pi \tau_{MZ}) \right) \]

\[ + 8 \text{Cos}(2 \pi \tau_{MZ}) \cdot \text{Cos}(\pi \tau_{MZ}) + 2 e^{-4 \pi^2 \sigma^2 \tau_{MZ}^2} \cdot \text{Cos}(4 \pi \tau_{MZ}) \]

Equation 3-32

This expression shows that the highest phase resolution is obtained for a well-balanced (quadrature-biased) interferometer, i.e. \( \tau_{MZ} \approx 0 \), since, in that instance, the visibility \( V \) equals unity. In the case of a well-balanced MZI, the phase resolution can be written as:

\[ (\phi_s)_\text{min}^2 \approx \frac{1}{\sqrt{4 \pi \sigma^2}} \cdot e^{\frac{f^2}{4 \sigma^2}} \cdot 16 \]

Equation 3-33

This gives a phase resolution of around 0.33 \( \mu \text{rad}/\sqrt{\text{Hz}} \) in the low frequency regime.
(f < 1 MHz), when interrogated with a source with a bandwidth of 32 MHz. Plotted in Figure (3-6) below is the theoretical phase resolution of the MZI, given by Equation (3-32), as a function of the source bandwidth, for three different path-imbances. The interferometer was biased around its quadrature point (most sensitive point).

![Diagram](image)

**Figure 3-6:** Predicted phase resolution as a function of source bandwidth for a single quadrature-biased MZI, for three different path-imbances ($L_{MZ} = 15$ mm, $10$ mm and $5$ mm).

Figure (3-6) above shows that for a path-imbalance of 15 mm (indicated with ...,), a maximum phase resolution of around 75 $\mu$rad/$\sqrt{\text{Hz}}$ is obtained for a 3 GHz bandwidth. The phase resolution drops off as the bandwidth becomes broader (reduction in excess photon noise as the bandwidth becomes broader). This is due to the fact that, the signal visibility (see Figure (3-3)) decreases faster than the noise power does, (see Figure (2-8)), as the source bandwidth becomes broader for relatively large path-imbalance. A decrease in phase resolution is also observed with a decrease in source bandwidth, i.e. an increase in excess photon noise as the bandwidth reduces. It is clear that there exists an optimum source bandwidth for a fixed path-imbalance of the
interferometer. For a more practical path-imbalance of 10 mm (indicated with —), the phase resolution reaches a maximum of around 62 μrad/√Hz at a bandwidth of 6.2 GHz. Better matching of the interferometer arms to, say, 5 mm (indicated with ·), gives a maximum phase resolution of around 44 μrad/√Hz at a bandwidth of around 10 GHz. A quadrature-biased, well-balanced interferometer (to within one fringe), (interrogated with a source with a 10 GHz bandwidth) has a phase resolution of around 33 μrad/√Hz. Reduction of the source bandwidth to 1 GHz reduces the phase resolution to around 103 μrad/√Hz. For a source bandwidth of 5 THz, the phase resolution is increased to around 1.46 μrad/√Hz. The phase resolution is plotted against received optical power, for a quadrature-biased MZI with a 10 mm path-imbalance in Figure (3-7) below. A source bandwidth of 6.2 GHz was used in the calculations.

![Graph](image)

**Figure 3-7**: Predicted phase resolution as a function of the received power for a quadrature-biased MZI with a 10 mm path-imbalance. The source bandwidth in the calculation was 6.2 GHz.

At relatively high optical power the phase resolution can not be improved by increasing the optical power, since the SNR is limited by the excess photon noise from the source. That is, the phase resolution of the interferometer is limited to around 62 μrad/√Hz, for
a quadrature-biased MZI with a path-imbalance of 10 mm and interrogated with a 6.2 GHz source bandwidth. This can be seen from Figure (3-7) shown above. In the calculation, an optimum source bandwidth of 6.2 GHz was chosen to give a maximum phase resolution for this particular path-imbalance (see Figure (3-6)). At relatively low optical power, the detector noise becomes the dominant noise factor and, hence, the system's phase resolution decreases rapidly with decreasing received power.

3.2.5. A Mach-Zehnder Interferometer incorporating a Frequency Shifter

The heterodyne technique is frequently used in interferometer systems to avoid the high noise level at low frequency and thereby to achieve high sensitivity. When a modulator is employed in the interferometer, the modulation changes not only the signal distribution in the spectral domain, but also the noise distribution. The drawing below shows a schematic of an MZI with a FS (or Acousto-Optic Modulator (AOM)) in one of its arms.

\[ \frac{1}{2} E(t-T_{MZ}) \cdot e^{2\pi i f_h t} \]

**Figure 3-8:** A schematic of the MZI incorporating a FS in one of its arms.

One path is directed to the FS, where the light experiences a shift in frequency of \( f_h \), i.e. the optical frequency, \( \nu_0 \), after passing through the FS is \( (\nu_0 + f_h) \). The exponential term \( e^{2\pi i f_h t} \) represents the phase shift from the FS (an incremental shift in frequency is equivalent to a continuous shift of phase in time, i.e. the angular frequency \( \omega_h = 2\pi f_h = \frac{d\phi_h}{dt} \)).
3.2.6. The covariance function of the output of a Mach-Zehnder Interferometer incorporating a Frequency Shifter

*Polarisation Preserving case*

In this case, due to the incorporation of the FS, the output electric field (Equation (3-1)) can be written as:

\[
E_{\text{out}}(t) = \frac{1}{2} \left( E(t) + E(t - T_{\text{MZ}}) \cdot e^{2\pi f_s t} \right)
\]

Equation 3-34

The covariance function given by Equation (2-9) in the second chapter is not applicable in this case due to the frequency shift term. Instead, we have to use the original definition of the covariance function given by Equation (2-8). We have to calculate \( \langle I(t)I(t + \tau) \rangle \) and \( \langle I(t) \rangle^2 \) separately. For the average output intensity, given by Equation (3-17), we get:

\[
\langle I(t) \rangle = \frac{1}{2^2} \left( E(t) \cdot E^*(t) + E(t) \cdot E^*(t - T_{\text{MZ}}) \cdot e^{-2\pi f_s t} + E^*(t) \cdot E(t - T_{\text{MZ}}) \cdot e^{2\pi f_s t} + E(t - T_{\text{MZ}}) \cdot E^*(t - T_{\text{MZ}}) \right)
\]

Equation 3-35

The intensity correlation function \( \Gamma_I(\tau) = \langle I(t)I(t + \tau) \rangle \) comprises \( 2^4 \) terms, each represents a fourth-order moment of the form \( \langle E(t_1)E^*(t_2)E(t_3)E^*(t_4) \rangle \), i.e. we get:
\[
\langle I(t)I(t+\tau) \rangle = \frac{1}{2^4} \left\{ E(t) \cdot E^*(t) \cdot E(t+\tau) \cdot E^*(t+\tau) \\
+ E(t) \cdot E^*(t) \cdot E(t+\tau) \cdot E^*(t+\tau-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t) \cdot E^*(t) \cdot E(t+\tau) \cdot E(t+\tau-T_{MZ}) \cdot e^{2nf_s(t+\tau)} \\
+ E(t) \cdot E^*(t) \cdot E(t+\tau-T_{MZ}) \cdot E^*(t+\tau-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t-T_{MZ}) \cdot E^*(t-T_{MZ}) \cdot E(t+\tau) \cdot E^*(t+\tau) \\
+ E(t-T_{MZ}) \cdot E^*(t-T_{MZ}) \cdot E(t+\tau-T_{MZ}) \cdot E^*(t+\tau-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t-T_{MZ}) \cdot E^*(t-T_{MZ}) \cdot E(t+\tau-T_{MZ}) \cdot E^*(t+\tau-T_{MZ}) \cdot e^{2nf_s(t+\tau)} \\
+ E(t+\tau) \cdot E^*(t+\tau) \cdot E^*(t) \cdot E^*(t-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t+\tau) \cdot E^*(t+\tau-T_{MZ}) \cdot E^*(t-T_{MZ}) \cdot e^{2nf_s(t+\tau)} \\
+ E(t+\tau) \cdot E^*(t+\tau-T_{MZ}) \cdot E^*(t) \cdot E^*(t-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t+\tau) \cdot E^*(t+\tau-T_{MZ}) \cdot E^*(t) \cdot E^*(t-T_{MZ}) \cdot e^{2nf_s(t+\tau)} \\
+ E(t+\tau-T_{MZ}) \cdot E^*(t+\tau-T_{MZ}) \cdot E^*(t) \cdot E^*(t-T_{MZ}) \cdot e^{-2nf_s(t+\tau)} \\
+ E(t+\tau-T_{MZ}) \cdot E^*(t+\tau-T_{MZ}) \cdot E^*(t) \cdot E^*(t-T_{MZ}) \cdot e^{2nf_s(t+\tau)} \right\}
\]

Equation 3-36

Using Gaussian statistics, according to Equation (3-8) we can write the fourth-order moments appearing in Equation (3-36) as second-order electric field correlation functions, i.e.:
\[
\left\langle E(t) \cdot E^*(t) \cdot E(t+\tau) \cdot E^*(t+\tau) \right\rangle = \left\langle \Gamma(0) \cdot \Gamma(0) + \Gamma(\tau) \cdot \Gamma^*(\tau) \right\rangle \\
\left\langle E(t) \cdot E^*(t) \cdot E(t+\tau - T_{MZ}) \cdot E^*(t+\tau - T_{MZ}) \right\rangle = \left\langle \Gamma(0) \cdot \Gamma(0) + \Gamma(\tau - T_{MZ}) \cdot \Gamma^*(\tau - T_{MZ}) \right\rangle \\
\left\langle E(t - T_{MZ}) \cdot E^*(t - T_{MZ}) \cdot E(t+\tau) \cdot E^*(t+\tau) \right\rangle = \left\langle \Gamma(0) \cdot \Gamma(0) + \Gamma(\tau + T_{MZ}) \cdot \Gamma^*(\tau + T_{MZ}) \right\rangle \\
\left\langle E(t - T_{MZ}) \cdot E^*(t - T_{MZ}) \cdot E(t+\tau - T_{MZ}) \cdot E^*(t+\tau - T_{MZ}) \right\rangle = \left\langle \Gamma(0) \cdot \Gamma(0) + \Gamma(\tau) \cdot \Gamma^*(\tau) \right\rangle \\
\left\langle E^*(t+\tau) \cdot E(t+\tau - T_{MZ}) \cdot E(t) \cdot E^*(t - T_{MZ}) \cdot e^{2\eta_1\tau} \right\rangle = \left\langle \Gamma(\tau) \cdot \Gamma^*(\tau) + \Gamma(T_{MZ}) \cdot \Gamma^*(T_{MZ}) \cdot e^{2\eta_1\tau} \right\rangle \\
\left\langle E(t+\tau) \cdot E^*(t+\tau - T_{MZ}) \cdot E(t) \cdot E(t - T_{MZ}) \cdot e^{2\eta_1\tau} \right\rangle = \left\langle \Gamma(\tau) \cdot \Gamma^*(\tau) + \Gamma(T_{MZ}) \cdot \Gamma^*(T_{MZ}) \cdot e^{2\eta_1\tau} \right\rangle
\]

Equation 3-37

Where we used the fact that by definition $\Gamma^*(0) = \Gamma(0)$. Note that the fourth-order moments represent cosine functions. The time average of a cosine function is zero. Hence, the time dependent terms (exponential function from the FS) in the expression for the covariance function, reduce to zero [37]. This can be written as follows:

\[
\left\langle E(t_1)E^*(t_2)E(t_3)E^*(t_4) \cdot e^{2\eta_1\tau} \right\rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left\langle E(t_1)E^*(t_2)E(t_3)E^*(t_4) \right\rangle \cdot e^{2\eta_1\tau} dt = 0
\]

Equation 3-38

According to this expression, the second, third, sixth, seventh, ninth, tenth, twelfth, thirteen, fifteenth and the sixteenth terms, in Equation (3-36) reduce to zero. The remaining terms can be replaced with the correlation functions given by Equation (3-37). Equation (3-36) reduces to:

\[
\left\langle I(t)I(t+\tau) \right\rangle = \frac{1}{2^4} \left\langle 4|\Gamma(0)|^2 + 2|\Gamma(\tau)|^2 + |\Gamma(\tau - T_{MZ})|^2 + |\Gamma(\tau + T_{MZ})|^2 \\
+ 2\left[|\Gamma(\tau)|^2 + |\Gamma(T_{MZ})|^2\right] \cdot \cos(2\eta_1\tau) \right\rangle
\]

Equation 3-39

which is real as expected, since light intensity is real valued. Using Equations (3-3), we can write the average output intensity given by Equation (3-35) as:
\[
\langle I(t) \rangle = \frac{1}{2^2} \left( 2\Gamma(0) + \Gamma(T_{MZ}) \cdot e^{-2\pi f_r \tau} + \Gamma^*(T_{MZ}) \cdot e^{2\pi f_r \tau} \right)
\]

Equation 3-40

The last two exponential terms between brackets represent a cosine function. Since the time average of a cosine function is zero, we find for the average output intensity:

\[
\langle I(t) \rangle = \frac{1}{2^2} \left( 2\Gamma(0) \right)
\]

Equation 3-41

After substitution of Equations (3-39) and (3-41) into the covariance function given by Equation (2-8) in the previous chapter we get:

\[
\text{Cov}_I(\tau) = \frac{1}{2^4} \left( 2|\Gamma(\tau)|^2 + |\Gamma(\tau - T_{MZ})|^2 + |\Gamma(\tau + T_{MZ})|^2 + 2|\Gamma(\tau)|^2 + |\Gamma(T_{MZ})|^2 \right) \cdot \cos(2\pi f_r \tau)
\]

Equation 3-42

**Polarisation Flipping case**

In the case of detection of two linearly orthogonal polarised beams, we get incoherent summing of the two optical signals. In other words, orthogonal polarisations add on an intensity basis and the detected output intensity is the sum of the two individual intensities, \( \langle I_x(t) \rangle \) and \( \langle I_y(t) \rangle \). The average output intensity, given by Equation (3-17) can then be written in the following form:

\[
\langle I(t) \rangle = \left( |e_x E_x(t)|^2 + |e_y E_y(t)|^2 \right)
\]

Equation 3-43

This becomes:
\[ \langle I(t) \rangle = \frac{1}{2^2} \left( |E(t)|^2 + |E(t - T_{MZ})|^2 \right) \]

Equation 3-44

Again, using the modified definition for the covariance function, given by Equation (3-9), we obtain:

\[ Cov_x(\tau) = \frac{1}{2^4} \left( |\Gamma(\tau)|^2 + |\Gamma(\tau - T_{MZ})|^2 + |\Gamma(\tau + T_{MZ})|^2 + (|\Gamma(\tau)|^2 + |\Gamma(T_{MZ})|^2) \cdot \cos(2\pi f_s \tau) \right) \]

Equation 3-45

3.2.7. Noise Spectral Density of the output of a Mach-Zehnder Interferometer incorporating a Frequency Shifter

Polarisation Preserving case

The noise power spectral density \( N \) at a frequency \( f \) is given by Equation (2-7) in the second chapter. After solving each of the five Fourier integrals within brackets, we find for the total power spectrum (using Equation (3-42)):

\[ N_{MZ}(f) = (g\alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi \sigma^2}} \left\{ e^{-\frac{f^2}{4\sigma^2}} \left( 2 + 2\cos(2\pi f T_{MZ}) \right) \right\} e^{-\frac{(f-f_s)^2}{4\sigma^2}} \left( e^{-\frac{(f-f_s)^2}{4\sigma^2}} \right) \]

Equation 3-46

In the incoherent regime \( (T_{MZ} \gg \tau_c) \), i.e. a strongly unbalanced interferometer, this becomes:

\[ N_{MZ}(f) \approx (g\alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi \sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 2 + 2\cos(2\pi f T_{MZ}) \right) \]

Equation 3-47

We notice that the two exponential terms from the heterodyne technique cease to be of
any significance. Comparing this expression for the NSD, with Equation (3-14), obtained earlier for the MZI without the FS, one sees that the FS has no effect other than a small offset on the redistribution of noise power in the spectral domain, for a strongly unbalanced MZI. For a well-balanced interferometer we obtain:

\[
N_{M_Z}(f) \approx (g\alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot \left( \frac{f^2}{4\sigma^2} + e^{\frac{(f+f_0)^2}{4\sigma^2}} + e^{\frac{(f-f_0)^2}{4\sigma^2}} \right)
\]

Equation 3-48

Again we note that there is no periodic structure in the spectral domain for a well-balanced MZI.

**Polarisation Flipping case**

In the case of two mutually perpendicular polarisation states we find (using Equation (3-45)):

\[
N_{M_Z}(f) = (g\alpha)^2 R \frac{I_0^2}{2^4} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot \left( \frac{f^2}{4\sigma^2} \left[ 1 + 2\cos(2\pi f T_{M_Z}) \right] + e^{-4\pi^2 \sigma^2 T_{M_Z}} \left( e^{\frac{(f+f_0)^2}{4\sigma^2}} + e^{\frac{(f-f_0)^2}{4\sigma^2}} \right) \right)
\]

Equation 3-49

The two noise spectra, given by Equations (3-46) and (3-49) are plotted in Figure (3-9) below both for the PP and the PF configuration respectively. The received optical power was set to –14 dBm and the source bandwidth to 6.2 GHz in the calculations.
**Figure 3-9:** Calculated source NSD after being filtered by the unbalanced MZI incorporating a FS, both for the PP and the PF implementation. The path-imbalance of the interferometer was 25.22 metres. The FWHM bandwidth of the source was 6.2 GHz. The received optical power was set to -14.0 dBm.

From Figure (3-9) above it is seen that in the PF case the noise power is 10 dB lower compared to the PP case at specific frequencies. Note that the incorporation of the FS causes the noise spectrum for the PF implementation to be much more pronounced (10 dB extinction compared to 5 dB in Figure (3-2) in the previous section). Again recall from Figure (2-7) that the noise power from the source itself at a -14 dBm power level is around -86 dBm/Hz within this 100 – 120 MHz frequency range.

### 3.2.8. Phase resolution of a Mach-Zehnder Interferometer incorporating a Frequency Shifter

After substitution of the electric field correlation function (Equation (2-18) in the previous chapter) and some algebraic manipulation we find for the average output intensity given by Equation (3-40):
\[
\langle I(t) \rangle = \frac{I_0}{2^3} \left( 2 + 2 e^{-2 \pi^2 \sigma^2 \tau_{me}} \cos(2 \pi v_o T_{MZ} + 2 \pi f_s t) \right)
\]

Equation 3-50

Note that the phase bias has become time dependent, i.e. \( \phi_b = (2 \pi v_o T_{MZ} + 2 \pi f_s t) \). The phase bias of the interferometer is continuously swept through 0 and \( 2\pi \) in time, due to the incorporation of the FS. If you recall the transfer function of this interferometer, it is trivial to see that, in time, on average, the phase bias is equal to \( \pi/2 \). This technique, referred to as heterodyne detection, also provides a relatively stable phase measurement, against slow drift from e.g. temperature fluctuations. Introducing a small phase modulation signal this becomes:

\[
\langle I(t) \rangle = \frac{I_0}{2^3} \left( 2 + 2 e^{-2 \pi^2 \sigma^2 \tau_{me}} \cos(2 \pi v_o T_{MZ} + 2 \pi f_s t + \phi_s \sin(2 \pi f_s t)) \right)
\]

Equation 3-51

Using this expression we obtain for the signal spectrum of a quadrature-biased interferometer (according to Equation (3-26)):

\[
S(f) = (g \alpha)^2 R \frac{I_0^2}{2^4} \left[ 4 \delta(f) + V^2 \cdot \left\{ \delta(f + f_h) + \delta(f - f_h) \right\} 
+ V^2 \frac{\phi_s^2}{2^2} \cdot \left\{ \delta(f + (f_h + f_s)) + \delta(f + (f_h - f_s)) + \delta(f - (f_h + f_s)) + \delta(f - (f_h - f_s)) \right\} \right]
\]

Equation 3-52

Here we assumed the signal to be small so \( \cos(\phi_s \sin(2 \pi f_s t)) \approx 1 \) and \( \sin(\phi_s \sin(2 \pi f_s t)) \approx \phi_s \sin(2 \pi f_s t) \). An illustration of this (amplitude) electrical signal spectrum produced by a single balanced MZI with FS, as seen on the receiver, is shown in Figure (3-10) below. The fundamental or DC component of the spectrum, represented by the Delta function \( \delta(f) \) is situated at zero frequency, with the two Delta
functions $\delta(f \pm f_h)$ originating from the heterodyne technique centred around the fundamental. Both heterodyne signals comprise of harmonics (appearing as modulation sidebands), represented by $\delta(f \pm (f_h \pm f_s))$ in Equation (3-52).

Figure 3-10: Illustration of the amplitude signal frequency spectrum as seen at the receiver output of a single balanced MZI incorporating a FS. Note that on the RF spectrum analyser only the positive frequencies will be shown.

Note that the signal is shifted away from the zero frequency by the carrier frequency. Also note the flat noise power across the full detection bandwidth, since there is no source noise filtering in a balanced interferometer (Equation (3-48)). After substitution of the noise spectrum, given by Equation (3-46), and the visibility, given by Equation (3-21), the phase resolution can be expressed as follows:

$$\left(\phi_s\right)_{\text{min}}^2 = \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{\frac{f^2}{\sigma^2}} \cdot e^{-4\pi^2\sigma^2 T_{\text{MZ}}^2} \left( e^{-\frac{(f + f_h)^2}{4\sigma^2}} + e^{-\frac{(f - f_h)^2}{4\sigma^2}} + 2 \cos(2\pi f T_{\text{MZ}}) \right)$$

Equation 3-53

For a well-balanced interferometer this becomes:
\[
(\phi_f)_{\text{min}} \approx \frac{1}{\sqrt{4\pi \sigma^2}} \left( 4e^{-\frac{f_r^2}{4\sigma^2}} + e^{-\frac{(f_r-f_s)^2}{4\sigma^2}} + e^{-\frac{(f_r+f_s)^2}{4\sigma^2}} \right)
\]

Equation 3-54

3.2.9. Experiments and Discussion

To validate the theory discussed above, we built a MZI that incorporated a FS. The MZI was formed with two directional fibre couplers, each with an equal power splitting ratio (3 dB couplers). The entire interferometer was made up of standard Single-Mode telecommunication Fibre (SMF-28). The two ports of each coupler were fusion spliced together, along with a longer length of fibre in one arm of the interferometer (to get the path-imbalance \( L_{MZ} \approx 25.22 \) metres). The path-imbalance was chosen to minimise the noise power around the heterodyne frequency defined by the FS \( (f_r = 110 \text{ MHz}) \). This is illustrated in Figure (3-9) where the path-imbalance is chosen so that a minimum in the noise spectrum coincides with the frequency carrier as defined by the FS in the MZI (110 MHz in this case). The FS used was a bulk optical device, i.e. it was connectorised on both ends with standard FC/PC (Fibre Connector / Physical Contact) connectors for incorporation in the MZI. Insertion loss of FC/PC connectors is typically around 0.2 dB. The FS used in the long arm of the MZI had a moderate insertion loss of about 5.1 dB. To balance the power in the two arms, the light in the short arm was attenuated by the same amount (~ 5.5 dB). The power imbalance was stable during the measurements and its value was typically around 0.2 dB. The heterodyne frequency was observed to be reasonably stable, with a slow drift of around \( \pm 4 \text{ Hz per minute} \). Figure (3-11) below shows a schematic of the set-up used to measure the source NSD after being filtered by the MZI. The output of the interferometer is directed via a variable attenuator (not shown in the figure) to a DC coupled receiver (125 MHz low-noise receiver, make New Focus, InGaAs PIN diode, model 1811), the output of which is fed directly into an RF spectrum analyser.
Figure 3-11: Schematic of the experimental set-up for the measurement of the optical source noise (NSD) after being filtered by a MZI incorporating a FS. The path-imbalance was $L_{MZ} = 25.22$ metres. The FWHM bandwidth of the source was 6.2 GHz. The receiver is a 125 MHz low-noise photo-receiver. Noise Spectral Density (NSD) was measured with an RF spectrum analyser at a constant receiver power of $-14.0$ dBm and a 1 Hz bandwidth.

NSD spectra were taken at a constant received optical power (controlled by the variable attenuator from JDS Fitel model no. VA 5505-506). The interferometer incorporated a PC to conveniently set the SOP within the interferometer to be PP or PF. The PC was set as follows for the PP and PF cases. The paddles on the PC in the interferometer were rotated such that the spectral noise power was either minimised, (or maximised). This results in the output polarisation states of the light passing through the two different paths in the interferometer (i.e. paths $n = 0$ and $n = 1$) to be either mutually parallel, (or orthogonal), resulting in the PP (or PF) case. Each EDFA in the source was operated at its maximum pump power, far above threshold, to ensure stable operation. NSD spectra were taken around the heterodyne frequency range of 110 MHz on the Marconi RF spectrum analyser in a 1 Hz resolution bandwidth. An averaging function was used on the spectrum analyser for the measurement of the NSD spectra. The spectra were stored on the RF spectrum analyser as traces. Each trace was then sampled at 1024 data points and subsequently fed into a personal computer via a GPIB data acquisition card. The two measured spectra (i.e. for PP and for PF implementation) in the frequency range of 100 MHz to 120 MHz are shown in Figure (3-12) below, together with the theoretical plot. For the measurements, the received optical power was set to be $-14.0$ dBm throughout.
Figure 3-12: Predicted and measured source NSD after being filtered by the unbalanced MZI incorporating a FS. The path-imbalance of the interferometer was 25.22 metres. The FWHM bandwidth of the source was 6.2 GHz. The received optical power was set to –14.0 dBm.

The experimental NSD for the PP and PF case are indicated with the symbols ■ and □ respectively and the theoretical NSD is indicated with the lines — and —— for the PP and the PF configuration respectively. The noise redistribution properties of the MZI are immediately apparent, recall from Figure (2-7) in the second chapter that the source noise itself is uniform over this frequency range for a fixed optical power of –14 dBm. The NSD from the source is about –86 dBm/Hz for a received optical power of –14 dBm, for this source bandwidth of 6.2 GHz. As predicted, the NSD shows periodicity in the spectral domain, with a FSR of around 8.2 MHz, which corresponds to the inverse of the elapsed time for light to pass through the path-imbalance of the interferometer (Equation (3-13)). The agreement between experimental and theoretical noise spectra is good. The noise power is highest (–83 dBm/Hz) for the PP (indicated with ■) case at the spectral maxima (e.g. at 105.9 MHz), and falls to its smallest value of –87 dBm/Hz at the noise minima (e.g. at 110 MHz). The maximum noise level
difference (maxima to minima) for the PP (■) case is about 5 dB and about 13 dB for the PF configuration (□). The noise power is seen to be lowest in the PF case compared to the PP implementation over the full detection bandwidth, reaching values of around 10 dB lower at specific frequencies.

3.3. The dual Mach-Zehnder Interferometer

The drawing below shows a schematic of a dual MZI. The first interferometer has a delay time $T_{MZ1}$, and the second interferometer has a delay time $T_{MZ2}$. In the following calculations, we will assume that the two time delays are much longer than the coherence time of the optical source. Light, entering via the first MZI can travel along four different routes through the system (paths $n = 0$ to $n = 3$), as illustrated in the Figures (3-13 b and c) below.

Figure 3-13: Schematic of the dual MZI used in this work (a). The delay time of the first and second interferometer is denoted with $T_{MZ1}$ and $T_{MZ2}$ respectively. The four possible paths through the system ($n = 0$ to $n = 3$) are shown both for the PP case (b) and for the PF case (c).
To get an understanding of the effect of SOP within the system, we again considered the two extreme cases, shown in Figure (3-13 b and c). In the first case, Figure (3-13 b), the polarisation of the light passing through the two paths of each MZI is the same (PP case). In the second case, the SOP of the light in one arm of each interferometer is changed to be orthogonal to the light in the other arm (PF case). Both cases result in maximum visibility for the interferometer. Paths \( n = 1 \) and \( n = 2 \) are nominally the same length and will produce the signal \( \phi_0 \).

**Polarisation Preserving case**

The output field \( E_{\text{out}}(t) \) of a dual MZI (with time delays \( T_{\text{MZI}} \) and \( T_{\text{MZ2}} \)) with polarisation maintaining paths (PP case in Figure (3-13 b)), can be expressed as:

\[
E_{\text{out}}(t) = \frac{1}{2^2} \left( E(t) + E(t - T_{\text{MZI}}) + E(t - T_{\text{MZ2}}) + E(t - T_{\text{MZI}} - T_{\text{MZ2}}) \right)
\]

Equation 3-55

The correlation function of the system output electric field, \( \Gamma_{\text{out}}(\tau) \), is given by Equation (3-2). It comprises of sixteen terms, each representing an electric field correlation function \( \Gamma(\tau) \) (Equation (3-3)). After substitution of these terms we get:

\[
\Gamma_{\text{out}}(\tau) = \frac{1}{2^4} \left( 4\Gamma(\tau) + 2\Gamma(\tau + T_{\text{MZI}}) + 2\Gamma(\tau + T_{\text{MZ2}}) + 2\Gamma(\tau - T_{\text{MZI}}) + 2\Gamma(\tau - T_{\text{MZ2}}) + \Gamma(\tau - (T_{\text{MZI}} + T_{\text{MZ2}})) + \Gamma(\tau + (T_{\text{MZI}} + T_{\text{MZ2}})) + \Gamma(\tau - \Delta T_{\text{MZ}}) + \Gamma(\tau + \Delta T_{\text{MZ}}) \right)
\]

Equation 3-56

where \( \Delta T_{\text{MZ}} \) is the relative time delay between the two MZI's, i.e. \( \Delta T_{\text{MZ}} = |T_{\text{MZI}} - T_{\text{MZ2}}| \).

After multiplying out all terms, and neglecting terms that give a insignificant contribution to the noise spectrum, we can write the covariance function of the output intensity, given by Equation (2-9) in the second chapter, in the following form:
\[ \text{Cov}_f(\tau) = \frac{1}{2^8} \left( 16|\Gamma(\tau)|^2 + 4|\Gamma(\tau + T_{MZ2})|^2 + 4|\Gamma(\tau - T_{MZ1})|^2 + 4|\Gamma(\tau - T_{MZ2})|^2 ight. \\
+ |\Gamma(\tau + (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau - (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau + \Delta T_{MZ})|^2 + |\Gamma(\tau - \Delta T_{MZ})|^2 \\
+ 4\Gamma^*(\tau) \left( \Gamma(\tau + \Delta T_{MZ}) + \Gamma(\tau - \Delta T_{MZ}) \right) + cc \\
+ 4\Gamma^*(\tau + T_{MZ}) \Gamma(\tau + T_{MZ2}) + cc \\
+ 4\Gamma^*(\tau - T_{MZ}) \Gamma(\tau - T_{MZ2}) + cc \\
\left. + \Gamma^*(\tau + \Delta T_{MZ}) \Gamma(\tau - \Delta T_{MZ}) + cc \right) \]

Equation 3-57

**Polarisation Flipping case**

In this case (Figure (3-13 c)), the output electric field is given by:

\[ E_{\text{out}}(t) = \frac{1}{2^2} \left( e_x \left( E(t) + E(t - T_{MZ1} - T_{MZ2}) \right) + e_y \left( E(t - T_{MZ1}) + E(t - T_{MZ2}) \right) \right) \]

Equation 3-58

After some considerable algebra, and using Equation (3-9) for the covariance function, we get for the covariance function:

\[ \text{Cov}_f(\tau) = \frac{1}{2^8} \left( 8|\Gamma(\tau)|^2 + 2|\Gamma(\tau + T_{MZ})|^2 + 2|\Gamma(\tau + T_{MZ2})|^2 + 2|\Gamma(\tau - T_{MZ1})|^2 + 2|\Gamma(\tau - T_{MZ2})|^2 \\
+ |\Gamma(\tau + (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau - (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau + \Delta T_{MZ})|^2 + |\Gamma(\tau - \Delta T_{MZ})|^2 \\
+ 2\Gamma^*(\tau) \left( \Gamma(\tau + \Delta T_{MZ}) + \Gamma(\tau - \Delta T_{MZ}) \right) + cc \\
+ 2\Gamma^*(\tau + T_{MZ1}) \Gamma(\tau + T_{MZ2}) + cc \\
+ 2\Gamma^*(\tau - T_{MZ1}) \Gamma(\tau - T_{MZ2}) + cc \\
\left. + \Gamma^*(\tau + \Delta T_{MZ}) \Gamma(\tau - \Delta T_{MZ}) + cc \right) \]

Equation 3-59
3.3.1. Noise Spectral Density of the output of a dual Mach-Zehnder Interferometer

*Polarisation Preserving case*

The noise power spectral density $N$ at a frequency $f$ is given by Equation (2-7) in the second chapter. For the noise spectrum, using the covariance function given by Equation (3-57), we find:

$$
N_{\text{Dual MZ}}(f) = \left( g \alpha \right)^2 R \frac{I_0^2}{2^8} \frac{1}{\sqrt{4 \pi \sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left[ 4 \left( 2 + \cos(2\pi f T_{MZ1}) \right) \cdot \left( 2 + \cos(2\pi f T_{MZ2}) \right) \right] \\
+ 32 e^{-\pi^2 \sigma^2 (\Delta T_{MZ})^2} \cos(2\pi \nu_0 \Delta T_{MZ}) \cdot \cos(\pi f T_{MZ1}) \cdot \cos(\pi f T_{MZ2}) \\
+ 2 e^{-4\pi^2 \sigma^2 (\Delta T_{MZ})^2} \cos(4\pi \nu_0 \Delta T_{MZ})
$$

Equation 3-60

For a well-matched interferometer this may be written as:

$$
N_{\text{Dual MZ}}(f) \approx \left( g \alpha \right)^2 R \frac{I_0^2}{2^8} \frac{1}{\sqrt{4 \pi \sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left[ 4 \left( 2 + \cos(2\pi f T_{MZ1}) \right) \cdot \left( 2 + \cos(2\pi f T_{MZ2}) \right) \right] \\
+ 32 \cos(\pi f T_{MZ1}) \cdot \cos(\pi f T_{MZ2}) + 2 
$$

Equation 3-61

We can see that the noise spectrum is highly dependent on the relative phase bias (and on the individual time delays $T_{MZ1}$ and $T_{MZ2}$), $\Delta \phi_b = (2\pi \nu_0 \Delta T_{MZ})$, with $\Delta T_{MZ} = |T_{MZ1} - T_{MZ2}|$. For quadrature biasing, i.e. $\Delta \phi_b \approx \frac{\pi}{2}$, this becomes:

$$
N_{\text{Dual MZ}}(f) \approx \left( g \alpha \right)^2 R \frac{I_0^2}{2^8} \frac{1}{\sqrt{4 \pi \sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left[ 4 \left( 2 + \cos(2\pi f T_{MZ1}) \right) \cdot \left( 2 + \cos(2\pi f T_{MZ2}) \right) \right] \\
- 2 e^{-4\pi^2 \sigma^2 (\Delta T_{MZ})^2}
$$

Equation 3-62
The noise spectrum, given by Equation (3-60), is plotted in Figure (3-14 a) below for three different relative phase biases.

**Polarisation Flipping case**

For the PF case, i.e. using Equation (3-59) for the covariance function, we find for the noise spectrum:

\[
N_{\text{Dual MZ}}(f) = (g\alpha)^2 R \frac{I_0^2}{2^8} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 4\left[1 + \left(1 + \cos(2\pi f_{\text{MZI}})\right) \cdot \left(1 + \cos(2\pi f_{\text{MZII}})\right)\right] \right)
+ 16e^{-\pi^2(\Delta T_{\text{MZ}})^2} \cos(2\pi v_0 \Delta T_{\text{MZ}}) \cdot \cos(\pi f_{\text{MZI}}) \cdot \cos(\pi f_{\text{MZII}})
+ 2e^{-4\pi^2(\Delta T_{\text{MZ}})^2} \cos(4\pi v_0 \Delta T_{\text{MZ}})
\]

**Equation 3-63**

For a quadrature-biased interferometer system this becomes:

\[
N_{\text{Dual MZ}}(f) \approx (g\alpha)^2 R \frac{I_0^2}{2^8} \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 4\left[1 + \left(1 + \cos(2\pi f_{\text{MZI}})\right) \cdot \left(1 + \cos(2\pi f_{\text{MZII}})\right)\right] \right)
- 2e^{-4\pi^2(\Delta T_{\text{MZ}})^2}
\]

**Equation 3-64**

This spectrum is plotted in Figure (3-14 b) below for three different relative phase biases. Once, again, a periodically modulated noise spectrum is obtained, however, the detailed form of the noise is actually significantly different relative to the PP case. For the PF case (Figure (3-14 b)) the general features of the noise redistribution were much the same, although there are slight differences in the detailed spectral shape. The most important difference is that the maximum noise power is about 2.5 dB lower in this instance (0 phase bias). This agrees with the 2.5 dB noise reduction found earlier for the single MZI (see Figure (3-2)). The main conclusion of the analysis is that the PF case has reduced noise levels relative to the PP case in an appropriately designed interferometer system.
Figure 3-14: Predicted source NSD after being filtered by a dual MZI, for three different relative phase biases ($\Delta \phi = 0, \pi/2$ and $\pi$) and for the PP (a) and the PF (b) polarisation settings. Both MZI's had a path-imbalance of around 29.6 metres. The length mismatch was 1.1 mm. The source bandwidth was 7.7 GHz. The received optical power was set to $-23$ dBm.

3.3.2. Fringe visibility of a dual Mach-Zehnder Interferometer

For the time averaged output intensity of a dual MZI we get:

$$\langle I(t) \rangle = \frac{1}{2^4} \left( E(t) + E(t - T_{MZ1}) + E(t - T_{MZ2}) + E(t - T_{MZ1} - T_{MZ2}) \right)^2$$

Equation 3-65

Replacing each second-order term with the corresponding correlation function, we get:

$$\langle I(t) \rangle = \frac{1}{2^4} \left( 4 \Gamma(0) + 2 \Gamma(T_{MZ1}) + 2 \Gamma(T_{MZ2}) + 2 \Gamma(T_{MZ1} + T_{MZ2}) + \Gamma(T_{MZ1} + T_{MZ2}) + \Gamma(\Delta T_{MZ}) + \Gamma(\Delta T_{MZ}) \right)$$

Equation 3-66
After substitution of the electric field correlation function, given by Equation (2-18) from the previous chapter, we get for the time averaged output intensity of the dual MZI:

\[
\langle I(t) \rangle = \frac{I_0}{2^4} \left( 4 + 2V \cdot \cos\left(2\pi\nu_0 \Delta T_{MZ}\right) \right)
\]

Equation 3-67

where \( V \) is given by:

\[
V = \frac{1}{2} e^{-2\pi^2(\Delta T_{dc})^2} \left| \int_{-\infty}^{\infty} \psi(x) e^{-2\pi i \Delta T_{dc}} dx \right|
\]

Equation 3-68

The visibility is half the value obtained earlier for the single MZI. This is due to the fact that in case of a dual MZI, only two of the four paths contribute to the signal (i.e. paths \( n = 1 \) and \( n = 2 \) in Figure (3-13)). The other two paths just add to the mean intensity at the receiver.

3.3.3. Phase resolution of a dual Mach-Zehnder Interferometer

After introducing the periodic phase modulation, we can write the time averaged output intensity, given by Equation (3-67) as follows:

\[
\langle I(t) \rangle = \frac{I_0}{2^4} \left( 4 + 2V \cdot \cos\left(2\pi\nu_0 \Delta T_{MZ} + \phi_s \sin\left(2\pi f_s t\right)\right) \right)
\]

Equation 3-69

The signal spectrum for a small modulation signal is found to be:

\[
|S(f)| = (g\alpha)^3 R \frac{I_0}{2^8} \left( 16\delta(f) + 4V^2 \cdot \frac{\phi_s^2}{2^2} \left[ \delta(f + f_s) + \delta(f - f_s) \right] \right)
\]

Equation 3-70
This signal spectrum is illustrated in Figure (3-15) below. The fundamental or DC component of the spectrum, represented by the Delta function $\delta(f)$ is situated at zero frequency, with the two other Delta functions $\delta(f \pm f_s)$, representing the modulation signal, centred around the fundamental.

![Amplitude signal frequency spectrum](image)

**Figure 3-15**: Illustration of the amplitude signal frequency spectrum as seen at the receiver output of a dual MZI. The signal is superposed on the filtered source noise spectrum.

Note, that in this instance the noise floor is not constant across the frequency range plotted in Figure (3-15), indeed it shows a periodicity which is due the filtering of the source noise by the two unbalanced MZI's (Equation (3-60)). The signal is superposed on the filtered source noise spectrum and coincides with a maximum in the noise spectrum, since the FSR is generally much larger than the signal frequency. Employing heterodyne detection, it is possible to appropriately shift the signal frequency to a minimum in the filtered noise spectrum.

In Equation (3-70) $V$ is twice the signal visibility and is given by Equation (3-68). After substitution of the noise spectrum $|N(f \pm f_s)|$ given previously by Equation (3-62), and the visibility, given by Equation (3-68), we obtain for the phase resolution:
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\[
(\phi_y)_{\min}^2 = \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{T^2}{4\sigma^2}} \cdot e^{4\pi^2\sigma^2(\Delta T_{MZ})^2} \left( 4 \left[ 2 + \cos(2\pi T_{MZ1}) \right] \cdots \left[ 2 + \cos(2\pi T_{MZ2}) \right] \right) - 2e^{-4\pi^2\sigma^2(\Delta T_{MZ})^2}
\]

Equation 3-71

The phase resolution of the quadrature-biased dual MZI given by Equation (3-71) is plotted in Figure (3-16) below as a function of source bandwidth, for three different length mismatches and \( L_{MZ} = 29.60 \) metres.

![Graph showing phase resolution vs. source bandwidth for a quadrature-biased dual MZI](image)

**Figure 3-16:** Predicted phase resolution as a function of source bandwidth for a quadrature-biased dual MZI, for three different length mismatches (\( \Delta L_{MZ} = 15 \) mm, 10 mm and 5 mm) and \( L_{MZ} = 29.60 \) m. Calculations were done for a detection frequency of 0 Hz.

For the case where the two MZI's are well-matched, the phase resolution can be written
in the following form:

\[
(\phi_2)_{\text{min}}^2 \approx \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-\frac{f^2}{4\sigma^2}} \cdot \left( 4 \left[ 2 + \cos(2\pi f_{MZ1}) \right] \cdot \left[ 2 + \cos(2\pi f_{MZ2}) \right] + 32\cos(\pi f_{MZ1}) \cdot \cos(\pi f_{MZ2}) + 2 \right)
\]

Equation 3-72

For quadrature-biased, well-matched interferometers, i.e. \((2\pi\sigma \Delta f_{MZ}) \ll 1\), the phase resolution in the low frequency region \(f < 1\) MHz becomes:

\[
(\phi_2)_{\text{min}}^2 \approx 4 \cdot 34 \cdot \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{4\pi^2 \sigma^2 (\Delta f_{MZ})^2}
\]

Equation 3-73

For example for a source bandwidth of 6.2 GHz we find a phase resolution of 121 \(\mu\text{rad}/\sqrt{\text{Hz}}\) at a detection frequency of 0 Hz, when biasing at the quadrature point.

### 3.3.4. Experiments and Discussion

In order to validate the above analysis we constructed a dual MZI and examined its filtering properties when interrogated with the narrowband ASE source described in the second chapter (incorporating FBG3, see Table (2-1)). The delay length in each of the interferometers was \(\sim 29.6\) metres. The coupling ratio of all couplers was 50:50, and PC’s were placed in each interferometer to enable us to set the SOP throughout the system for the PP and PF cases previously described. One PC allowed the polarisations of the two paths carrying the signal (paths \(n = 1\) and \(n = 2\) in Figure (3-13 b and c)) to be aligned so that the modulation depth was maximised. The other PC allowed the shortest and the longest paths (paths \(n = 0\) and \(n = 3\) in Figure (3-13 b and c)) to have polarisations either parallel (PP case, Figure (3-13 b)) or perpendicular (PF case, Figure (3-13 c)) to that of the signal paths. A PZT was included within the first interferometer to allow us to adjust the relative phase bias of the two interferometers. We measured the noise spectrum in the frequency range of \(0 \sim 20\) MHz for the PP and PF cases, for two values of length mismatch between the interferometers (1.1 and
9.3 mm), and for three values of phase bias. Since the output power was a function of the phase bias the output power at the phase bias of \( \pi/2 \) was set to be \(-23 \) dBm for all measurements.

The PC’s were set as follows for the PP and PF cases. First, we introduced a large bend loss in one of the arms of the second interferometer, and set the PC in the first interferometer such that the spectral noise power was either maximised, (or minimised). This allowed us to set the output polarisation states of the light passing through the two different paths in the first interferometer (i.e. paths \( n = 0 \) and \( n = 1 \)) to be either mutually parallel, (or orthogonal). Next, we removed the bend loss in the second interferometer, and set the PC within this interferometer to maximise the signal visibility (i.e. set the SOP of paths \( n = 1 \) and \( n = 2 \) to be the same), resulting in either the PP, (or PF) configuration.

![Figure 3-17: The calculated and measured NSD after being filtered by a dual MZI, for three different relative phase biases and for the PP case (a) and the PF case (b). (\( \Delta \phi = 0, \pi/2 \) and \( \pi \)). Both MZI's had a path-imbalance of around 29.6 metres. The length mismatch was 1.1 mm. The source bandwidth was 7.7 GHz. The received optical power was set to \(-23 \) dBm.](image)

Figure (3-17 a and b) show the noise spectra for the three different phase bias points, for the length mismatch of 1.1 mm, and for the different polarisation cases. The noise redistribution properties of the interferometer are immediately apparent, recall from Figure (2-7) in the previous chapter that the source noise itself is a uniform \(-105.6 \) dBm/Hz over this frequency range for a fixed optical power of \(-23 \) dBm. The
agreement between experimental and theoretical noise spectra is good. For the $\pi/2$ phase bias in Figure (3-17 a) (--- and $\circ$) the noise spectrum has a FSR of 6.92 MHz, which corresponds to the inverse of the elapsed time for light to pass through the path-imbalance of the individual interferometers. The noise power is highest ($-99$ dBm/Hz) at 0 relative phase bias ($-$ and $\Box$) at the spectral maxima (e.g. at 7.62 MHz), and falls to its smallest value of $-115$ dBm/Hz at the noise minima (e.g. at 3.46 MHz) for a phase bias of $\pi/2$. The maximum noise level difference (maxima to minima) at fixed phase bias is about 12 dB for the $\pi/2$ phase bias.

For the PF case (Figure (3-17 b)) the general features of the noise redistribution were much the same, although there are slight differences in the detailed spectral shape. The most important difference is that the maximum noise power is about 2.5 dB lower in this instance (0 phase bias). This agrees with the 2.5 dB noise reduction found earlier for the single MZI (see Figure (3-2)). The experimental results for the length mismatch of 9.3 mm showed the same features, although the dependence of the phase bias became weaker due to the lower signal visibility ($\sim 30\%$), compared to the former case ($\sim 48\%$).

From these results, it is clear that a higher overall sensitivity might be achieved for such an interferometer system by using the periodicity in the noise spectrum, simply by moving the signal frequency (typically in the low frequency regime), to a spectral region with a minimum in the noise distribution. This was recognised earlier [47], and can be readily achieved by incorporating a FS into one of the MZI's, and operating in a heterodyne fashion, as will be demonstrated in the next sub-section.

It has to be appreciated that, due to the reasonably long interferometer paths $\sim 29.6$ metres, there was an increased sensitivity to temperature fluctuations and other environmental factors. This manifests itself in the phase detection as a low frequency noise drift. Mechanical vibrations are also another potential source of noise. In order to minimise these factors, an isolating enclosure was set-up for the whole arrangement, providing a homogeneous environment for the interferometer and greatly improving the system stability.

3.3.5. The dual Mach-Zehnder Interferometer incorporating a Frequency Shifter

A schematic of a dual MZI (with time delays $T_{MZ1}$ and $T_{MZ2}$) that incorporates a FS in
the first interferometer is shown in Figure (3-18) below.

![Diagram of a dual MZI with FS and PC]

**Figure 3-18:** A schematic of the dual MZI used in this work. The FS is incorporated in one arm of the first MZI.

---

**Polarisation Preserving case**

With the incorporation of the FS in one of the arms of the first interferometer, we can write the output field $E_{\text{out}}(t)$, expressed earlier by Equation (3-55), as:

$$E_{\text{out}}(t) = \frac{1}{2} \left( E(t) + E(t - T_{MZ1}) \cdot e^{2\alpha g_{fs} t} + E(t - T_{MZ2}) + E(t - T_{MZ1} - T_{MZ2}) \cdot e^{2\alpha g_{fs} t} \right)$$

**Equation 3-74**

For the output intensity, given by Equation (3-65), we find:

$$\langle I(t) \rangle = \frac{1}{2} \left( E(t) + E(t - T_{MZ1}) \cdot e^{2\alpha g_{fs} t} + E(t - T_{MZ2}) + E(t - T_{MZ1} - T_{MZ2}) \cdot e^{2\alpha g_{fs} t} \right)^2$$

**Equation 3-75**

The covariance function of the output intensity is given by Equation (2-8) in the second chapter. After multiplying out all terms, and neglecting terms that give insignificant contributions to the noise spectrum, we can write this as follows:
\[ \text{Cov}_T(\tau) = \frac{1}{2^8} \left( 8|\Gamma(\tau)|^2 + 2|\Gamma(\tau + T_{MZ1})|^2 + 2|\Gamma(\tau - T_{MZ1})|^2 + 2|\Gamma(\tau + T_{MZ2})|^2 + 2|\Gamma(\tau - T_{MZ2})|^2 \\
+ |\Gamma(\tau + (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau - (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau + \Delta T_{MZ})|^2 + |\Gamma(\tau - \Delta T_{MZ})|^2 \\
+ 2 \left( 4|\Gamma(\tau)|^2 + |\Gamma(\tau + T_{MZ1})|^2 + |\Gamma(\tau - T_{MZ1})|^2 + |\Gamma(\Delta T_{MZ})|^2 \right) \cdot \cos(2\phi_0 \tau) \right) \]

**Equation 3-76**

**Polarisation Flipping case**

In this case (Figure (3-13 c)) the output field \( E_{\text{out}}(t) \) of the dual MZI, can be expressed as (see Equation (3-58)):

\[ E_{\text{out}}(t) = \frac{1}{2^4} \left( \begin{array}{c} e_x \left( E(t) + E(t - T_{MZ1} - T_{MZ2}) \cdot e^{2\phi_{0} t} \right) \right) + e_y \left( E(t - T_{MZ2}) + E(t - T_{MZ1}) \cdot e^{2\phi_{0} t} \right) \]

**Equation 3-77**

According to Equation (3-43), the output intensity can be expressed as:

\[ \langle I(t) \rangle = \frac{1}{2^4} \left( \left| E(t) + E(t - T_{MZ1} - T_{MZ2}) \cdot e^{2\phi_{0} t} \right|^2 + \left| E(t - T_{MZ2}) + E(t - T_{MZ1}) \cdot e^{2\phi_{0} t} \right|^2 \right) \]

**Equation 3-78**

Performing some tedious algebra, the covariance function, given by Equation (3-9) in this chapter can be written in the following form:

\[ \text{Cov}_T(\tau) = \frac{1}{2^8} \left( 4|\Gamma(\tau)|^2 + 2|\Gamma(\tau + T_{MZ1})|^2 + 2|\Gamma(\tau - T_{MZ1})|^2 + 2|\Gamma(\tau + T_{MZ2})|^2 + 2|\Gamma(\tau - T_{MZ2})|^2 \\
+ |\Gamma(\tau + (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau - (T_{MZ1} + T_{MZ2}))|^2 + |\Gamma(\tau + \Delta T_{MZ})|^2 + |\Gamma(\tau - \Delta T_{MZ})|^2 \\
+ 2 \left( 4|\Gamma(\tau)|^2 + |\Gamma(\tau + T_{MZ1})|^2 + |\Gamma(\tau - T_{MZ1})|^2 + |\Gamma(\Delta T_{MZ})|^2 \right) \cdot \cos(2\phi_0 \tau) \right) \]

**Equation 3-79**
3.3.6. Noise Spectral Density of the output of a dual Mach-Zehnder Interferometer incorporating a Frequency Shifter

**Polarisation Preserving case**

The NSD at frequency \( f \) is given by Equation (2-7) in the previous chapter. Substitution of the covariance function given by Equation (3-76) gives the following noise spectrum:

\[
N_{\text{DualMZ}}(f) = (g\alpha)^2 R \frac{I_0^2}{2^8 \sqrt{4\pi\sigma^2}} \left[ 4e^{-\frac{f^2}{4\sigma^2}} \left\{ 1 + \cos(2\pi f T_{M21}) \right\} \left\{ 2 + \cos(2\pi f T_{M22}) \right\} \right] \\
+ 2e^{-\frac{(f+f_h)^2}{4\sigma^2}} \left[ 4 + 2\cos(2\pi f T_{M22}) \right] + 2e^{-\frac{(f-f_h)^2}{4\sigma^2}} \left[ 4 + 2\cos(2\pi f T_{M22}) \right] \\
+ \Gamma(\Delta T_{MZ})^2 \left\{ \delta(f + f_h) + \delta(f - f_h) \right\}
\]

**Equation 3-80**

**Polarisation Flipping case**

For the PF case we find for the noise spectrum (using Equation (3-79)):

\[
N_{\text{DualMZ}}(f) = (g\alpha)^2 R \frac{I_0^2}{2^8 \sqrt{4\pi\sigma^2}} \left[ 4e^{-\frac{f^2}{4\sigma^2}} \left\{ 1 + \cos(2\pi f T_{M21}) \right\} \left\{ 2 + \cos(2\pi f T_{M22}) \right\} \right] \\
+ 2e^{-\frac{(f+f_h)^2}{4\sigma^2}} \cdot \cos(2\pi f T_{M22}) + 2e^{-\frac{(f-f_h)^2}{4\sigma^2}} \cdot \cos(2\pi f T_{M22}) \\
+ \Gamma(\Delta T_{MZ})^2 \left\{ \delta(f + f_h) + \delta(f - f_h) \right\}
\]

**Equation 3-81**

The terms containing \( e^{-\frac{f^2}{4\sigma^2}} \) in the expression for the noise power imply noise redistribution due to the frequency shift \( f_h \) and the terms containing \( \delta(f \pm f_h) \) correspond to the carrier signal in the heterodyne method. The noise spectrum is plotted in Figure (3-19) below both for the PP (---) and the PF (-----) configuration.
Figure 3-19: Calculated noise spectra for the dual MZI incorporating a FS, both for the PP (---) and the PF (----) configuration. In the calculations the source bandwidth, path-imbalance, length mismatch and received optical power was set to be 7.7 GHz, 26.52 metres, 1.3 mm and -23 dBm respectively.

Again the noise spectrum is found to exhibit periodic structure ($FSR \sim 8$ MHz). Although in this instance the issue of relative phase bias between the two interferometers ceases to be of any significance (due to the incorporation of the FS). Note that the PF configuration produces the lowest noise power over the full detection bandwidth. The noise power for the PF configuration is around 5 dB lower compared to the PP case at specific frequencies.

3.3.7. Phase resolution of a dual Mach-Zehnder Interferometer incorporating a Frequency Shifter

After introducing the signal with a small modulation depth in Equation (3-75), we find for the signal spectrum (according to Equation (3-26)):
\[
|S(f)| = (g\alpha)^2 R_0^2 \frac{J_0^2}{2^2} \left\{ 16\delta(f) + V^2 \left[ \delta(f + f_h) + \delta(f - f_h) \right] \right\}
+ V^2 \frac{\phi_1^2}{2^2} \left[ \delta(f + (f_h + f_s)) + \delta(f + (f_h - f_s)) + \delta(f - (f_h + f_s)) + \delta(f - (f_h - f_s)) \right]
\]

Equation 3-82

This signal spectrum is shown schematically in Figure (3-20) below. The fundamental or DC component of the spectrum, represented by the Delta function \(\delta(f)\) is situated at zero frequency, with the two other Delta functions \(\delta(f \pm f_h)\) centred around the fundamental, representing the heterodyne signals. The modulation signals are centred around the two heterodyne signals.

![Diagram](image)

**Figure 3-20:** Illustration of the amplitude signal frequency spectrum as seen at the receiver output of a dual MZI employing heterodyne detection. The signal is shifted to a minimum in the noise spectrum by the heterodyne technique. Note that on the RF spectrum analyser only the positive frequencies will be shown.

Again the noise floor shows a periodicity with an FSR related to the inverse of the time of flight through the MZI's. Note that the carrier frequency, as defined by the FS in the first MZI, has been appropriately chosen to coincide with a minimum in the filtered noise spectrum produced by the two unbalanced MZI's. Equating the signal spectrum given by Equation (3-82) to the NSD given previously by Equation (3-80), and
substitution of the visibility given by Equation (3-68), we obtain for the phase resolution of the dual MZI:

\[
(\phi_p)_{\text{mm}}^2 = \frac{1}{\sqrt{4\pi \sigma^2}} \cdot e^{4\pi^2 \sigma^2 (\Delta T_{MZ})^2} \cdot \left( 4e^{\frac{-f_p^2}{4\sigma^2}} \cdot \left\{ 1 + \cos(2\pi f T_{MZ1}) \right\} \cdot \left\{ 2 + \cos(2\pi f T_{MZ2}) \right\} + 2e^{\frac{-f_p^2}{4\sigma^2}} \cdot \left\{ 4 + 2\cos(2\pi(f + f_h) T_{MZ2}) \right\} + 2e^{\frac{-f_p^2}{4\sigma^2}} \cdot \left\{ 4 + 2\cos(2\pi(f - f_h) T_{MZ2}) \right\} \right) + \left| \Gamma(\Delta T_{MZ}) \right|^2 \left( \delta(f + f_h) + \delta(f - f_h) \right)
\]

Equation 3-83

For well-matched MZI’s, the exponential term \(e^{4\pi^2 \sigma^2 (\Delta T_{MZ})^2}\) representing the visibility can be ignored from the expression above. The phase resolution of the dual MZI incorporating a FS, is plotted in Figure (3-21) below as a function of the source bandwidth (for two length mismatches and both the PP and PF configurations).
Figure 3-21: Predicted phase resolution as a function of source bandwidth for a dual MZI employing heterodyne detection, for two different length mismatches and two different polarisation settings. Calculations were done for optimum signal frequency, i.e. at 27.12 MHz (for $L_{MZ1} \approx L_{MZ2} \approx 26.52$ metres).

Clearly, the calculations presented in Figure (3-21) above show that for a practical length mismatch of around 1 cm, maximum phase resolution is obtained for a source bandwidth of around 5 GHz. When the system’s polarisation setting is configured for the PP case a phase resolution of around 105 $\mu$rad/$\sqrt{\text{Hz}}$ (---) is obtained and for the PF configuration this figure is increased to around 62 $\mu$rad/$\sqrt{\text{Hz}}$ (--)..

3.3.8. Experiments and Discussion

We built the dual MZI incorporating a FS shown in Figure (3-18) to validate the above theory. The delay length of each MZI was 26.52 metres, a length chosen to minimise the noise around the heterodyne frequency of 27.12 MHz, as defined by the FS. A PZT was inserted in one of the arms of the second interferometer to allow us to apply a well-defined level of phase modulation. PC’s were included in the system to allow us to set
the SOP within the system for either PP or PF implementations. A picture of the PZT in one of the interferometers is shown in Figure (3-22) below.

**Figure 3-22:** A close-up picture of the piezoelectric transducer (PZT) tube in one of the interferometers, as shown in Figure (3-18). The PZT tube is shown on the left. The fibre wound around the PZT can be clearly seen. The remaining length of fibre of the interferometer arm is wound around half a Coke can, which is shown on the right. The twisted red and black wire on the top of the PZT is electrical wire for applying a voltage across the inner and outer wall of the PZT. The PZT and the can were placed in a small box, in order to try to shield the rest of the sensing system from the acoustic waves generated by the PZT.

The stability of the sensing system is mainly affected by a change in SOP within the system or vibrations due to acoustic perturbations. Therefore, the system was shielded from any environmental effects, like thermal noise and acoustic noise, by enclosing it in two perspex boxes with lids on top (see Figure (3-23) below). The two boxes were placed on four foam legs on an optical bench, to prevent any vibration from the bench entering the boxes. The system was further shielded by placing bubble wrap all around each box.
Figure 3-23: A digital picture of the full experimental sensing system. The two perspex boxes on the optical bench contain the sensors, which are shielded from environmental noise (mainly acoustic noise). The left box has its lid removed, and Figure (4-15) gives an inside view. The ASE source for sensor interrogation can be seen on the left. An RF spectrum analyser for detection purposes is situated in the rack, in far-left corner of the picture. Bubble wrap, around the boxes, was used to further shield the system.

We performed experiments for length mismatches of 1.3 and 10.9 mm. The length mismatches were estimated from the $FSR$'s of the measured noise spectra of the individual interferometers as measured to a resolution of $\pm$ 0.6 mm. The average optical power was set to $-23$ dBm throughout. Figure (3-24) below shows the source NSD after being filtered by system, both for the PP and the PF implementation.
Figure 3-24: Calculated and measured NSD after being filtered by a dual MZI employing heterodyne detection, for two different polarisation settings. Each interferometer had a path-imbalance of around 26.52 metres, and the length mismatch was 1.3 mm. The source bandwidth was 7.7 GHz. The received optical power was -23 dBm.

Figure (3-24) above shows the noise spectra for the source bandwidth of 7.7 GHz obtained from both theory and experiments for the different polarisation settings, and for a length mismatch of 1.3 mm. The noise level of the PF case (----- and •) is seen to be smaller than of the PP case (— and □) over the full detection bandwidth, falling to -112 dBm/Hz at the spectral minima. In both cases the signal power is the same for a fixed level of phase modulation (via the PZT). The noise level is thus substantially lower (~ 5 dB) for the PF case. Similar patterns and levels of noise redistribution were obtained for other values of source bandwidth and length mismatch. In the following part, we will present results on the phase resolution of this system. To get an accurate phase reading from an interferometer system, the system first needs to be calibrated.
Sensor Calibration

Sensor calibration was done as follows. Earlier we saw that, in general, the transfer function of an interferometer system can be given by a raised cosine function. When we apply a small modulation signal to one of the interferometers, i.e. we modulate one arm of the MZI with length $l_0$ periodically with period $f_s$. The arm length is then $l(t) = (l_0 + \Delta l(t))$. The change in length is given by $\Delta l(t) = \Delta l \cdot \sin(2\pi f_s t)$. That is the length is $l(t) = (l_0 + \Delta l \cdot \sin(2\pi f_s t))$, so the system's transfer function can be written as:

$$
\langle I(t) \rangle = \frac{I_0}{2} \left( 1 + |V| \cdot \cos \left( \phi_s \sin(2\pi f_s t) + \phi_0 \right) \right)
$$

Equation 3-84

In this equation, $\phi_s = (\beta \cdot l_0 + 2\pi f_s t)$ represents the phase bias of the interferometer system being swept continuously in time due to the heterodyne technique, and $\phi = \beta \cdot \Delta l$ is the modulation depth, with $\beta$ a constant. Using the trigonometric identity $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, we can write this in the following form:

$$
\langle I(t) \rangle = \frac{I_0}{2} \left( 1 + |V| \cdot \left\{ \cos(\phi_s \sin(2\pi f_s t)) \cdot \cos(\phi_s) - \sin(\phi_s \sin(2\pi f_s t)) \cdot \sin(\phi_s) \right\} \right)
$$

Equation 3-85

The cosine of a sine and the sine of a sine can be written as Bessel functions $J_n(x)$ with order $n$. The function $\cos(x\sin\theta) = \left( J_0(x) + 2J_2(x) \cdot \cos(2\theta) + 2J_4(x) \cdot \cos(4\theta) + \cdots \right)$ and converges to $J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x) \cdot \cos(2n\theta)$, where $J_n(x)$ is the value of $x$ of the $n$'th order. Replacing the sinusoidal function (the second term between the curly brackets), we can now write the system's transfer function as a Fourier expansion in terms of Bessel functions:
\[
\langle I(t) \rangle = \frac{I_o}{2} \left( 1 + |V| J_0(\phi_s) \cdot \cos(\phi_b) + 2|V| \sum_{n=1}^{\infty} J_{2n}(\phi_s) \cdot \cos(4\pi f_s t) \cdot \cos(\phi_b) \right) \\
- 2|V| \sum_{n=0}^{\infty} J_{2n+1}(\phi_s) \cdot \sin(2\pi(2n+1)f_s t) \cdot \sin(\phi_b)
\]

Equation 3-86

This expression shows that the output signal consists of a series of weighted harmonics of the signal frequency. The frequency spectrum has a component at the heterodyne frequency and an infinite number of sidebands separated from the heterodyne frequency by integer multiples of the signal frequency. The weighting factor for each sideband is a function of the modulation depth and the value of the corresponding Bessel function evaluated at the modulation depth. The first two terms \( \frac{I_o}{2} \left( 1 + |V| J_0(\phi_s) \cdot \cos(\phi_b) \right) \) represent the fundamental (i.e. the DC component) of the average output intensity, or the heterodyne signal. The two last terms (the summation of Bessel functions) represent the harmonics, i.e. the AC component of the intensity transfer function. If the interferometer system is quadrature-biased, i.e. \( \phi_b \approx \frac{\pi}{2} \), and \( \cos(\phi_b) \approx 0 \) and \( \sin(\phi_b) \approx 1 \) then the even harmonics drop from the transfer function. The intensity transfer function then comprises only the odd harmonics:

\[
\langle I(t) \rangle \approx \frac{I_o}{2} \left( 1 - 2|V| \sum_{n=0}^{\infty} J_{2n+1}(\phi_s) \cdot \sin(2\pi(2n+1)f_s t) \right)
\]

Equation 3-87

For small phase modulations, harmonics above the fundamental modulation frequencies, \( J_0(\phi_s) \), are small and these higher terms can therefore be neglected. If we only consider the first two odd harmonics to be of any significance, we get

\[
\langle I(t) \rangle \approx \frac{I_o}{2} \left( 1 - 2|V| \left( J_1(\phi_s) \cdot \sin(2\pi f_s t) + J_3(\phi_s) \cdot \sin(6\pi f_s t) \right) \right)
\]

Equation 3-88
The modulation depth, $\phi_{s}$, applied to the sensor, can be calculated from the difference in power from the first ($n = 0$) and third ($n = 1$) harmonics, which correspond to the first and third order Bessel functions, $J_1(\phi_s)$ and $J_3(\phi_s)$. The difference in electrical power is given by $(S_1 - S_3) = 10\log\left(\frac{J_3(\phi_s)}{J_1(\phi_s)}\right)^2$, where $S_1$ and $S_3$ represent the electrical power (in dBm) in the first and third harmonic respectively. From this, the modulation depth, $\phi_s$, can be readily solved:

\[
10 \frac{(S_1 - S_3)}{20} = \frac{J_3(\phi_s)}{J_1(\phi_s)}
\]

Equation 3-89

Another way of checking the modulation depth applied to the sensing interferometer, is by applying a specific voltage to the PZT which corresponds to a modulation depth of 2.405 rad. At this specific modulation depth the zeroth Bessel peak (or the heterodyne signal) disappears from the detected output frequency spectrum, as can be seen when we plot the zeroth Bessel function.
Figure 3-25: Behaviour of the zeroth-order Bessel function, \( J_0(\phi) \) as a function of the signal modulation depth. This function is zero at \( \phi = 2.405 \) radian.

When the DC heterodyne signal, \( J_0(\phi) \), disappears from the output spectrum, we know that a modulation depth of \( \phi = 2.405 \) radians is being applied, since the zeroth order Bessel function vanishes at this value, as can be seen from the figure above where this function is plotted. If we then assume a linear relationship of modulation depth with applied voltage (see Figure (3-26)), we can extrapolate to the required modulation depth by using the linear relationship, where we assume the modulation frequency \( f_s \) to be constant, i.e.:

\[
\left. \left( \frac{\phi_1}{V_{pp1}} \right) \right|_{f_s} = \left. \left( \frac{\phi_2}{V_{pp2}} \right) \right|_{f_s}
\]

Equation 3-90

Here \( V_{pp} \) is the voltage applied to the PZT in Volts peak-to-peak. Note, that if we
consider only the first harmonic, i.e. for \( n = 0 \), and that for a small modulation depth \( J_1(\phi) \approx \phi \). Equation (3-88) reduces to the original definition of the transfer function, i.e. Equation (3-84). Figure (3-26) below shows the modulation depth from the PZT versus the applied voltage to the PZT for two modulation frequencies.

![Graph showing modulation depth vs voltage applied to PZT](image)

**Figure 3-26:** The experimental sensor calibration plot, giving the phase modulation depth as a function of the applied voltage to the PZT in the sensor for two modulation frequencies. Up to \( 2\pi \)-radians modulation depth, the system behaves linearly. The straight lines are lines fitted to the two experimental data sets.

A linear fit has been performed on the experimental points. The slope of this plot is a measure of the phase shifting capability, or efficiency, of the PZT in the sensor [48]. This depends on the number of loops of fibre wound on the PZT and on the tension with which the fibre is wound. The PZT’s phase shifting capability was around \( (584 \pm 20) \text{ mrad/Vpp (at } f_s = 15.0 \text{ kHz)} \) for the sensor (indicated with □). The system’s response was observed to be linear with respect to the applied phase shift up to \( 2\pi \)-radians.
Measurement principle of the phase resolution

The phase resolution was measured by applying a known level of phase modulation to the PZT and observing the heterodyne signal and noise floor on an RF spectrum analyser. Figure (3-27) shows the spectrum with a resolution bandwidth of 300 Hz when a phase modulation of 25 mradian was applied for the PF case. The phase resolution was 44 μrad/√Hz in this instance.

![Spectral Power vs Frequency Deviation](image)

**Figure 3-27:** Experimental signal and noise corresponding to system parameters described within the text. The resolution bandwidth of the RF spectrum analyser was 300 Hz. The heterodyne frequency was 27.12 MHz.

Say we apply a 10 kHz phase modulation to the MZI with a 25 mrad modulation depth, as shown in Figure (3-27) above. We then measure the signal strength, S, with a 1 kHz resolution bandwidth and the optical noise floor, N, with a 1 Hz resolution bandwidth on an RF spectrum analyser. Now, we can calculate the SNR by simply subtracting the optical noise floor power in dBm/Hz from the signal strength in dBm (on a logarithmic basis). The minimum detectable phase modulation we are able to detect is when the signal is just buried in the optical noise floor, i.e. for $S = N$ or $\text{SNR} = 1$. We can now
use the linear relation between the applied phase modulation and the SNR for a constant modulation frequency \( (\phi_s)_1 + \text{SNR}_1 f_s = (\phi_s)_2 + \text{SNR}_2 f_s \). Where \( (\phi_s)_2 \) is the well defined phase modulation at a specific modulation frequency in radians, \( \text{SNR}_2 \) is the SNR calculated from the RF spectrum of the system output in a linear unit, and \( (\phi_s)_1 \) is the phase resolution of a particular sensor channel at a particular signal frequency in radians. If we now set the SNR to be 1, i.e. \( \text{SNR}_1 = 1 \), we can solve for the system’s phase resolution:

\[
(\phi_s)_\text{min} = \left. (\phi_s)_2 \right|_{\text{SNR}_2 = 1} \frac{\text{SNR}_2}{f_s}
\]

Equation 3-91

Example

We apply a 10 kHz modulation frequency to the sensor interferometer with a \( (\phi_s)_2 = 200 \) mrad modulation depth. The signal strength we measure is \( S = -24.3 \) dBm (measured with a 1 kHz resolution bandwidth) and the optical noise floor around 110 MHz is measured to be \( N = -91.35 \) dBm/Hz (1 Hz bandwidth), we get for the SNR, \( \text{SNR}_2 = -24.3 - (-91.35) = 67.05 \) dB at 1 kHz. We have to convert this to a linear unit, i.e. \( \text{SNR}_2 = 10^{\frac{67.05}{20}} \approx 2251.6 \). The phase resolution can now be obtained by using Equation (3-91), we find \( (\phi_s)_\text{min} \approx 89 \) μrad/√Hz at a modulation frequency of 10 kHz.

Measurements were made as a function of source bandwidth for two length mismatches of 1.3 and 10.9 mm, and for both PP and PF polarisation settings. The results are shown in Figure (3-28), superposed are theoretical curves derived using Equation (3-83) for the PP configuration and the phase resolution obtained for the PF configuration.
Figure 3-28: Theoretical and experimental phase resolution as a function of source bandwidth for a dual MZI incorporating a FS, for two different length mismatches and two different SOP's. In all cases the interferometer is automatically quadrature-biased by the incorporation of the FS. Calculations were done for optimum signal frequency, i.e. at 27.12 MHz (for $L_{MZ1}$ ≈ $L_{MZ2}$ ≈ 26.52 metres).

Good agreement between theory and experiment is obtained. Note that all parameters used in the theoretical curves are measured – there are no free-fit parameters in the theory. From Figure (3-28) it is seen that a maximum phase resolution of 62 $\mu$rad/$\sqrt{Hz}$ is obtained for a practical length mismatch of 10.9 mm ($\Delta$). The maximum value is obtained for a bandwidth of $\sim$ 5 GHz. At narrower source bandwidths the increased source noise reduces the sensitivity, whilst at broader bandwidths the rapidly reducing signal visibility limits performance. For lower (but impractical) length mismatches significantly higher phase resolutions can be achieved. For example, values as low as 25 $\mu$rad/$\sqrt{Hz}$ for a source bandwidth of 20 GHz are predicted (O). These results validate our theory and provide important information as how to design an optimised system for any level of practical limitation on length mismatch, source bandwidth, sensor length, etc.
3.4. Conclusions and Discussion

This chapter presented the results of a detailed theoretical and experimental study of the noise properties of various MZI based systems interrogated using narrowband spontaneous emission. The filtering effect of the interferometer is shown to introduce periodic structure in the optical noise spectrum with a period, level, and modulation depth that depends on the exact interferometer configuration and implementation, as well as the source bandwidth and spectral shape. Our theoretical analysis, based on the assumption of a Gaussian random process model for the inherent source noise is in good agreement with our experimental results. Finally, using a dual MZI incorporating a FS, we show that maximum phase resolutions of a few tens of μrad/√Hz can be achieved for practical values of length mismatch (~ 1 cm) by optimisation of the source bandwidth, heterodyne frequency, and SOP within the interferometer. For the two interferometer systems studied in this chapter I summarise the main conclusions as follows:

**Single Mach-Zehnder Interferometer**

The noise power is shown to be reduced by 2.5 dB over the full detection bandwidth when the light in the two arms of the MZI is orthogonally polarised (PF configuration), compared to mutually parallel polarisation states (PP configuration). For a path-imbalance of 1 cm this interferometer gives a visibility of 75 % and a phase resolution of 62 μrad/√Hz for a 6.2 GHz source bandwidth. Introducing a FS reduces the noise power by another 7 dB, then the PF configuration gives a 10 dB noise reduction compared to the PP configuration at specific frequencies defined by the path imbalance.

**Dual Mach-Zehnder Interferometer**

For a practical length mismatch of 1 cm, an optimum source bandwidth of 6.2 GHz gives a phase resolution of 50 μrad/√Hz and a visibility of 30 %, for quadrature-biased MZI's with a 29.6 metres path-imbalance. The fringe contrast in the noise spectrum is higher in the PP case compared to the PF case at specific frequencies for the π/2 phase bias (i.e. the quadrature phase bias). Introduction of the FS changes this and in that case the PF configuration produces 5 dB lower noise power. A phase resolution has been found of around 62 μrad/√Hz for interferometers with a 26.5 metres path-imbalance and a 1 cm length-mismatch when employing the heterodyne technique.
Chapter Four

Noise redistribution in Fabry-Perot Interferometer systems interrogated with Amplified Spontaneous Emission sources

Overview: In this part of the thesis, we will extend our discussion on the source noise redistribution to Fabry-Perot interferometer systems interrogated with ASE sources, and the effect that this has on the system performance.

4.1. System description

The use of FBG’s in FOAS is well established and documented. However, most of these FBG sensors are based on the measurement of the wavelength shift of a FBG caused by the measurand [49]. Miridonov et al. proposed a twin-grating sensor interrogated with a laser source [50]. This system employed both phase detection from the interference pattern produced by the light reflected from both FBG’s and FBG wavelength shift detection. Christmas et al. [51] and Choi et al. [52] both used low-coherence interferometry to interrogate FP sensing cavities. In the work described by Christmas the FP cavities were formed by two identical FBG’s, whereas Choi’s system was based on internal mirrors rather than FBG’s.

The FOAS system using a FP interferometer used in this work is drawn schematically in Figure (4-1) below. The system is different from the dual MZ system in the sense that the second MZI is now replaced with a FP interferometer (channel 1 and channel 2 in Figure (4-1) below). The FP interferometer is, basically, a cavity created by two inline FBG’s with a similar Bragg wavelength $\lambda_1$. A similar strain/temperature sensor system was previously developed by another group [31],
except that they did not spectrally slice the ASE source and relied on the FBG’s comprising the FP cavity to define the bandwidth of the detected light. Furthermore, a homodyne modulation technique was employed, instead of the heterodyne scheme employed in our system. In their other work [51] the heterodyne technique was employed. The primary differences between the FP and the MZI based systems are that in the FP systems the wavelength selectivity of the FBG’s makes the interferometers narrowband devices and hence suitable for WDM, and that the FP interferometers allow for multiple recirculations of light within the interferometer. I will first describe my experimental system and then review theoretically how the multiple-reflection nature of the FP devices effects the overall performance of the system.

![Schematic diagram](image)

**Figure 4-1:** Schematic of the dual-channel FOAS system employing Fabry-Perot interferometers.

The dual-channel narrowband ASE source used in our two-channel experiments is drawn schematically in Figure (2-12) in the second chapter, together with its output spectrum in Figure (2-13) (shown also inset (c) of Figure (4-1) above). The optical power in both source peaks at \( \lambda_1 \) and \( \lambda_2 \) was equalised by tuning the centre wavelength of the TF (160 GHz FWHM bandwidth) in the dual-channel narrowband ASE source.
The extinction ratio is seen to be around 40 dB, and 35 dB to the small lobe situated in between the two peaks. This small lobe is a genuine feature of the reflection spectrum of the pick-out filter FBG\textsubscript{p2} used in the dual-channel narrowband ASE source. For measurements on a particular channel, the other channel was appropriately filtered out prior to the receiver (inset (a) in Figure (4-1)). Inset (d) in the figure above shows the optical return from the two sensor channels to the receiver. Initially experiments were performed with a FP-TF prior to the receiver. The received optical spectrum is shown in inset (e). A small portion (5 \%) of the received light is coupled of to an optical power meter by a 95:5 coupler (C\textsubscript{4}) for received power monitoring purposes. Care was taken to eliminate optical feedback from loose fibre ends throughout the system, therefore all ends were terminated by angle cleaving and soaking them in index matching gel. Furthermore, index-matching oil was used in all FC/PC connectors to prevent feedback. Due to the non-ideal filter profile of the FP-TF (low extinction), the transmission spectrum still contains a contribution from channel 2 (see inset (e)). Reflection spectra of the four FBG's comprising the two FP sensing cavities are shown in Figure (4-2) below.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4-2}
\caption{Measured reflection spectra of the four FBG's employed in the two FP sensing cavities.}
\end{figure}
The reflection spectra of each of the four FBG’s were taken before they were fusion spliced in the system, by scanning a narrowband laser across the reflection peak of the grating. All FBG’s had a FWHM bandwidth of 25 GHz. For the experiments, care was taken that the reflection spectra of both FBG’s in each separate channel were spectrally aligned. This was verified by attenuating the light in the cavity and subsequently launching broadband light, first from one side and then from the other side of the cavity, thereby observing the centre wavelength of the reflection spectrum of FBG$_1$ and FBG$_2$ separately. This was done for both channels separately. The specifications of all the cavity FBG’s used in this work are listed in Table (4-1) below. The last FBG listed in the table (FBG$_{\text{drop}}$) was used as a drop-filter in crosstalk experiments on the dual-channel system shown in Figure (4-1) (inset (b) in this figure). These experiments will be presented in the last section of the next chapter.

**Table 4-1:** Sensor cavity and filter grating specifications listing the bandwidth (FWHM), peak reflectivity and centre wavelength.

<table>
<thead>
<tr>
<th>Grating</th>
<th>Channel</th>
<th>Centre wavelength$^a$ [nm]</th>
<th>Reflectivity$^b$ [%] / [dB]</th>
<th>FWHM [pm] / [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBG$_1^{(1)}$</td>
<td>1</td>
<td>1549.6</td>
<td>40 / 4.0</td>
<td>200 / 25</td>
</tr>
<tr>
<td>FBG$_2^{(1)}$</td>
<td>1</td>
<td>1549.9</td>
<td>80 / 1.0</td>
<td>200 / 25</td>
</tr>
<tr>
<td>FBG$_1^{(2)}$</td>
<td>2</td>
<td>1550.6</td>
<td>40 / 4.0</td>
<td>200 / 25</td>
</tr>
<tr>
<td>FBG$_2^{(2)}$</td>
<td>2</td>
<td>1550.8</td>
<td>60 / 2.2</td>
<td>200 / 25</td>
</tr>
<tr>
<td>FBG$_{\text{drop}}$</td>
<td>2</td>
<td>1550.7</td>
<td>80 / 1.0</td>
<td>200 / 25</td>
</tr>
</tbody>
</table>

$^a$ Centre wavelength has to be shifted to operation wavelength listed in Table (4-2).

$^b$ Peak power reflectivity

As previously stated in our initial experiments a FP-TF (make Queensgate, model DMF microfilter, 25 GHz FWHM bandwidth, scanning wavelength range 1526 nm – 1562 nm) was used to filter out one of the two returned signals (channel 1 and 2) from the sensors. This is shown in inset (a) of Figure (4-1). The FP-TF had a similar 25 GHz bandwidth (FWHM) to the FBG’s defining the two FP cavities. The transmission profile of this filter is shown in Figure (4-3) below (---). Also shown in the figure is the
reflection spectrum of the FBG drop-filter (0.2 nm, i.e. 25 GHz FWHM bandwidth and an 80 % peak power reflectivity), that was used in later experiments to achieve better crosstalk performance (—). The FBG drop-filter was fusion spliced on to a high isolation 3-port circulator and was mounted on a translation stage for strain-tuning (inset (b) of Figure (4-1)). The Bragg wavelength of the drop-filter was written around the wavelength of channel 2, therefore it was only possible to 'drop' channel 2 and not channel 1 (since it was impossible to shift the Bragg wavelength to the lower wavelength by cooling the FBG). The isolation of the circulator from port 1 to port 3 was checked experimentally and was found to be around 63 dB, which is high enough for our purpose (the optical extinction ratio of the dual-channel narrowband ASE source was around 40 dB, as seen in Figure (2-13) in the second chapter). In Figure (4-3) below I show the extinction ratio between channels for a channel spacing of 100 GHz (ITU grid standard [13, 53]) for both the FP and the FBG filter, the values are 21 dB and 42 dB respectively. However, there is a nasty side lobe on the FBG drop-filter, reducing the extinction ratio from 42 dB to about 32 dB within the bandwidth of channel 1 for the 100 GHz spacing (note, that this is still 11 dB better compared to the FP filter).
Figure 4-3: Measured transmission filter profile of the FP-TF (---) and reflection filter profile of the FBG drop-filter (FBGdrop ─), showing the extinction ratio of both filters for a standard 100 GHz channel spacing. Both filters have a 25 GHz FWHM bandwidth. The FBG drop-filter had an 80 % power reflectivity. The arrow callouts indicate where the two channels would be situated spectrally for a 100 GHz channel spacing.

Note the asymmetry of the FP transmission filter profile around its centre wavelength. This can be more easily seen in Figure (4-4) below where the same transmission spectrum of the FP filter is plotted. At 50 GHz from the peak transmission of the filter, there is a 3 dB difference in extinction ratio depending on which wavelength side of the peak transmission you are. This asymmetry has consequences for the achievable crosstalk level when tuning, either to channel 1, or tuning to channel 2. The impact of this in WDM (crosstalk) experiments will be presented in the next chapter of this thesis.
Figure 4-4: Measured transmission filter profile of the FP-TF showing the extinction at 50 GHz on either side of the peak transmission.

For the experiments presented later in this chapter, the filters prior to the receiver (FP-TF inset (a) and the FBG drop-filter inset (b)) were removed from the set-up. Experiments were performed on each channel separately by illuminating the system with the single-channel narrowband ASE source (Figure (2-1)). Measurements on channel 1 were done by isolating channel 2 from the system (by bending the fibre in-between the two sensor channels), and tuning the centre wavelength of the narrowband ASE source (spectrum is shown in Figure (2-5)) to this channel. Measurements on channel 2 were performed by removing the attenuation in-between the two channels and illuminating channel 2 with the single-channel narrowband ASE source. Channel 1 was thereby spectrally tuned so as to lie as far away as possible from channel 2. Table (4-2) below lists the key system parameters used in the experiments, together with a summary of the experimental performance in terms of experimental phase resolution and visibility.
Table 4-2: The experimental system parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Channel (1)</th>
<th>Channel (2)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source bandwidth $\Delta \nu_0$ (unchirped)</td>
<td>6.2</td>
<td>6.2</td>
<td>[GHz]</td>
</tr>
<tr>
<td>Source power $P_0$</td>
<td>+8.0</td>
<td>+7.0</td>
<td>[dBm]</td>
</tr>
<tr>
<td>Length mismatch $\Delta L$</td>
<td>(4.5 ± 0.1)</td>
<td>(1.4 ± 0.4)</td>
<td>[mm]</td>
</tr>
<tr>
<td>SOP</td>
<td>PP / PF</td>
<td>PP / PF</td>
<td>[-]</td>
</tr>
<tr>
<td>Modulation frequency $f_s$</td>
<td>10.0</td>
<td>15.0</td>
<td>[kHz]</td>
</tr>
<tr>
<td>Modulation depth $\phi_s$</td>
<td>(200 ± 5)</td>
<td>(200 ± 5)</td>
<td>[mrad]</td>
</tr>
<tr>
<td>Cavity loss $\delta_c$ (single pass)</td>
<td>0.66</td>
<td>2.2</td>
<td>[dB]</td>
</tr>
<tr>
<td>FBG peak reflectivity $R_1 / R_2$</td>
<td>40 / 80</td>
<td>40 / 60</td>
<td>[%]</td>
</tr>
<tr>
<td>Cavity length $L_{PP}$</td>
<td>12.5</td>
<td>12.5</td>
<td>[m]</td>
</tr>
<tr>
<td>Operational channel spacing</td>
<td>100</td>
<td></td>
<td>[GHz]</td>
</tr>
<tr>
<td>Operational centre wavelength $\lambda$</td>
<td>1550.0</td>
<td>1550.8</td>
<td>[nm]</td>
</tr>
<tr>
<td>Channel bandwidth $\Delta \nu$</td>
<td>25</td>
<td>25</td>
<td>[GHz]</td>
</tr>
</tbody>
</table>

**Measurements**

| Visibility                              | (50 ± 5)    | (30 ± 5)    | [%]     |
| Phase resolution $(\phi)_min$           | (80 ± 5)    | (110 ± 6)   | [µrad/√Hz] |
| PP                                      | (50 ± 5)    | (70 ± 6)    | [µrad/√Hz] |

(c) For the crosstalk experiments this was increased to 2π-radian.

d) For the system limited by source noise.

4.2. Theoretical modelling of Fabry-Perot and Mach-Zehnder interferometer systems

Due to the multiple-reflection nature of the FP interferometer, there are multiple paths through this system (i.e. $n = \infty$) which decay in amplitude after each subsequent reflection from FBG$_1$ and FBG$_2$ which define the cavity. Light entering the sensing cavity is partially reflected at FBG$_1$. A portion of the light though is also transmitted through FBG$_1$. This transmitted light will be reflected back and forth in the cavity between the two gratings. At each reflection of the two gratings, a portion of the light will escape from the cavity on either end of the cavity. This is illustrated schematically in Figure (4-5) below, where the first six consecutive paths the light can travel through the system are drawn (except for the transmission through the cavity, which has been omitted).
In our calculations we will only consider the first 10 consecutive paths (i.e. $n$ and $n' = 0$ to 9). This approximation is valid because the field amplitude drops exponentially with time when there is no round-trip gain. The round-trip optical path length of the FP interferometer, $2L_{FP}$, was adjusted to be almost equal to the path-imbalance of the compensating MZI, i.e. $L_{MZ} \approx 2L_{FP}$. Therefore, light passing through the long arm of the MZI and subsequently reflected back at FBG$_1$ (path $n' = 0$ in Figure (4-5) above) will be coherent with light that passes through the short arm of the MZI and is reflected back from FBG$_2$ (path $n = 1$ in Figure (4-5) above), since they will travel the same distance. These two beams will thus be coherent and produce interference, which contains information about the acoustic signal applied to the sensor cavity.

It is known that the transfer function of a FP cavity formed by e.g. two fibre ends approaches that of a two-beam interferometer [54]. However, the FP sensing cavities employed in our system are based on FBG’s to define the internal mirrors of the FP sensing cavity. The use of FBG-based interferometers introduces several issues.
that have to be addressed to assess the system performance, one of which is the reflectivity of both FBG’s comprising the sensing cavity, to give the system maximum phase resolution. The calculations, which will be presented in this chapter, will be used to predict these values. Note, from Table (4-2) that the power reflectivity of the first FBG of both cavities was chosen to be around 40 %, a value that is predicted with our theory to give our system maximum phase resolution and this agrees with the value found by Christmas et al. [51]. A second issue that has to be addressed is the multiple-reflection nature associated with the FP sensing cavity. To calculate the noise filtering properties of this system we need to characterise the FP cavity. First the covariance will be calculated.

4.2.1. Covariance function of the output intensity of a Fabry-Perot interferometer

The output field of a FP interferometer with an interacting infinite number of polarisation maintaining (PP case) paths, can be generally expressed as a superposition of delayed versions of the input field [42]:

\[ E_{out}(t) = \sum_{n=0}^{\infty} F_n E(t - nT_{FP}) \]

Equation 4-1

Here \( F_n \) represents the complex weighting factor of the electric field, which has experienced a group delay of \( nT_{FP} \) and \( n \) is an integer value giving the number of round-trips the light experiences in the FP interferometer. The time averaged optical intensity at the system output at time \( t \), given by Equation (3-17) in the previous chapter, can be written as:

\[ \langle I(t) \rangle = \left( \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n F_m^* \Gamma(mT_{FP} - nT_{FP}) \right) \]

Equation 4-2

Using the covariance function given by Equation (2-8) in the second chapter, and assuming Gaussian statistics according to Equations (3-8), we can write the fourth-
order moments appearing in the covariance function as second-order moments. We can then write the covariance function of the detected output intensity as:

\[
\text{Cov}_I(\tau) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \left( F_n F_m^* F_p F_q^* \langle \Gamma(mT_{FP} - nT_{FP}) \rangle \cdot \langle \Gamma(pT_{FP} - qT_{FP}) \rangle \right)
\]

\[
+ \langle \Gamma^*(\tau + (nT_{FP} - mT_{FP})) \rangle \cdot \langle \Gamma(\tau - (pT_{FP} - qT_{FP})) \rangle
\]

\[
- \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} F_n F_m^* F_p F_q^* \langle \Gamma(mT_{FP} - nT_{FP}) \cdot \Gamma^*(pT_{FP} - qT_{FP}) \rangle
\]

Equation 4-3

This can be simplified and be written in the following form:

\[
\text{Cov}_I(\tau) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} F_n F_m^* F_p F_q^* \langle \Gamma^*(\tau - (qT_{FP} - nT_{FP})) \cdot \Gamma(\tau - (pT_{FP} - mT_{FP})) \rangle
\]

Equation 4-4

If we consider that the term \( \langle \Gamma^*(\tau - (qT_{FP} - nT_{FP})) \cdot \Gamma(\tau - (pT_{FP} - mT_{FP})) \rangle \) will contribute to the covariance function only if \((n-q) = (m-p)\) and \(\tau = (n-q)T_{FP}\):

\[
\text{Cov}_I(\tau) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} F_n F_m^* F_p F_q^* |\Gamma(\tau - (nT_{FP} - qT_{FP}))|^2
\]

Equation 4-5

Once written in the previous form, we can write this covariance function as a convolution [42]:

\[
\text{Cov}_I(\tau) = |\Gamma(\tau)|^2 \oplus \sum_{M=-\infty}^{\infty} A_M \delta(\tau - MT_{FP})
\]

Equation 4-6

Here, \(\delta(\tau-MT_{FP})\) is the Dirac delta function centred around \(MT_{FP}\) and we define \(A_M\) as a
function of the $F$-factors given by:

$$A_M = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n F_m^* F_{m-M} F_{n-M}^*$$

Equation 4-7

Thus $\text{Cov}_1(\tau)$ is made up of sharp peaks, centred around all possible combinations of $MT_{FP}$. While the correlation function, $|\Gamma(\tau)|$ describes the coherence properties of the source (recall the second chapter), it is the term $\sum_{M=-\infty}^{\infty} A_M \delta(\tau - MT_{FP})$ whose form is completely determined by the system topology, i.e. by the FP interferometer. That is we have an infinite string of Delta functions, spread out uniformly (often called a 'comb function'). The NSD for the FP interferometer can now be calculated from the covariance function of the detected output intensity $\text{Cov}_1(\tau)$, given by Equation (4-6). Note that convolving in the time domain is equivalent to multiplying in the frequency domain. This means that with Equation (4-6) for the autocovariance function, we can transform the summation with the Delta function and the correlation function separately and subsequently multiply the two Fourier transforms with each other. Doing this, we obtain for the noise spectrum:

$$N_{FP}(f) = N_{ASE}(f) \cdot \sum_{M=-\infty}^{\infty} A_M \cdot e^{-2\pi f MT_{FP}}$$

Equation 4-8

Here we took the constant summation over $A_M$ out of the integral sign and integrated over the Delta function, which gives us the complex exponential function. The part $N_{ASE}(f) = (g\alpha)^2 R \int_{-\infty}^{\infty} |\Gamma(\tau)|^2 \cdot e^{2\pi f \tau} \, d\tau$ in the expression above is the NSD from the ASE source, found earlier in the second chapter (Equation (2-21) on page 47). Clearly, the power spectrum of the detected optical output intensity is a product of two terms. The first term, $N_{ASE}(f)$, is only source dependent and is just the input power spectral density of the source intensity fluctuations, found earlier in the second chapter. The last term, $\sum_{M=-\infty}^{\infty} A_M \cdot e^{-2\pi f MT_{FP}}$ is determined by the FP interferometer itself, and plays the
role of a transfer function, by filtering the source noise in the spectral domain. The RF response of a FP cavity based on two identical FBG's was already studied by Zhang et al. [55]. They showed that the structure's RF response was maximised when the first FBG had a 33 % reflectivity and the second FBG a 100 % reflectivity.

4.2.2. Calculation of the complex field-amplitudes for a Fabry-Perot interferometer

For all recirculating structures, the ratio between two consecutive fields is independent of \( n \), except, for the first two terms in the series. As will be seen, this is because the first field contributing to the output does not travel through the interferometer. The FP interferometer is shown schematically in Figure (4-6) below.

![Figure 4-6: Illustration of the FP recirculating structure (sensor cavity). The coefficients for the electric fields upon reflection and transmission are indicated by \( F \). The reflection and transmission coefficients for each grating are denoted by \( r \) and \( t \) respectively. The single pass cavity loss is denoted by \( \delta_c \).](image)
Figure (4-6) above shows the electric field upon reflection and transmission of the two FBG’s in the FP interferometer. The electric field incident on the FP interferometer is shown on the top left corner. The FBG’s are assumed to be point reflectors. We will consider the case for the FP interferometer in reflection mode, i.e. we will calculate the complex field amplitudes $F_n$ shown on the left in the figure (indicated by 'Reflection'). Upon reflection, we can write $F_n$ as [42]:

$$F_n = TR^n \quad (n \geq 1)$$

Equation 4-9

$T$ is a constant and $R$ is the round-trip loss, which is generally complex and includes the real loss, per round-trip, as well as the acquired phase shift on reflection and transmission. We can characterise a recirculating structure when we know the parameters $F_0$, $T$, $R$ and $n$. Let us consider the FP structure, which is shown in Figure (4-6). We assume the loss ($\phi$) at the reflective device is independent of the input fields, i.e. the loss is a single real number. The cavity loss ($\delta_c$) is defined as the loss in optical power experienced by the light when travelling through the cavity on a single pass. We define the complex amplitude reflectances $r$ and transmittances $t$, from both sides, as $r_{1a}$, $r_{1b}$, $r_{2a}$ and $t_{1b}$. In the reflection mode, the $F_n$’s are (see Figure (4-6)) given by:

$$
\begin{align*}
F_0 &= r_{1a} \\
F_n &= (\delta_c)^2 t_{1a} (r_{2a})^n (r_{1b})^{n-1} t_{1b} \\
&\quad (n \geq 1)
\end{align*}
$$

Equation 4-10

We assume the two gratings comprising the cavity to be uniform point reflectors, and assume that there is no phase shift experienced by the light when reflecting off the grating from either side. Therefore we can say that $r_{1a} = r_{1b} = r_1$ and $r_{2a} = r_{2b} = r_2$. Comparing the previous two expressions (4-9) and (4-10), we find:
\[ \begin{align*}
F_0 &= r_{ia} = r_i \\
T &= \frac{t_{ia} t_{ib}}{r_{ib}} = \left(\frac{t_1}{r_1}\right)^2 \\
R &= \left(\delta_c\right)^2 r_{2a} r_{1b} = \left(\delta_c\right)^2 r_2 r_i
\end{align*} \]

Equation 4-11

Where \( \delta_c = e^{\frac{1}{2} \ln(10)x} \) represents the loss of optical power in the FP cavity, and \( x \) is the real loss of optical power in dB in the cavity on a single pass. Using Equations (4-7), (4-9) and (4-11) we can calculate the \( A \)-factors, and which result from two nested infinite geometrical series as we will show next. First we will calculate \( A_0 \), i.e. for \( M = 0 \):

\[ A_0 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n F_m^* F_m F_n^* = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} |F_n|^2 |F_m|^2 \]

Equation 4-12

Writing out the first summation over \( n \), and after some algebraic manipulation, we can write this in the following form:

\[ A_0 = 2 \sum_{m=0}^{\infty} |F_m|^2 \sum_{n=1}^{\infty} |F_n|^2 \]

Equation 4-13

If we first consider only the second summation over \( n \) in Equation (4-13), we get, after substitution of \( F_n \) given by Equation (4-9):

\[ \sum_{n=1}^{\infty} |F_n|^2 = \sum_{n=1}^{\infty} |T|^2 |R|^{2n} = |T|^2 |R|^2 + |T|^2 |R|^4 + |T|^2 |R|^6 + \cdots = \frac{|T|^2 |R|^2}{1 - |R|^2} \]

Equation 4-14
Hence, we can write (4-13) as:

\[ A_0 = 2 \sum_{n=0}^{\infty} |F_n|^2 \left( \frac{|T|^2|R|^2}{1 - |R|^2} \right) \]

\[ = 2|F_0|^2 \left( \frac{|T|^2|R|^2}{1 - |R|^2} \right) + 2|F_1|^2 \left( \frac{|T|^2|R|^2}{1 - |R|^2} \right) + 2|F_2|^2 \left( \frac{|T|^2|R|^2}{1 - |R|^2} \right) + \cdots \]

Equation 4-15

After some rearranging, and taking into account that the infinite geometric series \((1 + |R|^4 + |R|^8 + |R|^{12} + \cdots)\) converges to \(\frac{1}{1 - |R|^4}\), we can write \(A_0\) as:

\[ A_0 = 2|F_0|^2 |T|^2 \left( \frac{|R|^2}{1 - |R|^2} \right) + 2|T|^4 \left( \frac{|R|^4}{(1 - |R|^2)(1 - |R|^2)} \right) \]

Equation 4-16

Performing a similar calculation for \(M \neq 0\) (for Equation (4-7)) we get:

\[ A_m = \left( 2 \text{Re}[F_0 T^m] |T|^2 \left( \frac{|R|^2}{1 - |R|^2} \right) + 2|T|^4 \left( \frac{|R|^4}{(1 - |R|^2)(1 - |R|^2)} \right) \right) |R|^{2|M|} \]

Equation 4-17

For the NSD of the reflection output of the FP interferometer we find, according to Equation (4-8):
\[ N_{FP}(f) = N_{ASE}(f) \cdot \left( 2|F_0|^2|T|^2 \frac{|R|^2}{1 - |R|^2} + 2|T|^4 \frac{|R|^4}{(1 - |R|^2)^2} \right) \]

\[ + \sum_{M=0}^{\infty} 2 \text{Re}[F_0T] \cdot |T|^2 \left( 2 \frac{|R|^2}{1 - |R|^2} + 2|T|^4 \frac{|R|^4}{(1 - |R|^2)^2} \right) \cdot |R|^{2|M|} \cdot e^{-2\pi M T_{FP}} \]

**Equation 4-18**

The total NSD of the system can be calculated by simply multiplying the noise spectrum found for the FP interferometer (Equation (4-18)) with the NSD for an MZI incorporating the FS (Equation (3-46) from the previous chapter). For the total noise spectrum, given as \( N(f) = N_{MZ}(f) \cdot N_{FP}(f) \), we obtain:

\[ N(f) = (g\alpha)^2 \frac{T_0^2}{2} \frac{1}{\sqrt{4\pi\sigma^2}} \left( e^{-\frac{f^2}{4\sigma^2}} \left( 2 + 2\cos(2\pi T_{MZ}) \right) + e^{-4\pi^2 \sigma^2 T_{MZ}^2} \left( e^{\frac{(f-f_0)^2}{4\sigma^2}} + e^{\frac{(f-f_0)^2}{4\sigma^2}} \right) \right) \]

\[ \times \left( 2|F_0|^2|T|^2 \frac{|R|^2}{1 - |R|^2} + 2|T|^4 \frac{|R|^4}{(1 - |R|^2)^2} \right) \]

\[ + \sum_{M=-\infty}^{\infty} \left( 2 \text{Re}[F_0T] \cdot |T|^2 \left( 2 \frac{|R|^2}{1 - |R|^2} + 2|T|^4 \frac{|R|^4}{(1 - |R|^2)^2} \right) \cdot |R|^{2|M|} \cdot e^{-2\pi M T_{FP}} \right) \]

**Equation 4-19**

It is very interesting to note that \( N_{MZ} \) and \( N_{FP} \) are, respectively, the NSD of the MZI and the FP interferometer, acting separately and independently. Thus, we can conclude that the total NSD at the output of the system reflects both noise filtering effects. The calculated NSD given by Equation (4-19) is plotted in Figure (4-7) below against frequency for channel 1.
Figure 4-7: Predicted Noise Spectral Density for the system (channel 1). The system’s SOP was set to the PP case. The received optical power, path-imbalance, source bandwidth and the length mismatch were set to be -20 dBm, 25.22 m, $\Delta v_0 = 6.2$ GHz and 4.5 mm respectively (see Table (4-2)).

4.2.3. Fringe visibility of the system

The visibility of the FP interferometer is dependent on the reflectivities of both FBG’s comprising the FP cavity, and on the length mismatch between the MZI and the FP interferometer, $\Delta L = |L_{FP} - L_{MZ}|$. The average output intensity was found to be equal to Equation (4-2). For coherent interference we consider only three cases: $m = n$, $m = (n+1)$, $m = (n-1)$. The sum of those three terms gives for the average output intensity:

$$
\langle I(t) \rangle = \frac{1}{2^2} \left( 2 \sum_{\pi=0}^{\infty} F_\pi |^2 \Gamma(0) \right) + \left( \sum_{\pi=0}^{\infty} F_\pi F^*_n \Gamma(T_{FP} - T_{MZ}) \right) + \left( \sum_{\pi=0}^{\infty} F^*_\pi F^*_{n+1} \Gamma^*(T_{FP} - T_{MZ}) \right)
$$

Equation 4-20

Evidently, contributions to $\langle I(t) \rangle$ which come from terms for which $\Delta T = |T_{FP} - T_{MZ}|$ is
much larger than the coherence time of the source, will be negligible. That is, \( \Gamma(T_{FP} - T_{MZ}) \) will only contribute to \( \langle I(t) \rangle \) if \( nT_{FP} \equiv T_{MZ} \). Substituting the correlation function of the electric field at the output of the ASE source, given by Equation (2-18), and using Euler formula \((\cos(x) - i\sin(x)) = e^{-ix}\), and \((\cos(x) + i\sin(x)) = e^{ix}\), we can write this in the following form:

\[
\langle I(t) \rangle = \frac{I_0}{2^2} \left(2 \sum_{n=0}^{\infty} |F_n|^2 + V' \sum_{n=0}^{\infty} F_n^* F_{n+1} \left(\cos(2\pi n \nu_0 \Delta T - i\sin(2\pi n \nu_0 \Delta T)\right) + V' \sum_{n=0}^{\infty} F_n^* F_{n+1} \left(\cos(2\pi n \nu_0 \Delta T + i\sin(2\pi n \nu_0 \Delta T)\right)\right)
\]

Equation 4-21

Where \( V' = e^{-2\pi^2 \sigma^2 (\Delta T)^2} \). Now we will introduce the heterodyne phase shift \( (2\pi f_s t) \) from the FS in the MZI and the signal \( \phi_s \sin(2\pi f_s t) \) or phase modulation due to the sinusoidal modulation of the cavity length of the FP interferometer. The time averaged output intensity, given by Equation (4-21) can now be written as follows:

\[
\langle I(t) \rangle = \frac{I_0}{2^2} \left(2 \sum_{n=0}^{\infty} |F_n|^2 + V' \sum_{n=0}^{\infty} F_n^* F_{n+1} \left(\cos(2\pi n \nu_0 \Delta T + 2\pi f_s t + \phi_s \sin(2\pi f_s t)\right) - i\sin(2\pi n \nu_0 \Delta T + 2\pi f_s t + \phi_s \sin(2\pi f_s t)\right)\right) + V' \sum_{n=0}^{\infty} F_n^* F_{n+1} \left(\cos(2\pi n \nu_0 \Delta T + 2\pi f_s t + \phi_s \sin(2\pi f_s t)\right) + i\sin(2\pi n \nu_0 \Delta T + 2\pi f_s t + \phi_s \sin(2\pi f_s t)\right))
\]

Equation 4-22

Replacing \((2\pi n \nu_0 \Delta T + 2\pi f_s t + \phi_s \sin(2\pi f_s t))\) with \((\alpha + \beta t)\), where \(\alpha = (2\pi n \nu_0 \Delta T)\) and \(\beta t = (2\pi f_s t + \phi_s \sin(2\pi f_s t))\), and using \(z + z^* = 2 \Re[z]\) and \(z - z^* = 2i \Im[z]\), the average intensity becomes:
\[
\langle I(t) \rangle = \frac{I_0}{2}\left(2\sum_{n=0}^{\infty} |F_n|^2 + 2V' \cdot \text{Re}\left[\sum_{n=0}^{\infty} F_n F_{n+1}^*\right] \cdot \cos(\alpha + \beta t) + 2V' \cdot \text{Im}\left[\sum_{n=0}^{\infty} F_n F_{n+1}^*\right] \cdot \sin(\alpha + \beta t)\right) \]

Equation 4-23

Here Re and Im represent the real and imaginary part of the function between square brackets respectively. If we assume the signal to be small, and we assume the interferometer to be quadrature-biased, then after some trigonometry, we can write the cosine and sine terms in the expression for the average output intensity as follows:

\[
\cos(\alpha + \beta t) = -\frac{\phi_i}{2} \left(\sin(2\pi(f_h + f_s)t) - \sin(2\pi(f_h - f_s)t)\right) - \sin(2\pi f_s t)
\]

\[
\sin(\alpha + \beta t) = \frac{\phi_i}{2} \left(\cos(2\pi(f_h + f_s)t) + \cos(2\pi(f_h - f_s)t)\right) + \cos(2\pi f_s t)
\]

Equation 4-24

After substitution of these two equations into the time averaged output intensity given by Equation (4-23), we obtain:

\[
\langle I(t) \rangle = \frac{2I_0}{2}\left(\sum_{n=0}^{\infty} |F_n|^2 \right) + V' \cdot \text{Re}\left[\sum_{n=0}^{\infty} F_n F_{n+1}^*\right] \cdot \left(-\frac{\phi_i}{2} \left(\sin(2\pi(f_h + f_s)t) - \sin(2\pi(f_h - f_s)t)\right) - \sin(2\pi f_s t)\right) \\
+ V' \cdot \text{Im}\left[\sum_{n=0}^{\infty} F_n F_{n+1}^*\right] \cdot \left(\frac{\phi_i}{2} \left(\cos(2\pi(f_h + f_s)t) + \cos(2\pi(f_h - f_s)t)\right) + \cos(2\pi f_s t)\right)
\]

Equation 4-25

The modulation signal, applied to the FP sensor cavity, does not contribute to the system's fringe visibility, so we can put the signal strength to zero and we get for the time-averaged output intensity:
\[ \langle I(t) \rangle = \frac{2I_0}{2^2} \left( \sum_{n=0}^{\infty} |F_n|^2 + V' \cdot \text{Re} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \cdot \left( -\sin(2\pi t) \right) \right) + V' \cdot \text{Im} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \cdot \cos(2\pi t) \]

Equation 4-26

We can now calculate the system's fringe visibility \( V \) by using the definition given by Equation (1-2) in the first chapter. For the minimum value of the averaged output intensity we find \( \langle I(t) \rangle_{\text{min}} = \frac{2I_0}{2^2} \left( \sum_{n=0}^{\infty} |F_n|^2 + V' \cdot \text{Re} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \right) \), and for the maximum output intensity we get \( \langle I(t) \rangle_{\text{max}} = \frac{2I_0}{2^2} \left( \sum_{n=0}^{\infty} |F_n|^2 + V' \cdot \text{Im} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \right) \).

After substitution of \( V' = e^{-2\pi^2 \sigma^2 (\Delta \tau)^2} \), this gives us for the visibility of this particular interferometer system:

\[ V = \frac{e^{-2\pi^2 \sigma^2 (\Delta \tau)^2} \left( \text{Im} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] - \text{Re} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \right)}{2 \sum_{n=0}^{\infty} |F_n|^2 + e^{-2\pi^2 \sigma^2 (\Delta \tau)^2} \left( \text{Im} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] + \text{Re} \left[ \sum_{n=0}^{\infty} F_n F^*_n+1 \right] \right)} \]

Equation 4-27

Here we assumed the power imbalance to be insignificant (i.e. we assumed \( I_1 = I_2 \)). From this expression we can clearly see that the fringe visibility of this system is highly dependent on the reflectivities of the both FBG's comprising the FP interferometer, via the term \( \sum_{n=0}^{\infty} F_n F^*_n+1 \). Our calculations using the theory presented above showed that maximum phase resolution and signal visibility is obtained for \( R_1 \approx 40 \% \). This can be seen from Figure (4-8) below. In this figure the visibility is plotted against the power reflectivity of the input grating of the FP cavity, for five different reflectivity values of the second grating in the FP cavity, as predicted with the theory presented in this chapter (i.e. Equation (4-27)).
Figure 4-8: The calculated visibility of the FP cavity sensor, as a function of the power reflectivity $R$ of the input grating of the FP cavity (i.e. FBG$_1$), for five different reflectivity values of the second grating (i.e. FBG$_2$).

From Figure (4-8) we can clearly see that there is zero visibility for the input grating having reflectivities of 0 % and 100 %. This is expected, since in that case there is effectively no FP cavity anymore. We can also observe that the visibility peaks at 55 % and 68 % for reflectivity values of the input FBG between 10 % and 40 % respectively. This maximum, at which the visibility peaks, shifts from $R_1 = 10 \%$ to $R_1 = 40 \%$, the higher the reflectivity of the second FBG is. A maximum visibility of around 68 % is obtained for the input FBG and the second FBG having reflectivities of 40 % and 100 % respectively. Next we wish to see how the phase resolution and the visibility of this system varies with $R_2$. The plot below shows the system's visibility as a function of the power reflectivity $R_2$ of FBG$_2$. Plotted are the theoretical curves for both channels.
Figure 4-9: Predicted visibility of both channels plotted against $R_2$, for $R_1 = 40\%$.

Figure (4-9) above shows that the highest visibility is obtained for as high as possible a reflectivity $R_2$, and the visibility drops to zero for $R_2 = 0$ (this is expected since in this instance there is effectively no cavity left). The visibility for both sensor channels is plotted against source bandwidth in Figure (4-10) below.
Figure 4-10: Predicted visibility as a function of the source bandwidth for both sensor channels.

Figure (4-10) above tells us that there is no significant change in visibility for source bandwidths ranging from near zero Hz to 10 or 20 GHz. According to the predictions, for a source bandwidth in this range, channel 1 and channel 2 should have a fringe visibility of around 60 % and 40 % respectively. For channel 2 the visibility drops to almost zero for bandwidths approaching 140 GHz or over. Note that due to the slightly worse length mismatch of the first channel (4.5 mm) the visibility falls off earlier, approaching zero visibility for bandwidths of 40 GHz or over.
4.2.4. Phase resolution of the system

The (electrical) amplitude signal spectrum is found to be:

\[
|S(f)| = (g\alpha)^2 \frac{R}{2^4} \frac{I_0^2}{2} \sum_{n=0}^{\infty} |F_n|^2 \cdot \delta(f) \\
+ 2\nu \cdot \text{Re} \left[ \sum_{n=0}^{\infty} F_n F_{n+1}^* \right] \cdot \left\{ \frac{\phi_s}{2} \left( \frac{\delta(f - (f_h + f_s))}{2} - \frac{\delta(f + (f_h - f_s))}{2} \right) \\
+ \frac{\phi_s}{2} \left( \frac{\delta(f - (f_h - f_s))}{2} - \frac{\delta(f + (f_h + f_s))}{2} \right) \\
+ 2\nu \cdot \text{Im} \left[ \sum_{n=0}^{\infty} F_n F_{n+1}^* \right] \cdot \left\{ \frac{\phi_s}{2} \left( \frac{\delta(f - (f_h + f_s))}{2} + \frac{\delta(f + (f_h - f_s))}{2} \right) \\
+ \frac{\phi_s}{2} \left( \frac{\delta(f - (f_h - f_s))}{2} + \frac{\delta(f + (f_h + f_s))}{2} \right) \right\}^2
\]

Equation 4-28

The terms containing \( \delta(f \pm f_s) \) correspond to the carrier signal from the heterodyne method. After some algebraic manipulation using the facts that \( (i\text{Re}[z] + \text{Im}[z])^2 = (i\text{Re}[z])^2 + (\text{Im}[z])^2 + 2(i\text{Re}[z] \cdot \text{Im}[z]) \), and that we can drop the cross term \( 2(i\text{Re}[z] \cdot \text{Im}[z]) \) since we are only interested in the amplitude spectrum and not the phase spectrum, we can write

\( (i\text{Re}[z] + \text{Im}[z])^2 = (i\text{Re}[z])^2 + (\text{Im}[z])^2 = \left| \sum_{n=0}^{\infty} F_n F_{n+1}^* \right|^2 \). Here \( z \) denoted a complex number. The amplitude signal spectrum can then be written in the following form:
\[ |S(f)| = (g\alpha)^2 R \frac{I_0^2}{2} \left\{ 4 \left( \sum_{n=0}^{\infty} |F_n|^2 \right)^2 \cdot \delta(f) + \left( \sum_{n=0}^{\infty} F_n F_{n+1}^* \right)^2 \cdot \frac{\delta^2}{2} \right\} \left( \delta(f + f_h) + \delta(f - f_h) \right) \]

\[ + \left( \sum_{n=0}^{\infty} \delta\left( f + \left( f_h + f_s \right) \right) + \delta\left( f - \left( f_h - f_s \right) \right) \right) \]

Equation 4-29

Where we took the common factor (the infinite summation over \( n \)) out of the brackets. This amplitude signal spectrum given by Equation (4-29) is illustrated in Figure (4-11) below. The fundamental or DC component of the spectrum, represented by the Delta function \( \delta(f) \) is situated at zero frequency, with the two other Delta functions \( \delta(f \pm f_h) \) centred around the fundamental, represent the heterodyne signals. The modulation signals \( \delta(f \pm f_h \pm f_s) \) are centred around the two heterodyne signals.

![Figure 4-11](image)

**Figure 4-11**: Illustration of the amplitude signal frequency spectrum as seen at the receiver output of the system.

As expected this signal spectrum is similar to the one produced by the dual MZI system with heterodyne detection, presented in the previous chapter. Equating the amplitude electrical signal spectrum, given by Equation (4-29), to the electrical noise spectrum, gives us the phase resolution:
\[
\left( \phi_s \right)_{\text{min}}^2 = \frac{2^4}{I_0^2} \cdot 2 \cdot \frac{N(f \pm (f_h \pm f_s))}{(g \alpha z)^2 R} \left( \nu' \right)^2 \left| \sum_{n=0}^{\infty} F_n F_{n+1}^* \right|^2
\]

Equation 4-30

Where the total NSD of the detected output intensity of the system \( N(f \pm (f_h \pm f_s)) \) is given previously by Equation (4-19). After substitution of the noise spectrum and the expression for the visibility \( \nu' \) given by Equation (3-68) we obtain:

\[
\left( \phi_s \right)_{\text{min}}^2 = 2 \cdot \frac{1}{\sqrt{4\pi \sigma^2}} \cdot e^{4\pi^2 \left( \Delta f \right)^2} \left( e^{-\frac{f^2}{4\sigma^2}} \left( 2 + 2\cos(2\pi f T_{\text{MZ}}) \right) + e^{-4\pi^2 \sigma^2 f^2 T_{\text{MZ}}^2} \left( e^{-\frac{(f+y)^2}{4\sigma^2}} + e^{-\frac{(f-y)^2}{4\sigma^2}} \right) \right)
\]

\[
\cdot \left( 2\left| F_0 \right|^2 \left| T \right|^2 \left( \frac{|R|^2}{1-|R|^2} \right) + 2\left| T \right|^4 \frac{|R|^4}{(1-|R|^2)^2} \left( 1+|R|^2 \right) \right)
\]

\[
+ \sum_{M=0}^{M=M_{\text{max}}} \frac{2\Re\left[F_n T \right] \left| T \right|^2 \left( \frac{|R|^2}{1-|R|^2} \right) + 2\left| T \right|^4 \frac{|R|^4}{(1-|R|^2)^2} \left( 1+|R|^2 \right) \right] \left| R \right|^{2M} e^{-2\pi M \theta_{\text{MZ}}}
\]

\[
\cdot \frac{1}{\left( \sum_{n=0}^{\infty} F_n F_{n+1}^* \right)^2}
\]

Equation 4-31

The system's phase resolution is plotted in Figure (4-12) below as a function of source bandwidth for both sensor channels. The predicted higher phase resolution of channel 1 compared to channel 2 (for source bandwidths narrower than around 15 GHz) is due to the higher reflectivity of FBG2 (and the lower cavity loss).
Figure 4-12: Predicted phase resolution as a function of optical bandwidth for both channels. The SOP was set for the PP configuration.

At narrow source bandwidths there is a sharp decrease in phase resolution. This is caused by the dramatic increase in source noise as the bandwidth becomes smaller. There is a decrease in phase resolution for broader bandwidths which is due to the fact that the visibility drops off more quickly than the reduction in source noise, i.e. as the bandwidth becomes broader you start to loose coherence. The phase resolution of channel 1 drops off earlier than channel 2 does as the bandwidth becomes broader, this is due to the slightly worse length mismatch (4.5 mm compared to 1.4 mm). Clearly, for a specific length mismatch, an optimum bandwidth of the source light must exist. From Figure (4-12) it is clear that for nearly matched interferometers the maximum phase resolution is obtained for source bandwidths of around 40 GHz. However, to maximise the WDM efficiency we want the channel bandwidth to be as narrow as possible. Since a 1 mm mismatch for a 25 metre path-imbalance is not a practical
length mismatch, we chose a source bandwidth of around 6 or 7 GHz. This bandwidth gives a good phase resolution but offers a less stringent length matching requirement of 1 cm.

Note that when the FP cavity and MZI are well-matched (i.e. \( V' \approx 1 \) in Equation (4-30)), then the exponential term \( e^{4\pi^2r^2(\Delta r)^2} \) becomes unity and can therefore be ignored from expression (4-31). A high phase resolution is obtained when the noise term \( |N(f \pm (f_s \pm f_r))| \) given by Equation (4-19) is small, but also when the factor in the denominator, i.e. \( \left| \sum_{n=0}^{\infty} F_n F_{n+1}^* \right|^2 \), is large. This factor is only dependent on the FP interferometer (on the reflectivities of both FBG's, on the cavity loss, etc.) and is directly related to the finesse of the FP interferometer.

The cavity finesse determines how many times the light will recirculate in the cavity. If we calculate the weighting factor (or complex amplitude) for the electric field (assuming the first and second FBG of the cavity to have a reflectivity of 40 % and 80 % respectively), \( A_M \), for the first 10 \( A \)-values, we see that the light remaining in the cavity after four round-trips does not significantly contribute to the output. This is illustrated in Figure (4-13) below. Similarly due to the lower cavity finesse of channel 2, the electric field will reduce in strength (represented by \( A_M \)) earlier compared to electric field circulating in channel 1. After only 2 round-trips in the cavity of channel 2, there is almost no light recirculating in this cavity, all of it being dissipated from either end of the cavity. Whereas, for channel 1 this only occurs after four round-trips. The early dissipation of light from cavity 2 is also caused by the relatively high cavity loss (power loss of 2.2 dB on a single pass) associated with this cavity, compared to the loss in sensor cavity 1 (0.66 dB single pass). This high power loss degrades the finesse of this cavity even more.
Figure 4-13: Calculated weighting factor (or complex amplitude) for the detected electric field, $A_M$, as a function of the number of round-trips $n$ in both FP cavities.

The transfer function of a FP interferometer is a periodic series of sharp reflection peaks, instead of the cosine function for a MZI. The maximum of each peak is centred on zero phase difference (modulo $2\pi$) between the interfering light paths. That is, the maxima occur when $\Delta \phi = 2\pi m$, where $m = (0, \pm 1, \pm 2, \pm 3, \ldots)$. The finesse of the FP interferometer is a measure of the sharpness of the reflection peaks, and characterises the phase resolution of the device [56]. The higher the finesse of the FP interferometer the higher and sharper the peaks of the transfer function. This can be seen from Figure (4-14) below, where the normalised output intensity, $\langle I(t) \rangle / I_0$, is plotted as a function of the length mismatch of the FP interferometer, for different values of $R_2$ (for $R_1 = 40\%$). (i.e. different cavity finesse; the combinations $R_1 = 40\%$ and $R_2 = 5\%$ giving the lowest finesse and the combination $R_1 = 40\%$ and $R_2 = 100\%$ providing the highest finesse for the FP interferometer).
Figure 4-14: Calculated system's output intensity as a function of length mismatch between FP sensor cavity and compensating MZI, for four different values of $R_2$ with fixed $R_1 = 40 \%$.

The four curves shown in Figure (4-14) above are typical transfer functions for a multiple-beam interferometer, i.e. the FP interferometer. Note that the slope of the transfer function can be much higher compared to the slope of the transfer function for a two-beam interferometer (see Figure (1-6) in the first chapter). Therefore, the phase resolution of a FP interferometer can be much higher than the resolution of an MZI. The higher the reflectivity of both FBG's, the sharper the reflection peaks shown in Figure (4-14) above. A high reflectivity for both gratings provides a high finesse interferometer, however a price is paid in terms of reduced output intensity, and which gives reduced signal strength. Later in this chapter I will present results of experiments performed to verify the, with the theory presented above, calculated optimum reflectivities of both FP cavity FBG's.
4.2.5. Experiments and Discussion

The Fabry-Perot Interferometer

To validate the theory presented above, we built an FP grating-based interferometer and used it in conjunction with the MZI. The FP cavities were made up of SMF-28 fibre. Both FBG’s in each individual sensor cavity were temperature tuned to fine-tune their reflection spectra to overlap each other, around a wavelength of 1550 nm. The relationship between the normalised Bragg wavelength shift $\lambda_B$, to applied temperature $T$, for a constant strain $\varepsilon$, is given by [1]:

$$\frac{1}{\lambda_B} \left| \frac{d\lambda_B}{dT} \right|_{\varepsilon} \approx 10 \cdot 10^{-12} \cdot C^{-1}$$

Equation 4-32

This means that, when there is no dynamic strain applied to the grating, for every degree temperature increase/decrease, the Bragg wavelength of the grating shifts by around 10 pm to a higher/lower wavelength respectively (at 1550 nm). Just as with straining the grating, the wavelength shift obtained by heating or cooling is a fully reversible effect, i.e. after removing the heat from the grating, its Bragg reflection wavelength returns to its original value. A picture of one of the two sensor channels can be seen below.
Figure 4-15: A digital picture of one of the acoustic sensor channels. The two Bragg gratings are underneath the white insulation material on top of a heat sink, which can be seen at the back end of the box. The PZT can be seen in the smaller box in the bottom left hand corner of the box. The two polarisation controllers, one inside the sensing cavity and one in front of the cavity can also be seen. The sensor was acoustically isolated in the box by placing insulation material all around the sides in the box.

For tuning purposes each of the four cavity gratings were placed on Peltier elements, or Thermal Electric Coolers (TEC’s), with some thermal paste to improve the thermal contact between the bare glass fibre and the Peltier element. The Peltier elements were 30 mm wide and long and 5.6 mm thick. They had a heat capacity of 18.8 Watt, i.e. a maximum temperature difference of 70 °C could be achieved (at 9.0 Ampere and 3.75 Volt). Close to the gratings, also on the Peltier element, a thermistor was placed for temperature sensing and controlling purposes. Insulating material was placed over each FBG to shield it from environmental temperature fluctuations. This can be seen in Figure (4-15) above shown by the two white blocks on top of the two heat sinks in the far end of the box. Both the Peltier and the thermistor were connected to a temperature controller (make Newport, model 325). With the controller it was possible to set the temperature of the FBG to a specific value (± 0.01 °C) within a range of about 80 °C.
With this capability it was possible to shift the Bragg wavelength of each of the FBG’s by about 0.8 nm (100 GHz). Typically the FBG’s were heated to a temperature of around 40 °C. The peak intensity reflectivity’s $R_1^{(1)}$ and $R_2^{(1)}$ were chosen to be close to 40 % and 80 % respectively for channel 1, and close to $R_1^{(2)} = 40 %$ and $R_2^{(2)} = 60 %$ respectively for channel 2.

When the PZT is driven with a harmonic electric signal, the fibre in the cavity is periodically strained and a phase change is generated. This simulates an acoustic signal at a certain frequency (modulation frequency) present at the sensor location, and upper and lower sidebands are produced around 110 MHz (see Figure (3-27) on page 136 of the previous chapter). These sidebands are separated from the heterodyne signal by integer multiples of the modulation frequency. The weighting factor for each sideband is a function of the amplitude of the acoustic signal and the order of the corresponding Bessel function (sideband). The PZT’s were made of PZT–5A ceramic, manufactured by Morgan Matrox and had the following dimensions, diameter 38.1 mm, height 38.1 mm, wall thickness 3.18 mm. 12 loops of fibre were wound around the PZT tubes. Using this implementation, it was possible to induce a $2\pi$ radians modulation to the interferometer system. The resonance frequency of both PZT’s was around 24 kHz. The PZT’s phase shifting capability was about $(618 \pm 60)$ mrad/Vpp ($@ f_s = 10.0$ kHz) for channel 1 and around $(584 \pm 20)$ mrad/Vpp ($@ f_s = 15.0$ kHz) for channel 2. The system's response was observed to be linear with respect to the applied phase shift up to $2\pi$-radians.

The sensor lengths were matched to the compensating MZI by measuring their individual optical path length from the beat spectrum (for broadband light) observed on a fast RF spectrum analyser (Tektronix 2782). This beat spectrum is illustrated in Figure (4-16) below.
Illuminating the MZI (or the FP cavity) with broadband light, a beat spectrum as shown in Figure (4-16) above can be observed at the receiver comprising spectral fringes with an FSR determined by the path-imbalance of the MZI. To get an accurate measurement of the path-imbalance of each interferometer. We measured the frequency of a high frequency fringe and used fringe counting to measure the fundamental frequency. As many fringes as possible were counted. For this length matching an AC-coupled 1 GHz receiver was used (New Focus Low Noise Photoreceiver Model 1611 comprising an InGaAs PIN photodiode followed by a low-noise amplifier with a current gain of 750 V/A, and a 3 dB bandwidth of 30 kHz – 1.0 GHz). The frequency span over which the number of fringes was counted was then divided by the number of fringes, giving an accurate value for the FSR of the interferometer. For this purpose 250 fringes were counted on the spectrum analyser. With Equation (3-13) in the previous chapter, we then obtained the path-imbalance of the interferometer. The path-imbalance of the MZI and of the two FP cavities were measured individually and subsequently subtracted from each other to give the length mismatch of the particular sensor channel relative to the compensating MZI. Sensor channel 1 was matched to the MZI to within (4.5 ± 0.1) mm and channel 2 to within (1.4 ± 0.4) mm. This corresponds to 178 ppm for channel 1 and 56 ppm for channel 2. The slightly larger error of ± 0.4 mm in the length mismatch of channel 2 is due to the lower visibility (fringe contrast) of the beat pattern, as observed on the RF spectrum analyser. There were four splicing points inside both sensing cavities. The power loss for a single pass through the cavity was measured to
be 0.66 dB and 2.2 dB for channel 1 and channel 2 respectively. This is the loss from the splicing points, from the incorporation of a PC (insertion loss around 0.5 dB), and from a PZT in the cavity (insertion loss was measured to be around 0.05 dB). The loss was measured after each subsequent splice was made when constructing the cavity. This was done by measuring the transmission loss through the splicing points, by launching laser light with set power through the splices and measuring the transmitted optical power (also ensuring that the wavelength of the launched light was far enough away from the Bragg wavelength of the two FBG’s comprising the cavities). This cavity loss then includes the insertion loss from the PZT and PC. The relatively high cavity loss of channel 2 was caused by a bad splicing point in this particular sensor channel. To reduce this high splice loss, it would have been necessary to re-splice the entire cavity. We chose not to re-splice the second cavity since experiments were already performed on this particular channel, and it would be nearly impossible to length match this particular FP cavity to the MZI to the same value as before. The PZT’s were driven by a 15 MHz function/arbitrary waveform generator (make Hewlett Packard, model 33120A).

Setting of the State Of Polarisation within the system

To set the SOP within the system, tri-wave plates PC’s were incorporated within the FP cavities and before both cavities. A tri-wave plate PC was also inserted in one of the arms of the MZI. It was possible to conveniently set the polarisation state of the light in this arm to be either parallel (PP case) or orthogonal (PF case) to the polarisation state in the other arm of the interferometer. The PC in the FP cavity was used to increase the signal visibility after either the PP or the PF state was set in the MZI. During the experiments, caution was taken not to interfere with the fibres at the input of the system so as not to induce any birefringence, which would change the input SOP to the MZI.

Setting of the SOP for the measurements was done as follows. First, the output of the MZI was directed to the receiver (see Figure (3-11) on page 107 in the previous chapter). The paddles on the PC in one of the MZI arms were set, so that the spectral noise power (observed on the RF spectrum analyser) showed minimum fringe contrast or maximum fringe contrast. This then corresponded to the polarisation states of the light in the MZI arms being either mutually parallel or orthogonal, resulting in either the PP or the PF configuration respectively. Next, the sensors were connected and the PC’s in the sensor cavities and between the sensors were set to maximise the signal.
visibility.

Note that the noise spectrum is generated separately by the MZI and each sensor channel, but the signal is from the combination of the MZI and the sensor cavities. This is why the noise power is reduced periodically in the frequency domain but the signal visibility is maximised.

*Noise redistribution*

The noise redistribution by the system $N(f)$, as predicted with Equation (4-19), is shown both theoretically and experimentally in the plot below for the PP configuration (for channel 1). Channel 2 was appropriately attenuated by bending the fibre in-between the two channels. The FP-TF was removed from the system and the system output was directed directly to the receiver. The received optical power was set to $-20$ dBm. The noise spectrum has a FSR of around 8.2 MHz, corresponding to the path-imbalance of around 25.22 metres. A minimum in the noise power can be seen at the heterodyne signal frequency of $f_h = 110$ MHz, and at $(m + \frac{1}{2})$ multiples of the FSR (e.g. at 101.8 MHz, 110 MHz, 118.2 MHz, etc.).
Figure 4-17: Theoretical and experimental Noise Spectral Density for the system (channel 1). The system's SOP was set to the PP configuration. The received optical power, path-imbalance, source bandwidth and the length mismatch were set to be \(-20\,\text{dBm}, 25.22\,\text{m}, \Delta \nu_0 = 6.2\,\text{GHz}\) and \(4.5\,\text{mm}\) respectively (see Table (4-2)).

The agreement between theory (—) and experiment (indicated with ■) is seen to be good. The noise power is around \(8\,\text{dB}\) lower at the spectral minima.

Visibility measurement

The visibility is an important parameter determining the performance of the system. The critical factor determining the visibility is the reflection coefficients \(R_1\) and \(R_2\) of both gratings in each sensor channel. The theoretical model described previously in this chapter was used to estimate optimum values for both \(R_1\) and \(R_2\). Experiments were then performed to verify the model for the visibility as a function of both reflectivity coefficients \(R_1\) and \(R_2\). In these experiments the reflectivity \(R_2^{(1)}\) was changed by attenuating the light just in front of FBG2\(^{(1)}\) (by bending the fibre), thus changing the effective reflectivity \(R_2^{\text{eff}}\) of FBG2\(^{(1)}\). The effective reflectivity \(R_2^{\text{eff}}\) was obtained by
measuring the difference in power reflected back from FBG$_1^{(1)}$ (which is proportional to $R_1^{(1)}$) and the power reflected back from both FBG$_1^{(1)}$ and FBG$_2^{(1)}$ (which is a function of both $R_1^{(1)}$ and $R_2^{\text{eff}}$). The visibility can be obtained by measuring the maximum output intensity, $\langle I(t) \rangle_{\text{max}}$, and the minimum output intensity, $\langle I(t) \rangle_{\text{min}}$, and using Equation (1-2) the visibility can be calculated. These two intensities can be measured from the fringe pattern observed on an oscilloscope. In practice at low intensities it is difficult to accurately determine the peak and trough values from the oscilloscope and in addition we developed a second method to measure the visibility. The visibility was obtained by measuring the DC-level, i.e. $\frac{1}{2}(\langle I(t) \rangle_{\text{max}} + \langle I(t) \rangle_{\text{min}})$, at the system output on an oscilloscope using a low-pass filter (integrator) and measuring the AC-level, i.e. $\langle I(t) \rangle_{\text{max}} - \langle I(t) \rangle_{\text{min}}$ on an RF spectrum analyser. This is explained in more detail in the figure below, where the fringe pattern, as observed on an oscilloscope, is drawn.
Figure 4-18: Illustration of the system’s fringe pattern as observed on an oscilloscope. The root-mean-square (rms) power \( AC' = \left( \frac{1}{2} \cdot \frac{AC}{\sqrt{2}} \cdot a \right) \) is what is measured on the RF spectrum analyser. The average power \( DC' = \langle DC \cdot a \rangle \) is measured on the oscilloscope.

Since the powers measured on an RF spectrum analyser in a linear scale are root-mean-square (rms) values, the values measured have to be multiplied by \( \sqrt{2} \). Also, spectrum analysers are capable of measuring only positive frequency components, hence the power in the negative frequency component is neglected such that we have to multiply the measured value by a further factor of 2. What is really measured for the \( AC \) power is \( AC' = \frac{a}{\sqrt{2}} \cdot \left( \frac{1}{2} AC \right) \) in mV_{rms}. The average power measured on an oscilloscope is
\[
DC' = \frac{b}{2} \cdot \left( \langle I(t) \rangle_{\text{max}} + \langle I(t) \rangle_{\text{min}} \right) = b \cdot DC \text{ in mV.}
\]
Here \( a \) and \( b \) are correction factors for the non-flat frequency response of the receiver \((a = 3.85, b = 3.02)\). According to Equation (1-2), we find for the visibility:
\[ V = \frac{AC}{2DC} = \left( \frac{2AC' \cdot \sqrt{2}}{a} \right) = \left( \frac{AC' \cdot \sqrt{2}}{b} \right) \]

Equation 4-33

The two measurement methods were compared at high power levels and were found to give good agreement. The visibility, given by Equation (4-27), is plotted as a function of the source bandwidth for the two sensor channels (see Figure (4-19) below). It can be seen that as the source bandwidth increases, there is a reduction in coherence and the visibility therefore drops.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Exp. Theory</th>
<th>( \Delta L )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>●</td>
<td>4.5 [%]</td>
<td>40 [%]</td>
<td>80 [%]</td>
</tr>
<tr>
<td>2</td>
<td>□</td>
<td>1.4 [%]</td>
<td>40 [%]</td>
<td>60 [%]</td>
</tr>
</tbody>
</table>

Figure 4-19: Measured and predicted visibility as a function of the source bandwidth for both channels.

The data point for the visibility at 25 GHz bandwidth in Figure (4-19) above was taken with the single-channel narrowband ASE source, with the source pick-out filter
(FBG<sub>p1</sub>) removed. The bandwidth of the source light launched into the system was therefore defined by the bandwidth of the FBG's defining sensor cavity 1 (i.e. 25 GHz FWHM). The received optical power was set to -20 dBm throughout. For received powers above a certain value (i.e. for optical noise higher than receiver noise) there is, as expected, no power dependence for the visibility, as can be seen from Figure (4-20) below where the visibility is plotted as a function of received optical power.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Exp.</th>
<th>Theory</th>
<th>ΔL</th>
<th>R&lt;sub&gt;1&lt;/sub&gt;</th>
<th>R&lt;sub&gt;2&lt;/sub&gt;</th>
<th>SOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.5</td>
<td>40</td>
<td>80</td>
<td>PP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.4</td>
<td>40</td>
<td>60</td>
<td>PP</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4-20: Predicted and measured visibility as a function of received optical power for the two sensor channels.

Received power was tuned by tuning an optical attenuator that was placed at the source output, while operating the narrowband ASE source at maximum power. Both experimental results and theoretical predictions indicate no power dependence of the visibility, as expected. Also note that both the PP (●) and the PF (O) configuration give the same visibility, in accord with expectation. Though, there is a slight discrepancy between the experimental points for both channel 1 and channel 2 (indicated with ●
and □ respectively) and the theory (--- and ···· respectively).

Optimum reflection coefficients of both sensor cavity gratings

Theoretical predictions for the power reflectivities of both FBG's comprising the sensor cavity (to get maximum phase resolution) showed that $R_1$ should be close to 40 % and $R_2$ near 100 % (see Figures (4-8) on page 161 and (4-9) on page 162). To validate these predictions we performed experiments whereby we varied $R_2$ (for fixed $R_1$) while observing the system's phase resolution and visibility. Setting the value of $R_2$ was done by attenuating the light in the cavity by bending the fibre in sensor cavity 1 prior to the second reflector. This has the effect that a portion of the light is coupled out of the fibre, thereby reducing the light intensity reflected back from the second FBG (and thus effectively reducing its reflectivity). We will therefore denote the reflectivity of FBG$_2$ by the 'effective' reflectivity $R_2^{\text{eff}}$.

We can calculate $R_2^{\text{eff}}$ if we assume the reflectivity of the first FBG of the sensor (FBG$_1^{(1)}$) to be fixed at a certain value, by assuming incoherent light at the output of the system. This condition is readily satisfied by taking the compensating MZI out of the system so there is only the linear addition of intensities (i.e. no interference). If we consider only the paths $n$ (and not $n'$ since we took the MZI out) shown in Figure (4-5) on page 148, we can write down the magnitudes of the first and subsequent electric fields at the receiver. All cross terms can be ignored from the expressions, since they show insignificant contribution to the output intensity, because the coherence length of the ASE light is much shorter than the distance travelled by the light in the cavity. Hence, the average output intensity comprises only terms like $\langle E_n(t)E_n^*(t) \rangle$, i.e.:

$$\langle I(t) \rangle = \left( \sum_{n=0}^{\infty} |E_n(t)|^2 \right)$$

Equation 4-34

Using the amplitudes of the electric fields, we can write each term as follows:
\[
\begin{align*}
|E_0(t)|^2 &= I_0 |\rho_1|^2 \\
|E_n(t)|^2 &= I_0 \left( |\rho_1|^2 |\rho_2|^2 \right)^n \left( \left( |\rho_1| |\rho_2| \right)^2 \right)^{(n-1)} \quad (n \geq 1)
\end{align*}
\]

Equation 4-35

After substitution of each term back into Equation (4-34), and noting that the infinite series \(1 + \left( |\rho_1| |\rho_2| \right)^2 + \left( |\rho_1| |\rho_2| \right)^4 + \left( |\rho_1| |\rho_2| \right)^6 + \cdots\) converges to \(\frac{1}{1 - \left( |\rho_1| |\rho_2| \right)^2}\), we can write the average output intensity as follows:

\[
\langle I(t) \rangle = I_0 |\rho_1|^2 + I_0 \left( |\rho_1|^2 |\rho_2|^2 \right)^2 \cdot \frac{1}{1 - \left( |\rho_1| |\rho_2| \right)^2}
\]

Equation 4-36

When we replace the field reflectivity with the power reflectivity \(R = |\rho|^2\), we find for the relative output intensity:

\[
\frac{\langle I(t) \rangle}{I_0} = R_{\text{overall}} \quad \text{(for } l_c \ll L_{FP})
\]

Equation 4-37

Where we define the 'overall' reflectivity of the FP cavity as:

\[
R_{\text{overall}} = R_1 + (1 - R_1)^2 R_{\text{eff}} \cdot \frac{1}{1 - R_1 R_{\text{eff}}}
\]

Equation 4-38

Here we used the assumption that \(|\epsilon|^2 + |\rho|^2 = 1\) (i.e. there is no loss of power on reflection and transmission of the gratings). We can now calculate the reflectivity of the last grating in the sensor by measuring the optical power or intensity, \(P_1\), in Watts from both gratings, which gives \(P_1 = P_0 \cdot \beta \cdot R_{\text{overall}}\), where \(\beta\) is the power coupling coefficient.
of the system output coupler and $P_0$ is source output power. And for the power reflected from only the first grating, $P_2$, we find $P_2 = P_0 \cdot \beta \cdot R_1$. Since now $R_2^{\text{eff}} = 0$ in the equation for $R_{\text{overall}}$. This means that if we know the value of $R_1$, we are able to calculate the value of $R_2^{\text{eff}}$ by using both previous equations and the relation $P_1 + P_2 = R_1 \times R_{\text{overall}}$. Solving for $R_2^{\text{eff}}$ we obtain (with the optical power $P$ in the linear unit Watt):

$$R_2^{\text{eff}} = \frac{\left( \frac{P_2}{P_1} \right) R_1 \times R_1}{\left( \frac{P_2}{P_1} \right) R_1^2 - R_1^2 + (1 - R_1)^2}$$

Equation 4-39

**Example**

First, we measure the optical power reflected of both the gratings in the cavity, $P_1$. Then, we totally attenuate the light in the FP cavity by bending the fibre, so we only measure the optical power reflected from the first grating, $P_2$. We measure the following values $P_1 = -14.63$ dBm $\approx 34.43$ µW and $P_2 = -17.17$ dBm $\approx 19.19$ µW. And if we assume that $R_1^{(1)} = 40 \%$, using Equation (4-39) we find for the power reflectivity of the last grating, $R_2^{\text{eff}} \approx 65 \%$.

Our predictions using the theory presented above showed that the maximum phase resolution and signal visibility is obtained for $R_1 \approx 40 \%$. Figure (4-21) below shows the system's visibility as a function of the power reflectivity $R_2^{\text{eff}}$ of FBG2. Plotted are the theoretical curves for both channels, using Equation (4-27), and the experimental points for channel 1 and one experimental point for channel 2 obtained using Equation (4-33).
The received optical power was set to $-20$ dBm throughout these experiments. Measurements on channel 1 (indicated with $\bullet$) were performed whereby channel 2 was attenuated by bending the fibre between the two channels. As predicted by the theory presented above, maximum visibility is obtained for a high as possible a value of reflectivity $R_2^{\text{eff}}$, and the visibility drops to zero for $R_2^{\text{eff}} = 0$ (this is expected since in this instance there is effectively no cavity left). It can be seen that there is a slight discrepancy between the experimental points and the theoretical predictions. The phase resolution is plotted in Figure (4-22) below.
Figure 4-22: The experimental and theoretical phase resolution of both sensor channels as a function of $R_2^\text{eff}$, for $R_1 = 40\%$. The SOP was set for the PP configuration. The inset shows the same plot only for a different scale for the phase resolution.

As seen from Figure (4-22) above a maximum phase resolution is obtained for a high as possible reflectivity of the FBG$_2$. Reasonable agreement between experiment and theory is obtained, as can be seen in the inset of Figure (4-22). Figure (4-23) below shows the system's phase resolution predicted with Equation (4-31) as a function of the source bandwidth, using the chirped narrowband ASE source described in the second chapter. The received power was $-20$ dBm throughout.
Figure 4-23: Measured and predicted phase resolution of both channels as a function of optical bandwidth for two different length mismatches. The SOP was set for the PP configuration.

Figure (4-23) above shows that for a length mismatch of around 1 mm (channel 2 ---), the sensitivity goes up as the bandwidth becomes broader (reduction in excess photon noise as the bandwidth becomes broader). For a slightly longer length mismatch of 4.5 mm (channel 1 --), the phase resolution reaches a maximum of around 70 μrad/√Hz for a source bandwidth of 10 GHz. This is due to the fact that the signal visibility decreases faster than the noise power, as the bandwidth becomes broader for relatively large length mismatch. The experimental point (●) at 25 GHz was obtained for the narrowband ASE source, with the pick-out filter FBG21 removed. Henceforth, the bandwidth of the received light was ultimately defined by the bandwidth of the two FBG's comprising the FP cavity (i.e. FWHM ~ 25 GHz). The phase resolution of both channels is plotted against received power in Figure (4-24) below.
Figure 4-24: The theoretical and experimental phase resolution as a function of received power for the two sensors. The open symbols denote SOP is PP and the solid ones denote the PF configuration.

Figure (4-24) above shows that at relatively high optical power (> −30 dBm) the phase resolution of the system can not be improved by increasing the optical power, since the SNR is limited by the excess photon noise from the source. At relatively low optical power, the receiver noise becomes the dominant noise factor and hence the system's phase resolution decreases rapidly with decreasing received power. The phase resolution of channel 1 (— and ○) drops off at slightly lower received power than for channel 2 (····· and □). This is probably caused by the higher finesse of cavity channel 1, due to the higher reflectivity of the last FBG comprising this cavity ($R_2^{(1)} \sim 80\%$). The higher cavity loss (2.2 dB) for sensor channel 2 also contributed to the reduced effective reflectivity of the last FBG in this cavity ($R_2^{(2)} \sim 60\%$). The system shows a relatively flat frequency response within the signal bandwidth of about 200 kHz, for both channels.
4.3. Conclusions and Discussion

In conclusion, we have studied in detail, both experimentally and theoretically, the noise characteristics and phase resolution limits of a MZI-FP sensing system interrogated using a narrowband fibre ASE source. Using the model described in this chapter, we have found optimum reflectivities for both gratings in the FP sensing cavity. A choice of input reflector and the second reflector reflectivities of 40 % and nearly 100 % respectively, give the sensor maximum phase resolution. This has been verified with experiments and good agreement with theory has been obtained. Furthermore, the system does not seem to suffer from its multiple-reflection nature as similar phase resolutions were achieved with both this multiple-reflection FBG-based system and a single path dual MZI based system (~ 50 μrad/√Hz at 10 kHz).

We have shown that the system performance is limited by the excess photon noise of the source and that the detailed system noise spectrum is dependent on the filtering effect and SOP in the optical paths of the interferometer system. By employing a heterodyne technique and utilising these noise redistribution effects we have shown that it is possible to reduce the noise level in the signal frequency region. We consider the approach we have demonstrated to be highly practical, providing good phase resolution while remaining consistent with interferometer manufacturing tolerances and requiring relatively simple demodulation methods. We have achieved a phase resolution of 50 μrad/√Hz at 10 kHz signal frequency, with an optimised SOP setting, heterodyne frequency, and for a practical length mismatch of 1 cm.
Chapter Five

Experimental investigation of the system robustness

Overview: The development and performance testing of a FOAS system will be discussed in this chapter. The system's robustness to drift of source wavelength, grating drift and optical power fluctuations on the phase resolution and fringe visibility will be presented. The final two sections show the results from crosstalk experiments on a 2-channel system and system improvement using the source incorporating a GS-SOA.

5.1. Influence of drift on the system performance

Employing FBG's to define narrowband FP interferometers introduces the issue of spectral drift of the gratings centre wavelength (on the system performance), due to temperature [57] and strain fluctuations. The influence of temperature and strain puts stringent specifications on proper packaging of the FBG's when they are employed in large streamers where temperature fluctuations in the sea and induced strains due to the towing of the array through the water can be substantial. Furthermore, a directly related issue with the employment of FBG's is how the phase response of the gratings effects the system performance. Systematic experiments were performed where the influence, if any, of these two issues on the system performance was investigated. These experiments were performed on channel 1. Channel 2 was removed from the system by breaking the fibre between the two sensor cavities. The FP-TF and the FBG drop-filter were also removed from the system shown in Figure (4-1) on page 141 in the previous chapter. For the following experiments the single-channel narrowband ASE source, as depicted in Figure (2-1) on page 50 in the second chapter was used.
5.1.1. Drift of the source wavelength

There has been considerable focus on designing WDM filters with ideal amplitude characteristics, for example flat-top passband (to allow for wavelength drift of filter position or source) and steep edges for high channel isolation in telecommunication applications [13]. The ideal filter profile would of course be a square filter. Experiments were performed whereby the source spectrum (incorporating pick-out filter FBG$_{p1}$) was tuned across the reflectivity spectrum of the sensors. The reflection spectra of both gratings of the sensor were kept spectrally overlapping with each other at the operational wavelength of 1550 nm. This is illustrated in Figure (5-1) below.

![Diagram of reflection spectra showing tuning range and detuning](image)

**Figure 5-1:** Illustration of tuning the source spectrum across the two gratings in channel 1. The source spectrum (---) is being tuned across the spectra of the two gratings (FBG$_1$ (1) --- and FBG$_2$ (1) ---). Tuning direction is indicated with the arrow $\rightarrow$. The two side lobes on FBG$_2$ are denoted by the words 'lobe$_1$' and 'lobe$_2$'.

Figure (5-1) above shows the two reflection spectra from the gratings in sensor channel 1 (FBG$_1$ (1) --- and FBG$_2$ (1) ---) together with the source spectrum (---). The two spectra
of FBG\textsubscript{1} and FBG\textsubscript{2} were recorded before they were fusion spliced in the system, by scanning a narrowband laser across their reflection peaks. In the experiment the source spectrum was tuned away from the centre wavelength of the two cavity gratings, within the specified tuning range. The arrow \(\Rightarrow\) indicates the tuning direction of the source spectrum, which was tuned by strain-tuning the (source) pick-out filter. Note that each spectrum has been normalised (i.e. normalised reflectivity = reflectivity – peak reflectivity) for ease of comparing the spectra. The received optical power, for the situation whereby the two cavity FBG’s and the source spectrum were spectrally aligned, was set to \(-20\) dBm. This received power is high enough for the system to be limited by source noise, i.e. operated within the linear regime (received power above \(-28\) dBm), shown in Figure (4-24) on page 188. The SOP within the single-channel system was set for the PF configuration. The channel was modulated at 10 kHz with a 200 mrad modulation depth. Care was taken to eliminate optical feedback from all loose fibre ends by terminating them (angle cleaving and soaking in index matching oil). The unused port of the output coupler \(C_2\) of the MZI was used to monitor the centre wavelength of the source light during the experiments by directing the light from this port to an Optical Spectrum Analyser (OSA).

The source spectrum was tuned over intervals of around 20 pm within the 0.5 nm tuning range indicated in Figure (5-1) above. At each point the system's phase resolution and visibility was measured. The received optical power was recorded throughout by coupling 5% of the light of before the receiver by a 95:5 coupler (as can be seen in Figure (4-1) on page 141).

Below is a graph showing the results of the experiment. The solid and dashed lines are smoothed curves fitted to both experimental data sets. It shows that detuning the source across the stopband (3 dB bandwidth of both gratings in the cavity is around 0.2 nm, i.e. 25 GHz) of the gratings has no significant effect on the sensitivity (denoted by \(O\)). The inset in Figure (5-2) below shows a 5 \(\mu\text{rad/\sqrt{Hz}}\) fluctuation within the stopband. Detuning further, to outside the stopband, gives a dramatic decrease in sensitivity.
Figure 5-2: Experimental phase resolution of channel 1 (⊙) and measured reflectivity of both $\text{FBG}_1^{(1)}$ and $\text{FBG}_2^{(1)}$ ($\square = R_1^{(1)}$ and $\Delta = R_2^{(1)}$) as a function of source wavelength detuning. The solid and dashed lines are smoothed curves fitted to each experimental data set. The SOP was set for PF implementation.

Figure (5-2) above also shows the peak reflectivity of the first ($\square$) and the second grating ($\Delta$) in sensor cavity 1, when detuning the source. By tuning across the stopband one notes the expected fall-off in reflectivity of both gratings. Although there is a dramatic decrease in reflectivity, the phase resolution of channel 1 (⊙) does not seem to suffer significantly until the reflectivity drops to around 10%, as can be seen more clearly in the inset of Figure (5-2). The experimental phase resolution (□) for the source wavelength detuning is plotted against received optical power in Figure (5-3) below.
Figure 5-3: Experimental phase resolution of channel 1 (●) as a function of received optical power, for the experiment where the source wavelength was detuned. The SOP was configured as PF.

From Figure (5-3) we can clearly see that there is some sort of a 'hysteresis like' effect, i.e. two different values are observed for similar received powers. This can be explained as follows, observing the three insets (a), (b) and (c) in the figure above. Note that for negative detuning, i.e. from (a) to (b) in the figure above, the reflection spectra of FBG$_1^{(1)}$ (-----) and FBG$_2^{(1)}$ (····) are spectrally overlapping. But, for positive detuning, from (b) to (c), the reflectivities of FBG$_1^{(1)}$ and FBG$_2^{(1)}$ are slightly different. This means that for negative detuning, the finesse (determined by the relative reflectivities of both FBG's) of this particular cavity stays constant, whereas for positive detuning the finesse changes. Hence the phase resolution differs for either positive and negative detuning, producing the observed 'hysteresis like' effect seen in Figure (5-3). The same phenomenon was seen in the measurement of the visibility. Figure (5-4) below shows the visibility of channel 1 as a function of received optical power for the source wavelength detuning experiment.
Figure 5-4: Experimental visibility (□) of channel 1 as a function of received optical power, for the experiment where the source wavelength was detuned. The SOP was set as PF.

That is, for negative values of detuning, i.e. from (a) to (b) in the figure above, the visibility (□) is more or less constant (~ 55 %). This is in accord with expectation, since the visibility is power independent as we saw earlier in Figure (4-20) in the previous chapter. Whereas, detuning to positive values, gives a steady drop in the visibility due to the change in cavity finesse.

5.1.2. Drift of grating wavelength

To test the system's resilience to grating drift, experiments were performed where one of the FBG’s in sensor cavity 1 was spectrally tuned away from the spectrum of the other FBG comprising sensor cavity 1. The centre frequency of the narrowband optical source was left fixed. This was done by temperature tuning the grating (by either heating or cooling). This is illustrated in Figure (5-5) below.
Figure 5-5: Illustration of tuning of one of the gratings in sensor channel 1. FBG$_2^{(1)}$ (-----) is being tuned across FBG$_1^{(1)}$ (—). Tuning direction is indicated with the arrow $\rightarrow$. The narrowband ASE source spectrum is indicated with (—). The two side lobes on FBG$_2$ are denoted by the words 'lobe$_1$' and 'lobe$_2$'.

Figure (5-5) shows the two reflection spectra from the gratings in sensor channel 1 (— and -----) together with the source spectrum (—), as FBG$_2^{(1)}$ (-----) is tuned away from FBG$_1^{(1)}$ (—). The source spectrum (—) was kept spectrally aligned with the untuned grating (—). The arrow $\rightarrow$ indicates the tuning direction of FBG$_2^{(1)}$, within the 0.8 nm tuning range. The received optical power, for the situation whereby the two-sensor cavity FBG’s and the source spectrum were spectrally aligned, was set to $-20$ dBm. While detuning the grating, both the phase resolution and the visibility of the system were observed. For different measurement points of the spectrum of the tuned grating relative to the untuned grating, the system's phase resolution and visibility were measured. Both the phase resolution (O) and the visibility (Δ) from this experiment are plotted against detuning of FBG$_2^{(1)}$ in the graph below. The solid and dashed lines are smoothed curves fitted to both experimental data sets.
Figure 5-6: Measured phase resolution ($\varnothing$) and visibility ($\Delta$) of sensor channel 1 as a function of detuning FBG$_2^{(1)}$. The indication 'lobe' corresponds to the lobes in Figure (5-5), and is further explained in the text. The solid and dashed lines are smoothed curves fitted to both experimental data sets. The SOP was set to the PF case.

The inset graph of Figure (5-6) above shows a flat response in sensitivity ($\varnothing$) when detuning across the stopband of FBG$_1^{(1)}$, what indicates that there is no significant phase response of the gratings in the sensor channel. The maximum change in resolution observed while detuning within the stopband is probably only about 5 $\mu$rad/\sqrt{Hz}. Again a sharp decrease in sensitivity is observed when tuning outside the stopband to a lower wavelength (negative detuning). Detuning even further away from the stopband ($\approx \pm 0.3$ nm) you start to see the effect of two side lobes on FBG$_2^{(1)}$, where the sensitivity starts to increase again. This is indicated with the two arrows and the words 'lobe;' and 'lobe;' in the graph above. The effect of lobe$_2$ can also just be seen with a slight increase in visibility ($\Delta$) of a few percent when tuning FBG$_2^{(1)}$ over lobe$_2$ (from $-0.3$ nm to around $-0.4$ nm detuning). The influence on the system's phase resolution, from the smaller lobe on the other side of this grating (lobe$_1$) can also be
seen, though it is less significant. The maximum sensitivity measured is about 60 μrad/√Hz (note that this is for the optimum PF case). For detuning within the stopband there is a slight decrease in visibility (Δ) of about 8%. Further detuning outside the stopband, there is a much more significant drop in visibility.

The same hysteresis effect, as was observed with the source wavelength detuning, can be seen in the visibility when detuning FBG₂. The visibility is plotted against received power in Figure (5-7) below.

![Figure 5-7](image)

**Figure 5-7**: Measured visibility of channel 1 as a function of received optical power for the experiment with FBG₂ detuning.

A maximum visibility of around 50% is observed when all spectra are perfectly aligned (indicated by inset (b) in the figure) and the visibility drops to nearly zero for maximum (0.4 nm) negative and positive detuning of FBG₂. A slightly (~5%) lower visibility is observed for positive detuning compared to negative detuning, again due to a reduced cavity finesse for positive values of detuning.
5.1.3. Sensitivity of the phase resolution of the system to the detailed response of the grating

Finally, it should be mentioned that when starting the project we were uncertain as to whether the dispersive properties of the FBG's used to define the cavities were likely to have an impact on the performance of the system, e.g. in terms of phase resolution, visibility, etc. Our intuition led us to conclude that since we were using incoherent light that the grating phase response should not have an impact, however we wished at as early a stage as possible to confirm this.

The data presented in Figure (5-2) essentially goes a long way to doing this since from this graph we see that the phase resolution itself is essentially flat as we tune the narrowband ASE source across the grating bandwidth. However, as we tune the source we vary both the individual grating reflectivities (and hence the cavity finesse, returned power and potentially also visibility) as well as the actual grating phase/time delay responses. This makes it somewhat difficult to establish the relative insensitivity of the system response to either the amplitude or phase in isolation. To do this properly, we need to make measurements of the returned power at the receiver for fixed input power, interferometer visibility as a function of detuning, and of the system responsivity with respect to received optical power. This data is plotted in Figures (5-9), (5-4) and (5-8) respectively.
Figure 5-8: Measured and predicted system phase resolution versus received optical power for channel 1. — is the calculated phase resolution for the SOP = PP. The experimental points for the source detuning are indicated with ■. The experimental points for the source power detuning are indicated with ● for the SOP = PF and ○ for the SOP = PP. Point (b) corresponds to point (b) in Figures (5-3), (5-4) and (5-9).

Note, that in addition in Figure (5-9) we also plot a measurement of the grating time delay response measured using the standard electronic vector volt meter phase shift technique where it is seen that the grating exhibits a time delay variation of order 10 ps over the 25 GHz bandwidth of the grating. Points (a), (b) and (c), which represent the specific detunings of -0.25, 0 and 0.25 nm are shown on all plots. Inset within Figure (5-8) one can see that as one tunes from point (a) to (b) (over which the interferometer visibility is constant – see Figure (5-4)) the measured phase resolution follows precisely the same power dependency curve as obtained by keeping the source wavelength fixed to the centre of the FBG reflectivity bandwidth and varying the input source power. One can thus deduce that the system exhibits absolutely no grating dispersion dependency.
Figure 5-9: The received optical power as a function of source detuning for channel 1. Also plotted is the time delay of FBG$_2^{(1)}$.

Note, that when one tunes from point (b) to (c) however there is a slight variation of phase resolution with respect to received power and which we attribute to the observed decline in grating visibility observed in Figure (5-4). This in turn we attribute to the relative difference in grating reflectivity profiles at positive frequency detunings apparent from the grating spectra shown inset in Figure (5-4).

5.2. Crosstalk experiments

In WDM systems high channel isolation is of paramount importance to prevent crosstalk. Crosstalk results from our inability to perfectly filter out radiation in adjacent channels and which then interferes with the channel that we wish to measure. Crosstalk in our dual-channel system is determined mainly by two factors:
1. The filter used prior to the receiver having finite inter-channel extinction.

2. The influence of each neighbouring channel on each other caused by spectral overlap of the gratings of both cavities, reflecting in-band radiation into neighbouring channel.

We have made systematic measurements of crosstalk effects for cavity gratings and filters made by current fabrication technology. The dual-channel system used in crosstalk experiments is drawn schematically in Figure (5-10) below.

Figure 5-10: Schematic of the dual-channel sensing system used in the crosstalk experiments. The sensors are denoted with channel 1 and channel 2. The optical return from the two channels before and after the filter is shown in the insets (d) and (e) respectively.

Figure (5-11) below illustrates the first point, i.e. the inability of the receiver filter to completely filter out channels other than the one we want to measure. This figure also illustrates how the optical crosstalk level was measured. Shown is the transmitted optical spectrum through the FP-TF (→) used in our earliest experiment, together with
the transmission filter spectrum of the FP-TF (—), with the filter centre wavelength tuned to channel 1 (the required channel) in this instance. The power difference between the two adjacent channels after passing through the filter is defined as the (optical) crosstalk level.

**Figure 5-11:** Illustration of the definition of (optical) crosstalk due to non-ideal filter profile (as inset (e) in Figure (5-10)). This figure shows the optical spectrum at the input of the receiver (—) and the transmission spectrum of the FP filter (—). The filter is tuned to the required channel. Crosstalk is defined as the power difference between the required channel and the residual adjacent channel, for a particular channel spacing. Optical spectrum analyser had a resolution bandwidth of 0.02 nm, for both spectra.

The second cause of crosstalk due to spectral overlap of the reflectivity profiles of both cavity gratings is illustrated in Figure (5-12) below. This figure shows the reflection spectra of each individual sensor cavity FBG for a standard 100 GHz channel spacing. The degree of spectral overlap can be clearly seen from this figure. The constraints on achievable channel spacing was governed by the maximum and minimum achievable temperature for the Peltier elements on which the gratings were placed in order to tune
their Bragg wavelength. For the 100 GHz spacing as shown in Figure (5-12), the extinction ratio can be as low as 40 dB. However, note the sidelobe situated on the lower wavelength part of the reflectivity spectrum of the second FBG of sensor cavity 2 (i.e. on FBG\textsubscript{2} in Figure (5-12) below). From Figure (5-12) it can be seen that for channel 1 and a spacing of 100 GHz this lobe (from channel 2) is situated within channel 1 and reduces the extinction ratio to 30 dB.

![Figure 5-12](image.png)

**Figure 5-12**: Measured optical spectra of each individual grating in the dual-channel system, showing the degree of spectral overlap for a 100 GHz channel spacing. All spectra were recorded individually before the gratings were spliced in the system (except for the source spectrum, which was adopted from Figure (2-13) in the second chapter).

To avoid crosstalk, the reflectivity profiles of the first and the second sensor must not spectrally overlap, as shown in Figure (5-12). Close spectral spacing of the reflection profiles of each sensor allows us to increase the number of sensors to be multiplexed, but the spacing is limited by crosstalk between adjacent channels.

To investigate the crosstalk between the two sensor channels, both channels
were modulated at the same time (2\pi-radian modulation depth). Channel 1 was thereby modulated at 10 kHz and channel 2 at a 15 kHz modulation frequency. The SOP within the dual-channel system was set following the same procedure described earlier in the previous chapter (section 4.2.5 on page 171). First the output of the MZI was directed to the receiver and the fringe contrast of the noise spectrum, as observed on the RF spectrum analyser, was either maximised or minimised by rotating the paddles on PC₁ in one of the arms of the MZI. This then resulted in either the PF or the PP configuration. Next, the output of the MZI was directed to the dual-channel sensing array and the output of the system was directed to the receiver (through the FP-TF). Then the centre wavelength of the transmission spectrum of the FP-TF was tuned to either one of the channels and the signal visibility of both channels was then subsequently optimised by rotating the paddles on PC₂ and PC₃ for channel 1 and PC₄ and PC₅ for channel 2 (see Figure (5-10) on page 202).

During the experiments optical and corresponding electrical spectra were recorded prior to and at the output of the receiver respectively. For measurement of the electrical signal powers, \( J_{1}^{(1)} \) and \( J_{1}^{(2)} \), the dynamic range (in the frequency domain) was maximised by modulating both channels at the same time with a 2\pi-radian modulation depth. By selecting a 3 Hz resolution bandwidth on the RF spectrum analyser, and at the same time using a 100 Hz span to keep the sweep time reasonably high, gave a good dynamic range. In the frequency spectrum, the power of the fundamental (heterodyne carrier) was around \( J_0 = -9.0 \text{ dBm} \) and the optical noise floor with a 3 Hz bandwidth was around \( N = -92 \text{ dBm} \), which gave the system a dynamic range in the electric domain of around 83 dB. To eliminate the influence of change in system sensitivity caused by power fluctuations, both channels were operated in the 'flat' optical power regime (as seen in Figure (5-8) at received powers above \(-28 \text{ dBm}\)), where the system sensitivity is limited by the optical source noise. The received optical power was set to \(-20 \text{ dBm}\) throughout. For the crosstalk experiments the system was configured for the PP case, since this provides a better defined system from an analysis perspective.

A typical measurement of the electrical crosstalk can be seen in Figure (5-13) below. The detected frequency spectrum is shown, for (a) a minimum channel spacing of 65 GHz and for (b) a maximum of 180 GHz. Both spectra are for the situation whereby the centre wavelength of the transmission spectrum of the FP-TF was tuned to channel 1, as shown in Figure (5-11) above.
Figure 5-13: Measured frequency spectra from channel 1 for two different channel spacings. Figure (a) is for a 65 GHz channel spacing and (b) for a 180 GHz spacing. Channel 1 was modulated at 10.0 kHz and channel 2 at 15.0 kHz, both channels with a 2π radian modulation depth. Both spectra are for the FP-TF tuned to channel 1. Spectra were taken with a 30 kHz resolution bandwidth. The superscript between brackets, $J^{(1)}$ and $J^{(2)}$ denotes either channel 1 or 2, respectively. The fundamental (heterodyne) frequency was 110 MHz. The optical crosstalk levels corresponding to graphs (a) and (b) are $-12$ dB and $-25$ dB respectively.
Within Figure (5-13) the first harmonic of channel 1 and channel 2 are denoted by $J_1^{(1)}$ and $J_1^{(2)}$ respectively. We define the electrical crosstalk level by the power difference ($J_1^{(2)} - J_1^{(1)}$). Crosstalk measurements were performed for different channel spacings.

The crosstalk data in Figure (5-14) below shows the optical (open symbols) and electrical (closed symbols) crosstalk as a function of channel spacing. Figure (5-14) clearly indicates the decrease in crosstalk with increasing channel spacing. This is in accord with expectation, since the further the channels are spectrally apart the smaller the influence of both channels on each other (see Figure (5-12)).

![Graph showing crosstalk vs channel spacing](chart.png)

**Figure 5-14:** Experimental optical and electrical crosstalk as a function of channel spacing for both channels. The solids and dashed lines are smoothed curves fitted to each of the four experimental data sets. Crosstalk for the experiment with the FBG drop-filter is indicated with △ and ▲. The indication 'limited' is explained in the text. The symbol ▲ does not represent a real measurement, instead it indicates the expected value. The received optical power was set to -20 dBm throughout.
Optical crosstalk decreases from about −12 dB optical (−24 dB electrical) for 65 GHz spacing, to around −25 dB optical (−54 dB electrical) for a 180 GHz channel spacing. Electrical crosstalk is double the optical value since a square law receiver was used in our experiments. Note from Figure (5-14) that the level of crosstalk for channel 1 and 2 is different (this is more apparent for the electrical crosstalk). This is due to the fact that the transmission filter profile of the FP-TF is asymmetric around its centre wavelength (see Figure (4-4) on page 146 in the previous chapter), which causes the extinction, and therefore the level of optical crosstalk, to be different when either tuning the filter to channel 1 or tuning to channel 2.

Better resilience to crosstalk is obtained with a FBG based drop-filter like the one shown in Figure (4-3) on page 145 in the previous chapter. Using this filter, the optical crosstalk could be reduced to be better than −40 dB such that in the optical domain only channel 1 could be observed and channel 2 was completely filtered out (see Figure (5-15) below). In the electrical domain the crosstalk level was thus expected to be around −80 dB (△), however when we measured electrical crosstalk we obtained a level of only −60 dB (denoted by △ and the word 'limited' in Figure (5-14) above). With the FP-TF, the crosstalk is limited by the optical extinction of the filter, as can be seen in the Figures (4-3) and (5-11). Whereas with the greater extinction of the FBG drop-filter the crosstalk is not limited by the filter itself anymore, and systematic observations have been made to try to find the origin of the limitation. These investigations will be discussed next.

A first thought would be that the crosstalk would be an aspect of the spectrum analyser (e.g. electrical noise pickup, or that the measurement might be limited by the dynamic range of our measurement technique). However we measured the dynamic range and found it to be 83 dB far in excess of the crosstalk level and made extensive tests to confirm that the crosstalk we observed was genuinely associated with our optical signal. A second thought would be the probability of transfer of acoustic waves directly from one channel to the other via air. However this probability was low, since care was taken to acoustically isolate both sensor channels from each other by placing them in two separate boxes (see picture in the Figures (3-23) on page 129 and (4-15) on page 172).
A third possibility we investigated was that the signal might have been generated by cavities formed by out of band reflections. To test this, using the dual-channel narrowband ASE source, modulating only channel 1 and dropping channel 2, the received frequency spectrum (see e.g. Figure (5-13)) consisted only of the modulation signal from channel 1, i.e. $J_{1}^{(1)}$ ($\sim -70$ dBm). This signal was unaffected when spectrally detuning channel 1 (by making sure $\text{FBG}_1^{(1)}$ and $\text{FBG}_2^{(1)}$ do not spectrally overlap so there is effectively no cavity anymore). However, the signal disappeared when detuning channel 2 (or by removing channel 2 by breaking the fibre in-between the two channels). This told us that the signal $J_{1}^{(1)}$ was not from channel 1 but instead originates from light reflecting in channel 2. It was next thought that the electrical crosstalk might be limited by an acoustic wave running along the fibre core from one cavity to the other [23, 58]. To prove or rule out this possibility, we tried to eliminate (or at least reduce) any possibility of a propagating acoustic perturbation by splicing a long length of SMF (around 300 metres) in-between the two sensing cavities.
(approximately 4 metres of SMF separated the two channels initially). The findings from these tests are still inconclusive, though the acoustic wave hypothesis remains the most likely explanation of the increased electrical crosstalk of −60 dB (−80 dB is expected).

However, another possible explanation was found when the frequency spectra observed were closely examined. The PZT's in both channels were driven so hard (2π-radians) that a 5 kHz signal appeared in the frequency spectrum. There are two possible causes for this signal, one of which is the beating between the 10.0 kHz and the 15.0 kHz signals applied to channel 1 and channel 2 respectively. The other possibility was found earlier by Zervas et al. [63] and was caused by the sub-harmonic contribution to the RF spectrum due to the way the fibre is wound on the PZT. If the additional frequency component is due to sub-harmonic generation then the additional component would be at \( f_s^{(1)} / N \) or \( f_s^{(2)} / N \) (where \( N = 3 \)), i.e. at 3.3 or 5.0 kHz. The number of sub-harmonics would then depend on e.g. the tension with which the fibre is wound on the PZT fibre stretcher. If the cause is beating, the additional frequency component would be at \( (f_s^{(2)} - f_s^{(1)}) \), i.e. at \( (15 - 10) = 5 \) kHz. Unfortunately, no frequency spectra were taken for cavity modulation frequencies other than 10.0 and 15.0 kHz. It is therefore, without further study of this effect, impossible to conclude whether beating or sub-harmonic behaviour is the cause of the limited crosstalk. This additional signal can be seen in the power spectrum shown in Figure (5-16) below, marked by \( J_i^{(1,2)} \) in the two callouts. Note that this signal is only observed when tuning to channel 2, which is at a higher wavelength than channel 1.
Figure 5-16: Measured frequency spectrum showing the appearance of a 5 kHz beat or sub-harmonic signal, indicated with $J_1^{(1,2)}$. The FP-TF was tuned to channel 2, and the channel spacing was 180 GHz. Spectrum was taken with a 30 kHz resolution bandwidth.

The measured phase resolution of both channels under WDM operation is plotted in Figure (5-17) below as a function of channel spacing. The solid and dashed lines are smoothed curves fitted to both experimental data sets. For the measurement of the phase resolution, the modulation depth of both channels was reduced from $2\pi$ radians to 200 mrad. For channel 1 (●) there is a decrease in resolution observed as the channel spacing becomes narrower, below around 100 GHz. This is expected since channel 1 starts to 'feel' the presence of channel 2 the closer they are spaced (see Figure (5-12)). For spacing wider than 100 GHz, the resolution is more or less constant. The dependence on channel spacing for channel 2 (□) is seen to be less significant. The dependence of the phase resolution of channel 1 on the channel spacing is probably higher due to asymmetry of the FP transmission filter profile.
Figure 5-17: Experimental phase resolution of both sensor channels as a function of channel spacing. For both experimental data sets the SOP was configured for PP. The solid and dashed lines are smoothed curves fitted to both data sets. In these experiments the FP-TF was used. Both channels were modulated at the same time with a 200 μradian modulation depth and a 10 and 15 kHz modulation frequency for channel 1 and channel 2 respectively.

The reduced phase resolution (by around 20 μrad/√Hz reduction for each channel) of this dual-channel system, compared to the single-channel system, is due to the increased noise floor, caused by the signal from the modulation of the second channel. For the single-channel system the phase resolution was measured to be around 80 and 110 μrad/√Hz, for channel 1 and 2 respectively (for the SOP configured for the PP case). The visibility of both channels is shown in Figure (5-18) below as a function of channel spacing. Again, the two lines in the plot are smoothed curves fitted to both experimental data sets.
Figure 5-18: Experimental visibility of both sensor channels as a function of channel spacing. The solid and dashed lines are smoothed curves fitted to both data sets. In these experiments the FP-TF was used. Both channels were modulated at the same time with a 200 mradian modulation depth and a 10 and 15 kHz modulation frequency for channel 1 and channel 2 respectively.

In accordance with the behaviour observed for the phase resolution of channel 1 (Figure 5-17), we can see that a reduction in fringe visibility is observed for channel spacings narrower than around 100 GHz (●). The visibility of channel 2 (□) stays more or less constant.
5.3. System improvement using a Gain-Saturated Semiconductor Optical Amplifier

In the second chapter of this thesis it was already mentioned that a substantial reduction (~ 20 dB) of source RIN could be obtained by incorporating a GS-SOA in the narrowband ASE source (see source configuration in Figure (2-14) on page 69). Earlier work documented on the use of the gain-saturation of a SOA was done by Sato et al. [59] and Koyama et al. [60]. We performed our experiments only on channel 1, channel 2 was removed from the system by breaking the fibre in-between the two sensor cavities (the fibre end was angle cleaved and soaked in index matching oil to prevent optical feedback). There was therefore no need anymore for pre-receiver filtering of one of the two channels, and the FP-TF and FBG drop-filter were therefore removed. Setting of the input SOP to the SOA (to the principal axis of the SOA), was performed following the same procedure as was described earlier in the second chapter (section 2.5.8 on page 72). The SOA was then operated in the gain-saturation regime, found in the second chapter. The received optical power was set to −23.0 dBm throughout, and the SOP was PP. Phase resolution measurements were made for four different drive currents for the SOA, for a fixed input power to the SOA of +3.0 dBm. Due to the non-linearity, the SOA tends to broaden the input spectrum [61]. The final source bandwidth, i.e. the degree of chirp of the output spectrum (see Figure (5-20) below), can thus be controlled by the drive current. To verify whether the GS-SOA improved the system performance we compared this data with data obtained using the chirped narrowband ASE source operating with the same final optical bandwidth at the receiver. This was done by replacing the narrowband ASE source employing the GS-SOA with the chirped narrowband ASE source described earlier in the second chapter (section 2.5.4 on page 64). This phase resolution data, obtained employing the chirped source, is indicated with • in Figure (5-19) below. The phase resolution for channel 1 with the GS-SOA implemented in the narrowband ASE source is indicated with (□).
Figure 5-19: Experimental and predicted phase resolution of channel 1 as a function of the source bandwidth. The experimental points (○) are for the narrowband ASE source that incorporates the GS-SOA, for four different drive currents for the SOA (a) = 50 mA, (b) = 100 mA, (c) = 150 mA, (d) = 200 mA. For all measurements and theory the SOP was set to be PP. Experimental points obtained with the chirped narrowband ASE source are denoted by ●.

The following four drive currents were used, (a) 50 mA, (b) 100 mA, (c) 150 mA, and (d) 200 mA, as indicated in Figure (5-19) above, giving source bandwidths of 7, 8, 9 and 10 GHz FWHM respectively. The source output spectra for the experiments with the source with the GS-SOA and for the chirped ASE source can be seen in Figures (5-20) and (5-21) below respectively.
Figure 5-20: Measured output spectra of the narrowband ASE source with the GS-SOA incorporated for four different drive currents for the SOA. The output spectrum from the narrowband ASE source without the GS-SOA (e) is also shown. The resolution bandwidth of the optical spectrum analyser was 0.02 nm throughout.
For comparison the optical spectrum of the un-chirped narrowband ASE source (denoted by (e) ⋯⋯) has been added to Figures (5-19) and (5-20) above. Also note the less noisy ASE background of the source spectra incorporating the GS-SOA compared to the spectrum from the narrowband ASE source (spectrum (e) ⋯⋯). This indicates the intensity noise suppression from the GS-SOA. The output spectrum of the chirped narrowband ASE source is shown in Figure (5-21) (denoted by (f) and ―). The bandwidth was 10 GHz FWHM and the extinction ratio to the ASE background is seen to be around 38 to 40 dB.

Also shown in Figure (5-21) is the output spectrum of the source with the GS-SOA implemented, with a similar 10 GHz chirped bandwidth (denoted by (c) ⋯⋯). Again a much less noisy ASE background can clearly be seen for the spectrum incorporating the GS-SOA. Note the different spectral shape of the narrowband ASE source spectrum (denoted by (f) ―) compared to the source spectrum with the SOA implemented (denoted by (c) ⋯⋯), caused by the non-linearity of the SOA.
The letters (a), (b), (c), (d), (e) and (f) indicating the spectra in Figures (5-20) and (5-21) above correspond to the same letters indicating the phase resolution in Figure (5-19) above. The experimental phase resolution for the filtered ASE source with the GS-SOA incorporated are in reasonable accord with the theoretical curve for a length mismatch of 4.5 mm and the values are relatively insensitive to source bandwidth over the range 6 – 15 GHz.

There seems to be a slight improvement in phase resolution when implementing the SOA (compare 70 μrad/√Hz □ for the source with the SOA implemented and 80 μrad/√Hz ● for the chirped source in Figure (5-19) above). However, the improvement, of around 10 μrad/√Hz only accounts for a 1.2 dB source noise reduction (i.e. 1.2 dB increase in the SNR). This does not agree with the 20 dB reduction of the source RIN previously observed, which would increase the system's phase resolution by a factor 10, i.e. to 8 μrad/√Hz (though the detector noise floor, –119.5 dBm/Hz, would then limit this at a resolution of about 16 μrad/√Hz). Measurements are not limited by the measurement equipment, since resolutions as low as 50 μrad/√Hz have been recorded for channel 1 (for the PF configuration). The optical extinction ratio is still seen to be around 37 to 40 dB in Figures (5-20) and (5-21). Note that bandwidths in excess of 25 GHz cannot be used, since the FWHM bandwidth of the FBG’s in each sensor cavity is 25 GHz.

5.4. Conclusions and Discussion

An FBG based FOAS system was considered the system to be employed for acoustic sensing. Employing FBG technology gives great scope for WDM multiplexing. For this system to be proven viable, several more issues were addressed in this chapter. These issues were:

- Spectral drift of the FBG’s due to temperature fluctuations
- Phase response of the FBG’s
- Crosstalk issues related to WDM technology

This FBG based FOAS system was found to be relatively insensitive to FBG drift.
Furthermore, the issue of phase response when employing FBG technology is non-existent. WDM experiments were performed on a 2-channel FOAS system. These results showed a discrepancy between the $-40$ dB crosstalk level in the optical domain and $-60$ dB (instead of the expected $-80$ dB) crosstalk in the electrical domain, the origin of which is not yet clear. However, there are two possible explanations for this. Since an additional signal at 5.0 kHz was discovered in the RF spectrum, we were led to believe that this had two possible origins:

1. Beating between the 10.0 kHz and the 15.0 kHz signals applied to channel 1 and channel 2 respectively.
2. Sub-harmonic generation as a result of the way the fibre is wound on the PZT fibre stretcher [63].

If beating in the RF power spectrum is the cause then the additional frequency components would exist at $(f_0^{(2)} - f_0^{(1)})$ i.e. at $(15.0 - 10.0) = 5.0$ kHz. If sub-harmonic generation in the RF power spectrum is the cause then the additional frequency components would be at $f_0^{(1)}/N$ or $f_0^{(2)}/N$ (where $N = 3$) i.e. at 3.3 or 5.0 kHz. Unfortunately, at this later stage, when writing my thesis, it is impossible to tell whether beating or sub-harmonic generation is the cause since I only took frequency spectra, at the time I did these experiments, for cavity modulation frequencies of 10.0 and 15.0 kHz for channel 1 and channel 2 respectively.

This system, when optimised SOP and heterodyne frequency, showed a phase resolution of 50 μrad/√Hz at 10 kHz. We have presented results of system performance using the narrowband ASE with the GS-SOA implemented. However, these results are inconclusive since the anticipated 20 dB improvement in the SNR of the system was not found. Instead the SNR increased by only 1.2 dB. Further work is required to understand the origin of this discrepancy.

With proper fusion splicing techniques (low splice loss) this system shows great scope for introducing more channels via WDM (i.e. by slicing the spectrum of the ASE source into more WDM channels).
Chapter Six

Conclusions and Future Directions

Overview: The conclusions from the complete thesis will be drawn in this final chapter. Also recommendations for future directions will be given.

6.1. Conclusions

In conclusion, in this thesis we have presented the results of a study into the design of a multiplexed Fibre-Optic Acoustic-Sensor (FOAS) array based on Fibre Bragg Grating (FBG) technology. A FOAS array of this type has a number of potential applications in areas that require sensors with a high acoustic sensitivity and low crosstalk. These areas may include military underwater surveillance systems and seismic systems for the oil industry (e.g. streamers or towed arrays).

This thesis presented a FOAS system with sensing sections of around 12.5 metres that are defined by two identical FBG’s acting as in-line point reflectors. We have evaluated several interferometer schemes based on both the Mach-Zehnder Interferometer (MZI) and the Fabry-Perot (FP) Interferometer configurations interrogated using narrowband fibre Amplified Spontaneous Emission (ASE) sources. The use of FBG-based interferometers allows the use of Wavelength Division Multiplexing (WDM) technology, which in turn allows us to multiplex large number of sensors in the system. Conventionally FBG-based systems are interrogated using laser sources. However, interrogation using lasers introduces phase-induced intensity noise, which then limits the performance of these systems. Employing low-coherence ASE sources for sensor interrogation eliminates this kind of noise. Moreover, one master source can be used to address many sensors, by spectrally slicing the full optical bandwidth of the ASE source. Also, the coherence properties of the light can be used as
a further means of multiplexing. This should further improve crosstalk.

The noise characteristics and sensitivity limits of these systems have been studied both experimentally and theoretically. In order to proof the viability of such a system we initially focused our study on less complex interferometer systems based on the MZI configurations interrogated using a narrowband ASE source.

*Narrowband Amplified Spontaneous Emission source for sensor interrogation*

FOAS systems employing ASE source for sensor interrogation are generally limited in terms of sensitivity by the inherent intensity noise from the ASE source (termed excess photon noise). To appreciate the level and influence of this source on the sensor system, we have fully characterised theoretically the noise properties of the narrowband ASE source used for sensor interrogation. These results on the narrowband ASE were presented in chapter (2). The model, based on a Gaussian random process, was verified with experiments and found to agree well. It was found that the source did behave as a thermal source as expected, i.e.:

1. The source noise scales with the inverse source bandwidth.
2. The source noise is proportional to the square of its output intensity.

The scaling of the source noise with the square of the intensity means that the system's Signal to Noise Ratio (SNR) can not be improved beyond a certain optical power level. To increase the number of sensors that can be multiplexed in the system, the channel bandwidth has to be as narrow as possible. However, this has consequences for the ultimate sensitivity of the system since narrowing the source bandwidth brings an associated increase in source noise. Clearly an optimum source bandwidth must exist. We believe the approach to be suitable for a broad range of sensing applications.

In the last part of the second chapter we have shown that the inherent source intensity noise could be reduced by as much as 20 dB by incorporation of a Gain-Saturated Semiconductor Optical Amplifier (GS-SOA) in our narrowband ASE source, to act as a 'noise eater'. The intensity noise is reduced by the fast gain (carrier) dynamics of the SOA.

*Single Mach-Zehnder Interferometer*

The first part of chapter (3) dealt with the study of the impact of a single MZI on the
source noise. This study revealed that, in case of a strongly unbalanced interferometer, there is periodicity in the spectral domain. This well-known effect is referred to as the 'filtering effect' of an interferometer. Furthermore, we have shown that the system performance is limited by the excess photon noise of the source and that the detailed system noise spectrum is dependent on the filtering effect and the State Of Polarisation (SOP) in the optical paths of the interferometer. For a strongly unbalanced MZI (25 metres) it was seen that the noise power could be reduced by 2.5 dB when configuring the SOP to two mutually orthogonal beams (denoted the Polarisation Flipping or PF configuration). For a balanced MZI with a sensing arm length of around 25 metres and a practical path-imbalance of 1 cm we have shown that an optimum phase resolution of around 62 μrad/√Hz is obtained.

_Dual Mach-Zehnder Interferometer_

The last part of _chapter (3)_ described a dual MZI system. By employing a heterodyne technique and utilising these noise redistribution effects, we have shown that it is possible to reduce the noise level in the signal frequency region and have achieved a phase resolution of 50 μrad/√Hz, for a practical path length mismatch of ~ 1 cm. We consider the approach we have demonstrated to be highly practical, providing good phase resolution while remaining consistent with interferometer manufacturing tolerances and requiring relatively simple demodulation methods.

_Mach-Zehnder Fabry-Perot Interferometer_

_Chapter (4)_ presented results on a system employing FP interferometers based on FBG's. The introduction of FBG-based FP interferometers introduces several issues that were addressed in this thesis, knowing:

1. Optimum reflectivities of the cavity FBG's giving the system maximum phase resolution.
2. Multiple-reflection nature of the FP cavities.
3. Spectral drift of the FBG's due to temperature (or strain) fluctuations.
4. Phase response of the FBG's.
5. Crosstalk issue related to WDM technology.

The first two points were addressed in _chapter (4)_. It was found both experimentally
and using our theory that the system's phase resolution reaches a maximum for the input FBG having a 40 % reflectivity and the second reflector having a 100 % reflectivity. It was also found that the system does not suffer from the multiple reflections created by the FP cavities since similar phase resolutions were obtained (i.e. around 50 μrad/√Hz) with this system and the dual MZI system.

Finally chapter (5) presented results on the robustness of the system. Spectral drift of the FBG's and phase response of the FBG's was highlighted in this chapter. It was found that the system is relatively insensitive to spectral drift of the FBG's due to temperature (or strain) fluctuations. It was also discovered that there is no phase response of the FBG's. Another issue that was addressed had to do with WDM technology and that was crosstalk. Intra-channel crosstalk imposes a lower limit on the channel spacing that can be used. We have presented experiments on a two-channel WDM sensor system and have shown that a crosstalk level of −60 dB is achieved. The crosstalk level should be around the −80 dB level, since a −40 dB optical crosstalk was observed. This crosstalk level is probably limited by either beating between the 10.0 kHz and the 15.0 kHz signals or by sub-harmonics generated by the PZT used in the sensor cavity to simulate the acoustic signal. To conclude which of these two explanations is the cause of the limited crosstalk, frequency spectra have to be taken for modulation frequencies other than the 10.0 and the 15.0 kHz signals for both channels.

We have also presented results on using a GS-SOA to reduce the inherent intensity noise from the narrowband ASE source. Although a 20 dB reduction in source RIN was observed, the results on the improvement of the system's phase resolution are still inconclusive since a 1.2 dB gain in the system's SNR was measured.

*Acoustic pressure sensitivity of the sensor*

It would be useful to mention some of the basic acoustic levels and units of measure. Fibre-optic acoustic hydrophones are usually calibrated in terms of pressure, with the normalised sensitivity defined as the relative optical phase shift induced by a given acoustic pressure at a specific acoustic frequency. The pressure sensitivity of the optical phase in a fibre is defined as:

$$M_o = \frac{\Delta \phi}{\phi \cdot \Delta P}$$

Equation 6-1
where $\Delta \phi$ is the shift in the optical phase $\phi$ due to a pressure change $\Delta P$. A typical polyester (trade name Hytrel) coated optical fibre has a normalised pressure sensitivity of $2 \cdot 10^{-11} \text{Pa}^{-1}$.

One frequently encountered acoustic reference pressure level used in air acoustics is 0.0002 dynes/cm$^2$ (20 $\mu$Pa). This is the accepted average minimum detectable sound pressure for humans at 1 kHz. Another is the currently accepted 1 $\mu$N/m$^2$ or 1 $\mu$Pa reference pressure for underwater acoustics. To compare these on a decibel scale, recall first that for a pressure ratio, $P_1/P_2$, the number of decibels, $H$, in dB, is defined by:

$$H = 20 \log_{10} \left( \frac{P_1}{P_2} \right)$$

Equation 6-2

Since 0.0002 dynes/cm$^2$ is equal to 20 $\mu$Pa, the air acoustic reference pressure is 26 dB above the underwater reference pressure. A pressure of 1 dyne/cm$^2$ is at a level of 74 dB re 0.0002 dyne/cm$^2$$\sqrt{\text{Hz}}$ and at a level of 100 dB re 1 $\mu$Pa/$\sqrt{\text{Hz}}$. Finally, a pressure of 1 Pa corresponds to 120 dB re 1 $\mu$Pa/$\sqrt{\text{Hz}}$. 
The minimum detectable acoustic pressure level (detection threshold) can be calculated using the following expression [62]:

\[
P_{\text{min}} = \frac{\lambda_0 \left( \phi_{\text{s}} \right)_{\text{min}}}{2\pi m M_0 I_{\text{min}}}
\]

Equation 6-3

Here \( \lambda_0 \) is the wavelength of the light, \( n \) is refractive index of the fibre core, \( M_0 \) is the normalised phase resolution given by Equation (6-1). For the experimentally obtained phase resolution of 50 \( \mu \text{rad}/\sqrt{\text{Hz}} \) of our system we find a minimum detectable acoustic pressure level of 17 mPa/\( \sqrt{\text{Hz}} \) (where we used \( \lambda_0 = 1550 \text{ nm} \), \( n = 1.46 \), \( M_0 = 2 \cdot 10^{11} \text{ Pa}^{-1} \)). This corresponds to a normalised acoustic resolution of around 85 dB re 1 \( \mu \text{Pa}/\sqrt{\text{Hz}} \). In order to be able to obtain a pressure sensitivity of around 13 \( \mu \text{Pa}/\sqrt{\text{Hz}} \) (15 dB above the underwater acoustic reference level), we would need a

---

**Figure 6-1:** Illustration of the reference pressure levels and units. The reference level for underwater acoustics is 26 dB below the air reference level at 1 \( \mu \text{Pa}/\sqrt{\text{Hz}} \).
32 km sensing arm in the interferometer! Although the induced optical path change in an optical fibre is very high for anisotropic strain, it is many orders of magnitude less for isotropic strain changes (i.e. pressure). Kilometres long lengths of fibre would make these systems both bulky and expensive. Therefore considerable research effort has been directed toward the study of fibre coatings and other means (like mandrels) to improve the pressure sensitivity of the optical fibre. It has been shown that coatings can provide ~ 20 dB improvement in acoustic sensitivity and a suitably designed mandrel up to ~ 40 dB improvement.

6.2. Future Directions

The following points serve as recommendations for possible future work:

- Following the investigation of crosstalk levels on the 2-channel system it is worth pursuing the origin of the discrepancy in the optical crosstalk (~40 dB) and the crosstalk level in the electrical domain (~60 dB). Frequency spectra have to be taken when modulating the two sensor cavities with modulation frequencies other than 10.0 kHz and 15.0 kHz, to confirm whether fibre winding plays a role here.
- It would be worth pursuing the further scaling of WDM based FOAS systems based on our approach.
- With the WDM system investigated in detail it would then be interesting to see how a FDM or perhaps even a TDM system would behave.
- Since the system is supposed to be employed in a towed hydrophone array, it would make sense calibrating the system in a controlled and calibrated acoustic environment.
- It would be worth pursuing the discrepancy between the anticipated improvement in phase resolution for the GS-SOA and the experimentally obtained value.

Finally I would like to mention that this thesis did not discuss the effect that Polarisation Induced Fading (PIF) has on the performance of FOS. Interferometric sensors built using standard low-birefringence fibre are often prone to variations in the output signal caused by random fluctuations in the light’s SOP. This phenomenon, known as PIF, affects the performance, sensitivity, and therefore practicality, of many
types of interferometric sensors. When light travels in a circular fibre, as opposed to propagating in free space, its polarisation state distorts due to the random birefringence induced by thermal stress and mechanical stress and irregularities of the fibre core. At any given point along the fibre, the optical beam is generally elliptical polarised with various degrees of ellipticity and orientation. Practical devices are built using High Birefringence (Hi-Bi) Polarisation Maintaining (PM) fibre, eliminating the need for polarisation controllers. To realise Hi-Bi fibre, the fibre is designed so that the two orthogonal modes of polarisation have different propagation constants. This can be done, by making the core of the fibre elliptical instead of circular or by the incorporation of stress rods.
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