

The Impact of Inflation Uncertainty on Interest Rates^{*}

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Abstract

This paper examines the impact of inflation uncertainty on interest rates for the case of the U.S. three-month Treasury bill rate. We emphasize how consistent OLS estimators can be obtained from an extended Fisher equation which includes a proxy variable of inflation uncertainty measured by an ARCH-type model. The empirical results show a significantly negative effect of inflation uncertainty on the nominal interest rate. This evidence is not consistent with the hypothesis of inflation risk premium that inflation uncertainty positively affects the nominal interest rate.

Keywords: ARCH; Consistent OLS estimation; Measured variable

JEL classification: C13, E31, E43

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1. Introduction

Recently, some researchers argue that the current low level of the nominal interest rate in the United States is generally attributed by reduced inflation risk premiums, due to the improved inflation performance of the U.S. economy (see, for example, Mehra (2006)). This argument is based on the hypothetical posit that an increased inflation leads to a higher inflationary uncertainty and the uncertainty in turn affects interest rates positively. Under consideration of savings and investment in real economic activities, this conventional view of inflation risk premiums in financial markets is called into question in this study.

Inflation uncertainty can affect economic activities by distorting both inter-temporal and intra-temporal allocation efficiency of resources (see Friedman (1977)). The former affects outputs through changes in the level of interest rates, while the latter works through the changes in relative prices. To examine the first hypothesis, a number of macroeconomists have investigated how inflation uncertainty affects interest rates. The empirical evidence is generally mixed. For examples, Barnea et al (1979), Brenner and Landskroner (1983), Ireland (1996), Mehra (2006), and Berument et al (2007) all provide positive relationships between inflation uncertainty and interest rates and interpret their findings as evidence that risk-averse investors try to avoid increased uncertainty about future inflation by simply adding risk premiums to interest rates. On the other hand, Levi and Makin (1979), Bomberger and Frazer (1981), and Hartman and Makin (1982) provide evidence of negative relations between inflation uncertainty and interest rates. These authors explain their findings in the framework of the IS-LM model as evidence that, reduced investment in the side of the demand for investment funds is larger than reduced savings in the supply side of the funds, given that inflation uncertainty simultaneously affects investment and savings negatively. Meanwhile, Lahiri et al (1988) and Shome et al (1988) provide statistically insignificant evidence and conclude that there seems to be no direct relation between inflation uncertainty and interest rates.

The empirical mixture of the above earlier studies might stem from econometric problems. One of the difficulties in this kind of research is that inflation uncertainty is not directly observable. Most previous studies commonly use survey data to measure inflation uncertainty and apply OLS estimation. However, it is often argued in the literature that, the proxy is a disagreement about the forecasted level of inflation rather than an explicit measure of inflation uncertainty, since the measurement is calculated by the standard deviation of respondents' point forecasts of the inflation rate (see Zarnowitz and Lambros (1987), and Rich and Butler (1998)). Another problem with this generated variable is that, if conventional OLS technique is applied to estimate an empirical interest rate equation which includes a measured variable of inflation uncertainty, the obtained estimators suffer from inconsistency (see Pagan and Ullah (1988)). Thus, a considerable econometric care in this line of research is required to have valid inference from the estimators obtained by OLS estimation.

In this paper, we re-examine the possible impact of inflation uncertainty on

interest rates by focusing on econometric issues being able to occur from a measured risk variable. Instead of survey data, we use an ARCH-type model to generate inflation uncertainty. Since it well captures the clustering patterns of uncertainty, various ARCH-type models have been popularly applied in the recent literature to measure inflation uncertainty. Considering econometric problems with this measured uncertainty, we first discuss the existence of an orthogonality condition between the measured proxy and error term in the underlying estimation equation, and show that the orthogonality condition can be used to derive a consistent covariance matrix of OLS estimators by applying the Newey and West (1987) procedure. The latter correction adjusts the non-scalar covariance matrix of OLS estimators, which mainly stems from the composite error term involving noises in the auxiliary equation used to generate the variable of uncertainty. Applying this procedure to the U.S. Treasury bill rate, we provide evidence of a significantly negative relationship between inflation uncertainty and interest rates. The result obtained from this simple OLS estimation is also consistent with those of ML and IV methods commonly recommended by the literature. Our finding is noteworthy in that, in spite of a large body of the literature, few papers provide apparently convincing evidence on the negative impact of inflation uncertainty on interest rates. This evidence is contrasted to the conventionally dominant premium view that inflation uncertainty might positively affect interest rates in financial markets.

The paper is organised as follows. Section 2 discusses the orthogonality condition for the consistent OLS estimation of an extended Fisher equation, which includes the variable of inflation uncertainty measured by an ARCH class auxiliary equation, and explains that the condition can be used to derive OLS-based GMM estimators. Then, the procedure is applied to the case of the U.S. Treasury three-month bill rate in Section 3. Section 4 provides alternative IV-GMM estimators and compares them with our OLS estimators. Finally, conclusions are provided in Section 5.

2. OLS estimation with the uncertainty measured by an ARCH model

Consider a reduced-form equation for the nominal interest rate (R_t) based on an information set \mathfrak{I}_t :

$$R_t = \beta x_t + \gamma h_t + e_t, \quad (1)$$

where x_t is a vector of exogenous regressors including a constant term, h_t denotes inflation uncertainty, and e_t is a disturbance term with $E(e_t | \mathfrak{I}_t) = 0$. Since h_t is not directly observable in practice, it is assumed that the uncertainty variable is replaced by the conditional variance of the inflation rate π_t , of which generating process is characterised as the following standard ARCH model,

$$\begin{aligned} \pi_t &= \eta \psi_t + \varepsilon_t, & \varepsilon_t | \mathfrak{I}_{t-1} &\sim N(0, h_t), \\ h_t &= \rho z_t + v_t, \end{aligned} \quad (2)$$

where η is a vector of regression parameters, ψ_t is a vector of independent variables

for the inflation rate, $\rho' = (\rho_0, \rho_1, \dots, \rho_s)$, $z_t = (1, \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots, \varepsilon_{t-s}^2)$, v_t is a white noise with $v_t \sim IID N(0,1)$ and $cov(e_t', \varepsilon_t) = 0$. A usual two-step procedure for the estimation of equation (1) is first to obtain \hat{h}_t , where $\hat{h}_t = \hat{\rho}z_t$, from (2) and then to replace h_t with \hat{h}_t . With this procedure, an estimable form of equation (1) can be rewritten as:

$$R_t = \beta x_t + \gamma \hat{h}_t + u_t = \phi w_t + u_t, \quad (3)$$

where $u_t = e_t + \gamma(h_t - \hat{h}_t)$, $w_t = (x_t, \hat{h}_t)$, and $\phi = (\beta, \gamma)'$.

From the above two-step procedure with the inflation uncertainty measured by the ARCH model (2), uncorrelated conditions between explanatory variables and error term in (3) can be established by using the ARCH generating equation $\varepsilon_t = v_t \sqrt{h_t}$, $v_t \sim IID N(0,1)$. This fundamental property entails that, \hat{h}_t is the consistent estimator of h_t by the law of iterated expectations, $h_t = E(\varepsilon_t^2 | \mathfrak{I}_t) = E(h_t | \mathfrak{I}_t) = E(v_t^2 | \mathfrak{I}_t) = E(h_t | \mathfrak{I}_t) = \hat{h}_t$, since $E(v_t^2 | \mathfrak{I}_t) = 1$. In practice, the fitted \hat{h}_t is not exactly equal to h_t , but the difference between the two is disappeared as the sample size is increased by the law of large numbers. Nelson and Forster (1994) show that, even an ARCH model is misspecified, the conditional variance estimates produced by the ARCH model converge in probability to the true conditional variances.¹ This strong property $\hat{h}_t \xrightarrow{a.s.} h_t$ leads $(x_t, \hat{h}_t)'$ and u_t to be uncorrelated with $E(x_t' \cdot \gamma(h_t - \hat{h}_t)) = 0$ and $E(\hat{h}_t \cdot \gamma(h_t - \hat{h}_t)) = 0$, and consequently establishes orthogonality conditions between explanatory variables and error term in (3), such that, $T^{-1} \sum x_t' [(h_t - \hat{h}_t) \gamma + e_t] \xrightarrow{p} 0$ and $T^{-1} \sum \hat{h}_t [(h_t - \hat{h}_t) \gamma + e_t] \xrightarrow{p} 0$, because $E(x_t' \cdot e_t) = 0$ by the initial assumption of exogeneity in equation (1) and $E(\hat{h}_t \cdot e_t) = 0$ by the assumed uncorrelation of e_t with ε_t in equation (2). Thus, the interested parameters β and γ of equation (3) can be consistently estimated by using OLS technique directly.

If h_t is generated by moving averages of the variances or standard deviations calculated from the mean value of π_t , the generated \hat{h}_t has a weak property with $E(\varepsilon_t^2 | \mathfrak{I}_t) = \hat{h}_t$ in the sense of Pagan and Ullah (1988). In this case, $E(x_t' u_t) = 0$ but $E(\hat{h}_t u_t) \neq 0$ because the latter involves $\gamma(h_t - E(\hat{h}_t))$. The term $T^{-1} \sum \hat{h}_t [(h_t - \hat{h}_t) \gamma +$

¹ On the other hand, see McKenzie and McAleer (1994) who discuss some possible impacts of misspecification in an auxiliary equation used to generate an unobservable variable on the estimators of the primary equation in a two-step procedure.

$e_i] \xrightarrow{p} 0$ converges to $\lim_{T \rightarrow \infty} T^{-1} \sum E(\hat{h}_i h_i - \hat{h}_i^2) \gamma = \lim_{T \rightarrow \infty} T^{-1} \sum [h_i^2 - E(\hat{h}_i^2)] \gamma$ by the law of large numbers for independently distributed random variables, which is only degenerate when $h_i^2 = E(\hat{h}_i^2)$. However, since ε_i is usually assumed to be normal with its fourth central moment $E(\hat{h}_i^2) = 3h_i^2$, $(h_i^2 - E(\hat{h}_i^2))$ becomes $-2h_i^2$, such that $T^{-1} \sum \hat{h}_i [(h_i - \hat{h}_i) \gamma + e_i] \neq 0$. The correlation of the generated regressor \hat{h}_i with the error term u_i leads the OLS estimators to be inconsistent.

Note that, even though \hat{h}_i generated by an ARCH type model is not correlated with the error term and so the OLS estimation delivers consistent estimators, the variance of the estimators is not equal with that of usual OLS estimators, since $Var(\hat{\phi} | w) = E[(w'w)^{-1} w' u u' w (w'w)^{-1}] = Q^{-1} \Omega Q^{-1}$, where $Q = w'w$, $\Omega = w' u u' w$, and $E(u u')$ is non-spherical disturbances. The former is larger than the latter, mainly due to the composite error u_i which includes noises in the ARCH generating equation (see Appendix A). This indicates that an application of the conventional statistical inferences based on usual OLS standard errors can be misled. To correct this problem, the procedure suggested by Newey and West (1987) can be applied to obtain the consistent covariance matrix of $\sqrt{T}(\hat{\phi} - \phi)$ by adjusting the inconsistency in the element of Ω , using the sample information in frequency domains, such that $\hat{\Omega} - \Omega \xrightarrow{p} 0$. Then, the non-scalar covariance matrix $V = Q^{-1} \Omega Q^{-1}$ of $\sqrt{T}(\hat{\phi} - \phi)$ is consistently estimated by $\hat{V} = \hat{Q}^{-1} \hat{\Omega} \hat{Q}^{-1} \xrightarrow{p} Q^{-1} \Omega Q^{-1}$ with the property of the non-singular matrix $\hat{Q} \xrightarrow{p} Q$. The true value ϕ is asymptotically approximated by the adjusted variance of the OLS estimator $\hat{\phi}$ as $\hat{\phi} \approx N(\phi, (\hat{V}/T))$. The square root of the diagonal elements of the covariance matrix \hat{V}/T can be treated as the heteroscedasticity and autocorrelation consistent standard error for the OLS estimators and used to test the underlying hypothesis as usual. The adjusted OLS estimators in this way are a special case of just-identified GMM estimators (see Appendix B).

3. Empirical results

3-1. Measuring inflation uncertainty

Since the variable of inflation uncertainty can not be directly observable, economists have used a proxy measured by moving averages of standard deviations or variances of the inflation rate; by the residuals obtained from time series models; by survey data; or by ARCH-type models.² The first two measures are obtained by quantifying either the range of observations or the degree of dispersions around the mean of the

² See Oxley and McAleer (1993) for a comprehensive survey of parametrically generated uncertainty. Alternatively, Hong and Pagan (1988), Pagan and Ullah (1988), and Baltagi and Li (2001) consider nonparametrically generated risk measurements.

inflation rate. These proxies assume the constant conditional variance, consequently ignoring information about the underlying stochastic process by which inflation uncertainty is generated in a non-constant way. Meanwhile, a proxy measured by survey data, particularly with the Livingston survey popularly applied in the literature, is computed by the standard deviation of respondents' point forecasts of the inflation rate. A problem with this measurement is that the proxy is a disagreement about the forecasted level of inflation rather than an explicit measurement of inflation uncertainty (see Zarnowitz and Lambros (1987)). On the other hand, a proxy measured by an ARCH-type model allows non-constant time varying conditional variance to be captured and thus are useful in describing the feature of uncertainty clustering and other characteristics of uncertainty, such as excess kurtosis and fat-tailedness (see Engle (1982)). Given that inflation uncertainty can be generally characterized as the clustering of large shocks to the conditional variance of the inflation rate, a GARCH model is used to measure inflation uncertainty in this study.

To formulate a GARCH model, we used the annualised inflation rate (π_t) of the monthly changes obtained from the first differenced series of the seasonally adjusted consumer price index (CPI) in logarithm. The sample period covers from 1953(1) to 2008(12). Initially, a twelve-order autoregressive (AR) model was estimated for the conditional mean equation, of which its error term is assumed to follow a student t -distribution with some degrees of freedom rather than a Gaussian normal distribution (see Bollerslev (1987)). Then, insignificant lags were sequentially reduced by using the encompassing test proposed by Mizon and Richard (1986). A conditional mean equation finally obtained together with the underlying GARCH model is

$$\begin{aligned}\pi_t = & 0.36 + 0.32\pi_{t-1} + 0.11\pi_{t-2} + 0.15\pi_{t-6} + 0.08\pi_{t-7} + 0.14\pi_{t-9} + 0.10\pi_{t-10} \\ & (0.13) (0.04) \quad (0.04) \quad (0.04) \quad (0.04) \quad (0.04) \quad (0.04) \\ & + 0.12\pi_{t-11} - 0.13\pi_{t-12} \\ & (0.04) \quad (0.04) \\ h_t = & 0.28 + 0.15\varepsilon_{t-1}^2 + 0.83h_{t-1} + 5.15\theta \\ & (0.14) (0.04) \quad (0.04) \quad (1.19),\end{aligned}\tag{4}$$

$$T = 1953(1) - 2008(12), \quad \text{Log-likelihood} = -1505.08,$$

where θ denotes the degree of freedom of the student t -distribution and the values in parentheses represent standard errors. Herewith, the degree of freedom to capture the student t -distribution was not specified in advance. Rather, it was treated as a parameter along with others in the model. All the estimated GARCH coefficients exceed zero, and their stationarity condition is satisfied with $0.15 + 0.83 = 0.98 < 1$. These results ensure that the conditional variance is strictly positive, thus satisfying the necessary conditions of the GARCH model. The statistical significance of the ARCH effect in the model is confirmed by a Wald statistic $\chi^2(4) = 3914.02$ for the joint null hypothesis that all the coefficients in the GARCH equation are zero. It is

noticeable that the estimated parameter of the student t -distribution is statistically significant. The conditional kurtosis calculated from the estimated value is $3(\hat{\theta}-2)(\hat{\theta}-4)^{-1}=8.22$, which considerably differs from the normal value of three. The null hypothesis of the zero coefficient on the degree of freedom is also rejected at the 5% level with a Wald statistic $\chi^2(1)=18.78$.

3-2. Estimated results

Using the inflation uncertainty generated by equation (4), an extended version of the Fisher equation was considered as a special case of equation (1),

$$R_t = r + \beta\pi_t^e + \gamma h_t, \quad (5)$$

where R_t denotes the nominal interest rate; r is a constant term representing the real interest rate; π_t^e is expected inflation; and h_t is inflation uncertainty. Most of earlier studies use this reduced-form equation as a bench-mark model. While inflation uncertainty was measured by the GARCH model estimated in the above sub-section, the 3-month Treasury-bill secondary market rate was used to eliminate the effects of default risk premiums. The alternative rates, such as the commercial paper rate and various corporate bond rates, are largely contaminated by default risk premiums, so that it is difficult to identify the genuine impacts of inflation risk on the interest rate. Expected inflation was measured by the annualised rate of growth of the seasonally adjusted consumer price index (CPI) in logarithm under the assumption of rational expectations. Sargent (1973) and Summers (1986) argue that the errors-in-variable problems being able to occur from the common use of the actual inflation as a proxy for the expected rate of inflation can be circumvented by using a dynamic model (7) below. This is because lagged variables representing the evolution of the actual rate of inflation effectively approximate the source of generating rational expectations forecasts. Following Hendry (1995, p.591) who suggests to use the original series of the interest rate without changing it into logarithm, all the variables were transformed into logarithms for analysis, except the interest rate. The monthly data series taken from FRB St. Louis FRED II covers the sample period from 1953(1) to 2008(12).

The separate impacts of the expected inflation rate and its uncertainty on the nominal interest rate can be clarified in the line of Knight (1921, p.321) who distinguishes between risk and uncertainty. Expected inflation π_t^e captures the risk related to the central tendency of the frequency distribution in the future inflation rate. This is a known risk which can be observed by using the first moment of inflation-generating processes. Meanwhile, there is also another risk that the existing frequency distribution of expected inflation is not exactly coincided with the exact distribution of expected inflation. This risk is referred to inflation uncertainty associated with what is not known that might affect what is known about future possibilities. The uncertainty h_t is generally captured in the second moment of expected inflation. Following Fisher, it is expected that the known risk related to expected inflation has a positive impact on interest rates, but the unknown factor in expected inflation can lead

the nominal rate of interest to rise or fall with almost equal probability. Thus, the impact of inflation uncertainty on the interest rate is an empirical issue to be examined.

Before we investigate the interrelationships between the nominal interest rate, expected inflation, and inflation uncertainty, augmented Dickey-Fuller (ADF) tests were first conducted to examine stationarity of the data series used. With an initial maximum lag of twelve, the auxiliary lags were selected on the basis of the Akaike information criterion (AIC). In the case of the variables in levels, the testing equation included an intercept and a linear trend, while the differenced variables included only an intercept term. The test results reported in Table 1 reveal that all the variables appear to be $I(1)$ at the 5% significance levels, respectively. Since the variables are non-stationary, the cointegration test proposed by Johansen (1988) was applied with maximum likelihood (ML) estimation to check the co-movements between the variables. With twelve lags on the basis of the AIC criterion, the testing equation included a constant term but no trend. The intercept was restricted to lie in the cointegration space. The testing results reported in Table 2 (A) show that the null hypotheses of cointegration rank $\lambda=0$ in both maximum eigenvalue and trace statistics are rejected at the 5% levels by the critical values taken from Osterwald-Lenum (1992). Thus, it seems that there is only one cointegrating vector in the system.

Table 2 (B) reports the statistics of the weak exogeneity tests proposed by Engle et al (1983) and extended by Johansen (1992) into the cointegrated system. The results show that the variables of π_t and h_t can be treated as weakly exogenous ones at 1% levels in the system but R_t is an endogenous variable. This evidence suggests that, in the long run, the causality between the variables seems to run from π_t and h_t to R_t . Based on this property, an extended Fisher equation under uncertainty was obtained by normalizing the cointegrating vector as follows

$$R_t = 2.92 + 1.15\pi_t - 0.27h_t \quad (6)$$

(0.88) (0.18) (0.11),

where the values in parentheses denote the standard errors. The obtained parameters show expected signs with reasonable magnitudes at the conventional 5% significance levels, respectively.

As argued by Engle et al (1983), an important methodological implication of the weakly exogenous properties of expected inflation and inflation uncertainty is that, even though we examine the Fisher effect under uncertainty through a single equation conditioned on the weakly exogenous variables, we do not lose any information about the entire system. Based on the statistical properties of weak exogeneity and cointegration, an error correction model (ECM) which is equivalent to a thirteen-order autoregressive distributed lag (ADL) model was initially formulated,

$$\Delta R_t = d_0 + \sum_{i=1}^{12} d_{1i} \Delta R_{t-i} + \sum_{i=0}^{12} d_{2i} \Delta \pi_{t-i} + \sum_{i=0}^{12} d_{3i} \Delta h_{t-i} + d_4 R_{t-1} + d_5 \pi_{t-1} + d_6 h_{t-1} + e_t$$

$$\begin{aligned}
&= \sum_{i=1}^{12} d_{1i} \Delta R_{t-i} + \sum_{i=0}^{12} d_{2i} \Delta \pi_{t-i} + \sum_{i=0}^{12} d_{3i} \Delta h_{t-i} + d_4 [R + (d_5 / d_4) \pi + (d_6 / d_4) h \\
&\quad + (d_0 / d_4)]_{t-1} + e_t.
\end{aligned} \tag{7}$$

An advantage of this dynamic model is that it matches the lag reactions of the static model (5) to the autocorrelation structure of the observed time series by considering the presence of common factors (see Hendry (1995, p.233) and Mizon (1995)). Thus, this model is compared to those of earlier studies that are mostly variant forms of the static model (5) and so often subject to the problem of residual autocorrelations due to dynamic misspecification. For analysis, instead of a restricted way in which the identified equation (6) in the cointegration vector is used, herewith we estimated the level terms in an unrestricted way to fully retrieve data regularity and to later compare with the ML estimators in (6). Since the over-parameterisation of the unrestricted model (7) often captures accidental features of the sample, to reduce the sample dependence we sequentially simplified the general model by eliminating statistically insignificant parameters, using the encompassing test suggested by Mizon and Richard (1986).

A finally derived, parsimonious model is

$$\begin{aligned}
\Delta R_t &= 0.26(\sum \Delta R_{t-i}) - 0.09(\sum \Delta \pi_{t-i}) - 0.06(\sum \Delta h_{t-i}) \\
&\quad (0.13) \quad (0.03) \quad (0.02) \\
&\quad - 0.03(R - 0.73\pi + 0.15h - 3.41)_{t-1}, \\
&\quad (0.01) \quad (0.27) \quad (0.10) \quad (0.97)
\end{aligned} \tag{8}$$

$$R^2 = 0.30, \sigma^2 = 0.38, DW = 1.96, F_{AR(5,608)} = 2.50,$$

$$\begin{aligned}
&\text{lags included in } \sum \Delta R_{t-i} : \{i = 1, 2, 5, 6, 9\}, \sum \Delta \pi_{t-i} : \{i = 0, 2, 3, 4, 5, 6\}, \\
&\sum \Delta h_{t-i} : \{i = 1, 2, 3, 7, 9, 12\},
\end{aligned}$$

where R^2 denotes the coefficient of determination; σ^2 is the standard error of the regression; DW is the Durbin-Watson statistic; F_{AR} indicates the Lagrange multiplier (LM) test for five-order autocorrelations; and the values in parentheses denote the standard errors adjusted by the Newey-West method. To weight the covariance matrix, the Bartlett kernel was used with 6 bandwidth parameters. The obtained results show that most of the estimated coefficients are significant at the 5% levels and the measure of inflation uncertainty is marginally significant at the 10% level.

In the short run, the interest rate is negatively affected by expected inflation and inflation uncertainty, indicating the non-existence of the Fisher effect. This would be due to the liquidity effect of monetary shocks in the short run with the rises in expected inflation and uncertainty. In a recent study, Carpenter and Demiralp (2008) find that a negative correlation between short-run movements in money growth and the federal fund rate and interpret it as evidence that the liquidity effect dominates the

anticipated inflation effect at least in the short run. On the other hand, the measured feedback coefficient -0.03 is significant at the 5% level but indicates a slow adjustment to the past disequilibrium in the level of interest rates. By placing zero restrictions on all the coefficients of the short-run growth rates in (8), a long-run static-state relation between the variables used can be obtained as follows,

$$R_t = 3.41 + 0.73\pi_t - 0.15h_t. \quad (9)$$

(0.97) (0.27) (0.10)

This result corresponding to equation (5) is a solved Fisher equation under uncertainty at equilibrium. In terms of signs and magnitudes, the estimated coefficients are almost similar with the ones obtained from the cointegration space of the multivariate dynamic system, using Johansen's (1988) ML estimation.

Unlike the case of the short-run, the expected inflation in equation (9) positively affects the nominal interest rate with less than unit elasticity, as is often found in most of the existing literature.³ This Fisher effect puzzle in the long run is interpretable as evidence of the Mundell-Tobin effect that an increase in the expected inflation rate results in a fall in real money balances and the resulting decline in wealth in turn leads to an increased saving bringing downward pressure on the real interest rate. The adjustment of the nominal interest rate is therefore less than one-to-one increase with the expected rate of inflation.⁴ On the other hand, the long-run coefficient of the uncertainty measure, which is our major concern in this study, shows a negative sign. In the literature, Levi and Makin (1979) and Bomberger and Frazer (1981) provide evidence of the negative relationship, using semi-annual Livingston survey data to measure the unobservable market perceptions about expected inflation and inflation uncertainty. However, their actual findings are positive relations between inflation uncertainty and nominal interest rates, if serial correlations in Levi and Makin's (1979) estimators are corrected (see Taylor (1981)) and if lagged exogenous variables in Bomberger and Frazer's (1981) model are ignored (see Shome et al (1988)). Using a transfer function model, Hartman and Makin (1982) also find a negative impact of inflation uncertainty on interest rates. However, their finding is too weak to support the negative hypothesis, since the uncertainty variable lagged one quarter is only significant in their model. Considering there is no empirical work that provides convincing evidence of the negative effect of inflation uncertainty on the interest rate, it is worthy to notice that our study provides an evident result for the negative relationship between the two variables.

³ See Koustas and Serletis (1999), Atkins and Coe (2002), Rapach (2003), and Jensen (2009).

⁴ Some economists explain this puzzle with fiscal illusion (Tanzi, 1980), peso problem (Evans and Lewis, 1995), and nonlinearity (Christopoulos and Leon-Ledesma (2007)). On the other hand, Darby (1975) argues that the nominal interest rate rises by more than one in response to expected inflation in the presence of a positive tax on nominal interest income.

The negative impact of inflation uncertainty on the nominal interest rate could be interpretable in the line of Levi and Makin's (1979) view that the negative impact of inflation uncertainty on investment is bigger than the impact on savings and so the reduced demand for investment funds tends to exert downward pressure on interest rates at equilibrium. To further see this explanation in details, consider a textbook-version Keynesian IS-LM model, which consists of the following four equations by ignoring government expenditures, taxes, and foreign factors;

$$S_t = a_0 + a_1 r_t - a_2 h_t + a_3 (PY/M)_t + e_{1t}, \quad (a_1, a_2, a_3 > 0) \quad (10)$$

$$I_t = b_0 - b_1 r_t - b_2 h_t + e_{2t}, \quad (b_1, b_2 > 0) \quad (11)$$

$$(M/P)_t = c_0 + c_1 Y_t - c_2 R_t - c_3 h_t + e_{3t}, \quad (c_1, c_2, c_3 > 0) \quad (12)$$

$$R_t = r + \pi_t^e \quad (13)$$

where S_t denotes real savings; Y_t is real income; I_t is real investment; P_t is price levels; and M_t is a nominal money stock. Under the assumptions that the income velocity of the money demand equation (12) is unity and money demand is always equated by money supply, combining equations (10)-(13) yields an extended Fisher equation under uncertainty, which is equivalent to (5),

$$R_t = r + \beta \pi_t^e + \gamma h_t + e_t,$$

where $r = [(b_0 - a_0 - a_3 c_0)/(a_1 - b_1 + a_3 c_2)]$, $\beta = [(a_1 - b_1)/(a_1 - b_1 + a_3 c_2)]$, $\gamma = [(b_2 - a_2 - a_3 c_3)/(a_1 - b_1 + a_3 c_2)]$, and $e_t = [-(e_{1t} - e_{2t} + a_3 e_{3t})/(a_1 - b_1 + a_3 c_2)]$. On the basis of the Fisher theory, it is expected that (i) $r > 0$ with the constant real interest rate, (ii) $\beta = 1$ with the Fisher effect, but (iii) the coefficient of inflation uncertainty is ambiguous with either $\gamma > 0$ or $\gamma < 0$, depending on the relative importance of the negative impact of inflation uncertainty on investment and savings, respectively.

In the framework of this simple macroeconomic model, if there is a high inflation uncertainty, risk-averse investors may curtail investment or require a higher expected return to undertake a project. This will reduce the overall demand for investment funds and lead interest rates in the markets to fall.⁵ The downward effect of inflation uncertainty on interest rates is measured by b_2 in equation (11). On the opposite side, risk-averse economic agents may reduce savings and in turn the supply of funds at given levels of income and real balances, because of uncertainty in future price changes. The reduced funds in supply consequently boost the nominal interest rate to rise. This uncertainty premium is measured by a_2 in equation (10). As indicated by the sign on γ in the reduced-form interest rate equation, the net impact upon the equilibrium nominal rate is ambiguous, depending on the relative magnitude of the negative impact of inflation uncertainty on investment and savings. Our results obtained in equation (9) suggest that the negative impact of inflation uncertainty on investment seems to be larger than its impact on savings and so that the reduced

⁵ See Holland (1988) and Grier and Perry (2000) who show the inverse relationship between inflation uncertainty and output growth.

demand for investment funds leads the nominal interest rate to be lowered at equilibrium.

Overall, our study provides evidence of a significantly negative relationship between inflation uncertainty and the nominal interest rate. The result is interpretable in the framework of reduced aggregate demand at equilibrium under the IS-LM model. Most of the previous studies used the reduced-form model (5), without appropriate consideration of dynamics in specification, find positive impacts of inflation uncertainty on interest rates and interpret the evidence as either a result of inflation premium or reduced aggregate supply. However, this view is not supported by our result obtained from a general-to-specific dynamic modelling approach.

4. Comparison with IV-GMM estimators

In the literature, it is often recommended to use IV-GMM estimation for this line of research, particularly in the case of a theory-based study. To compare our results obtained from a general-to-specific model with this alternative specific-to-general estimation, we reestimated the extended Fisher relationship using an IV-GMM technique. For this, we first eliminated the endogeneity of the measured π_t and h_t , using past lags of the two variables. Nelson (1976) suggests using lag variables as instrumental variables, since they are closely correlated with endogenous variables and can insulate explanatory variables from being correlated with error term. In practice, there are so many candidates for this kind of role. However, increasing the number of instruments leads to more efficient instrumental variable estimators but can lead to increase the biasness. Angrist and Kruger (2001) argue that, if k instruments are used to estimate the effect of s endogenous variables, the bias is proportional to $(k - s)$.

Taking into account of this trade-off between bias and efficiency, twelve lags of each variable were initially used and then sequentially reduced by using the explanatory power of each lag, following Hahn and Hausman (2002) who recommend to apply parsimonious lag lengths as a rule of thumb. A quadratic spectral kernel with seven bandwidths was applied to weight the autocovariances in computing the weighting matrix. The estimated results are

$$R_t = 2.57 + 0.75\pi_t - 0.06h_t, \quad (14)$$

(0.31)(0.08) (0.03)

$$T = 1953(1) - 2008(12), R^2 = 0.04, \text{ Hansen's } J \text{ statistic: } \chi^2(9) = 9.71(0.36),$$

Instrumental variables: $\{\pi_{t-1}, \pi_{t-2}, \pi_{t-7}\} \{h_{t-2}, h_{t-3}, h_{t-5}, h_{t-6}, h_{t-7}, h_{t-9}, h_{t-10}, h_{t-12}\}$.

All the estimated parameters are statistically significant at the 5% levels. The validity of the over-identified restriction $(k - s)$ is accepted by Hansen's (1982) J statistic $\chi^2(9) = 9.71$ at the 5% significance level, satisfying the orthogonality conditions between instrumental variables and error terms. Considering the overidentified degree of freedom in the GMM estimation, the obtained results are almost similar to those of the OLS-based estimators in (9). It is noticeable that all of the three alternative

estimators obtained by applying OLS, IV-GMM, and ML methods show the same evidence of the negative effect of inflation uncertainty on the interest rate.

5. Conclusions

In this paper, we have investigated the possible effect of inflation uncertainty on interest rates. For analysis, we emphasized the derivation of consistent and efficient OLS estimators, when the estimation equation of the interest rate includes a risk variable of inflation uncertainty measured by an ARCH-type model. In this context, we discussed that there exists an orthogonality condition between the uncertainty variable measured by an ARCH model and the error term in the estimation equation and explained that the uncorrelated condition can be used to have a consistent covariance matrix of the OLS estimators with the application of a correction method. This suggests that, in addition to the FIML and IV methods commonly suggested by the literature, the procedure discussed in this study can be alternatively used to derive consistent and efficient estimators, given that an ARCH class model is used to measure the unobservable uncertainty. The OLS estimation has an advantage in easy implementation, compared to alternative FIML and IV estimations.

In investigating the impact of inflation uncertainty on nominal interest rates, unlike most the previous studies that mainly apply theory-based specific-to-general modelling approach, we used a general-to-specific modelling approach suggested by Hendry (1995). The results show that inflation uncertainty negatively affects the U.S. 3-month Treasury-bill secondary market rate. This evidence is also consistent with the results obtained by applying ML and IV-GMM methods. Earlier studies in this line mostly provide positive relationships between the two variables and interpret their findings as evidence that, if there is considerable uncertainty about future inflation, people demand compensation for bearing the inflation risk and hence interest rates contain risk premiums (see, for example, Mehra (2006)). However, our evidence here does not support this view of inflation risk premiums. The inflation uncertainty increased by expected inflation in general tends to depress nominal interest rates through its negative impacts on investment and savings in economic activities. In equilibrium, this indirect negative impact of inflation uncertainty on interest rates through real economic activities seems to dominate its direct positive impact on interest rates in financial markets.

Appendix A:

Assume that $\hat{\rho} \xrightarrow{p} \rho$, $T^{-1} \sum z_i' e_i \xrightarrow{p} 0$, and $\sqrt{T}(\hat{\rho} - \rho)$ has a limiting normal distribution with a covariance matrix D_ρ and is independent of e_i (see Engle (1982)). Under this construction, the asymptotic property of the OLS estimators for equation (3) is:

$$(\hat{\phi} - \phi) = (\sum w_i' w_i)^{-1} (\sum w_i' u_i) = (T^{-1} \sum w_i' w_i)^{-1} (T^{-1} \sum w_i' u_i). \quad (A-1)$$

While the first term converges to $(T^{-1} \sum w_i' w_i)^{-1} \xrightarrow{p} Q_T^{-1}$, the second term becomes

$$T^{-1} \sum w_i' u_i = T^{-1} \sum w_i' (e_i + \gamma(h_i - \hat{h}_i))$$

$$\begin{aligned}
&= T^{-1} \sum w_t' (e_t + \gamma_t(\rho - \hat{\rho})) \\
&= T^{-1} \sum w_t' e_t + T^{-1} \sum w_t' \gamma_t(\rho - \hat{\rho}).
\end{aligned}$$

Since $\hat{\rho} \xrightarrow{p} \rho$ by construction and $T^{-1} \sum w_t' e_t \xrightarrow{p} 0$ by the exogeneity assumption in equation (1), the second term converges to $T^{-1} \sum w_t' u_t \xrightarrow{p} 0$. Hence, (A-1) leads to $(\hat{\phi} - \phi) \xrightarrow{p} Q_T^{-1} \cdot 0 = 0$, verifying the consistency of the OLS estimators.

On the other hand, the asymptotic distribution of ϕ is

$$\sqrt{T}(\hat{\phi} - \phi) = (T^{-1} \sum w_t' w_t)^{-1} (T^{-1/2} \sum w_t' u_t). \quad (\text{A-2})$$

Since the first term converges in probability to Q_T^{-1} , $\sqrt{T}(\hat{\phi} - \phi)$ mainly depends on the distribution of $T^{-1/2} \sum w_t' u_t$. Using $u_t = e_t + \gamma_t(\rho - \hat{\rho})$, the second term can be rewritten as

$$T^{-1/2} \sum w_t' u_t = T^{-1/2} \sum w_t' e_t + \gamma(T^{-1} \sum w_t' z_t) T^{1/2}(\rho - \hat{\rho}).$$

Under the initial assumptions that (a) $(\rho - \hat{\rho})$ and e are independently distributed; (b) $T^{-1} \sum w_t' e_t \xrightarrow{p} 0$; and (c) $\sqrt{T}(\hat{\rho} - \rho)$ has a limiting normal distribution with a covariance matrix D_ρ , the limiting distribution of $T^{-1/2} \sum w_t' u_t$ becomes

$$T^{-1/2} \sum w_t' u_t \xrightarrow{d} N(0, \Omega_T)$$

where $\Omega_T = \sigma_e^2 (p \lim T^{-1} \sum w_t' w_t) + \gamma^2 (p \lim T^{-1} \sum w_t' z_t) D_\rho (p \lim T^{-1} \sum z_t' w_t)$.

Hence, the asymptotic distribution of the OLS estimate can be obtained by

$$\sqrt{T}(\hat{\phi} - \phi) \xrightarrow{d} N(0, Q_T^{-1} \Omega_T Q_T^{-1}). \quad (\text{A-3})$$

This shows that the variance of conventional OLS estimators, $\sigma_e^2 (p \lim T^{-1} \sum w_t' w_t)$, understates the true standard errors of the estimators in the two-step procedure.

Appendix B:

Since the regression residual u_t in equation (3) is uncorrelated with explanatory variables, let's represent a moment condition for the GMM estimation of the equation as follows,

$$E(f(\phi, \bar{w}_t)) = E(w_t(R_t - w_t \phi)) = 0, \quad (\text{B-1})$$

where $f(\cdot)$ is a q -dimensional vector-valued function of $\bar{w}_t = (R_t, x_t, \hat{h}_t)$ for an unknown $((p+1) \times 1)$ vector of parameters (see Hansen (1982)). Then, the GMM estimate $\hat{\phi}$ is the value of ϕ that minimizes a criterion function,

$$g(\phi; \bar{R}_t) \hat{\Omega}_T^{-1} g(\phi; \bar{R}_t), \quad (\text{B-2})$$

where $g(\phi; \bar{R}_t) = (1/T) \sum_{t=1}^T w_t (R_t - w_t' \phi)$ is the sample moment of $f(\phi, \bar{w}_t)$ and $\hat{\Omega}_T$ is an estimate of $\Omega = \lim_{T \rightarrow \infty} (1/T) \sum_{t=1}^T \sum_{v=-\infty}^{\infty} E\{[f(\phi, \bar{w}_t)][f(\phi, \bar{w}_{t-v})]'\}$. Since the moment condition $(B-1)$ has $(p+1)$ orthogonality conditions, of which the number is the same as the number of unknown parameters in ϕ , the whole system is just-identified with $q = p+1$.

Unlike the case of an over-identification where the number of orthogonality conditions exceeds the number of parameters to be estimated, the objective function $(B-2)$ under the just-identification can be minimized by setting

$$g(\hat{\phi}; \bar{R}_t) = (1/T) \sum_{t=1}^T w_t (R_t - w_t' \hat{\phi}) = 0. \quad (B-3)$$

Solving $(B-3)$ for $\hat{\phi}$ becomes

$$\hat{\phi} = (\sum_{t=1}^T w_t w_t')^{-1} (\sum_{t=1}^T w_t R_t), \quad (B-4)$$

which is the usual OLS estimator. If the weighting matrix is calculated by using the Newey and West method, the estimator can be treated as if $\hat{\phi} \approx N(\phi, \hat{V}/T)$, where $\hat{V} = (\hat{Q}^{-1} \hat{\Omega} \hat{Q}^{-1})$ which is the GMM approximation of the variance-covariance matrix $\hat{\phi}$, and $\hat{Q} = \partial g(\phi; R_T) / \partial \phi' |_{\phi=\bar{\phi}_T} = (1/T) \sum_{t=1}^T \partial w_t (R_t - w_t' \phi) / \partial \phi' = (-1/T) \sum_{t=1}^T w_t w_t'$.

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[Table 1]

ADF unit roots test statistics

Variables	π_t	R_t	h_t	$\Delta\pi_t$	ΔR_t	Δh_t
Lags	12	12	12	10	12	11
<i>t</i> -statistics	-2.69	-2.06	-2.14	-8.79*	-6.48*	-3.96*

Notes: 1. The critical values of the ADF test are -3.41 for the level variables and -2.87 for the differenced variables at the 5% levels, respectively. 2. * indicates the statistical significance at the 5% level.

[Table 2]

Cointegration test statistics

(A) Cointegrating statistics					
Eigenvalues	Hypothesis	Max statistics	5% c.v.	Trace statistics	5% c.v.
0.04	$\lambda = 0$	29.22*	22.00	41.14*	34.91
0.02	$\lambda \geq 1$	13.75	15.67	14.92	19.96
0.01	$\lambda \geq 2$	1.17	9.24	1.17	9.24
(B) Weak exogeneity tests under the restriction of one cointegrating vector.					
$R_t: \chi^2(1)=8.73 [0.00] \quad \pi_t: \chi^2(1)=4.55 [0.03] \quad h_t: \chi^2(1)=4.68 [0.03]$					
(C) The Fisher equation under uncertainty by normalization					
$R_t = 2.92 + 1.15\pi_t - 0.27h_t$ (0.88) (0.18) (0.11)					

Notes: 1. λ indicates the number of cointegrating vectors. 2. * indicates a statistical significance at the 5% level. 3. The values in parentheses and square brackets represent standard errors and ρ values, respectively.