# Demographics and the politics of capital taxation in a life-cycle economy

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#### Abstract

This article studies the effects of demographics on the mix of tax rates on labour and capital. It uses a quantitative general-equilibrium overlapping-generations model where tax rates are voted without past commitments in every period and characterized as a Markov equilibrium. In the U.S., the younger voting-age population in 1990 compared to 1965 accounts for the observed decline in the relative capital tax rate between those two years. A younger population rises the net return to capital, leads voters to increase their savings, and results in a preference for lower taxes on capital. Conversely, ageing might increase capital taxation.

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The objective of this paper is to assess the effect of changing demographics on the mix of capital and labour taxes. Two considerations motivate this study. First, as a political choice, the tax structure must reflect the voter's interests in taxing different factors. By affecting the economy as well as the age profile of voters, demographic changes are likely to alter these interests. Second, observed sharp changes in the tax mix have coincided with substantial shifts in the population structure. In the U.S., between 1965 and 1990 there is a marked reduction in the tax rate on capital relative to the tax rate on labour income at the same time as the adult population becomes noticeably younger. This paper's main finding is that changing demographics were the major driver of the changing tax mix over that period.

The analysis is based on the preferences over tax rates held by voters within a general equilibrium overlapping generations model. The economy is represented by the neoclassical model of capital accumulation, with households who make decisions about savings and labour supply, and a government that implements fiscal policy. The structure of tax rates on labour and capital reflect the aggregation of these tax preferences through a political process. I consider a form of representative democracy where voters of different ages may enjoy different degrees of influence in the political outcome. In equilibrium the tax mix is the one preferred by the typical voter of the decisive age group. I adopt the politico-economic equilibrium concept introduced by Per Krusell, Vincenzo Quadrini and José-Víctor Ríos-Rull (1997). Households vote in every period, without commitment to future policy decisions. Tax outcomes are time consistent and described as a Markov equilibrium.

The model's parameters, including those of the political influence function, are calibrated to match U.S. observations assuming the 1990 age structure of the population. I then use the

calibrated model to analyse the steady-state implications of changing the age structure of the population to match the 1965 values and the projected 2025 values. Two main findings emerge. First, the change in demographic structure can account for much of the change in tax mix between 1965 and 1990. Second, the model predicts a significant increase in taxes on capital income between now and 2025.

Although the focus of the analysis is quantitative, a simple two-period model is also studied in order to help articulate the intuition. The age structure of the population influences equilibrium taxation in two opposing ways. First, it directly shifts the age of the decisive voter. Since younger individuals rely less on capital than on labour, a younger population will tend to increase capital taxation through this channel. Second, the structure of population has a general-equilibrium effect through its impact on factor supplies and prices. Specifically, holding tax rates constant, as the population becomes younger the return to capital increases, thereby increasing the savings rate. As a consequence, a given decisive voter will hold more capital and prefer to tax capital less heavily.

The quantitative analysis finds that the general equilibrium effect dominates the effect of a change in the age of the decisive voter. One of the reasons for this is that the calibrated political influence function turns out to attenuate the impact of changes in demographics on the age of the decisive voter. In sensitivity analysis, I show that the rise in the overall size of government between 1965 and 1990 and the increased share of government revenues being allocated to transfers reinforces the increase in the relative tax on capital.

The main contribution of this paper is to analyse the implications of demographic change

for taxation, using a quantitative life cycle model with a rich demographic structure, and a voting process that results in time-consistent policies. There is a long literature on the political economy of factor taxation. Torsten Persson and Guido Tabellini (1994a,1994b) and Alberto Alesina and Dani Rodrik (1994) are early examples based on tractable models of the economy and the voting process. More recently, Paul Klein, Quadrini and Ríos-Rull (2005), like the present paper, study factor taxation in a quantitative model where voting delivers policies that are time consistent. However, by construction, these works cannot analyse the role of demographic change. Demographic factors have been considered in other papers with dynamic voting, including Assaf Razin, Efraim Sadka and Phillip Swagel (2004) which addresses a question similar to that of the present paper. These works contain simplifications, notably that of two-period lifetimes, which limit their suitability for quantitative evaluation.

The rest of the paper is organised as follows. Section I reports the U.S. facts object of the analysis. Section II studies a simple model in order to develop an intuitive interpretation. Section III presents a more general quantitative model, and then section IV defines and discusses the equilibrium. Section V calibrates a benchmark version of the model and describes features of the associated equilibrium. Section VI studies the consequences of demographic changes. The final remarks in Section VII conclude the paper.

<sup>&</sup>lt;sup>1</sup>The approach to voting as a Markov equilibrium was espoused in Krusell, Quadrini and Ríos-Rull (1997) and applied to quantitative analysis in Krusell and Ríos-Rull (1999). In a similar vein, Klein and Ríos-Rull (2003) and Klein, Krusell and Ríos-Rull (2008) study optimal policies.

<sup>&</sup>lt;sup>2</sup>See also Gregory W. Huffman (1996), Kennet R. Beauchemin (1998), Marco Bassetto (1999), John Hassler, José-Vicente Rodríguez-Mora, Kjetil Storesletten, and Fabrizio Zilibotti (2003), and Razin, Sadka and Swagel (2002).

#### I. FACTS FOR THE U.S.

In this section I document two facts for the U.S economy. First, between 1965 and 1990, the tax rate on capital decreased relative to the tax rate on labor. Second, the voting population became younger between 1965 and 1990, and it is projected to become older between now and 2025.<sup>3</sup>

Figure 1 depicts annual values for the ratio of the tax rate on capital income to the tax rate on labour income between 1965 and 1996. There is a sharp decline in the relative tax rate on capital throughout the period as the ratio drops from 2.23 in 1965 to 1.35 in 1996. Most of the reduction takes place in the central years between 1970 and 1983. The tax rate on capital declines not only in relative terms but also in absolute terms over this period, averaging 0.383 over the period 1965-1970, and 0.326 over the period 1983-1996. The corresponding tax rates on labor income were 0.174 and 0.237.

#### INSERT FIGURE 1 ABOUT HERE

Figure 2 displays the median age at five-year intervals since 1965 of the U.S. population aged between 20 and 80. The figures after the year 2000 are based on projected fertility and mortality trends. The median age of the adult population declines steadily from 43.78 in 1965 to 40.74 in 1990 when it shows the first indications of the marked upward trend which is to follow. The fall in the median age of the adult population until the 1990's is precisely an echo effect of the baby-boom ending in the mid 1960's. The more recent rise shows this echo

<sup>&</sup>lt;sup>3</sup>The sources for the figures shown are described in Appendix A.

<sup>&</sup>lt;sup>4</sup>Average effective tax rates from aggregate data may be sensitive to measurement choices but the overall pattern remains always consistent with Figure 1. See further discussion in appendix A.

effect is dying off after about one generation.<sup>5</sup> The projections anticipate a continuation of this rising trend. The median age of adults will rise from 40.74 in 1990 to 46.42 in 2025, and to 46.48 in 2050. To appreciate the fine details of these demographic changes, Figure 3 shows the size distribution of age groups in the population for 1965, 1990, and the 2025 projection.

### INSERT FIGURE 2 ABOUT HERE

#### INSERT FIGURE 3 ABOUT HERE

# II. DEMOGRAPHICS AND THE TAX MIX

The evidence thus suggests a positive association between the population age and the weight of capital taxation in the U.S.<sup>6</sup> This section presents the intuition for the connections between demographics and the tax mix. The argument has two parts. First, the tax outcome depends on the voters' relative income, and second, the voters' relative income may depend on demographics.

#### A. Voter's relative income and the tax mix.

Consider the following simple model. The economy has two periods. The population consists of I types of individuals, indexed by i, with  $\mu^i$  denoting the fraction of the population that is of type i. Individuals are endowed with income in period 1, and the only difference among individuals is the size of this endowment, which is denoted by  $y_i^i$ . The agent's income

<sup>&</sup>lt;sup>5</sup>In contrast, the median age of total population aged 0-80 rises since 1965 from 27.52 up to 32.08 in 1990, and further to the present. I have chosen to report the 20+ age group since it is the adult population that makes relevant economic and political choices.

<sup>&</sup>lt;sup>6</sup>Comparable data for twenty one OECD countries over 1960-1995 lend support to the wider relevance of this link (available from the author upon request). Razin, Sdaka and Swagel (2004) run regressions with international data and find a significant positive coefficient for the old dependency ratio on the capital tax rate.

then can be divided between current consumption  $c_1^i$  and capital for the second period  $k_2^i$ . Capital depreciates completely within one period. In the second period, the individual works the unit endowment of labour at the wage rate  $w_2$  and earns the rate of return  $r_2$  on the capital invested  $k_2^i$ . The income from capital in the second period is taxed at a rate  $\tau^k$  and the income from labour is taxed at rate  $\tau^l$ . The agent can spend the disposable income in the second period on consumption  $c_2^i$ . An agent's utility depends only on her lifetime consumption, and is represented by  $\log c_1^i + \log c_2^i$ . Firms produce output in the second period with a Cobb-Douglas production function of labour and capital, with a capital share of 1/2. There is a government in the second period who uses the tax revenues to fund an amount of government spending equivalent to a share 1/2 of output. Given a capital share of 1/2 and a government expenditure share of 1/2, the government budget constraint reduces to  $\tau^k + \tau^l = 1.7$  Markets for inputs and output are competitive.

Consider now the determination of total investment,  $K_2$ , assuming an already determined tax rate on capital,  $\tau^k$ , and given initial incomes  $y_1^i$ . The individual decision problem for a typical member of group i consists of the choice of investment that maximises utility for given factor prices and taxes, subject to the budget constraints  $c_1^i = y_1^i - k_2^i$  and  $c_2^i = (1-\tau^l)w_2 + (1-\tau^k)r_2k_2^i$ . The standard Euler equation  $c_2^i = (1-\tau^k)r_2c_1^i$  describes the optimal intertemporal allocation of consumption. Individual investment thus is  $k_2^i = (1/2)y_1^i - (1/2)((1-\tau^l)/(1-\tau^k))(w_2/r_2)$ . Using the conditions for market clearing  $K_2 = \sum \mu^i k_2^i$ , the factor prices as marginal products  $w_2 = (1/2)k_2^{1/2}$  and  $r_2 = (1/2)k_2^{-1/2}$ , and the government

<sup>&</sup>lt;sup>7</sup>The equality of the capital share and the government expenditure share serves to simplify the exposition but is not essential to the analysis. The results in this section also follow in a more general model where utility is  $\log c_1 + \beta \log c_2$ , with  $\beta > 0$ , the share of government spending is G in the two periods, there are given initial tax rates  $\tau_1^k$  and  $\tau_1^l$ , and the capital share is  $\alpha$ . See Appendix to the working paper.

constraint  $1 = \tau^k + \tau^l$ , it becomes

$$k_2^i = \frac{1}{2}y_1^i - \frac{1}{2}\frac{\tau^k}{1 - \tau^k}K_2. \tag{1}$$

Aggregating this individual investment over the total population, a relation between the capital tax rate, aggregate income  $Y_1 = \sum \mu^i y_1^i$ , and aggregate investment emerges:

$$K_2 = s(\tau^k)Y_1$$
 where  $s(\tau^k) \equiv \frac{1 - \tau^k}{2 - \tau^k}$ , (2)

where  $s(\tau^k)$  is the investment (or savings) rate over total income. Note that  $s(\tau^k)$  is a decreasing function of the tax rate on capital.

Consider now the effect of a marginal change in capital taxation on an individual's utility. First note that the induced response of individual investment  $k_2^i$  leaves utility unchanged since consumption across the two periods will be reallocated optimally (i.e., the envelope theorem holds). Therefore, all consequences for utility are accounted for by the response of disposable income and consumption in the second period  $c_2^i$ , for a given value of the individual's investment  $k_2^i$ . Upon using the government constraint  $\tau^k + \tau^l = 1$ , consumption in period 2 can be written as  $c_2^i = \tau^k w_2 + (1 - \tau^k) r_2 k_2^i$ . There are two channels through which a change in  $\tau^k$  affects  $c_2^i$ . The first, and direct effect, is to increase after tax labor income and reduce after tax capital income. The second, and indirect effect, is the general equilibrium effect, since a change in  $\tau^k$  will impact on the equilibrium values of  $w_2$  and  $r_2$ . Using the determination of aggregate investment in (2), the equilibrium input prices can be written as functions of the tax rate as  $w_2 = \frac{1}{2}s(\tau^k)^{1/2}Y_1^{1/2}$  and  $r_2 = \frac{1}{2}s(\tau^k)^{-1/2}Y_1^{-1/2}$ . The general

equilibrium effect of an increase in  $\tau^k$  is to decrease the savings rate, thereby decreasing  $w_2$  and increasing  $r_2$ .

In order to examine the net tax impact on consumption it will be convenient to define the absolute value of the proportional change in the investment rate  $s(\tau^k)$  from (2) as  $e_s(\tau_k) = 1/[(1-\tau^k)(2-\tau^k)]$ . It measures the sensitivity of the investment rate to the capital tax rate. With this notation, the effect of a higher  $\tau^k$  on labour income is given by  $w_2 - w_2 \frac{1}{2} e_s(\tau^k) \tau^k$ . The first term is the positive direct effect and the second term is the negative general equilibrium effect, which will eventually dominate when the tax rate and the elasticity of investment become sufficiently large. On the other hand, the effect of a higher  $\tau^k$  on capital income is given by  $-r_2k_2^i + r_2k_2^i \frac{1}{2}e_s(\tau^k)(1-\tau^k)$ . The first term is the negative direct effect and the second term is the positive general equilibrium effect. The negative effect always dominates. The response of capital income is the one that may have a differential impact across individual types. The negative effect of capital taxation on the returns to capital matters more for agent with larger investment  $k_2^i$ .

An agent of type i seeking to maximise her utility will choose the tax rate  $\tau^k$  to the point where the sum of the effects on the two sources of income discussed above becomes zero. Using the implication of constant factor shares that  $w_2 = r_2K_2$ , this condition then reads:

$$\[1 - \frac{1}{2}e_s(\tau^k)\tau^k\] - \frac{k_2^i}{K_2} \left[1 - \frac{1}{2}e_s(\tau^k)(1 - \tau^k)\right] = 0$$
(3)

Given that the individual type i chooses the tax rate on capital  $\tau^k$ , an equilibrium satisfies the condition for the optimal tax choice (3), with the voter's relative investment  $k_2^i/K_2$ 

determined by (1) and (2). Using (1), the marginal value to  $\tau^k$  in (3) can be written more explicitly as a function of the tax rate on capital and the ratio of individual initial income to aggregate investment. I denote this function

$$\Omega(\tau^k, y_1^i/K_2) \equiv \left[1 - \frac{1}{2}e_s(\tau^k)\tau^k\right] - \frac{1}{2}\left[\frac{y_1^i}{K_2} - \frac{\tau^k}{1 - \tau^k}\right]\left[1 - \frac{1}{2}e_s(\tau^k)(1 - \tau^k)\right]. \tag{4a}$$

Using (2) to determine aggregate investment, the equilibrium capital tax rate  $\tau^k$  then solves:

$$\Omega\left(\tau^k, \frac{1}{s(\tau^k)} \frac{y_1^i}{Y_1}\right) = 0. \tag{4b}$$

The crucial property is that the marginal value to taxing capital represented by the function  $\Omega$  is decreasing in its second argument  $y_1^i/(s(\tau^k)Y_1)$ , the voter's individual income relative to aggregate income and investment.<sup>8</sup> A richer voter saves more and will prefer to tax capital less. A lower aggregate income leads to smaller aggregate investment, a higher interest rate follows given the initial tax mix and, as a consequence, the voter invests more and prefers to tax capital less. More specifically, (4) says that a higher relative income  $y_1^i/Y_1$  leads to a lower capital tax rate  $\tau^k$ .

# B. The role of demographics.

From equation (4), the structure of population can matter for the tax mix only as long as it affects the decisive voter's relative income in the first period,  $y_1^i/Y_1$ . To see how demographics impinges on income, the model has to be further specified. First, I extend the model of production and competitive markets also to the first period. So an individual, instead of receiving a given income endowment, works the unit endowment of labour the wage rate

<sup>&</sup>lt;sup>8</sup>As a reference, when  $y_1^i/Y_1 = 1$  the equilibrium has  $\tau_2^k = 1/2$ ; with  $y_1^i/Y_t > 1$ ,  $\tau_2^k < 1/2$ .

 $w_1$  and earns the rate of return  $r_1$  on his initial endowment of capital  $k_1^i$ . Second, in order to be specific, the population is divided in only two types of individuals, i, old and young, with group sizes  $\mu^{old}$  and  $1 - \mu^{old}$ , respectively. The old and young individuals live for the two periods and will only differ in their initial capital endowment.<sup>9</sup> In particular I assume  $k_1^{old} > k_1^{young}$ .

The first period income for an individual in group i is  $y_1^i \equiv r_1 k_1^i + w_1$ . Similarly, aggregate income is  $Y_1 \equiv r_1 K_1 + w_1$ . In equilibrium, factor markets clear so  $K_1 = \mu^{old} k_1^{old} + (1 - \mu^{old}) k_1^{young}$ , and factor prices equal their marginal products so  $w_1 = (1/2) K_1^{1/2}$  and  $r_1 = (1/2) K_1^{-1/2}$ . It follows that:

$$y_1^i = \frac{1}{2} K_1^{1/2} \left( \frac{k_1^i}{K_1} + 1 \right) \tag{5}$$

and

$$Y_1 = K_1^{1/2} \tag{6}$$

Thus an agent's initial income is a proportion of per capita output that depends on his relative endowment of capital.

There are two consequences of a shift to a younger population that results from a decrease in  $\mu^{old}$ . The first is that the age of the decisive voter may decrease. The second is that initial aggregate income is decreased relative to individual incomes. I analyse each of these effects in turn. First, the decisive voter's type i may change from old to young. The assumption made that the old type owns initially more capital than the young type implies, by (5) and (6), that

<sup>&</sup>lt;sup>9</sup>Although this economy has no explicit age structure, I refer to the two types as old and young since they are distinguished by their endowment of capital, and in the life-cycle model that I analyze later in the paper, a key distinguishing feature between old and young individuals will be their accumulated capital stocks.

the old has a higher initial income too so  $y_1^{old} > y_1^{young}$ . For a given demographic structure,  $\mu^{old}$ , a change in the type of the decisive voter from old to young leads to a reduction in the decisive voter's relative initial income  $y_1^i/Y_1$  and, along the lines of the previous discussion of condition (4), causes a rise in the tax rate on capital.

**Proposition 1** Suppose the decisive voter's type i changes from old to young for a given  $\mu^{old}$ . Then  $\tau^k$  increases.

Second, the consequences of a lower  $\mu^{old}$  for the optimal choice of  $\tau^k$  for a given decisive voter i, can be characterised in two steps. As the first step, consider the effect of a younger population on initial incomes  $y_1^i$  and  $Y_1$ . Given that  $k_1^{old} > k_1^{young}$ , the initial impact of this shift is a reduction in the initial aggregate supply of capital and output. The relative income of each individual relative to the aggregate,  $y_1^i/Y_1$ , naturally rises as can be seen from (5) and (6). As the second step, these changes in disposable incomes affect taxation  $\tau^k$  as in the discussion of (4) above. The increased decisive voter's relative initial income,  $y_1^i/Y_1$ , reduces the marginal utility to capital taxation and leads to a lower tax rate.

**Proposition 2** Suppose a lower  $\mu^{old}$  for a given decisive voter's type i. Then  $\tau^k$  decreases.

To visualise the results, it is useful to express (4) as two conditions, one for investment,  $K_2 = y_1 s(\tau^k)$ , and another for taxation,  $\Omega(\tau^k, y_1^i/K_2) = 0$ . They can be represented graphically as two curves that describe  $k_2$  as a function of  $\tau^k$ . The 'investment' curve has clearly a negative slope. The 'taxation' curve will have also a negative slope if, for the sake of argument, an old decisive voter is assumed. The reason is that this is a situation where, holding aggregate investment constant, the marginal utility to capital taxation is increasing or, more precisely,

 $\Omega_1(\tau^k, y_1^i/K_2) > 0.^{10}$  Intuitively, the negative response of the decisive voter's individual investment to a higher tax on capital breeds further support to this tax. An equilibrium is given by the intersection of the two curves. Figure 4 below depicts one such situation. Proposition 1 says that the decisive voter turning young means a reduction in the decisive voter's initial income  $y_1^i$  that shifts the taxation curve to the left causing the capital tax rate to increase. Proposition 2 says that the reduction in the after-tax aggregate income  $Y_1$ , which follows from a younger population composition  $\mu^{old}$ , shifts the investment curve downwards and leads to a new equilibrium with a lower capital tax rate. Incidentally, with the negative slope of the taxation curve, a higher investment results in the end. The possible rise in the individual decisive voter's income  $y_1^i$  would also shift the taxation curve upwards but, even if it were to fall, the result survives.

#### INSERT FIGURE 4 ABOUT HERE

Summing up, capital taxation can decline with a younger population as a response to the induced shift in the initial relative aggregate supply of inputs (see Proposition 2). However, for this to be case the change in the age of the decisive voter must be small since it tends to push taxation in the opposite direction (see Proposition 1). This analysis helps intuition but has limitations. It relies on exogenous assumptions regarding the decisive voter's endowment of capital and the change in her age. Besides, the individual labour supply is exogenous and the economic and political choices are essentially made in a static environment. The rest of the paper builds a quantitative dynamic general equilibrium model in order to study more rigorously the connection between demographic changes and the tax mix.

<sup>&</sup>lt;sup>10</sup>Note that, on the other hand, capital taxation also raises the return to capital through aggregate investment. This makes the marginal total utility decreasing and renders the tax-mix choice problem well defined.

<sup>&</sup>lt;sup>11</sup>The flatter taxation curve expresses the required second order condition for a maximum, which can be shown to hold generally.

# III. THE QUANTITATIVE MODEL

The model is the standard general equilibrium neoclassical model of capital accumulation, with overlapping generations, a government that implements fiscal policy, and a political constitution that specifies the process through which the fiscal policy parameters are set.

Demographics - Agents live for J periods. Age is denoted by  $j \in \{1, 2, ..., J\}$ . Let the vector  $\mu = [\mu_1, ..., \mu_J]' \in \mathbb{R}^J$  denote the age distribution of the population.

Preferences - Preferences of an agent are defined over life-time consumption and leisure streams. Denote by  $c_j$  the consumption of this household and by  $l_j$  the hours of leisure at age j. Then preferences are represented by

$$\sum_{j=1}^{J} \beta^{j} [(1-\gamma) \log c_j + \gamma \log l_j],$$

where  $\beta$  and  $\gamma$  are positive parameters.

Output technology - Aggregate output is produced by combining labour services H and capital K inputs into a neoclassical production function F(K,H). Assume the production function is Cobb-Douglas with  $\alpha$  the capital share. This output can be consumed or invested in capital. The rate of depreciation of capital is  $\delta$ . The value of the labour services per unit of time supplied by an individual of cohort j is the product of a fixed age-specific labour productivity parameter,  $\epsilon_j$ , and an index of aggregate labour-augmenting productivity which grows at a constant proportional rate  $\lambda$ .

Government (fiscal constitution) - The government collects proportional taxes on capital and labour income, at rates  $\tau_l$  and  $\tau_k$  respectively, to fund unproductive government consumption, and lump-sum transfers to households. The government cannot borrow nor lend. The balanced budget constraint is:

$$G + T = \tau_l \frac{Hw}{F(K, H)} + \tau_k \frac{K(R - \delta)}{F(K, H)},$$

where G is the share of public spending on GDP, T the share of transfers on GDP, w is the wage per unit of labour services, R is income earned per unit of capital, and H and K are the aggregate supplies of the labour and capital inputs. The lump-sum amount of transfers perceived by an individual is independent of age j and is thus determined as the constant fraction T of output, or  $F(K, H) \times T$ .

Markets and Ownership - Each agent is endowed with one unit of time per period that can be divided between leisure and hours of work. Individual agents also own all the assets in the economy; in the first period of life the household starts off with zero wealth. There is a competitive market for one-period bonds with a return r where agents can freely borrow and lend. There are competitive markets for inputs where households rent their labour services and capital to firms at rates w and R. These firms produce output using the available technology and sell it in a competitive market at a price that is normalised to unity.

Political constitution - The political constitution specifies the set of rules that determine the policies in place: tax rates, and government consumption and transfers. The output shares of government consumption, G, and transfers, T, are taken to be fixed and constant.

Tax rates are instead decided one period in advance. I will consider two alternative regimes for the determination of next-period tax rates. In one case policies are determined by the preferences of the typical household of an exogenously specified age. The other rule translates popular voting into political outcomes, but accounts for different levels of political influence across age groups of voters. In general, this may reflect the inner details of the political process leading up from the polls to policy decisions. I represent this idea in a simple way and define a 'political-influence' factor,  $I_j$ , which may vary with age j, and use it to weight the size of each cohort,  $\mu_j$ , to obtain the distribution of effective votes over cohorts,  $\tilde{\mu}_j$ , as follows:

$$\tilde{\mu}_j = \mu_j I_j / \sum_{i=1}^{J} (\mu_i I_i).$$

The tax rates are the outcome of pair-wise contests between alternatives. There will be a decisive voter who is the median voter according to this distribution  $\tilde{\mu}$ . The particular case where  $I_j$  is constant corresponds thus to direct democracy under majority rule. Under either regime, the age of the decisive voter, which I denote m, will be a real number in the interval [1, J] and may therefore differ from any of the integer age groups  $j \in \{1, ..., J\}$ . Policy decisions will appropriately average the preferred choices of the two cohorts j closest to m.<sup>12</sup>

# IV. POLITICO-ECONOMIC EQUILIBRIUM

I use this model to study the determination of policies and macroeoconomic variables. I will focus on outcomes that arise in equilibria where markets clear, and agents make optimal choices. I will assume that households have perfect foresight. Under the assumed political constitution thus next-period endogenous tax rates will reflect the current preferences of ra-

 $<sup>^{12}</sup>$ This will help strike a balance between affordable computational costs – which require the age groups j to lie relatively far between – and empirical relevance – which must account for relatively modest changes in the age of the decisive voter.

tional forward-looking agents. These preferences will take into account the future economic and political consequences of current policy choices. To make the definition and characterisation of equilibria manageable, I will follow the practice common in some of the literature on dynamic voting models by restricting attention to equilibria that satisfy a Markov property as follows.

If tax rates were to be given exogenously then a competitive equilibrium could be characterised recursively in terms of mappings from the aggregate state of the economy into the endogenous variables of the model. The state of the economy at a given point in time would be fully described by the current exogenous tax rates,  $(\tau_l, \tau_k)$ , and the distribution of individual asset wealth over the life cycle,  $A = [A_1, ..., A_J]'$ . With endogenous policies, I will consider equilibria where next-period tax rates only depend on this minimal state as well. In equilibrium the state (tax rates and the distribution of wealth) will be governed by a Markov law of motion. Moreover, by virtue of the government budget constraint, the labour tax rate becomes redundant and the aggregate state can be completely characterised by A and  $\tau_k$ . More formally, I will study situations where policies are governed by a transition function  $\Psi^{\tau}$ :

$$\tau_k' = \Psi^{\tau}(A, \tau_k).$$

That is, tomorrow's capital tax rate depends on today's state as summarised by the current distribution of assets across age groups and the value of the tax rate on capital.

In the rest of this section I will first define an equilibrium for a given law of motion of the

<sup>&</sup>lt;sup>13</sup>Since all the growing variables will be normalised by aggregate growth, the level of labour-augmenting productivity can be removed from the list of aggregate states.

tax rate  $\Psi^{\tau}$ . Then I can obtain the preferences over tax rates by agents in different cohorts associated with this equilibrium. A political equilibrium is then defined as the  $\Psi^{\tau}$  that, under the assumed political constitution, is consistent with those tax preferences.

# A. Economic equilibrium for given policy transition

An agent's individual type is characterised by her asset holdings a and age j. Age is important since it determines the agent's planning horizon and labour productivity. Individual asset holdings at time t by one such agent have been normalised by the level of technology existing in her first lifetime period t - (j - 1). Households decide on individual asset accumulation and labour supply to the market and these choices will depend on the individual state (j, a) and the aggregate state  $(A, \tau_k)$ .

For a given transition for policies  $\Psi^{\tau}$ , a recursive competitive equilibrium can be defined as a set of age-specific individual value functions and decision functions for asset holdings and leisure,  $v_j(A, \tau_k, a; \Psi^{\tau})$ ,  $\psi_j^a(A, \tau_k, a; \Psi^{\tau})$  and  $l_j(A, \tau_k, a; \Psi^{\tau})$ , an aggregate law of motion for the distribution of wealth,  $\Psi^A(A, \tau_k; \Psi^{\tau})$ , and an aggregate function for the supply of labour services,  $H(A, \tau_k; \Psi^{\tau})$ , such that competitive firms and households behave optimaly and all markets clear.<sup>14</sup>

#### B. Equilibrium with endogenous policy transition

The type of political constitution assumed establishes that the next-period tax rate is determined in the current period by the preferences of some cohorts of agents. To derive these preferences over tax rates, agents think through all the current and future equilibrium consequences of alternative choices of  $\tau'_k$ . These preferences are defined over  $\tau'_k$ 's that will in

<sup>&</sup>lt;sup>14</sup>More formal definitions are in an appendix available from the author.

general differ from the value dictated by the equilibrium law of motion  $\Psi^{\tau}(A, \tau_k)$ . These one-period deviations in the tax rate will be evaluated taking into account their initial effect on the labour supply and savings decision rules across cohorts, provided that future outcomes will be those associated with the equilibrium law of motion for policies  $\Psi^{\tau}$ . Denote by  $\tilde{v}_j(A, \tau_k, a, \tau'_k; \Psi^{\tau})$  the utility function of  $\tau'_k$  that represents these preferences for an agent of age j = 1, ..., J.

The aggregation of these preferences over tax rates  $\tilde{v}_j(...; \Psi^{\tau})$  yields a political outcome. The particular aggregation rule  $\Gamma(A, \tau_k; \Psi^{\tau})$  depends on the prescriptions of the political constitution. The two regimes that I will explore imply that the outcome reflects the preferred policy of households associated with a particular age  $m \in [1, J]$ , using linear interpolation when m does not coincide with a cohort  $j \in \{1, ..., J\}$ . In the first regime this decisive age is determined exogenously. If  $m \in [j^* - 1, j^*]$  is the age of the designated decisive age then

$$\Gamma(A, \tau_k; \Psi^{\tau}) = (j^* - m) \times \arg\max_{\tau'_k} \tilde{v}_{j^* - 1}(A, \tau_k, A_{j^* - 1}, \tau'_k; \Psi^{\tau}) +$$

$$(m - (j^* - 1)) \times \arg\max_{\tau'_k} \tilde{v}_{j^*}(A, \tau_k, A_{j^*}, \tau'_k; \Psi^{\tau}) \quad (7)$$

The second regime is democracy under majority voting rule in pair-wise tax rate contests where the weight of each cohort is given by  $\tilde{\mu}_j$  – its demographic size  $\mu_j$  adjusted by its relative political power or influence  $I_j$  as defined above in section II. It will be assumed that individual preferences over taxes are single-peaked so that the median voter theorem applies. In this case, the policy rule is as above in (7) but with the decisive age m determined in equilibrium as a function of the state,  $m(A, \tau_k; \Psi^{\tau})$ , which locates the median age based of the policy preferences,  $\tilde{v}_j$ , and effective political weight,  $\tilde{\mu}_j$ , of the different cohorts.

An equilibrium can be now defined as the policy transition consistent with the choices expressed through the political process. Formally, it is a fixed point  $\Psi^{\tau}$  of the following mapping:

$$\Gamma(A, \tau_k; \Psi^{\tau}) = \Psi^{\tau}(A, \tau_k) \tag{8}$$

I will focus on steady states, which adds the requirement that the system be consistent with a constant  $\tau_k$  and a stationary distribution of wealth A:

$$A = \Psi^{A}(A, \tau_{k}; \Psi^{\tau})$$

$$\tau_{k} = \Psi^{\tau}(A, \tau_{k})$$
(9)

# C. Computation and representation of equilibrium

This equilibrium involves fixed points of laws of motion of entire distributions. As in Krusell and Ríos-Rull et al. (1999), the solution will be approximated by linear aggregate and individual policy functions. This is achieved by computing the equilibrium for a linear-quadratic version of the model around the steady state.<sup>15</sup> Then iterations on the law of motion for the tax rate  $\Psi^{\tau}$  can be implemented to solve the equilibrium fixed-point problem.

The main steps of the computation procedure are the following:

- 1. Choose one decisive voter's age m. Choose a tax rate  $\tau_k$ .
- 2. For the given  $\tau_k$ , solve for the steady state of the original economy to obtain A. Approximate return functions around the steady state by a quadratic form.

 $<sup>^{15}</sup>$ The choice of leisure for some agents (i.e., those in retirement ages) may be non-interior in which case it will be locally treated as exogenous.

- 3. Choose one linear  $\Psi^{\tau}$ .
- 4. For the given  $\Psi^{\tau}$ , solve for the  $\Psi^{A}$  and  $v_{j}$ 's by successive approximations to the (quadratic) value function.
- 5. Compute one-period deviation tax preferences for the cohorts j-1 and j if  $m \in [j-1, j]$  as  $\tilde{v}_{j-1}$  and  $\tilde{v}_j$ , and update  $\Psi^{\tau}$  according to (7) and (8). Start over in step 4 until convergence in  $\Psi^{\tau}$  as in (8).
- 6. Given  $\Psi^{\tau}$ , solve for the tax rate implied by the steady state A found in step 2 and the  $\tau_k$  assumed in step 1. Update  $\tau_k$  and start over in step 2 until convergence in  $\tau_k$  as in (9).
- 7. Given the steady state, compute one-period deviation tax preferences for all voters  $j \in \{1, ..., J\}$ , and the implied preferred tax rates given A and  $\tau_k$ . Verify that m is the decisive age. Otherwise start over in step 1 with a new guess on m. With an exogenous m this last step 7 is obviously omitted.

The procedure for finding steady states admits a graphical representation which will prove helpful to present the quantitative analysis. With some abuse of notation, let  $\Psi^{\tau}(\tau_k)(.,.)$  denote the function that solves the equilibrium fixed point in (8) for the economy approximated around the steady state associated with the specific tax rate  $\tau_k$ , that is:  $\Gamma(.,.;\Psi^{\tau}(\tau_k)) =$  $\Psi^{\tau}(\tau_k)(.,.)$ . Now the policy decision aggregator for the next period tax rate in (7) can be evaluated at this specific current tax rate as  $\Gamma(A(\tau_k), \tau_k; \Psi^{\tau}(\tau_k))$ , where  $A(\tau_k)$  denotes the wealth distribution in the steady state associated with the tax rate  $\tau_k$ . The values returned by this calculation for alternative  $\tau_k$ 's result in a mapping in the tax rate that can be represented graphically. Note that for each different  $\tau_k$ , this evaluation requires solving a new fixed-point problem in the function  $\Psi^{\tau}(\tau_k)$ .<sup>16</sup> Which particular  $\tau_k$  characterises a steady-state equilibrium? For an exogenously given decisive voter's age m, an equilibrium is a  $\tau_k$  solving  $\tau_k = \Gamma(A(\tau_k), \tau_k; \Psi^{\tau}(\tau_k))$ . This is a scalar fixed point that can be identified using this graphical representation of the tax mapping. When the decisive voter is endogenous, there is one mapping for each possible candidate to decisive age. Any fixed point will be an equilibrium if the candidate assumed coincides with the decisive vote.

# V. CALIBRATION AND BASELINE EQUILIBRIUM

The benchmark model is meant to represent the U.S. economy in 1990. The calibration is done in two main steps. In the first step, I will assume that the age of the decisive voter, m, is an exogenous parameter so that the influence profile  $I_j$  plays no role. The value of m and parameters other than those of  $I_j$  are set so that the equilibrium matches a number of targets for economic, demographic and policy variables. In the second step of the calibration, I will specify a form for  $I_j$  and parameterise it so that the equilibrium decisive voter coincides with the value of m calibrated in the first step. This section will end by describing some properties of the benchmark equilibrium associated with the calibration.

# A. Calibration with exogenous m

Tables 1 and 2 summarise the procedure and results when the decisive age m is regarded as exogenous. In this first step of the calibration, certain parameters are directly determined from the choice of targets. The number of cohorts J is set so that one model's period corresponds to 5 years, which is consistent with the typical format of demographic data and, on the other hand, keeps computational costs reasonable. The parameters  $\alpha$ ,  $\delta$ ,  $\lambda$ , and

<sup>&</sup>lt;sup>16</sup>If we had instead a global method for finding the equilibrium that did not rely on local approximations, then the fixed-point problem for  $\Psi^{\tau}$  would have to be solved only once.

Table 1. Calibration with exogenous decisive age

		9	
parameter	value	target to match	source
$\overline{J}$	12	5 years per period	
$\alpha$	0.33	capital share $33\%$	NIPA post WW2
$\delta$	0.341	annual depreciation $8\%$	NIPA and BEA
$\lambda$	1.069	productivity growth $1.34\%$	NIPA
G	0.15	gov. consumption/GDP 15%	OECD Outlook
$\epsilon$	Table 2	age-productivity	PSID
$\mu$	Table 2	population 1990	U.N. database
$\beta$	0.895	investment/output 18%	NIPA
$\gamma$	0.588	time at work $40\%$	PSID
T	0.07	labour tax 1983-96 $24\%$	Section I
m	5.360	capital tax 1983-96 $33\%$	Section I

Table 2. U.S. age productivity and 1990 demographic profile

				, <u>.</u>							•	
Age	1	2	3	4	5	6	7	8	9	10	11	12
$\epsilon$	5.95	7.91	9.35	10.3	10.9	11.0	10.9	10.5	9.99	8.34	5.77	3.85
$\mu$	.110	.126	.129	.118	.103	.081	.067	.062	.062	.058	.046	.036

G are set directly to match average 1965-2000 values for the capital share of GDP, the growth rate of productivity, the ratio of government consumption expenditures to GDP, and the depreciation rate. The productivity age profile  $\epsilon$  matches the age component of labour productivity estimated from the PSID by Jonathan Heathcote, Kjetil Storesletten and Giovanni L. Violante (2008) over the period 1967-1995. The age distribution of population,  $\mu$ , corresponds to the data displayed in Figure 1 for the U.S. in 1990 as supplied by the U.N. The remaining four parameters are T,  $\beta$ ,  $\gamma$ , and m. They have to be calibrated so that the equilibrium of the model matches certain targets. These targets are the average fraction of time worked by individuals estimated from the PSID by Heathcote *et al.* (2008) over the period 1967-1995, the average investment-to-output ratio over 1965-1996, an the average tax rates on capital and labour income over the period 1983-1996 reported in Section I. The impression from Figure 1 was that over the latter period the tax mix remains relatively stable after the sharp changes of the preceding period.

# B. Calibration with endogenous m

In the second step of the calibration, I choose the following specification for the age profile of political influence:

$$I_{j} = \exp\left[-\left(\frac{j - j^{*}}{\sigma}\right)^{2}\right], \text{ with } \sigma = \begin{cases} \sigma_{Y} & j < j^{*} \\ \sigma_{O} & j \geq j^{*} \end{cases}$$

where  $j^*$ ,  $\sigma_Y$ , and  $\sigma_O$  are three positive parameters. One can regard this function as a (Gaussian) distribution of influence over age groups. In this view the parameter  $j^*$  is the mode characterising the centredness (i.e., the age of the most politically influential group); the ratio  $\sigma_O/\sigma_Y$  is a measure of asymmetry to the right (i.e., how much mass of influence corresponds to ages older than the mode  $j^*$  relative to younger ages); the parameter  $\sigma_Y$  describes the level of dispersion of influence away from the mode. I choose to calibrate  $j^*$ ,  $\sigma_Y$ , and, in order to facilitate the interpretation, the ratio  $\sigma_O/\sigma_Y$  so that the implied age of the decisive voter in 1990, which I will denote  $m_{1990}$ , coincides with the m calibrated in the first step of the calibration and, in addition, the resulting influence profile is empirically plausible. To specify the five additional targets associated with this latter requirement, I use evidence on the age distribution of Congress representatives between 1985 and 1995 and voting participation by ages in the Presidential elections of 1994 and the Congress elections of 1992. For each of five age groups, I simply construct an index by multiplying the weight in Congress and the average percentage of electoral participation.

<sup>&</sup>lt;sup>17</sup>The case  $j^* = J$  and  $\sigma = +\infty$  delivers a flat profile so  $\tilde{\mu} = \mu$ . The decisive age is the population median age of 4.15, much lower than the 5.36 required in the calibration. Since there are many ways to hit this with the 3 free parameters of the influence function, I need to discipline the exercise by adding at least two more targets.

<sup>&</sup>lt;sup>18</sup>Empirical work finds that voting outcomes reflect to a large extent the legislators' personal motivations over those of their constituencies. See, for example, Stephen D. Levitt (1996), Elisabeth R. Gerber and Jeffrey B. Lewis (2004), and John Griffin (2008).

<sup>&</sup>lt;sup>19</sup>The data used is provided by the Census Bureau, Tables 395 and 405,

Table 3. Calibration with endogenous decisive age	ന വി	$\alpha$ 1.1 $\alpha$	• . 1	1	1	
Table 9. Calibration with chargenous accisive age	Lanie 3	. Calibration	with	endogenous	CHECKETAN	ace
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		`	_			
parameter	value	targets to match		source		
$\overline{j^*}$	5.561	- Decisive age m		Table 1		
$\sigma_O/\sigma_Y$	2.10	- Influence pattern from	weights	Census Bureau		
$\sigma_Y$	1.90	in elections and (	Congress			
Implications for decisive age: $m_{1990} = 5.360$ , $m_{1965} = 5.621$ , $m_{2025} = 5.823$						

parameter choices and also reports the implied value of the targeted decisive age  $m_{1990}$ , as well as the implied decisive ages under the population profiles displayed in Figure 3 earlier corresponding to the years 1965 and 2025, which are denoted  $m_{1965}$  and  $m_{2025}$  respectively. Figure 5 plots the calibrated influence profile as well as the pattern constructed from the data. The overall approximation is very close.

#### INSERT FIGURE 5 ABOUT HERE

#### C. Stationary equilibrium

This benchmark equilibrium implies an annual interest rate close to 7 per cent. Graphically, the equilibrium tax rate can be viewed as a fixed point of the mapping  $\tau'_k = \Gamma(A(\tau_k), \tau_k; \Psi^{\tau}(\tau_k))$ , defined at the end of section IV, corresponding to the decisive age m. Figure 6 represents this mapping graphically for the benchmark m and also for two other arbitrary values lying one year apart. For a given exogenous m, an intersection of the corresponding curve with the 45 degree line is an equilibrium. The crossing associated with m = 5.360 characterizes the benchmark equilibrium tax rate. For a given m, the steady state is unique and the possibility

http://www.census.gov/compendia/statab/elections/. Given the data format, I have sorted observations into the age groups 30-39, 40-49, 50-59- 60-69, and 70+. The corresponding weights in Congress are 0.121, 0.357, 0.307, 0.17, 0.046. The electoral weights are 0.427, 0.548, 0.63, 0.642, 0.654. The resulting combined indexes have been uniformly re-scaled to facilitate comparison with the influence function to be calibrated. The differences in voting-participation patterns play a very minor role.

of multiplicity will not be an issue in the present analysis. The figure also demonstrates that the capital tax rate declines with the decisive age, and quite sharply.

#### INSERT FIGURE 6 ABOUT HERE

When m is endogenous, its value must coincide with the age of the median voter calculated using the weights in  $\tilde{\mu}$ . With the calibrated parameters, this only happens for the benchmark m. None of the other candidates survives as an equilibrium with endogenous m. More specifically, in all the cases the preferred capital tax rate by any cohort younger (older) than the given candidate is higher (lower) than the one supported by this candidate. Therefore the only decisive age in equilibrium must be the median age calculated on the effective distribution of votes  $\tilde{\mu}$  which is precisely the value of m of the benchmark equilibrium.<sup>20</sup> Figure 7 depicts the cumulative distribution across age groups of demographic size,  $\mu_j$ , and effective votes,  $\tilde{\mu}_j$ , for the specification of  $I_j$  presented in Table 3. Comparison of  $\mu$  and  $\tilde{\mu}$  shows the role of the political influence profile  $I_j$  which is to make the decisive voter with 46.80 years of age (i.e., model's age 5.36) older than the median age in the voting population of 40.75 years (i.e., model's age 4.15).

# INSERT FIGURE 7 ABOUT HERE

The life cycle - Individuals go through a typical disaving-saving-disaving life cycle, while their labour supply remains fairly stable over the main age range until they effectively decide to fully retire at age and 70 (period 11 in the model). Figure 8 displays the individual working

<sup>&</sup>lt;sup>20</sup>For some non-decisive ages j the second-order condition fails as the sign of the second derivative of the indirect utility function with respect to the tax rate is positive. The extremum found is thus a minimum of the quadratic (parabolic) approximation to the 'true' utility function over tax rates  $\tilde{v}_j(.,...,\tau_k';.)$ . This is just indication that the 'true' utility function is convex around the proposed tax rate. A geometric argument suffices to establish that if the minimum rate delivered is lower (higher) than the proposed  $\tau_k$  then the voter prefers a higher (lower) tax rate.

hours and asset holdings over the life cycle in the baseline equilibrium. Individuals in the middle-age range and above are clearly wealthier than younger groups. The decisive voter aged 5.36 has a wealth of 1.027, which is very close to the economy's average of 1.036. This decisive voter is characterised as a weighted composite of individuals aged 5 and 6, where the latter owns a wealth of 1.49, clearly in excess of the average. Thus it is plausible that this will impart an asset-rich bend on the voter's tax preferences.

#### INSERT FIGURE 8 ABOUT HERE

# D. Local dynamics

Around the baseline steady state, one can study the dynamic interaction between the key variables. In the approximated equilibrium, the law of motion for the distribution of assets and the capital tax rate is described by the linear mappings between the current and next-period capital tax rate,  $\tau_k$ , and wealth distribution, A:

$$\tau_k' = \Psi^{\tau c} + \Psi^{\tau A} A + \Psi^{\tau \tau} \tau_k$$

$$A' = \Psi^{Ac} + \Psi^{AA}A + \Psi^{A\tau}\tau_k$$

The values of the entries in these matrices and vectors are shown in Appendix B.<sup>21</sup> In order to illustrate the dynamic interactions between fiscal policy and the economy in the short run, Figure 9 displays the time path for capital and the tax rate  $\tau_k$  following an exogenous generalized 40 per cent reduction in wealth at the steady state. The main message to retain is that a reduction in the economy's capital intensity leads to a decline in capital taxation

 $<sup>^{21}</sup>$ Stability can be established by computing the eigenvalues of this system or simulating the response to a deviation from the steady state.

before the economy returns to the steady state. The intuition – discussed in the Appendix to the working paper – is similar to the one for the long-run effects of demographic factors, to which now the rest of the paper turns to.

#### INSERT FIGURE 9 ABOUT HERE

#### VI. NUMERICAL EXPERIMENTS

In this section I will study the consequences of variation in the age composition of the population  $\mu$ . The starting point is the benchmark calibration which assumes the demographic structure of the U.S. in 1990. The two experiments will consist of finding the stationary equilibrium for a  $\mu$  characteristic of the U.S. population back in 1965 and projected for 2025, respectively. The demographic data underlying Figure 3 will be used. In order to help the interpretation of the results, each experiment is run with a constant decisive age m as well as with an endogenous decisive age. Table 4 summarizes the main results. For the population structure corresponding to the year indicated in the first column, the next three columns show values of the political variables: age of the decisive voter m, and the tax mix  $\tau_k$  and  $\tau_l$ . The last four columns display macroeconomic variables including the wage rate per efficient unit of labor w, the gross rate of return r, and the aggregate supplies of capital K and labor services H. The 1990 entries in the first row thus represent the benchmark equilibrium. The rest of rows are for situations where m is either determined endogenously or held constant at its benchmark value.

# A. From 1965 to 1990

I start by comparing the 1990 baseline and the 1965 results. With an endogenous decisive age, the 1990 tax rate on capital  $\tau_k$  of 0.33 is lower than the tax rate of 0.40 implied by

Table 4. Experiments with  $\mu$ 

				,			
$\mu$	m	$ au_k$	$ au_l$	r	$\overline{w}$	K	Н
1990	5.360	0.330	0.240	0.401	0.450	1.036	3.471
$\mathbf{Endo}$	genous	$\mathbf{s} m$					
1965	5.621	0.404	0.217	0.431	0.441	0.997	3.546
2025	5.823	0.662	0.111	0.680	0.384	0.640	3.452
Cons	$\mathbf{tant} \ m$						
1965	5.360	0.655	0.112	0.693	0.382	0.659	3.625
2025	5.360	0.847	-0.006	1.389	0.296	0.297	3.520

the 1965 demographic conditions. As reported in Section I on U.S. data, the capital tax rate dropped from 0.38 to 0.33 between the periods centered roughly around 1965 and 1990. These implications are thus comparable to the observed data and suggest that much of the observed shift in the tax mix could be explained by the age structure of the voting population.

This result expresses the interplay of two forces with contrary sign. One force is that the decisive age m falls between 1965 and 1990 by about 1.3 years. The decisive voter becoming younger between 1965 and 1990 would on its own have increased the tax rate on capital. To see this, consider the experiment with constant exogenous decisive age reported in Table 4. It shows that holding m constant an even larger decline in  $\tau_k$  between the 1965 and 1990 population would obtain. Therefore, there must be another force that explains the net fall in capital taxation. This second force must necessarily be found in a general equilibrium effect of a younger population which somehow shifts the political preference in favour of lower capital taxation, for a given median voter m. Figure 10 represents graphically these outcomes by showing the benchmark equilibrium tax mapping in 1990, also shown in Figure 6 earlier, and in 1965 for both the constant and the endogenous decisive age. The lower position of the mapping for 1990 compared to 1965 indicates the fall in the political support to capital taxation. What explains it?

#### INSERT FIGURE 10 ABOUT HERE

From the life-cycle profiles in Figure 8, the individuals in the middle-age range are clearly wealthier than younger groups. Now consider the demographic changes displayed in Figure 3. Relative to 1965, the 1990 population has a larger share of asset-poor young individuals at the expense of the asset-rich middle-age and older individuals. This has therefore a negative composition effect on the economy's capital stock. If the tax mix were held constant, there would be a reduction in the relative supply of capital, a rise in the rate of return and a subsequent increase in the households' asset holdings. But the voter becomes more reluctant to tax capital as a consequence, so the tax mix responds by featuring a reduction in the tax rate on capital income. In effect, across the steady states in Table 4, the change in demographic conditions between 1965 and 1990 brings about, alongside the reduction in capital taxation, a rise in the after-tax rate of return and, consequently, an increase in individual savings and asset holdings.<sup>22</sup> For the decisive voter, the marginal cost to taxing capital has increased and, as a consequence, the preference for the lower taxation of capital is sustained.

This interpretation is consistent with the analytical findings of Section II. The 1965-1990 demographic transformation corresponds to the shift towards a younger population studied in Proposition 2. More specifically, the quantitative experiment reported in Table 4 features the capital-supply reversal noted in the discussion of the graphical representation in Figure 4 there. The fact that households save more causes aggregate capital to ultimately increase – overturning the initial demographic composition effect – and the (pre-tax) interest rate to

<sup>&</sup>lt;sup>22</sup>From the figures in Table 4, with an endogenous voter m, the net return on capital  $(1 - \tau_k)r$  increases from 0.257 to 0.269. The net labour return  $w(1 - \tau_l)$  decreases from 0.345 to 0.341.

decline. According to that discussion, the fact that, even with lower pre-tax interest rate, a lower capital taxation is sustained must rest on the increasing-partial-marginal-utility property of the voter's derived preference over the tax rate on capital: a lower capital tax raises individual savings and breeds further support for lower capital taxation. The predominantly positive slope of the equilibrium mappings represented in Figure 10 renders this property for the present quantitative model. Furthermore, in the analytical model of Section II, this was necessarily the case if the voter was of the *old* wealthy type. Correspondingly, in the present quantitative setting, as discussed in Section V, the decisive age involves agents with wealth above average.

As for the decline in the decisive age m, Figure 10 and Table 4 show clearly that it mitigates the scale of the reduction in capital taxation during the period considered. A younger voter, all else equal, has less capital and is more inclined to taxing capital. This conforms the result in Proposition 1 for the analytical model. In the present analysis, however, this mechanism is quantitatively too weak to overturn the decline in the capital tax rate. The relatively modest variation in m, by an amount equivalent to roughly one year and a third, reflects that the change in the median age of the population  $\mu$ , by about three years, is dampened by the calibrated profile of political influence, I, shown in Figure 5. The reason is that it lends less weight to the changes in the numbers at age groups towards the tails of the distribution which are characteristic of the demographic transformation over 1965-1990 shown in Figure 3. As Figure 6 suggests, the tax outcome is highly sensitive to the decisive age, and the fact that the reduction in m is a mild one is important for the negative response of the tax rate on capital to obtain.

#### B. From 1990 to 2025

I turn now to comparing the 1990 benchmark and the results under the population structure projected for 2025. With an endogenous decisive age, the 2025 tax rate on capital of 0.66 is vastly higher than the 0.33 implied by the 1990 population conditions. At the same time, the decisive age increases by about 2.5 years. Therefore this analysis indicates that the ongoing ageing process might lead to a shift of taxation towards capital income over the next decades in spite of the decisive voter becoming older.

Like in the 1965-1990 experiments seen previously, these tax responses rest on the interaction of two forces with opposed sign, namely the change in the decisive age m and the general equilibrium impact of population on factor returns and individual asset holdings. To understand this, consider again the experiment with an exogenous decisive age. Holding m constant, the 2025 setting implies a vast rise in the capital tax rate to 0.85, up from the 0.33 of the 1990 benchmark. Thus a marked shift in the preferred tax mix by any decisive age must be the key to the heavy increase in capital taxation. Partly counteracting this general equilibrium effect, there is the increase in the decisive age. Figure 11 depicts these changes by comparing the equilibrium tax mapping in 1990, also shown in Figure 6 earlier, and in 2025 for both an exogenous and an endogenous decisive voter's age.

#### INSERT FIGURE 11 ABOUT HERE

The general-equilibrium effect takes the form of an increased support by a *given* median voter to heavier taxation of capital in 2025 relative to 1990. An intuition can be developed along the lines of the previous discussion for the 1965-1990 experiment. Consider again the

life-cycle pattern of individual labour supply and wealth displayed earlier in Figure 8. The change in the age structure of population between 1990 and 2025 must have a direct impact on the relative aggregate supply of capital and labour inputs. The rise in the share of the older asset-rich groups causes, for given policies, an increase in the supply of capital and a decline in the supply of labour, from which a higher wage rate and lower rate of return follow. This change in factor prices prompts a stronger support for higher taxes on capital and a reduction in individual savings. In the end, it leads to a net reduction in aggregate capital intensity thus overturning the initial demographic composition effect. Again, the type of increasing-partial-marginal-utility to taxing capital identified in the analysis of Section II must be at work too. As for the rise in the decisive age, it follows from the sizeable aging of the population. Quantitatively the effect is relatively mild since the political influence pattern of Figure 5 lends a modest weight to the shifts composition of the population shown in Figure 3 for 1990-2025. In the present case, this decisive-age effect is weaker and the general equilibrium adjustment dictates the increase in capital taxation.<sup>23</sup>

# C. Sensitivity to fiscal settings

This section presents some exercises intended to assess the robustness of the previous results by modifying some of the maintained quantitative assumptions on fiscal variables.

Government expenditures - In the earlier analysis, government spending G and residual transfers T were held constant in the 1965-1990 experiment. Here I allow these parameters to adjust along with the population in order to be consistent with two features in the data:

 $<sup>^{23}</sup>$ The assumption of a constant influence profile over time, while reasonable between 1965-1990, is more questionable for the succeeding 1990-2025 period. Congress appears to be growing substantially older since the 1990's. Raising accordingly the mode of the influence profile  $j^*$  from the benchmark 5.561 up to 5.70 would lead to a 2025 decisive age of 5.963, which is enough to effect a reduction in capital taxation.

Table 5. Adjusting 1965 fiscal parameters

$\mu$	m	G	T	$ au_k$	$ au_l$	$ au_k/ au_l$
1990	5.360	0.15	0.070	0.330	0.240	1.375
1965	5.621	0.18	0.006	0.373	0.177	2.107
1965	5.621	0.15	0.070	0.404	0.217	1.862

the fall in the GDP share of government spending from about 18 per cent down to the baseline 15 per cent between 1965 and  $1990^{24}$ ; the increase in the share of transfers during that period, starting from the lower share associated with the 1965 tax rates on labour and capital of 0.174 and 0.380 reported in Section I earlier. To this effect, I set G = 0.18 and T = 0.006 in the calculation of the equilibrium under the 1965 demographic structure. Note that the size of the government total expenditure that is to be funded through income taxes, G + T, increases from 0.186 to 0.22 between 1965 and 1990, since transfers grow to a larger extent than expenditures decline. This picture is consistent with the shifts in expenditure in the U.S.<sup>25</sup>

Table 5 shows the tax rates on capital and labour, including the tax ratio  $\tau_k/\tau_l$ , for various assumptions on the population structure  $\mu$ , the decisive age m, and the exogenous parameters for government spending and transfers G and T. The first row reproduces values for the benchmark 1990 calibration. The corresponding 1965 outcomes under the changed fiscal parameters are displayed in the second row of Table 5. The tax mix is again more tilted towards capital taxation in 1965 than in 1990. But with changing government expenditures, the implications provide an even closer quantitative match to the observed change in the tax mix. To see this, the third row of Table 5 reproduces again values for the 1965 experiment with constant fiscal parameters reported earlier in Table 4. Compared with the tax ratio of

<sup>&</sup>lt;sup>24</sup>See Penn Tables at http://dc2.chass.utoronto.ca/pwt/

<sup>&</sup>lt;sup>25</sup>In the BEA data base, the ratio of government current expenditures (serie GEXPND) – to be funded from all sources of revenues – to GDP increased from about 0.24 to 0.32 between 1965 and 1990.

1.862 in this benchmark 1965 experiment, the realistically smaller government used now for 1965 in the second row of Table 5 implies a larger tax ratio of 2.107, which is closer the observed 2.201 from Section I. In other words, the overall increase in the size of the government has contributed to the change in the tax mix. This negative connection between government size and relative capital taxation is precisely a central theme in Klein, Quadrini and Ríos-Rull (2005).<sup>26</sup> The present exercise indicates that both demographics and government size can be complementary driving forces for the tax mix. Demographic change is quantitatively more significant though, accounting for 2/3 of the change in the tax ratio. Note also that the change in government spending cannot account for the decline in the absolute value of the tax rate on capital.<sup>27</sup>

Measured effective tax rates - As indicated in footnote 4, estimated tax rates differ across different studies. For example, the seminal study of Mendoza et al. (1994) calculates higher average tax rates than in the updated and extended series used in the present 1990 calibration. More specifically, for the period 1983-1988 they estimate average tax rates on capital and income of 0.40 and 0.285 respectively. In order to match these figures as well as the rest of 1990 targets set out in Table 1, a number of parameters have to be recalibrated: T = 0.1122,  $\beta = 0.913$ ,  $\gamma = 0.565$ . Then the required decisive age becomes m = 5.320 which calls for new parameters of the influence function. I will consider one minimal departure from the benchmark calibration which adjusts the mode so  $j^* = 5.524$ . This 1990 setting is shown in the first row of Table 6. In this setup, the 1965 demographic structure leads to a

 $<sup>^{26}\</sup>text{Like}$  in that paper, the sign of this link holds for the two types of government current expenditures, government consumption and transfers. As an illustration, consider the benchmark 1990 equilibrium with G=0.15 and T=0.07. with the tax ratio  $\tau^k/\tau^l=0.330/0.241=1.375$ . A larger G=0.17 leads to a lower tax ratio 0.335/0.268=1.25. A larger T=0.10 leads to a lower ratio 0.371/0.272=1.360.

<sup>&</sup>lt;sup>27</sup>See the change from the second to the third row of Table 5.

tax rate on capital around 0.45 as shown in the second row of Table 6. These implications are also consistent with the observed change in the tax mix. Regarding the impact of the population projection for 2025, the last row of Table 6 reports a further increase similar to that seen earlier.

Table 6. Alternative 1990 tax-mix

$\overline{\mu}$	$\overline{m}$	$ au_k$	$ au_l$
1990	5.320	0.400	0.285
1965	5.588	0.452	0.268
2025	5.786	0.674	0.176

#### VII. CONCLUDING REMARKS

This paper studies the influence of the age structure of the population on the determination of capital income taxation through voting. A novel aspect is the use of a model with a rich demographic structure which can account for the fine details of a population's age makeup. Another distinctive feature is that voters express their preferences recurrently without commitment, in a fully rational and forward-looking manner, so that policies are time-consistent.

The analysis for the U.S. demonstrates that the younger voting-age population in 1990 relative to 1965 can account for much of the large decline in the capital tax rate observed between these two years. On the other hand, the older voting-age population expected in 2025 is shown to lead to a sharp increase in capital taxation. There is no unequivocal one-way relation between the population's age, which determines the age of the pivotal voter, and the tax outcome. It is true that a younger decisive voter tends to support a higher tax rate on capital. But the very demographic change that alters the age of the decisive voter also brings about aggregate equilibrium effects which cause the voter's saving to increase thereby shifting the political preferences against taxing capital. The ability to match the 1965-1990 drop in capital taxation rests precisely on the strong economic equilibrium effect. This equi-

librium effects are intuitive and can be understood with the help of a simple tractable model.

The methods developed in this paper may find fruitful application to the study of other important fiscal policy variables such as social benefits or the size and composition of public spending.<sup>28</sup> This paper has at least three limitations. First, the analysis focuses on stationary situations and assumes the demographic profile corresponds to a stable population. For many policy reforms, understanding the transition is important. Second, it is based on a useful yet crude reduced-form setting for the political process. Given the sensitivity of taxes to the pivotal age, more robust conclusions may require a political model with deeper foundations. Third, this investigation is based on a deliberately simple framework for policies and the economy. Bringing into consideration consumption taxes, government debt and international capital mobility, might prove important to understand better the differences in the tax mix over time and across countries.

<sup>&</sup>lt;sup>28</sup>Xavier Mateos-Planas (2008) studies, for example, unfunded Social Security.

#### **APPENDIX**

#### A. Data for section I

Figure 1 shows estimates of average effective tax rates (AERT's) in Boscá et al. (1999) for the period 1965-1996 based on the methodology in Enrique G. Mendoza, Razin and Linda L Tesar (1994). Section I contains the average tax rates based on the data represented in Figure 1.

The methodology for calculating AETR's relates tax revenues directly to the relevant macroeconomic variables in the national accounts. There are difficulties, both conceptual and practical, in obtaining precise estimates (for a critical study, see OECD 2000). The initial calculations by Mendoza, Razin and Tesar (1994) have been revised in various studies. The source used in the present paper extends the series forward. David Carey and Harry Tchilinguirian (2000) propose alternative criteria. Although the level of tax rates at a particular time may differ across these studies, they all share the conclusion of Figure 1 of a marked reduction in the relative tax rate on capital, beginning at least since 1970 and stopping by the early 1990's. A remaining important issue is whether this fall is part of an earlier trend. With the numbers in Figure 1, as well as in the original Mendoza, Razin and Tesar (1994), it is hard to elucidate a trend between 1965 and 1970. However, Cara McDaniel (2007), with alternative measurement choices, has calculated series that extend back to the 1950's which suggest an ongoing trend. In any case, across all the available measurement studies, the persistent fall in the absolute value of the capital tax rate is only observed after 1970.

Figure 2 shows calculations based on the U.N. population database. The observations are provided at five years intervals. In each of these years, the population is described in terms of the distribution across 5-year age groups. For the median-age calculations I have used linear interpolation. The data after 1995 are estimates based on the medium-variant assumptions on the path of fertility used by the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat. These data are available at http://www.un.org/popin/. Figure 3 is constructed on the same data used for Figure 2.

#### B. Computed linear mappings

The linear mappings for the benchmark equilibrium are the following:

$$\Psi^{Ac} = \begin{bmatrix} 0.000 \\ -0.177 \\ -0.046 \\ 0.224 \\ 0.609 \\ 1.078 \\ 1.586 \\ 2.070 \\ 2.458 \\ 2.712 \\ 2.398 \\ 1.180 \end{bmatrix}, \ \Psi^{A\tau} = \begin{bmatrix} 0.000 \\ 0.220 \\ 0.483 \\ 0.609 \\ 0.606 \\ 0.488 \\ 0.275 \\ 0.015 \\ -0.223 \\ -0.316 \\ -0.206 \\ -0.769 \end{bmatrix}$$

```
\Psi^{AA} =
  0.000
           0.000
                    0.000
                             0.000
                                      0.000
                                                0.000
                                                         0.000
                                                                  0.000
                                                                           0.000
                                                                                    0.000
                                                                                             0.000
                                                                                                       0.000
  0.938
          -0.089
                  -0.045
                            -0.008
                                      0.099
                                              -0.013
                                                       -0.040
                                                                          -0.017
                                                                                    -0.002
                                                                                             -0.001
                                                                                                       0.004
                                                                -0.029
 -0.121
           1.001
                  -0.022
                             0.013
                                      0.123
                                                0.001
                                                       -0.032
                                                                -0.022
                                                                          -0.009
                                                                                    0.006
                                                                                             0.003
                                                                                                       0.007
 -0.121
          -0.063
                    1.039
                             0.022
                                       0.142
                                                0.006
                                                       -0.031
                                                                -0.021
                                                                          -0.007
                                                                                    0.009
                                                                                             0.005
                                                                                                       0.009
-0.139
         -0.077
                  -0.024
                             1.057
                                      0.156
                                                0.005
                                                       -0.036
                                                                -0.026
                                                                          -0.010
                                                                                    0.007
                                                                                             0.004
                                                                                                       0.008
-0.178
         -0.105
                  -0.046
                             0.004
                                       1.183
                                              -0.001
                                                       -0.047
                                                                -0.035
                                                                          -0.019
                                                                                    0.001
                                                                                             0.000
                                                                                                       0.007
-0.232
         -0.153
                  -0.078
                            -0.017
                                      0.172
                                                0.977
                                                       -0.060
                                                                -0.048
                                                                          -0.030
                                                                                   -0.008
                                                                                            -0.005
                                                                                                       0.004
-0.236
         -0.217
                   -0.130
                            -0.046
                                      0.176
                                              -0.017
                                                         0.871
                                                                -0.060
                                                                          -0.043
                                                                                   -0.019
                                                                                            -0.011
                                                                                                       0.000
-0.240
         -0.198
                  -0.195
                            -0.093
                                      0.174
                                              -0.019
                                                       -0.077
                                                                  0.812
                                                                          -0.052
                                                                                   -0.028
                                                                                            -0.016
                                                                                                     -0.003
-0.184
         -0.163
                  -0.129
                            -0.158
                                      0.137
                                              -0.010
                                                       -0.061
                                                                -0.054
                                                                           0.718
                                                                                   -0.027
                                                                                            -0.017
                                                                                                     -0.005
-0.058
         -0.055
                  -0.056
                                                       -0.020
                                                                -0.018
                                                                          -0.016
                                                                                            -0.007
                                                                                                     -0.003
                            -0.040
                                      0.009
                                              -0.014
                                                                                    0.509
 -0.129
         -0.121
                  -0.105
                           -0.090
                                     -0.015
                                              -0.044
                                                       -0.048
                                                                -0.042
                                                                          -0.038
                                                                                   -0.032
                                                                                             0.582
                                                                                                     -0.010
```

$$\Psi^{\tau c} = \left[ \begin{array}{c} 0.167 \end{array} \right], \quad \Psi^{\tau \tau} = \left[ \begin{array}{c} 0.107 \end{array} \right]$$

 $\Psi^{\tau A} = \begin{bmatrix} 0.115 & 0.077 & 0.047 & 0.129 & -0.291 & -0.050 & 0.037 & 0.032 & 0.027 & 0.013 & 0.008 & 0.001 \end{bmatrix}$ 

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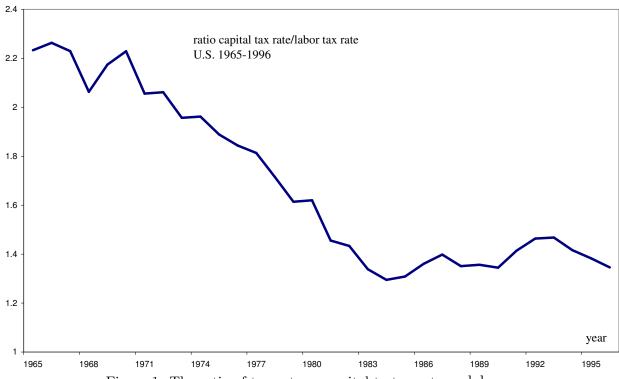
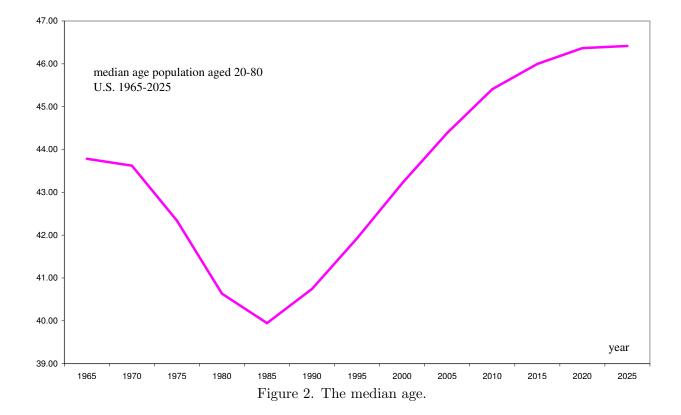
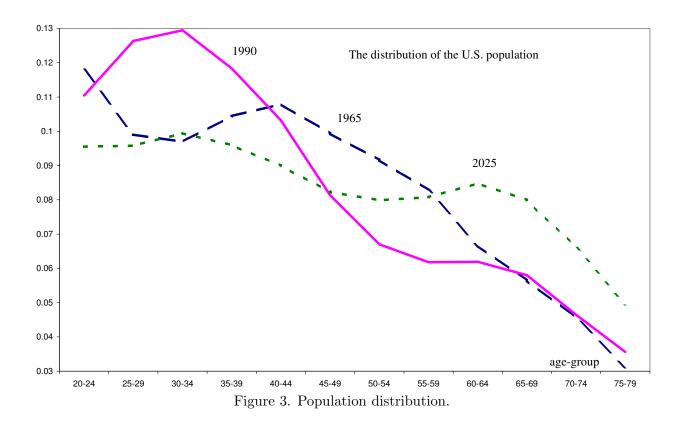


Figure 1. The ratio of tax rate on capital to tax rate on labour.





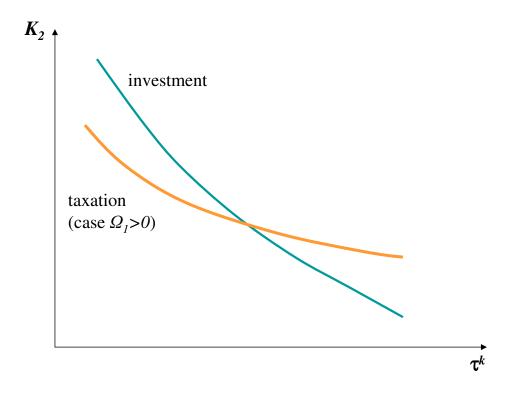


Figure 4. The equilibrium with an old voter.

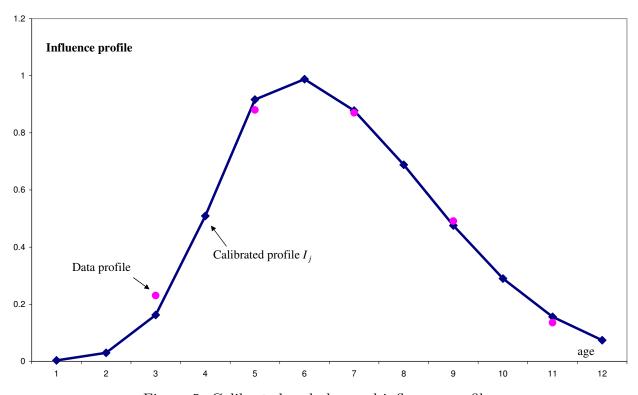


Figure 5. Calibrated and observed influence profile.

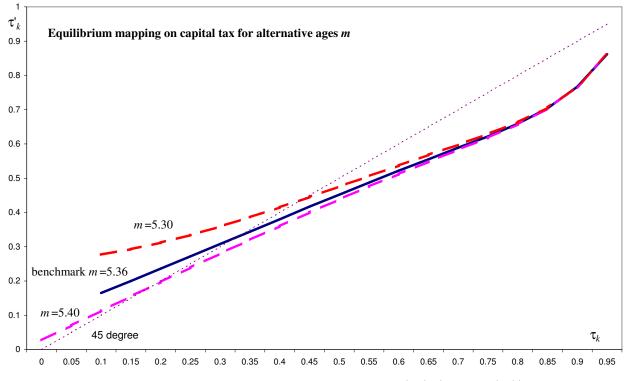


Figure 6. Equilibrium tax rate solves  $\tau_k = \Gamma(A(\tau_k), \tau_k; \Psi^{\tau}(\tau_k))$ .

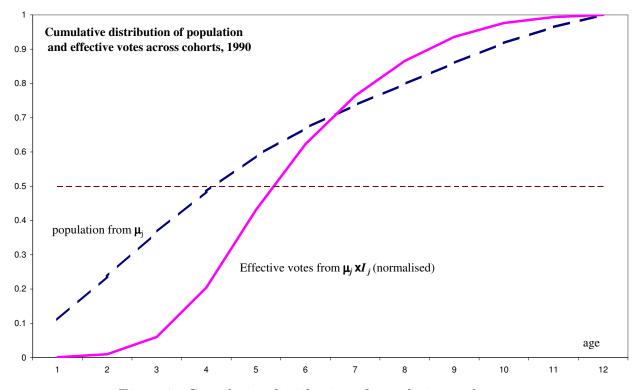


Figure 7. Cumulative distribution of population and votes.

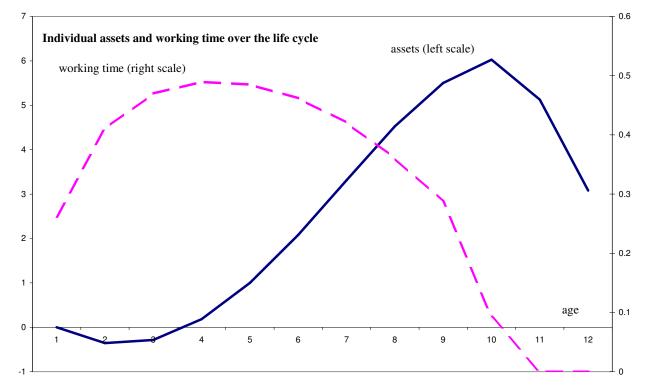
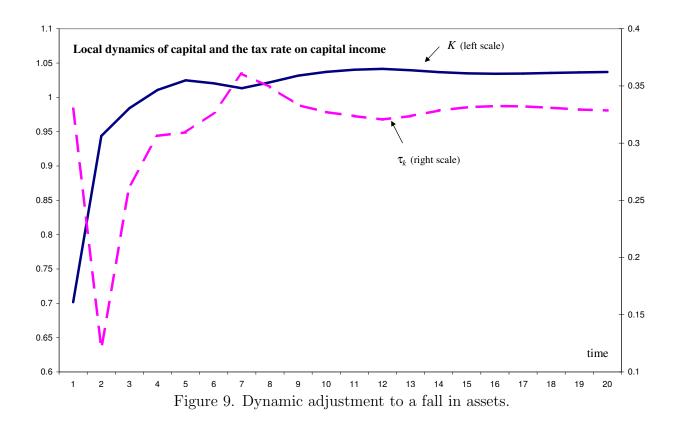


Figure 8. Life-cycle in the benchmark equilibrium.



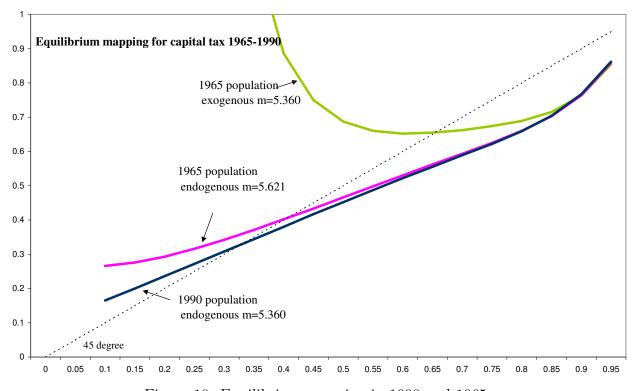


Figure 10. Equilibrium mapping in 1990 and 1965.

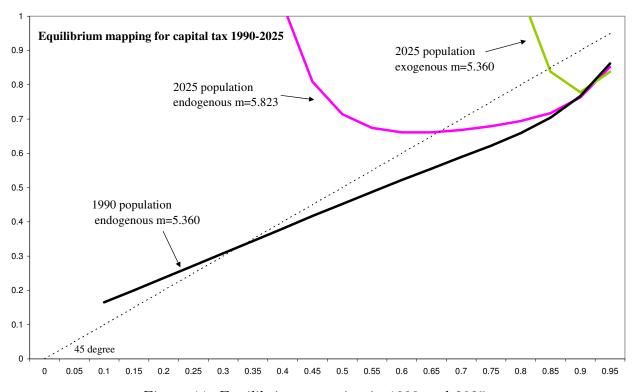


Figure 11. Equilibrium mapping in 1990 and 2025.