DESIGN AND OPTIMISATION OF A CORELESS SUPERCONDUCTING SYNCHRONOUS GENERATOR

by

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Constantly increasing demand for electrical power requires more efficient and more powerful machines to be built. The conventional technology cannot provide such machines. It cannot deliver machines that are smaller, lighter and provide larger torques and power ratings. The answer to these problems is believed to be in superconducting machines.

After short introduction to the phenomena of superconductivity and superconducting devices, practical superconducting tapes are described. The evolution and problems considered during the design of a coreless superconducting rotor for a synchronous machine are described. A few possible coreless rotor configurations are characterised and a simple formula is used to minimise the harmonic content.

Estimation of machine parameters and evaluation of losses is also conducted. The areas to which particular attention has to be paid are pointed out. All these are undertaken for a demonstrator size machine with BSCCO windings. But to achieve real benefits it is important to build a machine that more closely represents real machines. Hence an optimisation method is used to investigate the possibility of increasing the size of the machine.
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Declaration Of Authorship

I, Bartosz Lukasik, declare that the thesis entitled DESIGN AND OPTIMISATION OF A CORELESS SUPERCONDUCTING SYNCHRONOUS GENERATOR and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

• this work was done wholly or mainly while in candidature for a research degree at this University;

• where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;

• where I have consulted the published work of others, this is always clearly attributed;

• where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

• I have acknowledged all main sources of help;

• where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

• Parts of this work have been published as: [43], [42], [26], [41], [45], [11], [25]

Signed: ..............................................................................................................

Date

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Acknowledgements

I would like to thank, my supervisor, Professor Jan Sykulski for giving me the opportunity to conduct this research and for his, assistance and patience for the duration of this project. I would also like to thank Dr Kevin Goddard for his help and support.
To Sylwia
Chapter 1

Introduction and project overview

Constantly increasing demand for electrical power requires more efficient and more powerful machines to be built. Current densities in copper windings cannot be increased to more than about 10A/mm\(^2\) (even with water or similar type of cooling) due to excessive heating. Therefore conventional technology appears to be at a limit in terms of its ability to deliver machines that are smaller, lighter and at the same time provide larger torques and power ratings. One possible way to overcome these restrictions is believed to be development of superconducting machines, thanks to the much higher current densities that can be achieved in superconductors and significantly reduced winding losses. The task however is not straightforward. The initial attempts at building superconducting power devices using low temperature superconductors were not very successful, due to limitations imposed by the intrinsic material properties and high refrigeration costs, and such projects were quickly abandoned. However, low superconductivity designs continue to thrive in MRI, NMR, particle accelerators and many other applications. The discovery of high temperature superconductors (HTS) in 1986 reignited the interest in their potential for power applications. There are, or have been, several projects around the world that endeavour to build such devices (USA[18], Russia [11], Japan [71], Korea [23]) with many large companies becoming involved (Siemens [51], Converteem [2], ASC [18]). The Electrical Power Engineering Research Group, of the School of Electronics and Computer Science at the University of Southampton, has been interested in power applications of superconducting technology for the last thirty years with several successful research projects completed. The last of such projects involved designing and building a small (100kVA)
synchronous generator with a coreless HTS superconducting rotor working at temperatures between 65K and 77K. To date this is the only machine that can operate at temperatures above 65K. Other projects around the world all use BiSCCO material operating at temperatures below 40K, which improves the performance of the tape but significantly increases cooling costs.

The purpose of this work was to aid the team designing, building and testing the HTS generator in terms of conducting thorough electromagnetic analysis, design studies and multi-objective optimisation in support of the main design effort. These tasks were very challenging in view of the three-dimensional nature of the problem, significant levels of non-linearity encountered and material anisotropy. The problem was further exacerbated by various constraints imposed by the mechanical and thermal aspects of the design combined with the limitations of what could practically be achieved. The electromagnetic analysis was conducted using commercial Finite Element software Opera by Cobham Vector Fields. Since the software did not have the facility to include superconductor characteristics in the problem description a simple method was developed and applied to incorporate these into the models.

This thesis aims to develop methods that help designing such machines working at boiling temperatures of liquid nitrogen or liquid air. In some respects it was a continuation of previous programs run at University of Southampton on a superconducting transformer, and then a synchronous generator with an iron cored superconducting rotor. The latter project identified coreless rotor superconducting machine as the next step in the development of synchronous superconducting machines. Hence this thesis continues the previous work and expands the modelling and simulation concepts. Some of the methods are refined, like the optimization method used in Chapter 3, while others are developed, e.g. the SFFR modelling in Chapter 4, or application of kriging assisted response surface methodology to superconducting machines. In the thesis a number of possible rotor configurations are suggested and reasons for choosing the one that was build are explained. A number of difficulties in building a generator using BiSCCO tapes working at temperatures above 65K are described, and several possible solutions to those obstacles are presented. The thesis also looks at the losses in various components of the machine and investigates different configurations with the aim to find one with the lowest losses. Finally, a method of combining superconducting material properties and response surface modelling to optimize a next generation machine with YBCO (2G) winding is developed. This analysis identifies the best configuration recommended for the next generation of superconducting machines. The
objective of the thesis can be summarized as development of a magnetic design and optimisation methodology to assist the designers of electrical machines employing superconducting technology.
Chapter 2

Superconductivity

There are two kinds of superconductors and the distinction between them is based on their behaviour in the presence of magnetic fields. Type-I superconductors are mainly metals and Type-II superconductors are made of alloys and inter-metallic compounds. However both have the common characteristic that their resistance vanishes below certain critical temperature. To understand the superconductivity phenomenon it is essential to look at the electron transport mechanism and magnetic behaviour of these materials. In the following sections the basic concepts of low and high temperature superconductivity are presented. The general features of both types of superconductors together with the behaviour of these materials when used under AC operating conditions are also discussed.

2.1 Type I - superconductors

Type-I metallic superconductors, also called low temperature superconductors (LTS), were the first to be discovered. Mercury, aluminium, tin, titanium and others become superconducting when cooled below their critical temperature. All these materials are called Low Temperature Superconductors (LTS, Type-I superconductors) because superconducting properties appear only when they are cooled to temperatures below 7K. The low resistance observed for these materials in the superconducting state is described well by the BCS theory (Bardeen, Cooper and Schrieffer)\[13\], which is applicable only to low temperature superconductors. According to this theory, the lack of resistance comes from paired electrons (called Cooper pairs) that have an energy gap above. This prevents the kind of collision interaction that normally leads to resistivity. In very low temperatures (below
25K) the thermal energy of the lattice is so small that when the electron travels through a conductor it attracts nearby positive charges from the crystal lattice of the material. These charges will then attract an electron with opposite spin and then the travelling electron forms a pair with it. This pair can only be separated when certain conditions are met and then the superconductivity is lost.

Each superconductor is therefore characterised by a few parameters. The transition temperature ($T_c$) which constitutes the boundary of the superconducting state. The critical current density ($J_c$), as the name suggests describes the maximum resistanceless current density that can be carried by the metal. Finally, the critical magnetic field strength $H_c$, and - remembering that $B_c = \mu_0 H_c$ - the critical magnetic flux density $B_c$. When below $T_c$ the material is in a superconducting state, when above it loses its superconducting abilities and becomes ordinary again. And in the superconducting state when any of the other parameters is above its critical value the resistanceless state is lost as well.

Critical values of $B_c$ and $H_c$ are shown in Table 2.1. High refrigeration costs and poor tolerance of magnetic fields limit the wide use of these materials especially in power applications [57].

### Table 2.1: Critical $T_c$, $H_c$ and $B_c$ values for Type-I superconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ [K]</th>
<th>$H_c$ [A/m]</th>
<th>$B_c$ [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titanium (Ti)</td>
<td>0.4</td>
<td>0.42 \times 10^4</td>
<td>5.28 \times 10^{-3}</td>
</tr>
<tr>
<td>Aluminium (Al)</td>
<td>1.2</td>
<td>0.79 \times 10^4</td>
<td>9.93 \times 10^{-3}</td>
</tr>
<tr>
<td>Tin (Sn)</td>
<td>3.7</td>
<td>2.40 \times 10^4</td>
<td>10.5 \times 30.2^{-3}</td>
</tr>
<tr>
<td>Mercury (Hg)</td>
<td>4.2</td>
<td>3.30 \times 10^4</td>
<td>41.5 \times 10^{-3}</td>
</tr>
<tr>
<td>Pb</td>
<td>7.2</td>
<td>6.40 \times 10^4</td>
<td>80.4 \times 10^{-3}</td>
</tr>
</tbody>
</table>

### 2.2 Meissner Effect

The effect discovered in 1933 by Walter Meissner and Robert Ossenfeld is that the material in the superconducting state does not allow any magnetic flux to exist within itself ($B=0$). This effect shows that a superconductor is different from just being a perfect conductor. For the perfect conductor, if it is cooled with magnetic field present, the magnetic field penetrates the material and if then the magnetic field is removed the perfect conductor generates its own magnetic field (as an effect of resisting against the change of magnetic field, Lenz’s law). When
Chapter 2 Superconductivity

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Figure 2.1: Meissner Effect - The Magnetic behaviour of superconductor compared to perfect conductor

<table>
<thead>
<tr>
<th>Condition</th>
<th>Perfect Conductor</th>
<th>Metal Superconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T &gt; T_c$</td>
<td>$B_a &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$T &lt; T_c$</td>
<td>$B_a &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

![Cooling](image)

a different approach is taken and the magnetic field is applied after the perfect conductor has been cooled below its transition temperature the effect, as expected, is different and the magnetic field is expelled from the material. However, in the case of a superconductor something different happens and no matter when the magnetic field is applied, before or after, the material has crossed the transition temperature, the magnetic field is always expelled from it.

The comparison of the behaviour between a perfect conductor and a superconductor in the presence of a magnetic field is shown in Figure 2.1.

2.3 Superconductors and AC current

The fact that the superconductor has zero resistance is only applicable to a DC current. When an AC current is applied to the superconductor the electric field is developed and some power is dissipated. It has been proposed that below the transition temperature the conduction electrons divide themselves into two classes: superelectrons and normal electrons behaving like electrons in normal metals (two
fluids model). These can be scattered so they can experience resistance. In general the current in a superconductor might be carried by both types of electrons.

The special case is one where there is only a constant direct current present and then it is carried only by superelectrons since they can flow without being scattered - the material yields zero resistance. Therefore the constant DC current has to be carried by the superelectrons. In the presence of an alternating current normal electrons are accelerated and these scatter on the lattice of the material and therefore a resistance is observed in the material [17].

### 2.4 Type-II superconductors

Type-II superconductors may be composed of pure metals, metal alloys and different compounds, in contrast to Low Temperature Superconductors which are all composed of metallic elements. All superconductors with transition temperature higher then 30K are known as Type-II superconductors and those with transition temperature above 77K (boiling point of nitrogen) are called High Temperature Superconductors (HTS). Type-II superconductors are characterised by a lower critical value \( H_{c1} \) and a higher critical value \( H_{c2} \) of the magnetic field. Values for a few typical materials are shown in Table 2.2. When magnetic fields are weaker then the lower critical field \( H_{c1} \), the superconductor is in perfect diamagnetic state (Meissner state) expelling all magnetic field from its interior. However, when the field is raised above that value, the magnetic field starts to penetrate until it reaches \( H_{c2} \) when the full penetration is achieved and the superconducting state is destroyed. Figure 2.2 shows the dependence of both fields for the Type-I and Type-II superconductors. In the mixed state Type-II superconductors exhibit a hysteresis behaviour. Impurities in the material structure cause the flux to get trapped on them creating local vortices. This phenomenon, called the flux pinning, prevents travelling on the same branch of the magnetisation curve.

<table>
<thead>
<tr>
<th>Compound</th>
<th>( T_c ) [K]</th>
<th>( H_{c1} ) [A/m]</th>
<th>( B_{c2} ) [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NbTi</td>
<td>10</td>
<td>0.9 ( \cdot 10^7 )</td>
<td>12</td>
</tr>
<tr>
<td>Nb_3Sn</td>
<td>18</td>
<td>1.6 ( \cdot 10^7 )</td>
<td>22</td>
</tr>
<tr>
<td>Nb_3Gn</td>
<td>23</td>
<td>2.2 ( \cdot 10^7 )</td>
<td>30</td>
</tr>
<tr>
<td>Nb_3Al</td>
<td>19</td>
<td>2.3 ( \cdot 10^7 )</td>
<td>32</td>
</tr>
<tr>
<td>PbMo_6S_8</td>
<td>14</td>
<td>3.3 ( \cdot 10^4 )</td>
<td>45</td>
</tr>
</tbody>
</table>
2.5 Mixed state

Flux pinning occurs when the superconductor is in the mixed state. When the magnetic field is weak, below the lower critical value, the flux is totally expelled from the material (Meissner state). Above the lower critical value of magnetic field strength $H_{c1}$ it is energetically favourable for the material to go into the mixed state. In this state cores of normal state (parallel to the applied magnetic field) appear in the material. Each of these cores is surrounded by the current circling around it. The direction of this current is opposite to the current shielding the superconductor from the external magnetic field. Because these cores repel each other soon the equilibrium is achieved and the cores form a regular pattern called fluxon lattice (Figure 2.3). When the magnetic field is increased even more the cores join and then eventually the higher critical magnetic field strength ($H_{c2}$) is exceeded and superconductivity is lost.

While in the mixed state it can be expected that there will be a Lorentz force acting on those cores and at some point when the transport current is increased...
to the value when the force is strong enough (larger than pinning force) to move the cores the superconductor starts to experience losses (during the movement the energy is dissipated). The cores tend to be pinned on material imperfections, hence the more imperfections there are the higher the transport current needs to be to start moving these normal cores; so effectively impurities help to increase the critical current [57].

A different loss mechanism related to vortices movement is called *flux creep*. It occurs when a vortex is thermally activated and jumps to a different pinning location. During this process some energy has to be dissipated and therefore losses occur.

### 2.6 High Temperature Superconductors classification and development

The first high temperature superconductor was discovered by Bednorz and Müller in 1986. It was a lanthanum-barium-copper oxide ceramic material with transition temperature as high as 30K. It represented a new class of materials. Up until that year all superconductors were metallic with the critical temperatures not higher than 23.2 K for Nb$_3$Ge. The new superconductors were ceramic. This discovery led to the explosion of research on that class of materials since it was previously believed that no superconductivity was able to exist in temperatures higher than the one of Nb$_3$Ge. Ceramic compounds were also believed not to have superconducting capabilities. Currently the temperature record is held by (Sn$_{1.0}$Pb$_{0.5}$In$_{0.5}$)Ba$_4$Tm$_6$Cu$_8$O$_{22+}$ at 195K.

The most commonly used high temperature superconductors are: YBa$_2$Cu$_3$O$_x$ (Y-123), Bi$_2$Sr$_2$CaCu$_2$O$_x$ (Bi-2212) and Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_x$ (Bi-2223), with critical temperatures 90K, 95K and 107K, respectively, that allows them to be used at the temperature of liquid nitrogen (77K), which is a very easily obtainable and cheap coolant. They are fairly simple to use and to cool to their operating temperatures compared to LTS which require significant resources to be cooled.

#### 2.6.1 Crystal Structures

All High Temperature Superconductors have a structure that is a variation of the crystal structure known as perovskities (Figure 2.4) that is a simple cubic...
Chapter 2 Superconductivity

Figure 2.4: Perovskite unit cell structure

structure. Ba-K-Bi-O is one of the superconductors built out of cells with such organisation. For a brief period of time it was one of the most important materials in the history of high temperature superconductivity with the critical temperature of 30K. It is composed of the bismuth atom that is in the centre of the structure with barium or potassium atoms occupying the corners and oxygen atoms shared between every two cells.

The other type of superconductor structure is the Perovskite "Layered" structure (Figure 2.5). This is the structure of the La\(_{2-y}\)Sr\(_y\)CuO\(_4\) compound which is a close relative of the lanthanum barium copper oxide that was discovered by Bednorz and Muller. It is usually said that this compound has a 2-1-4 structure, 2 lanthanum atoms, 1 copper and 4 oxygens, while in reality, because every CuO\(_2\) plane is offset by one-half, hence the real structure is 4-2-8.

The yttrium barium copper oxide (Y\(_1\)Ba\(_2\)Cu\(_3\)O\(_{7-x}\)) was the first superconductor to be found with a critical temperature greater than 77K. It is also one of the most thoroughly studied compounds. The structure of this material is obtained by tripling the perovskite unit cell and its lattice looks almost like three cubes stacked on top of each other. It is composed of two CuO\(_2\) layers separated by one yttrium atom. The superconductivity takes place in those two layers.

Compounds based on the formula A\(_2\)B\(_2\)Ca\(_n\)Cu\(_{n+1}\)O\(_{2n+6}\), where A is Bi or Tl and B is Sr or Ba, have the critical temperature higher than 100K. The structure is similar to YBCO when n is one and, for example, when n=2 there are three CuO\(_2\)
layers in the material. Bi2212 and Bi2223, along with YBCO, are now the most advanced and most commonly used high temperature superconductors in power applications.

The high temperature superconductor crystal structure can be described in terms of planes (Figure 2.5). Since they consist of CuO\(_2\) layers in the a-b plane, stacked along the c plane, strong anisotropy can be expected. The current transport takes place in the a-b plane (CuO\(_2\) layers) therefore the critical current is higher in this direction compared to its value in the c plane direction. The crystal nature of these novel materials prevents creating wires like it is done with conventional conductors. Therefore a base is required in which the crystals will be placed. In case of 1G (1\(^{st}\) generation) superconductors (BiSCCO) as the matrix in which the superconducting crystals are embedded the most common metal is silver. Other metals that are suitable are: platinum and gold but they are more
expensive. 2G superconductor YBCO usually as the substrate stainless steel is used and the superconductor is deposited on insulating buffer layer. The most common shape of the superconducting wire is that of a tape.

2.6.2 A short review of the most important superconductors

2.6.2.1 Fabrication of BiSCCO

One of the most commonly used superconductors in modern applications of HTS technology is bismuth strontium calcium copper oxide known as BSCCO which belongs to the first generation of high-temperature superconductors.

![Diagram of production process](image)

**Figure 2.6:** Production of Ag/BSCCO tape by the OPIT method

The production process for this material is well established and currently a few companies around the world produce long length tapes composed of this material. For producing this material the Oxide Powder In Tube (OPIT) process is used. A schematic overview of this process is presented in Figure 2.6. BSCCO bars are placed in silver tubes that are later drawn into single filaments which are then placed again into silver tubes and drawn into a multifilament wire that can then be rolled to a form of tape and heat treated at a temperature around 830°C. The tape usually has the dimensions of width of 2-4 mm and thickness of 0.2-0.4 mm. During the production rolling is important as it makes the superconducting
grains align in order to have the best superconducting capabilities. Obtaining the appropriate shape of the rolled filaments is also important.

Silver is present in the superconducting material as a base and a strengthening material for the superconducting material, which is ceramic and thus has weak mechanical properties. Ag has been chosen because it does not react with the superconducting material during heat treatment and has good oxygen diffusion properties. It is also cheaper than platinum and gold which would also make a good base for a filament material.

Ceramic materials impose some important limitations on the superconducting tape that need to be taken into account when considering practical applications. For example the loss of current that can be transported by the tape starts to rise significantly after using a smaller bend radius then a certain value. The material is mechanically damaged and the loss in current carrying capacity cannot be recovered.

There are two commonly used types of BSCCO superconductors currently on the market: BSCCO-2212 with \( T_c \) around 90K and BSCCO-2223 with \( T_c \) around 110K. Although BSCCO-2212 can be operated at temperatures around 77K (boiling point of nitrogen) it is used by many in temperatures below 20K where it performs much better than BSCCO-2223 when it comes to \( J_c \) and operating in high external fields and thus outweighs the performance of Nb3Sn which is one of the most commonly used LTS materials.

BSCCO-2223, with its transition temperature equal to 110K, is the most popular HTS material. It is usually operated at the temperature of liquid nitrogen (77K). Cooling at this temperature is easy and cheap to implement and the material itself can easily carry high current densities and accept magnetic fields high enough to be used in electrical machines.

Presently, the best available tape with critical current values around 200A is produced by Sumitomo Electric. This superconducting tape is being used in the current generator project at the University of Southampton.

### 2.6.2.2 BiSCCO tape

When building a HTS generator there are various aspects that need to be considered. Electromagnetically the environment in the machine is a quite demanding one whereas the properties of the superconducting material put serious constraints
on the conditions in which they can provide the best performance. The structure of superconducting crystals is such that the preferred direction for current flow is along CuO planes. The plane of these CuO planes is called ab-plane. The plane orthogonal to it is called a c-plane. As mentioned it is easier to drive current in ab-planes, hence the rolling during the production process, as it aligns the ab-planes. The position of these is such that c-plane is the broad face of the superconducting tape and ab-planes are along the tape. Because of this the superconducting tape exhibits different magnetic field tolerance for the two directions.

The two limiting directions, in respect to tape orientation, are presented in Figure 2.7 and the superconducting carrying capacity depending on temperature for both directions is presented in Figure 2.8.

One can easily see that the tolerance of a superconducting tape is very different for the field directed perpendicular (normal) to the planes than for field parallel (tangential) to the planes.

It is necessary to consider the fact that the capacity for carrying a superconducting current is also limited by the strain limit for a particular tape. This characteristic is important since, during the coil winding process, the wire will be subjected to tensile and bending stress. During operation of the generator further stresses will arise, as well as centrifugal forces and presence of large magnetic fields. Because of all these parameters, the most important being the nominal (at 77K in zero field) current carrying capacity of 200A, the Sumitomo Electric tape has been chosen for the new design.
2.6.2.3 Fabrication of YBCO

The Yttrium Barium Copper Oxide, known as YBCO, is the ‘second generation’ (2G) of superconducting wires. The superiority of YBCO over first generation wires lies in the fact that it can achieve better electrical performance at higher temperatures. This is due to the irreversibility field of YBCO which is significantly higher than for BSCCO. The 2G wires are expected to play significant role in future HTS applications especially with the predicted prices below $10 \text{ kA/m}$, which is comparable to copper (with some predictions suggesting prices even lower than copper). It is believed that one of the methods of making this possible is to reduce the cost of producing YBCO wire by using a MOD/RABiTS process \[58\]. The
structure of the wire produced using this method is presented in Figure 2.9.

The schematic overview of the production process using RABiTS/MOD Rolling Assisted Bi-axially Textured Substrate and Metal Organic Deposition process is presented in Figure 2.10. In the RABiTS method the deformation textured metal or metal alloy substrate serves as a base for deposition of an epitaxial oxide buffer layer which is also a barrier stopping the diffusion between YBCO and metal substrate and a base for growing the YBCO layer. During the metal organic deposition, the YBCO forms itself from solution-based precursor present in the buffered substrate.
Apart for RABiTS, there are several other techniques for providing appropriate texture for the substrate layer. Inclined Substrate Deposition (ISD) and Ion Beam Assisted Deposition (IBAD) are good examples. IBAD-YZS (yttria-stabilized zirconia) and GZO (gadolinium zirconate) rely on a complicated and slow process but it is believed to give better quality superconducting material. On the other hand IBAD-MgO (magnesium oxide) is fast and gives good results. After establishing a good base for YBCO there are several methods that can be used for deposition, of which the most effective and cheap ones are the metal-organic deposition (MOD) and metal-organic-chemical vapour deposition (MOCVD) [58].

2.6.2.4 YBCO tape

The better properties of YBCO superconductor have been known for a while now. However, research into the use of these materials in electrical machines was held up by the difficulty of obtaining suitable lengths of such conductors. Only recently the tape manufacturing companies have begun to industrialise the production of such materials and announced that they are able to make tapes with the properties and lengths required. Hence the designers of electrical power machinery have started exploring the possibility of using these new tapes [46].

The main advantage of 2G superconductors is the much better tolerance to magnetic fields in directions different than parallel where the field tolerance is similar to that of BiSCCO superconductors; more improvement should be expected soon since advancing tapes performance is a very active area of research. Figure 2.11 shows the field dependence of the critical current of YBCO compared with that of BiSCCO. The critical current of the YBCO tape has been doubled in order to compare the tapes of the same rating, since according to most of the manufacturers it will not be long before they can offer more than 200A rated YBCO tapes. The YBCO tapes that are now commercially available in long lengths are mostly rated around 100A; the field dependence data used for YBCO in this chapter are for the tape produced by American Superconductor.

It can be easily seen that YBCO has a better performance than BiSCCO, although in case of the parallel field, especially at temperatures higher than 60K, it seems that it loses its advantage and sometimes, especially at lower magnetic fields, it is marginally worse than BiSCCO. However, lowering the temperature significantly improves the performance of a 2G superconductor where it can easily outperform 1G materials. When subjected to a significant perpendicular component of the magnetic field, YBCO outperforms BiSCCO at all temperatures and
Figure 2.11: Critical current dependedance on flux density for BiSCCO and YBCO superconductor a) perpendicular b) parallel component.

This is the reason for its superiority. This makes it easier to use it in building electrical machines, where controlling the direction of magnetic field is difficult. One more thing worth noting is that for YBCO, especially in higher magnetic fields, the slope of performance decrease is much flatter then for 1G superconductors.
2.6.2.5 Magnesium Diboride

Magnesium diboride ($\text{MgB}_2$) is considered to be a simple and inexpensive superconductor. Although it was first synthesised in 1953 its superconducting properties were discovered in 2001. Its critical temperature ($T_c$) is 39K which is substantially higher then LTS but lower then 77K required for relatively cheap cooling. This superconductor is the only superconducting material with two types of electrons at the Fermi level. One is much more superconducting then the other. The upper critical field in the parallel direction to the ab-plane is around 14.8T and in the perpendicular direction approximately 3.3T; for thin films the values reach 74T and for fibres 55T.

The structure of this interesting material is very simple and consists of hexagonal honey-combed planes of boron atoms separated by planes of magnesium atoms with the magnesium centred above and below the boron hexagons (Figure 2.12).

Magnesium diboride can be synthesised in a few ways with the simplest being a high temperature (650°C) reaction between magnesium and boron. Other possible ways of obtaining it are powder in tube method or hybrid physical-chemical vapour deposition [74].
2.7 Losses in superconducting tapes and cables

As it was already mentioned before superconducting materials are not always free of losses. There are various different mechanisms that can cause losses. Using these novel materials in devices subjects them to certain conditions under which additional loss mechanisms can occur.

2.7.1 Hysteresis Loss

The main loss occurring in high temperature superconductors is hysteresis loss [19] caused by magnetic flux entering and leaving the superconductor. If there are no transport currents flowing, in every cycle, the loss equals to the area of the hysteresis loop. The losses are eventually dissipated as heat from the superconductor so it is very important to keep them as low as possible. At the temperature of liquid nitrogen it is estimated that removing 1W of loss requires 15-25W of refrigeration power.

As the flux driven by an applied magnetic field starts to penetrate the material from the surface, induced currents occur, similar to the skin effect in a copper conductor. The two main differences are that the superconductor is infinitely conductive and that there is a critical current density for any superconductor which is a decreasing function of the magnetic field at a fixed temperature (in comparison to fixed frequency for a copper conductor). When the superconductor

![Figure 2.13: Hysteresis loss in the superconductor](image)

is penetrated, decreasing the magnetic field will not reverse the process since flux is pinned by the impurities of the material. Decreasing the applied field causes a new region to appear on the surface with reversed current density that propagates into the material as the field is decreased. When the applied field crosses zero there is still some remanent magnetisation present in the material. The field needs to be decreased further to make it zero. This way a hysteretic curve is traced.
This mechanism, illustrated in Figure 2.13, is responsible for hysteresis loss in a superconducting material.

### 2.7.2 Eddy current and coupling current loss

Varying magnetic field induces eddy currents both in the matrix and filaments of the superconductor. The magnitude of these currents depends on length $l$ of the filaments, $\rho_m$ (resistivity of the material) and the derivative of the applied magnetic field. Normally the filaments are able to partially shield themselves but once the length of the filaments is large enough the induced currents can reach the current carrying capacity of the filaments and as a result of that the filaments become coupled and behave like one large filament with averaged properties.

This problem is usually alleviated by twisting the filaments. It reduces the effective length and by preserving appropriate twist pitch it is possible to reduce the coupling to a level well below the critical current carrying capacity of individual filaments, hence they can screen themselves and the loss is therefore reduced. This loss is proportional to $\frac{l^2}{\rho_m}$.

A superconducting tape when subjected to changing magnetic field will have eddy currents induced in it. The highest currents will be induced when the field is in the perpendicular direction to the broad face of the tape which is usually 4 mm wide and just 0.25-0.5 mm thick. At 50Hz the penetration depth is larger than the thickness of the tape. A method for calculating these losses is suggested in [50]. The authors propose to treat the superconducting tape as a hollow cylinder and derive appropriate equations.
Chapter 3

Applications of High Temperature Superconductors

The discovery of high temperature superconductors in 1986 started a rapid development of new applications. By constant improvement of these novel materials the performance boundaries of many machines can be pushed further. Only recently Sumitomo Electric announced the arrival of the DI-BSSCO wire that has the critical current value of 200A [12]. Up until a few years ago the power industry was not much interested in using superconductors for everyday applications because of somewhat discouraging experience it had with the low temperature superconductors. Potential applications of high temperature superconductors include transmission cables, magnetic energy storage devices (SMES), fault current limiters, transformers, motors, generators and novel propulsion systems.

3.1 Superconducting Transmission Cables

First research and development of superconducting power cables [36] dates back to 1970s when the oil crises occurred and suddenly the governments realised that new technologies needed to be developed in order to meet constantly increasing demands of society and industry. Although these projects were soon abandoned, because the oil crisis was over, they inspired interest in this technology. These projects were concerned with using low temperature superconductors and therefore were complicated and expensive.

Nowadays, researchers concentrate on using HTS cables composed usually of
Chapter 3 Applications of High Temperature Superconductors

BSCCO-2223 wires, which are easier and cheaper to keep at the operating temperature. In certain conditions, when for example new ducts would have to be constructed to put a higher rated cable, the cables based on this technology can easily compete economically with standard cables. The advantage of a HTS cable is the much higher current density it can carry in comparison to a conventional cable of similar cross section. There are actually two main designs studied at the moment: a warm dielectric and a cold dielectric. The former is presented in Figure 3.1.

![Figure 3.1: Warm dielectric superconducting cable](image)

It is composed of a pipe, with a coolant inside, onto which the superconducting tape assembly and cryostat are installed and then everything is wrapped in the dielectric. This design is characterised by smaller dimensions compared to a cold dielectric design, it also allows use of standard dielectric materials as insulation because, as the name suggests, the dielectric works at room temperature and only the superconductor is kept in a cryogenic environment.
Chapter 3 Applications of High Temperature Superconductors

The cold dielectric coaxial design, shown in Figure 3.2, is composed of two HTS conductors, one for carrying phase current and the other serving as the return conductor, separated by suitable electrical insulation. When current flows in the phase conductor a magnetic field is generated. If this magnetic field was allowed to cut through neighbouring conductors, it would lead to increased AC losses (mainly hysteretic losses). Using coaxial return conductors prevents this by confining the magnetic field within the dielectric; this allows higher currents to be transported, decreases the load on refrigeration system and shields the surrounding space from the magnetic field. For three-phase cables there are two configurations that can be used, a single cryostat enclosing all the phase cables or individual cryostats for every single phase cable. One other interesting concept for building a HTS power cable is a tri-axial configuration shown in Figure 3.3.

Because this design does not require a superconducting shield layer, it requires only half as much tape as the 3-coaxial-cable design. Also the diameter of the cable is reduced, reducing the size of the cryostat and the cost. This type of cable is currently being tested in Columbus, Ohio [22], where it served a commercial customer for more than 22,000 hours. Another one is planned for installation in Amsterdam (Holland) in a 6000 m long configuration with cooling stations only.
Although the tri-axial cables are still under development there are reports on successful implementations of these in real life applications. One of them is the Detroit Edison Cable System \[35\] that was successfully installed. It operates the HTS cable at a nominal current of 2400A, voltage of 24kV and has the length of 120m. This cable has replaced three parallel sets of old oil-impregnated insulated cables. Most of the projects undertaken to evaluate the feasibility of using HTS in cable applications used BiSCCO as the superconducting material mainly because the production process for long lengths has been very well established. There however a project which is operating a YBCO based cable, although only 50m out of the total of 350 is 2G, the rest is still 1G. The cable is rated at 34.5kV and 800A \[47\]. This project has been running for over 9 months now, hence proving the technology. Another interesting project that successfully tested the possibility of using an HTS cable in real life applications is the Super-ACE project in Japan \[75\]. The tests were conducted on a 500 m cable installation that included a 77kV 1kA cable. There were two sections: one 90m subterranean section and one 10m high section simulating a bridge.

### 3.2 Superconducting Energy Storage

Superconducting magnetic energy storage devices are under continuous development all around the world and there are several successful installations. These
devices, consisting of a superconducting coil placed in a properly designed cryostat, a power conversion mechanism and a cryogenic system, can potentially store hundreds of megawatt-hours in the magnetic field produced in a coil. To date the highest reported rating of these devices oscillates around few MJ.

The potential application of Superconducting Magnetic Energy Storage (SMES) devices is very wide, they can be used for reducing power fluctuations and system oscillations, to balance loads, to defend critical loads from power drops and finally can serve as a UPS while ordinary generators are started. They can be loaded and discharged thousands of times without degradation of the magnet which is an advantage over battery systems or systems using any mechanical components.

Otonello and others have recently reported devices of this type that can supply energy of 1MW for 1s with a nominal current of 1100A. While most of them are built using LTS there are reports of using HTS. They can also be used to store the surplus energy produced by power plants during off-peak hours and release it during power shortage times.

SMES is not the only kind of energy storage device that utilises superconductors. The other type is flywheel energy storage (FES) which, in comparison to SMES, stores energy not in the form of the magnetic field but as the kinetic energy of a rotating mass and the superconducting material is used to construct a frictionless magnetic bearing. By using magnetic levitation it is possible to build a bearing for the flywheel that will practically eliminate friction and will therefore allow the rotor to continue its movement almost indefinitely.

### 3.3 Fault Current Limiters

A fault current limiter is another application for superconductors. These devices help to control the current when a fault occurs. Unlike other methods of limiting fault currents this type of devices can be installed on the network without introducing extra impedance during normal working conditions. Contrary to very fast switches (which are usually one time breakers) it also provides fast and automatic recovery when the fault conditions disappear. The principle of successful operation of a SFCL device comes directly from the properties of superconductors. When the critical current is exceeded the superconductor looses its properties, thus introducing resistance or impedance (depending on the configuration) to the circuit.
There are three basic types of SFCL configurations. While there are many possible configurations of superconducting fault current limiters, the two most popular ones from the industrial point of view are resistive and inductive.

### 3.3.1 Resistive SFCL

A resistive SFCL is connected to a circuit and when a fault occurs it introduces extra resistance, thus limiting the current and allowing the use of simpler circuits breakers if necessary (Figure 3.4). This type of fault current limiter relies on the quench of the superconductor which is usually in the circuit. Therefore when the fault occurs it is the superconductor as the element which can be expected to heat up rapidly. If this process is not stopped it leads to the destruction of the material; therefore usually there is a shunt resistor and some type of a fast switch that disconnects the superconductor. There are other designs of such a resistive SFCL where the superconductor (YBCO) is deposited on a special substrate and forms an elaborate pattern of paths. This arrangement helps dissipate heat more easily. Also, by appropriate structuring of the paths, the inductance can be adjusted. The main problem with this design is with the recooling of the device after quench. Usually during fault the temperature rise is quite significant and the time necessary for this type of device to recover to its operating condition is longer than few seconds.

![Resistive SFCL Operation Principle](image)
3.3.2 Inductive SFCL

There are a number of possible arrangements of inductive SFCL that can work. One such arrangements is based on two coaxially arranged coils which can produce opposing fluxes, therefore effectively introducing very small impedance into the circuit. One of them is powered from the circuit itself and the other is short circuited through a HTS. When a fault occurs the superconductor quenches introducing additional resistance in the secondary circuit of this air cored transformer, hence disrupting the balance of fluxes and increasing the inductance of the device [62].

A different configuration of this type might include just two coils, out of which one is superconducting and the other is a normal conductor.

In a similar type of design copper wires are placed around the yoke and between them a superconducting shield is inserted. In normal operation the yoke is screened from the magnetic field produced by the copper wire but, once the current exceeds critical value, the superconductor becomes resistive and no longer screens the yoke, hence introducing a much higher inductance into the circuit (Figure 3.5) [77]. A different alternative to the two described above is an inductive fault current limiter with a saturated core. In this configuration the superconducting DC coil is driving

![Figure 3.5: Principle configuration of the shielded inductive type SFCL](image)
the flux density in the transformer limbs, to which AC windings are connected, deep into saturated state. During the fault, in one half cycle the core is driven out of saturated state by high fault current and effectively introducing a much higher inductance into the circuit when compared to the normal operating state [27].

3.4 Superconducting Transformers

As one of the most common components of an electric system, a transformer was an obvious choice to be attempted as a superconducting device. While using low temperature superconductors had proved uneconomical because of huge refrigeration costs, HTS materials allow building machines economically competing with conventional transformers especially when other advantages are considered. Among these some are especially worth mentioning, like reduced size and weight of the device, lower environmental impact and better operation under overload. While the first one is obvious, the second one arises mainly because - in contrast to conventional transformers - superconducting devices use liquid nitrogen for cooling and therefore, in case of a leak, no harm is done since this gas exists in earth’s atmosphere. The last advantage comes from the fact that HTS windings might be designed to operate under overload conditions and, because of that, the increase in temperature does not have a significant impact on the operation of the transformer, except for some increased load on the cooling system. In a conventional transformer, an overload leads to the increase of temperature in the winding and therefore can cause insulation damage. To further improve the performance of superconducting transformers only the winding is kept in cryogenic temperatures, which helps reduce the cost of removing losses from the core.

Research is also being done in developing air-cored transformers which can offer much lower mass and can act as a shunt reactor correcting the reactive power developed by HV power lines [76].

There are various projects around the world aimed at building commercially successful superconducting transformers and very good results have been achieved. A group in Japan successfully built and tested a 66 kV/6.9 kV 2 MVA transformer [16].

In Korea there are two competing ideas for designing 100MVA 3 phase superconducting transformer. One is trying to build a 5MVA single phase transformer which would be a preliminary device before attempting the design of a 33MVA
single phase (100MVA) device which is supposed to replace, the current 60MVA (3 phase) conventional transformers while retaining the same size [29]. The other proposes a common magnetic core configuration for building 100MVA machine [21].

It also worth mentioning that the University of Southampton itself was the first to build and test a 10kVA HTS demonstrator transformer [69].

3.5 Superconducting Motors

Superconducting machines are particularly interesting for applications in environments where size and efficiency are of special importance, such as marine vessels and airplanes. Many companies around the world have been conducting research in that field for a number of years with good results. Although synchronous machines were the simplest choice there are attempts to build induction superconducting motors with a superconducting squirrel cage [49].

American Superconductor has been conducting research in this field and is now capable of delivering ship propulsion motors in, the power range between 5MW to 36.5MW and has recently successfully tested the latter [7]. These motors are up to 2/3 smaller then classical machines and at the same time deliver greater power density and higher efficiency which can reach 98%. It is worth mentioning that although this is usually only 0.1% to 1% better then for ‘copper‘ machines, even such a small amount gives enormous savings when the motor life span is taken into consideration [59]. Another advantage of superconducting motors is higher reliability, that is due to the constant temperature at which the rotor winding is operated and the fact that the constant current adjustments, which cause winding fatigue in traditional AC machines, are not necessary [6].

Kalsi [34] mentions other benefits of utilising superconducting machines:

- Improved reactive power (VAR) in under- and over-excited conditions,
- Small harmonics in the terminal voltage,
- Higher efficiency in virtually all operating conditions,
- Noise and vibrations smaller compared to conventional machines.
Figure 3.6 shows a typical superconducting motor configuration that consists of a cryogenically cooled rotor and a stator that is kept at room temperature; however, recently other configurations have been considered, designed and tested \cite{30, 52, 68}.

There are two types of stator configurations that are the most popular in motor design; one is a conventional stator arrangement with magnetic teeth for low reluctance flux path. This construction allows retrofitting a conventional machine with a superconducting rotor but, to limit the influence of the stator teeth on the rotor, the air gap usually needs to be increased. In the air gap of the superconducting motor the flux density in the air-gap can reach 1.5 - 2T, which usually leads to saturation of the stator steel. The other configuration which is also considered for building superconducting machines is one that assumes axial air-gap configuration instead of more conventional radial setup \cite{52}. These configurations can be implemented through high performance air-cored stator windings. There are no teeth and the main purpose of the steel is to enclose the magnetic field within the machine and provide flux path. This design also leads to a more compact device.
3.6 Superconducting Generators

Increasing demand for electrical energy favours building more powerful generators. Superconductivity, with its constantly increasing commercial competitiveness, is a technology that gives the most promising results. The advantages are similar to the ones mentioned before; among them, the most important are smaller size, higher efficiency and higher power output. These advantages encourage also different applications like ships or aircraft power generation [65].

The most interesting and probably the biggest project is the Japanese Super-GM started in 1988 with a goal to establish the technology and estimate feasibility of building a 200MW generator. The project ended in 1999 with full success. During the whole time three rotor designs were tested, all operating above expected values. The project also allowed the designers to evaluate the efficiency of a 200MW generator, which is expected to be 0.5% higher then for a conventional machine [28].

The current project at the University of Southampton is a direct descendant of the earlier project [8] which proved that building a superconducting generator operating at temperatures between 73-77K was possible. The aim of the current project is to build a rotor without an iron core that will reduce its weight significantly and will increase the competitiveness of this technology against conventional machines. As with motors, the smaller size, weight and higher torque are of particular importance in environments where there is very limited space, or when weight is important, e.g. in aircrafts, off shore wind turbines or ship propulsion.

3.7 Construction and principles of operation of synchronous machines

Basic laws of electromagnetism describe how converting mechanical energy into electrical energy is possible. The phenomenon quantified by Faraday’s law is the principle of operation of rotating machines. It says that the electromotive force is induced in the conductor either if it is moved through the magnetic field or if the magnetic field encompassed by this conductor is changing. This principle can be used to convert mechanical energy into electricity. In a synchronous machine such conditions are generated by two windings. The winding responsible for generating
the magnetic field is usually called the field winding whereas the armature winding is where the EMF is induced. Although either winding of an AC machine can be stationary, the field winding is usually on the rotor driven by external torque and the armature winding is placed on a stationary stator, since this reduces the amount of power that must be transferred to or from the rotor. The rotor is powered by a DC current and, depending on the method of transferring it into the machine, the generator is considered to be brushed or brushless. There are also other classification schemes for AC generators which divide them into salient pole and cylindrical (non-salient); these are shown in Figure 3.7.

They can also be characterised as single or poly-phase. The former are usually small systems generating power at a specific utilisation voltage while the latter generate alternating voltages in two, three or six phase configurations while the most common are three phase alternators. All synchronous AC generators need a separate DC source to power the field winding. A separate generator, called the exciter is often used for this. The torque necessary to drive it is usually taken from the main machine by drive-belt or it is installed directly on the main rotor shaft.
3.8 Advantages of a superconducting generators

Superconducting synchronous electrical generators have been under research around the world since 1970s. All the programs are aimed at evaluating economic and technical feasibility of building a machine that can operate in industrial conditions. The programs demonstrated that it was possible to build superconducting machines with power output range from single watts to hundreds of megawatts. Superconducting machines can convert mechanical energy into electricity more efficiently. Usually the overall improvement in efficiency is around 0.5\% and although this might seem not to be much, it could mean a 50\% reduction in generator’s losses. Smaller size and weight, the possibility of generating more volt-ampere-reactive (VAR) power, increasing grid stability and reducing capital and operating costs are the four ways by which superconducting power generators might improve electric power systems.

In addition to these advantages, superconducting generators have the possibility of eliminating step-up transformers by directly generating a high voltage. Currently the only classical design allowing producing high voltage directly is one produced by ABB and called Powerformer \([39]\). In their approach classical rectangular armature winding is replaced with high voltage cables with XLPE (Cross linked polyethylene) insulation. This evens the electrical field distribution around stator conductors effectively reducing any stresses on the insulation that would otherwise occur. This allows increasing the number of turns in the stator winding, hence removes the need for the step-up transformer. In case of a superconducting generator the increase may be even bigger because by removing the stator teeth we can utilise the fact that the superconducting winding can produce much stronger fields (above 2T) compared to 1T for an ordinary machine.

The power density of a generator is roughly proportional to the product of flux density (magnetic loading) produced by field winding and field strength produced by armature windings (electric loading), hence the superconducting generator of the same power output as a conventional might be significantly smaller. A sample comparison between two types of 1200MVA generators is presented in Table \([3.1]\). All these advantages, especially the smaller size and weight, suggest the possibility of upgrading power plants with new machines which at the same size will generate more power. Superconducting technology also pushes the limit for building big generators beyond today’s 1500MVA, the value which is currently mainly limited by transportation constraints, and problems with supporting and balancing the huge rotor mass.
Table 3.1: Potential superconducting 1200 MVA generator parameters compared with a conventional machine

<table>
<thead>
<tr>
<th>Property</th>
<th>Superconducting Generator</th>
<th>Conventional Generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase-to-phase voltage(kV)</td>
<td>26-500</td>
<td>26</td>
</tr>
<tr>
<td>Line current (kA)</td>
<td>27-1.4</td>
<td>27</td>
</tr>
<tr>
<td>Active length (m)</td>
<td>2.5-3.5</td>
<td>6-7</td>
</tr>
<tr>
<td>Total length (m)</td>
<td>10-12</td>
<td>17-20</td>
</tr>
<tr>
<td>Stator outer diameter(m)</td>
<td>2.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Rotor diameter(m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rotor length (m)</td>
<td>4</td>
<td>8-10</td>
</tr>
<tr>
<td>Field exciter power(kW)</td>
<td>6</td>
<td>5000</td>
</tr>
<tr>
<td>Generator weight (kg)</td>
<td>1600000-300000</td>
<td>6000000-700000</td>
</tr>
<tr>
<td>Total losses (MW)</td>
<td>5-7</td>
<td>10-15</td>
</tr>
<tr>
<td>Flux density(T)</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

### 3.9 Available superconducting machine designs

One of the easiest methods of building superconducting machines is to retrofit a conventional stator with a superconducting rotor. The advantage is that this eliminates the need to design the stator, thus eliminating the associated risk. This approach also utilises an iron core in the rotor which provides a good torque transmission. Using as many existing elements as possible in building a new machine is also the cheapest method to demonstrate technical feasibility of building such a device.

The previous project conducted at the University of Southampton [9] utilised that idea and achieved very promising results.

The other approach is called the air-core or coreless design in which a lot of magnetic and structural steel, mainly in the rotor, is eliminated, resulting in a more compact and lighter machine. This advantage can be utilised in environments imposing weight or space restrictions, like ships and aircraft. The concept of an air-core design is being used in the current project at the University of Southampton.

The idea can also be implemented to the stator winding. A HTS winding permits higher air gap fields which would saturate the stator iron of a conventional machine. Proper design of such a stator may lead to very compact designs much smaller compared to a standard machine. This idea has been used by American Superconductor and ALSTOM in their 5MW ship propulsion motor [6].

When building a superconducting rotor there are various issues that need to be addressed. One of them is an AC loss coming from the asynchronous fields
and torque pulsations. That is why usually the rotor is surrounded by an electromagnetic shield that also serves as a vacuum enclosure. Vacuum is what prevents the heat leaks from the superconducting winding. Furthermore, radiation shields are used in order to minimise the leak even further. Thermal shielding is very important, especially when the cost of refrigeration is taken into account. An additional way of reducing the thermal loss is by designing a proper connection of the rotor with the external world. Efficient designs of torque tubes (parts that transfer torque) can decrease the heat loss significantly.
Chapter 4

Machine modelling and optimisation

4.1 CAD software

Computer Aided Design (CAD) software has become a standard tool for designing various types of machines and devices. One of the unquestionable advantages of virtual prototyping is the ease of searching for the optimal solution. Creating and modelling a machine in computer software saves a lot of money and time, allowing analysis of the behaviour of a number of devices within the range of possible configurations. Currently available software packages are relatively easy to use. The constant improvement in computational power lead to the possibility of simulating full 3D models of actual machines. The designer however has to have a very good understanding of the process and the physics involved to understand and analyse the results critically because these sometimes might not be accurate enough. Flux [1] and Opera [4] are only two of many available software packages in which simulations of electrical machines can be done and they are successfully used by the industry [3]. The CAD system is usually composed of three elements: pre-processor, solver and post-processor (Figure 4.1), all sharing one database. The pre-processor and post-processor provide interfaces for the designer, whereas the solver does not require any such interaction. In the pre-processor, the geometry, loading and material properties are defined. The pre-processor also discretizes the model by defining a mesh, although the mesh may subsequently be refined by the solver to reduce the predicted errors to some specified value. The mesh might be built automatically by the software. The post-processor allows analysing the
Solving electromagnetic problems requires obtaining solutions to Maxwell equations for a certain set of boundary conditions. Analytical methods allow solving only simple problems and so usually require many simplifications assumed, which prevent them from being very accurate. Because analytical solutions use so-called design rules, which are supposed to help minimise the error but are based on empirical observations, they are not good when novel types of machines are being designed. In such a case an appropriate choice of numerical technique can give very accurate answer and allows for easier study of many possible variants. The most popular numerical techniques nowadays are: finite difference method (FDM) [66], finite integration technique (FIT), which is a generalised form of FD, method of moments (MOM) [24], finite-element method (FEM) and the boundary element method.

Figure 4.1: Flow chart for components of CAD system and heuristic design results of the simulation.
method (BEM) \[15\]. FDM and MOM yield simple algorithms that are easy to implement and are good in a number of applications, but are more problematic when dealing effectively with complex geometries and nonlinear materials. This type of problems is a domain where the FEM is the most effective, although the implementation of this technique is not as straightforward as the other two. BEM tries to combine the advantages of MOM and FEM but is not free from its own limitations. Therefore FEM is the most commonly used method for analysing electric machines because nonlinearities and geometry complexity is a problem for other methods and cannot be simplified because it has a significant influence on the solution.

### 4.1.1 Pre-preprocessor

The pre-processor is the input interface for the designer. The geometry of the problem, boundary conditions and the material data are defined during this step. Programs are usually equipped with a library of basic shapes and material descriptions which the designer can use for building the model of the machine. For more complicated geometries it is possible to use Boolean operations on the shapes provided by the software. The defined geometry can be divided into three classes of regions: materials, conductors and free space. The software can provide automatic procedures for testing the correctness of the geometry. The pre-processor may also allow the designer to define external circuits that are connected to the model. In most of the packages it is possible to impose movement on some regions of the model.

Usually the mesh is also generated by the pre-processor. Typically it is built automatically and the role of the designer is restricted to defining constraints on the mesh size in all the regions of the model. For meshing two dimensional regions the most common shapes used are triangles and quadrilaterals, while for a three dimensional mesh tetrahedra or hexahedra are used. To obtain the best solution the mesh should be denser in regions of most rapid field variation. The mesh transition between regions of different mesh size should be as smooth as possible. The Delaunay triangulation is today’s most popular algorithm for providing a mesh with appropriate parameters.
4.1.2 Solver

The solver is the most important part of the CAD software. At the same time it is the least visible one. It does not involve the designer in any way. The number of equations is related to the number of nodes in the discretized model and for most of the modern engineering problems it entails solving a large number of simultaneous equations. One of the concepts also available for solving a problem might be using an adaptive solver. This type of solver tries to estimate the error after a solution has been obtained and if the termination condition is not met the mesh is automatically refined and the problem is solved again, as many times as necessary for obtaining satisfactory accuracy. When the whole process is complete the results are stored in the database.

4.1.3 Post-processor

The Post-processor is a graphical interface to the obtained solution. It interprets the values stored in the database to physical quantities such as flux density, field strength or forces. It allows to visualise them and perform some mathematical operations on the date. The range of possible operations depends on the package.

4.2 The finite element method

Finite Element analysis was first introduced into electrical engineering by Winslow [73] in the form of discreet irregular grid of triangles. But it was Silvester [20] who in 1970 introduced a method that could be applied to rotating machines. Since then this method of designing electrical machines has been constantly progressing and now is used as one of the primary tools. In those 30 years the method has been successfully applied to the analysis of various problems in electrostatic, magnetostatic and time varying fields. FEM is used in analysing high-frequency problems, electromagnetic scattering, wave propagation phenomena and superconductor quenching.

The simulation software allows defining electric, magnetic and geometric properties of each part of the model independently. The best practise is to divide regions with complex geometry into smaller cells. This helps the meshing algorithm build a better mesh. Moreover the regions where big variations of electromagnetic...
field are expected should be meshed finer than those were a relatively uniform field is expected. It has to be noted that a higher number of elements increases the solution time of the model therefore a proper choice of mesh sizes in different parts of the model may help solve it more efficiently.

The idea behind the finite element method is to divide the modelled space into smaller sub-domains called elements. These may be, for example, triangles for 2 dimensional discretization or tetrahedra for 3 dimensional models. Then the unknown variable (typically a potential) is approximated by low order piecewise functions usually polynomials. Continuity is imposed on the value of the potential in adjacent elements at their shared nodes. The aim of the solver is to obtain the values of the unknown variables in each node that will minimise the error in satisfying the differential equations. Improved speed and accuracy can also be obtained by using edges or facets instead of nodes.

### 4.3 Types of models

Most of the Finite Element packages nowadays contain two principal modules 2D and 3D. Because 2D models have little or no capability to take into account the three dimensional (vector) nature of electromagnetic field, it is usually necessary to make substantial simplifications to the geometry before building the model. Consequently, their accuracy is limited when the analysed problem has many three dimensional features influencing the magnetic field.

In the case of standard 2D models of electrical machines the end winding regions are not taken into account, hence there is some loss of precision. It is possible to improve the accuracy of such models by connecting them to external circuit with the additional end winding inductance added. The value of this inductance is obtained from appropriate formulae which were empirically validated for a number of machine configurations. Obviously this is not practical when a novel design is considered since obtaining an appropriate formula would require 3D modelling or test results for a substantial number of machine design variants.

However, the most important advantage of using 2D models is their much smaller computational cost. This fact was used to obtain models of the machine and optimise its output waveform as much as it was possible using this simplified representation. The obtained shapes were then used as a good starting point for 3D models which are much more complicated. In some cases due to software
limitations it was not possible to use three dimensional models.

4.4 Analysis modules used

The finite element package used contains different types of solvers; steady state AC, demagnetisation, linear motion, rotating machines, stress analysis, space charge beam analysis, static field analysis, transient analysis, thermal analysis and velocity analysis.

In principle by using appropriate set of solvers a complete solution could be obtained. However, this did not prove possible for the machine considered.

Although the accuracy cannot be considered adequate in all of the cases, due to the inability to build the 3D models for all the cases requiring consideration, some inaccuracy had to be accepted. 2D non-linear models were used at the preliminary stage of building and optimising the machine mainly because of their small computational cost. The static solver was used to obtain the estimate of the basic parameters like air-gap flux density, and flux density distribution in the coils. The other solver, used to estimate the losses in different parts of the machine, was a rotating machine solver which is a transient solver extended to include the effects of a rigid body (rotating motion).

Figure 4.2: Grading of the mesh used in 2D models
When building a model, it is important to increase the mesh density in the area where the variation is expected to be the highest; in the case of the superconducting generator these are the areas surrounding the superconducting coils and in the airgap. An example mesh is shown in Figure 4.2.

In order to accommodate the revolving motion the rotating machine models required one additional region to be defined. This is a thin region in the machine’s air-gap which is re-meshed in each time-step. Ideally, to improve the accuracy of the solution, the user is advised to put two air regions on both sides of this region.

The 2D models analysed are usually full load models and due to the symmetry only half of the machine needs to be modelled with the flux lines at the polar axis taken as \((A \text{ at } 180^\circ) = (A \text{ at } 0^\circ)\). In a no load case this could be simplified further to a quarter of the model; however, the interaction of the rotor and armature fields makes the field distribution unsymmetrical therefore the whole pole pitch needs to be modelled.

### 4.5 3D models

The analysis of 3D electromagnetic problems is much more complicated and time consuming than 2D. The reason for this lies with not only the threefold increase in the number of unknowns but also a big increase in the number of nodes in the mesh. The increase in the number of unknowns is caused by taking into account the vector nature of the problems, therefore at each point there are now three variables instead of two or one. However, the 3D analysis allows building much more complicated models where nearly real life representations of the actual machine components are included, like real shape of nuts or arced field winding conductors as well as the end regions of the machine. Therefore, whenever it was possible full 3D models were built.

The need for 3D models arises because unlike the cored machines, the type of a coreless rotor considered has additional flux paths (around the ends of the diverter rings) which are believed to have a significant influence on the results, whereas in the cored designs these play a much lesser role. The use of 3D modelling provides a way to investigate the effects of these important features of the machine.

In principle the package that was used should allow building 3D rotating machine models with circuit windings. This type of winding is represented in the software as a set of cells (the number depends on the mesh size/filament number...
defined by the designer) current filaments and is placed in regions of total (Vector) potential. In regions of total vector potential, flux linkages can be calculated from line integrals of the Magnetic Vector Potential (A). Moreover, the voltage waveform induced in this type of winding can be obtained from the circuit results without any need for integrating the fields predicted by the model. Using models with circuit winding would greatly increase the chance to accurately predict the output waveform and parameters of the machine. However, due to the high complexity of the stator winding, it proved impossible for the program to mesh and therefore the approach had to be rejected. Instead the stator winding is modelled as a set of current source conductors which are defined independently of the mesh, so they do not complicate the meshing process.

The 3D models were solved using only static and steady state AC solvers, mainly due to much lower computational cost when compared with transient solvers. This type of a solver, however, requires the coils to be placed in a reduced scalar potential region. In the model $H$ is considered to be the sum of two components: the field of sources $H_s$ and the field of induced magnetisation $H_m$, where $H_m$ can be represented as, a gradient of, the scalar magnetic potential and $H_s$ is calculated using Biot-Savart’s law. Such division is used mainly to simplify the calculation and decrease computational effort. This is to avoid the potential becoming multi-valued. But due to lack of description how this is defined in the software it is not possible to use $\int A \cdot dl$ integration to obtain flux linkages of the coils. Instead, the flux linkages of the stator coils were calculated by integrating the normal component of $B$ over a series of quadrilateral patches, the edges of which define 6 current paths uniformly distributed over the cross section of each 3-turn coil.

A 3D model is shown in Figure 4.3, for reasons mentioned previously only a quarter of the machine needs to be modelled. In this figure half of the background region and half of the end winding region is hidden in order to better show the complexity of the model.
Figure 4.3: Meshed 3D model of the generator (note that $\frac{1}{2}$ of the background and end winding regions are hidden)

Table 4.1: Specification of the stator

<table>
<thead>
<tr>
<th>Stator Details</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated output, kVA</td>
<td>100</td>
</tr>
<tr>
<td>Power Rating, W</td>
<td>100</td>
</tr>
<tr>
<td>Rated current, A</td>
<td>193</td>
</tr>
<tr>
<td>Rated voltage, V</td>
<td>415</td>
</tr>
<tr>
<td>Bore, m</td>
<td>0.33</td>
</tr>
<tr>
<td>Diameter, m</td>
<td>0.54</td>
</tr>
<tr>
<td>Iron core length, m</td>
<td>0.325</td>
</tr>
<tr>
<td>Overall length, m</td>
<td>0.795</td>
</tr>
<tr>
<td>Number of turns in each coil</td>
<td>3</td>
</tr>
<tr>
<td>Number of poles</td>
<td>2</td>
</tr>
<tr>
<td>Number of slots</td>
<td>48</td>
</tr>
<tr>
<td>Slot size, mm</td>
<td>40x10.5 with 3x2 mm groove in sides and bottom</td>
</tr>
<tr>
<td>Winding Connection</td>
<td>Star, parallel circuits</td>
</tr>
<tr>
<td>Coils pitch</td>
<td>14/24</td>
</tr>
<tr>
<td>Working temperature</td>
<td>25 50oC</td>
</tr>
<tr>
<td>Cooling method</td>
<td>Air</td>
</tr>
</tbody>
</table>

4.6 Existing stator configuration

Like in the previous project, mentioned before, the new design will utilise a stator from a conventional 100kVA machine. It has a 330 mm bore diameter and 325 mm iron length, three phases and two poles. It has 48 three-turn coils that are fitted into the 48 slots and arranged to form a two layer winding. The short pitching of the winding (14/24 pitch coils) reduces the length of the coil-end connections.
The full configuration of the stator is shown in Table 4.1 and winding arrangement in Figure 4.4.

![Figure 4.4: The stator winding configuration](image)

### 4.7 First rotor design

2D modelling was used for all preliminary evaluations of possible designs of the generator built for this project. Although not very accurate (for the type of machine considered), 2D models have the advantage of a low computational cost and therefore allow for faster evaluation of a higher number of possible designs. The most promising designs, in terms of harmonic content, were then chosen and used as starting points for 3D analysis.

The shape of the first analysed model was inspired by the results obtained at the last stage of the previous project (Figure 4.5) [64]. It assumed a 20 coil rotor configuration, with the top-most coil having only 50 turns (whereas there were 60 in all the other coils). The middle six coils were assumed identical, mainly to simplify the construction. The 150A rated superconducting tape was to be used
in the construction of the new machine. The advancements in superconducting materials had been greater than anticipated and the tape actually used in the current project is rated at 180A. Each of the coils was placed between 9% nickel steel rings later called flux diverters. The idea of using these rings comes from the superconducting transformer project run at the University of Southampton [70]. The purpose of these is to redirect the perpendicular magnetic field around the coil. Each of them has an overhang of at least 3mm to lessen the effect of fringing fluxes on the perpendicular field.

The different tolerance of the tapes to magnetic fields in the direction parallel and perpendicular to the broad face of the tape is the reason for which these two components of the magnetic field have to within certain limits to achieve the desired current density. Those limits for the material used, and assumed current of 150A in the coils are 1.5T in parallel direction and 0.13T in perpendicular direction.

Even with the increased current density in the coils this model gave promising results. The magnetic field distribution in the tape is shown in Figure 4.6.

**Figure 4.5:** Preliminary design of the HTS rotor
Regrettably, this design had to be abandoned mainly due to the fact that the available tape required a higher bend radius on the inner side of the top-most coil. It had to be increased from the 18.1 mm as assumed in this model to the minimum of 28 mm. This required 20 turns to be removed from this coil, which decreased slightly the expected air-gap flux density and increased the harmonic content. It was then necessary to improve the waveform and since modification to the most top diverter was the simplest method the changes were concentrated at that region. Figure 4.7 shows these changes. The increased width of the top diverter then allowed the number of turns in the top-most coil to be increased to 50. This allowed the fundamental air-gap flux density component to reach 0.59T which is marginally higher than the estimation for the first described design (0.58T).
Figure 4.8: The magnetic field in the superconducting coils under load a) perpendicular component b) parallel component

It was then decided that it is possible to reduce the mass of the top-most diverter by removing the inner part of it. Figure 4.8 shows the distribution of the perpendicular and parallel components of the magnetic field after these modifications. It was found, however, that the main constraints were not met; hence it was necessary to address this problem before proceeding further. The perpendicular component in this configuration reached 0.144T which is much higher than acceptable 0.13T and the parallel component of the field reached 1.62T which is also much higher than the limiting 1.5T. In order to solve this problem the thickness of the second top-most diverter was reduced by 1 mm and the coil was moved 0.5 mm upwards to make space for moving the third diverter 0.5 mm up. Now, instead of 0.5 mm gaps, the gap above the three top-most coils became 1 mm. It was also necessary to decrease the inner diameter of the third top most diverter from 37 mm to 29.5 mm. The resulting dimensions are shown in Figure 4.9. These changes allowed the perpendicular component of the magnetic field to fall below the limit by symmetrising the field in the area around the coils. The increased distance helped mostly by moving the highly saturated diverters away from the coils. The field distribution after these changes is presented in Figure 4.9.
Superconducting machines with large air-gaps usually have a low synchronous reactance and consequently small voltage harmonics can drive significant currents in external circuits. That is why the above modelling has been aimed both at optimising the working conditions of the HTS tape and at the same time trying to minimise the harmonic content.

In order to analyse the effects of changes air-gap flux density was extracted from the model and then split into its harmonic components by applying Fourier analysis. Equations 4.1 and 4.2 are used to obtain $\alpha$ and $\beta$ coefficients of the series which, when fitted, allow reconstructing the voltage waveform. Both coefficients are required since most of the models analysed are of the machine on load so the symmetry of the field is reduced.

$$\beta = \frac{2}{\pi} \int_{0}^{\pi} B_r(\theta) \sin(n\theta)$$  \hspace{1cm} (4.1)$$

$$\alpha = \frac{2}{\pi} \int_{0}^{\pi} B_r(\theta) \cos(n\theta)$$  \hspace{1cm} (4.2)$$

where $n$ is the harmonic order, $\theta$ is the angle from the point on the circle to the local $x$ axis and $B_r$ is the radial flux density given by

$$B_r = -(B_x \cdot N_x - B_y \cdot N_y)$$  \hspace{1cm} (4.3)$$

where $B_x$ and $B_y$ are the components of the flux density and $N_x$ and $N_y$ are the
components of the normal vector to the circle at the considered point.

To obtain the harmonic contributions to the voltage waveform, the winding factors have to be included. These reflect the reduction in voltage caused by the short pitch of the coils and their distribution across the phase belts. The winding factor is the product of the coil pitch factor,

\[ K_{pn} = \sin \frac{n\pi \beta}{s} \]  

(4.4)

and the distribution factor,

\[ K_d = \frac{\sin(n\alpha S_{pm})}{S_{pm}(\sin \left( \frac{n\alpha}{2} \right))} \]  

(4.5)

where \( \beta \) is the coil pitch, \( s \) is the number of slots, \( S_{pm} \) is the number of slots occupied by a phase per pole, \( n \) is the harmonic order and \( \alpha \) is the slot pitch. The coil pitch factor is the ratio of the voltage induced in a full pitched coil (spanning over the full width of the pole) to the voltage induced in a coil whose span is shorter.

The distribution factor is the ratio of the voltage induced in the winding to the \( n \)-times voltage induced in one coil. The difference comes from the fact that coils occupy different spatial locations therefore the voltage induced in each of them is different.

The easiest way to compare alternative configurations of the rotor is by comparing the total resultant \( \text{rms} \) harmonic voltage (distortion factor \( FD \)) which is defined as the ratio of the square root of the \( \text{rms} \) amplitudes of all frequency components except the fundamental (\( \sum E_n^2 \)) to the \( \text{rms} \) value of the complete wave (including fundamental \( E_{\text{rms}} \)).

\[ F_d = \frac{\sqrt{\sum E_n^2}}{E_{\text{rms}}} \]  

(4.6)

Usually it is not necessary to analyse very high frequencies because in most cases the amplitude of harmonics decreases as the order of the harmonic increases. However, if the waveform shape indicates that there might be higher order harmonics, it is necessary to include these in the evaluations. The distortion factors for the models described above are presented in Table 4.5. It can be noticed from the table above that the distortion factor (overall content of harmonics) has been successfully reduced. Table 4.6 presents the sine and cosine coefficients of harmonics 1-50, while the harmonic content of the voltage waveform for the final shape is
Table 4.2: Distortion factors for different shapes of the top-most diverter

<table>
<thead>
<tr>
<th>Shape</th>
<th>Figure 4.5</th>
<th>Figure 4.5 + smaller top coil</th>
<th>Figure 4.7</th>
<th>Figure 4.9</th>
<th>Figure 4.7 + bigger top coil</th>
<th>Figure 4.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distortion Factor [%]</td>
<td>1.69</td>
<td>2.69</td>
<td>1.69</td>
<td>1.54</td>
<td>1.51</td>
<td>1.47</td>
</tr>
</tbody>
</table>

listed in Table 4.3. It has to be mentioned that the harmonic content of the voltage waveform is for the phase voltage, hence the existence of 3rd harmonic, which will not be present if the output circuit of the generator is star connected. The waveform is shown in Figure 4.10. Further, Figure 4.11 compares harmonics of the preliminary shape with the reduced coil size to the final shape obtained by 2D modelling.

Table 4.3: Harmonic components of the air-gap flux density

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Cosine Term</th>
<th>Sine Term</th>
<th>Harmonic Order</th>
<th>Cosine Term</th>
<th>Sine Term</th>
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<tr>
<td>1</td>
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<td>0.587901</td>
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<td>-1.37E-16</td>
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<td>2</td>
<td>1.22E-16</td>
<td>-1.62E-17</td>
<td>27</td>
<td>1.93E-05</td>
<td>-1.02E-04</td>
</tr>
<tr>
<td>3</td>
<td>-0.024753</td>
<td>2.53E-03</td>
<td>28</td>
<td>-2.04E-07</td>
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<td>5</td>
<td>8.42E-03</td>
<td>2.26E-03</td>
<td>30</td>
<td>4.14E-16</td>
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<td>24</td>
<td>3.21E-16</td>
<td>-9.86E-17</td>
<td>49</td>
<td>-5.50E-03</td>
<td>0.03047</td>
</tr>
<tr>
<td>25</td>
<td>-2.55E-05</td>
<td>3.16E-04</td>
<td>50</td>
<td>-1.65E-16</td>
<td>-2.76E-16</td>
</tr>
</tbody>
</table>
Table 4.4: Harmonic components of the stator voltage (FD = 1.4754%)

<table>
<thead>
<tr>
<th>Space Harmonic Order</th>
<th>Cosine Harmonic Magnitude</th>
<th>Sine Harmonic Magnitude</th>
<th>Winding Factor</th>
<th>Actual Harmonic</th>
<th>% Harmonic Voltage Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0774358</td>
<td>0.58790166</td>
<td>0.758138</td>
<td>0.44956</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>-0.0247536</td>
<td>0.00253</td>
<td>0.245196</td>
<td>0.006101</td>
<td>1.36</td>
</tr>
<tr>
<td>5</td>
<td>0.00842</td>
<td>0.00226</td>
<td>0.192777</td>
<td>0.001681</td>
<td>0.37</td>
</tr>
<tr>
<td>7</td>
<td>-0.00215</td>
<td>0.00565</td>
<td>0.018445</td>
<td>0.000112</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>-0.00209</td>
<td>-0.0114936</td>
<td>0.207867</td>
<td>0.002428</td>
<td>0.54</td>
</tr>
<tr>
<td>11</td>
<td>0.00115</td>
<td>0.000166</td>
<td>0.057705</td>
<td>6.70E-05</td>
<td>0.01</td>
</tr>
<tr>
<td>13</td>
<td>-0.00048</td>
<td>0.00211</td>
<td>0.050661</td>
<td>0.00011</td>
<td>0.02</td>
</tr>
<tr>
<td>15</td>
<td>-0.000259</td>
<td>-0.00319</td>
<td>0.138893</td>
<td>0.000445</td>
<td>0.1</td>
</tr>
<tr>
<td>17</td>
<td>0.000269</td>
<td>-0.000148</td>
<td>0.009096</td>
<td>2.79E-06</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>-0.000136</td>
<td>0.00082</td>
<td>0.065438</td>
<td>5.44E-05</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figure 4.10: Air-gap flux density waveform of the final 2D design rotor
Table 4.5: Distortion factors for different shapes of the top-most diverter

<table>
<thead>
<tr>
<th>Shape</th>
<th>Figure 4.5</th>
<th>Figure 4.5 + smaller top coil</th>
<th>Figure 4.7</th>
<th>Figure 4.9</th>
<th>Figure 4.7 + bigger top coil</th>
<th>Figure 4.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distortion Factor [%]</td>
<td>1.69</td>
<td>2.69</td>
<td>1.69</td>
<td>1.54</td>
<td>1.51</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Figure 4.11: Harmonic order of the preliminary design (with the coil size reduced) and the final 2D designs

\[
F_d = \sqrt{\sum \frac{E_n^2}{E_{rms}}} \tag{4.7}
\]

Usually it is not necessary to analyse very high frequencies because in most cases the amplitude of harmonics decreases as the order of the harmonic increases. However, if the waveform shape indicates that there might be higher order harmonics, it is necessary to include these in the evaluations. The distortion factors for the models described above are presented in Table 4.5.

It can be noticed from the table above that the distortion factor (overall content of harmonics) has been successfully reduced. Table 4.6 presents the sine and cosine coefficients of harmonics 1-50, while the harmonic content of the voltage waveform for the final shape is listed in Table 4.4. It should be noted that results are presented for a phase voltage, hence the presence of the 3\textsuperscript{rd} and triplen
harmonics. These harmonics are included to illustrate better the performance of the optimisation method used later in this chapter, but due to the cancellation effect - their practical significance would obviously disappear if a star connected three-phase winding were used.

The waveform is shown in Figure 4.10. Further, Figure 4.11 compares harmonics of the preliminary shape with the reduced coil size to the final shape obtained by 2D modelling.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Cosine Term</th>
<th>Sine Term</th>
<th>Harmonic Order</th>
<th>Cosine Term</th>
<th>Sine Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.077435</td>
<td>0.587901</td>
<td>26</td>
<td>-1.37E-16</td>
<td>1.71E-17</td>
</tr>
<tr>
<td>2</td>
<td>1.22E-16</td>
<td>-1.62E-17</td>
<td>27</td>
<td>1.93E-05</td>
<td>-1.02E-04</td>
</tr>
<tr>
<td>3</td>
<td>-0.024753</td>
<td>2.53E-03</td>
<td>28</td>
<td>-2.04E-17</td>
<td>-4.92E-17</td>
</tr>
<tr>
<td>4</td>
<td>1.93E-17</td>
<td>-4.24E-17</td>
<td>29</td>
<td>2.23E-06</td>
<td>-5.05E-05</td>
</tr>
<tr>
<td>5</td>
<td>8.42E-03</td>
<td>2.26E-03</td>
<td>30</td>
<td>4.14E-16</td>
<td>6.84E-16</td>
</tr>
<tr>
<td>6</td>
<td>-2.07E-17</td>
<td>-6.89E-17</td>
<td>31</td>
<td>1.99E-05</td>
<td>7.02E-05</td>
</tr>
<tr>
<td>7</td>
<td>-2.15E-03</td>
<td>5.65E-03</td>
<td>32</td>
<td>-1.30E-15</td>
<td>-7.01E-16</td>
</tr>
<tr>
<td>8</td>
<td>-4.61E-17</td>
<td>2.73E-17</td>
<td>33</td>
<td>-2.26E-05</td>
<td>1.71E-04</td>
</tr>
<tr>
<td>9</td>
<td>-2.09E-03</td>
<td>-0.01149</td>
<td>34</td>
<td>4.20E-16</td>
<td>-2.94E-16</td>
</tr>
<tr>
<td>10</td>
<td>6.29E-17</td>
<td>-2.24E-17</td>
<td>35</td>
<td>-1.73E-05</td>
<td>-9.87E-05</td>
</tr>
<tr>
<td>11</td>
<td>1.15E-03</td>
<td>1.66E-04</td>
<td>36</td>
<td>1.66E-16</td>
<td>3.89E-16</td>
</tr>
<tr>
<td>12</td>
<td>7.96E-17</td>
<td>5.91E-18</td>
<td>37</td>
<td>4.39E-05</td>
<td>-2.67E-05</td>
</tr>
<tr>
<td>13</td>
<td>-4.80E-04</td>
<td>2.11E-03</td>
<td>38</td>
<td>-2.60E-17</td>
<td>-1.10E-17</td>
</tr>
<tr>
<td>14</td>
<td>-1.28E-16</td>
<td>-7.55E-18</td>
<td>39</td>
<td>-1.58E-04</td>
<td>5.72E-04</td>
</tr>
<tr>
<td>15</td>
<td>-2.59E-04</td>
<td>-3.19E-03</td>
<td>40</td>
<td>1.31E-16</td>
<td>-2.61E-16</td>
</tr>
<tr>
<td>16</td>
<td>7.34E-17</td>
<td>-1.01E-17</td>
<td>41</td>
<td>-1.12E-04</td>
<td>-2.58E-04</td>
</tr>
<tr>
<td>17</td>
<td>2.69E-04</td>
<td>-1.48E-04</td>
<td>42</td>
<td>-1.36E-16</td>
<td>3.43E-17</td>
</tr>
<tr>
<td>18</td>
<td>5.26E-17</td>
<td>5.99E-17</td>
<td>43</td>
<td>3.90E-04</td>
<td>-1.07E-05</td>
</tr>
<tr>
<td>19</td>
<td>-1.36E-04</td>
<td>8.20E-04</td>
<td>44</td>
<td>-2.77E-17</td>
<td>-1.89E-16</td>
</tr>
<tr>
<td>20</td>
<td>-4.01E-17</td>
<td>7.30E-17</td>
<td>45</td>
<td>-1.30E-03</td>
<td>-2.33E-04</td>
</tr>
<tr>
<td>21</td>
<td>-4.15E-05</td>
<td>-7.46E-04</td>
<td>46</td>
<td>1.51E-16</td>
<td>2.43E-16</td>
</tr>
</tbody>
</table>

### 4.8 Early evolution of the rotor design

The first full design was produced using the last 2D model as a guide. All the coils were assigned the same overall length, and the shape of the rings was chosen to give similar minimum values of overhang at the ends and the chamfered corners as at the sides. The model includes the complete shape of the coils and flux diverters and a full 3D model of the armature winding which is in the majority of models excited with the sinusoidal current distribution with its MMF peaking at the quadrature axis, hence simulating a high-power-factor load on the machine.

In the first 3D models the field winding was modelled using racetrack coils, but later they were changed into more oval shapes by replacing the straight sides
of the coil by arcs of radius 1800 mm and the ends by arcs of radius 1200 mm. This change was made to reflect the proposed shape of the real coils; experience suggested that it would be easier to ensure the integrity of coils with this shape. The rotor of the first three dimensional - model is shown in Figure 4.12.

![Image](image.png)

**Figure 4.12:** Rotor of the first 3D model used to optimise the harmonic content of the voltage waveform: a) shape of the diverters b) field winding

This model represents a design in which the whole stack of coils, fibreglass formers and rings would be held together by the outer stainless steel tube with a fibreglass former inside with grooves machined in it to locate the rings and coils. However, different thermal contraction coefficients of rings, coils, and fibreglass would probably make everything loose once the rotor is cooled to its operating temperature. Although it would be possible to make them tight it would mean that they would push away the rings from their outer supports and that would make it difficult to transmit the acceleration torque [26]. Because at that time no alternative could be found that would offer safe solution, the design had to be abandoned and was soon replaced by the one presented in Figure 4.13, in which through bolts had been added to hold the whole rotor assembly together.

The load from these bolts must be carried by the pole pieces, which must therefore be thicker to make them strong enough; this required the removal of the top-most coil and its diverter ring.

This change required that the top-most hat material was changed to Invar because in this configuration it would serve as a primary structural element and therefore it required a material with toughness guaranteed at temperatures below 73K.
Mechanical calculations showed that stresses in the bolts would be higher than the allowed 200 MPa. This led to the conclusion that 12 bolts would not be adequate to safely hold the whole rotor assembly; this number was therefore increased to 14. At the same time the radius of the middle coils of the rotor had to be decreased and that required the inner diameter of the flux diverters to be decreased, effectively extending them towards the centre of the rotor. This provided not only flux redirection but also allowed regaining some of the flux lost by decreasing the area of the rotor (because it was thought that cooling pipes would be installed on the sides of the rotor). The shape of the rotor after these changes is shown in Figure 4.14.

Figure 4.13: Rotor configuration after adding the through bolts and Invar nuts field winding

Figure 4.14: Final rotor shape a) front showing the overall shape b) back showing bigger middle diverters
4.9 Field modelling and evaluations

The magnetostatic solver included in the Opera finite element package (TOSCA) was used to analyse the 3D models. Because the excited stator winding is used it is necessary to analyse a quarter of the model (otherwise only 1/8th would be sufficient). All the models were nonlinear with the appropriate BH curves defined for the magnetic materials within the rotor. The stator, however, is modelled as a linear material with anisotropic permeability of 200 in XY direction and 20 in Z direction (to account for the lamination of the stator). A typical field distribution under load is shown in Figure 4.15.

![Figure 4.15: Post processing results, flux density contours of the machine under load](image)

The first 3D model built was an equivalent to the 2D model and is shown in Figure 4.12a. In order to analyse the harmonic content an average of the radial field component, along the axial direction, on the same arc as in the 2D model was taken and the harmonics were extracted using Fourier analysis. This allows direct comparison between 2D and 3D models. When the results for the first 3D models are compared with the 2D results the influence of the elements not included in 2D evaluations becomes apparent. The individual harmonic components of the air-gap flux density up to the 19th order for 2D and 3D models are shown in Table 4.7 and Table 4.8 respectively.

The additional flux paths present in the 3D model change the distribution of the air-gap flux (decrease it), which substantially influences the waveform and its
### Table 4.8: Harmonic components of the air-gap flux density and phase voltage of the 3D model shown in Figure 4.12 (Total rms harmonic voltage 0.96%)

<table>
<thead>
<tr>
<th>Space Harmonic Order</th>
<th>Cosine Component</th>
<th>Sine Component</th>
<th>Amplitude Factor</th>
<th>Actual Harmonic Factor</th>
<th>Harmonic Contribution %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.51982</td>
<td>-0.11033</td>
<td>0.5314</td>
<td>0.75818</td>
<td>0.402875</td>
</tr>
<tr>
<td>3</td>
<td>2.60E-03</td>
<td>0.010235</td>
<td>0.01058</td>
<td>0.245196</td>
<td>0.002594</td>
</tr>
<tr>
<td>5</td>
<td>-2.24E-03</td>
<td>-7.09E-03</td>
<td>1.39E-03</td>
<td>0.192777</td>
<td>0.001125</td>
</tr>
<tr>
<td>7</td>
<td>-8.57E-03</td>
<td>1.62E-03</td>
<td>5.73E-03</td>
<td>0.018445</td>
<td>0.000161</td>
</tr>
<tr>
<td>9</td>
<td>0.012291</td>
<td>1.83E-03</td>
<td>0.012427</td>
<td>0.207867</td>
<td>0.002583</td>
</tr>
<tr>
<td>11</td>
<td>8.42E-04</td>
<td>-5.08E-04</td>
<td>9.83E-04</td>
<td>0.057705</td>
<td>5.67E-05</td>
</tr>
<tr>
<td>13</td>
<td>-3.92E-03</td>
<td>4.70E-04</td>
<td>3.94E-03</td>
<td>0.050661</td>
<td>0.0002</td>
</tr>
<tr>
<td>15</td>
<td>3.60E-03</td>
<td>5.45E-05</td>
<td>3.60E-03</td>
<td>0.138893</td>
<td>0.0005</td>
</tr>
<tr>
<td>17</td>
<td>2.38E-04</td>
<td>3.40E-04</td>
<td>4.15E-04</td>
<td>0.000996</td>
<td>3.77E-06</td>
</tr>
<tr>
<td>19</td>
<td>-3.87E-04</td>
<td>7.07E-05</td>
<td>3.93E-04</td>
<td>0.065438</td>
<td>2.57E-05</td>
</tr>
</tbody>
</table>

The changes to the mechanical model however required that the through bolts were added and that unfortunately increased the harmonic content of the air-gaps waveform by introducing more permanence variation in the air-gap. The rms harmonic voltage for the first model with through bolts was 2.237%. Table 4.9 contains the values of harmonic components of the air-gap flux density for this rotor configuration.

### Table 4.9: Harmonic components of the air-gap flux density and phase voltage of the 2D model shown in Figure 4.9 (Total rms harmonic voltage 1.512%)

<table>
<thead>
<tr>
<th>Space Harmonic Order</th>
<th>Cosine Component</th>
<th>Sine Component</th>
<th>Amplitude Factor</th>
<th>Actual Harmonic Factor</th>
<th>Harmonic Contribution %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.079095</td>
<td>0.588918</td>
<td>0.594206</td>
<td>0.758138</td>
<td>0.45049</td>
</tr>
<tr>
<td>3</td>
<td>-0.02572</td>
<td>1.78E-03</td>
<td>0.025783</td>
<td>0.245196</td>
<td>0.006322</td>
</tr>
<tr>
<td>5</td>
<td>8.03E-03</td>
<td>1.49E-03</td>
<td>0.008169</td>
<td>0.192777</td>
<td>0.001575</td>
</tr>
<tr>
<td>7</td>
<td>-1.54E-03</td>
<td>7.17E-03</td>
<td>0.007335</td>
<td>0.018445</td>
<td>0.000135</td>
</tr>
<tr>
<td>9</td>
<td>-2.25E-03</td>
<td>-0.01165</td>
<td>0.011866</td>
<td>0.207867</td>
<td>0.002457</td>
</tr>
<tr>
<td>11</td>
<td>9.27E-04</td>
<td>-9.45E-04</td>
<td>0.001324</td>
<td>0.057705</td>
<td>7.64E-05</td>
</tr>
<tr>
<td>13</td>
<td>-2.46E-04</td>
<td>2.77E-03</td>
<td>0.002777</td>
<td>0.050661</td>
<td>0.000141</td>
</tr>
<tr>
<td>15</td>
<td>-2.80E-04</td>
<td>-2.91E-03</td>
<td>9.002919</td>
<td>0.138893</td>
<td>0.000405</td>
</tr>
<tr>
<td>17</td>
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<td>-6.27E-04</td>
<td>0.000641</td>
<td>0.009699</td>
<td>5.83E-06</td>
</tr>
<tr>
<td>19</td>
<td>-5.62E-05</td>
<td>9.55E-04</td>
<td>0.000957</td>
<td>0.065438</td>
<td>6.26E-05</td>
</tr>
</tbody>
</table>

The high harmonic content required further investigation to reduce it to a lower value. This was done using the optimisation method described in the next section. The harmonic contribution for various designs is shown in Figure 4.16.
### Table 4.9: Harmonic components of the air-gap flux density and phase voltage of the 3D model shown in Figure 4.13 (Total rms harmonic voltage 2.237%)

<table>
<thead>
<tr>
<th>Space Harmonic Order</th>
<th>Cosine Component</th>
<th>Sine Component</th>
<th>Amplitude Factor</th>
<th>Winding Harmonic Factor</th>
<th>Actual Harmonic Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.48969</td>
<td>-0.11013</td>
<td>0.501922</td>
<td>0.758138</td>
<td>0.380526 100</td>
</tr>
<tr>
<td>3</td>
<td>-0.0283</td>
<td>0.016303</td>
<td>0.032656</td>
<td>0.245196</td>
<td>0.008007 2.104212</td>
</tr>
<tr>
<td>5</td>
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<td>-3.21E-03</td>
<td>0.018464</td>
<td>0.192777</td>
<td>0.003557 0.947114</td>
</tr>
<tr>
<td>7</td>
<td>-4.64E-03</td>
<td>-1.12E-03</td>
<td>0.018458</td>
<td>0.804E-05</td>
<td>0.003137 0.023137</td>
</tr>
<tr>
<td>9</td>
<td>-2.78E-03</td>
<td>2.62E-03</td>
<td>3.82E-03</td>
<td>0.207867</td>
<td>0.000094 0.208675</td>
</tr>
<tr>
<td>11</td>
<td>5.99E-03</td>
<td>1.53E-03</td>
<td>6.18E-03</td>
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<td>0.000057 0.093752</td>
</tr>
<tr>
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<td>-7.75E-04</td>
<td>1.75E-03</td>
<td>0.050061</td>
<td>8.87E-05 0.02331</td>
</tr>
<tr>
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<td>-8.73E-06</td>
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<td>0.138893</td>
<td>0.000221 0.058036</td>
</tr>
<tr>
<td>17</td>
<td>6.76E-04</td>
<td>1.03E-03</td>
<td>1.23E-03</td>
<td>0.009996</td>
<td>1.12E-05 0.002945</td>
</tr>
<tr>
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<td>9.61E-04</td>
<td>0.065438</td>
<td>6.29E-03 0.016521</td>
</tr>
</tbody>
</table>

**Figure 4.16:** Harmonic contributions up to 19th order for three different 3D designs

### 4.10 Field optimisation

When a classical optimisation process is considered it involves defining an objective function dependent on a number of variables. The optimiser then explores the space of the function and typically looks for appropriate gradients of the param-
eters defined in the function. Then it decides to follow a certain direction which
the algorithm evaluates to have the highest chance to meet the objective. When
further improvement is not possible the algorithm evaluates the objective function
gradients from the point it reached at the last step.

Such a process is usually very computationally expensive and when the model
in itself is very complicated the computational costs rise enormously and in many
practical cases cannot be accepted. The models built for the current machine
need to be 3D because of all the features in the rotor. Each of the models takes
approximately 90 hours to solve for a static/AC solution. Classical optimisation
process, as mentioned before, would try to explore the space of the objective
function by varying parameters and solving the models and that is obviously not
acceptable in the case of the models being solved.

To predict the area for optimisation a simple formula, first suggested in [63],
was used. It provides a very good guidance for optimising the rotor shape with
the aim to minimise the harmonic content in the air gap flux density waveform;
using

$$\frac{\partial V_i^2}{\partial B_r(\Theta)} = 2V_i \frac{\partial V_i}{\partial B_r(\Theta)}$$

and applying the quotient rule, yields

$$\frac{\partial}{\partial B_r(\Theta)} \left( \sum_{i>1} V_i^2 \right) = \frac{2}{V_1^2} \left[ \sum V_i \frac{\partial V_i}{\partial B_r(\Theta)} - \left( \sum_{i>1} V_i^2 \right) \left( V_1 \frac{\partial V_1}{\partial B_r(\Theta)} \right) \right]$$

where \( V_1 \) is the fundamental harmonic, \( V_i \) is the \( i^{th} \) harmonic of the voltage, and \( B_r \) is the radial flux density in the air-gap with the other symbols having the usual
meaning.

This function gives the designer a very good estimate of which parts of the rotor
should have the flux increased (by adding more ampere-turns or iron) and where it
should be reduced. Thus it allows the designer to control the optimisation process
and helps to significantly reduce the number of iterations required for minimising
the harmonic content. The method can be used with any type of a model, but
requires that the derivative of the objective function with respect to the air-gap flux
density can be estimated without the use of computationally expensive models.

In order to increase the accuracy of the harmonic voltage evaluations the flux
linking each coil was estimated from integrals of the normal component of B over a
large number of 8-node quadrilateral patches. These patches were arranged into 6
sets, such that each set formed a continuous surface with its outer edges following a current streamline within the coil. These 6 paths were uniformly distributed over the cross section of the conductor, and were defined using data generated while building the stator winding. Two of the patches used for flux density extraction are shown in Figure 4.17. Apart of one is removed to better show the one following the current streamline in the coil.

![Figure 4.17](image-url)  
**Figure 4.17:** 4 of the patches used for extraction of the flux linked by the coil

The results obtained by using the above formula (4.8) are presented in Figure 4.18. The graph is obtained for the first of the designs with through bolts. The graph suggests that there is not enough flux in the region below 30 degrees and above 70 region of the rotor and that in the remaining part of the rotor the harmonic content could be minimised by reducing the amount of flux crossing the air-gap. The angular position is well explained by Figure 4.19 showing the angular scale and the final shape of the rotor.
Figure 4.18: Air-gap flux harmonic contribution for the first design with through bolts

Figure 4.19: Cross section of the final shape of the rotor with the angle scale assumed for the optimisation formula
Since it was not possible to target the whole area because the reduction of the overhang of diverters over coils would doubtlessly lead to the increased magnetic field in the coils, the only other part of the rotor which could be targeted for optimisation was the top hat. The analysis of the graph lead to the conclusion that it would be welcome to increase the flux in the area between 70° and 90°. This leads to the conclusion that the only thing that can be done is to change the geometry of the top hat chamfers. They were removed in the next model shown in Figure 4.20.

![Figure 4.20](image)

**Figure 4.20:** The top-most diverter with the chamfers removed to reduce the harmonic content

This simple step helped to reduce the rms harmonic content to 0.75% from 2.23%. The results obtained by the optimisation formula for both shapes are presented in Figure 4.21. This design had to be changed later to the one with 14 bolts. At the same time the mechanical considerations required the middle six coils outer diameter to be reduced by 7mm. The loss in flux has been partially recovered by the reduction in the inner diameter of the middle flux diverters effectively making them extend 27, 24, 21, 17 mm (respectively from the middle plane of the rotor) beyond the radius of the coil instead of 5-10 mm for the other flux diverters.
Figure 4.21: The harmonic content in the first through bolt design (blue curve) and the improved design (red curve)

Figure 4.22: Harmonic contribution for the final of the through bolts design
The harmonic contribution for the final through bolts design is shown in Figure 4.22. Despite the frequent and major changes imposed by mechanical considerations, it was possible to reduce the harmonic content of the air-gap flux density in the final design to 0.52%. Although the graph obtained by the formula (4.9) shows that there is more room for improvement between the 45° and 80°, where less flux is required, and below 30°, where more flux would be welcome, it was believed not to be practical to improve the waveform further. The only way to add more flux in the lower region is to increase the number of turns in the coils or add more magnetic material in that region and this was rejected because it might cause difficulties in mechanically supporting the rings and because the increase in mass would be disproportionate. The other region where the obtained graph suggests there is room for improvement is where the top hat and the first diverter are located and when the magnetic flux in these two is analysed it becomes apparent that further changes in those regions would lead to the saturation of the diverters. The field distribution in the top diverter is shown in Figure 4.23. Figure 4.24 shows the distribution in all other rings and Figure 4.25 in the whole rotor.
Figure 4.24: Flux distribution in the 9% Nickel steel diverter rings

Figure 4.25: Flux distribution in the rotor under load
4.11 Alternative design

The design composed of the stack of metal flux diverters, fibreglass formers and superconducting coils faces a lot of problems mainly due to the thermal contraction of each of the materials. The design with through bolts had to be abandoned because different thermal contraction coefficients of the materials in the rotor would require a special assembly to absorb them and although a possible solution has been found it would be difficult to ensure its success. More detailed explanation can be found in [26].

The mechanical difficulties connected with building the coreless rotor of the type described previously forced the reconsideration of the design and required searching for a new arrangement that would mechanically be much simpler to construct. It was then decided that nearly all of the flux diverters should be removed from the rotor and the coils would be mounted on a stainless steel can and then enclosed in a second stainless steel can. These would be welded together to form a closed fluid vessel that would allow the winding to be flooded with liquid nitrogen, while the whole assembly is enclosed in the outer vacuum vessel. The final configuration of the rotor for this alternative design is shown in Figure 4.26.

![Figure 4.26: The final shape of the rotor for the alternative design](image)
This configuration of the rotor utilises the fact that the perpendicular component of the self magnetic field for each of the coils is counteracted by the neighbouring coils. However, the last coils at the top and bottom of the rotor do not have two neighbours, therefore it was necessary to retain the flux diverters in that region. The main disadvantage of this configuration is the fact that the field winding coils are not shielded from the armature field which can therefore reduce the critical current of the HTS coils. The path taken by the flux due to the quadrature axis current in the armature in the presence of diverters and without them is shown in Figure 4.27.

![Figure 4.27: Flux distribution in the coils a) for the first coreless design b) for the second coreless design](image)

The picture clearly shows the function of the flux diverters. The flux is diverted away from the coils and the main component of the magnetic field in the coils is parallel to the broad face of the tape. However, in the second design both flux components are clearly present within the coils. A number of possible configurations were analysed for such a rotor. In the first arrangement it was assumed that the coils would be simply placed between two pole pieces and held by the through bolts (Figure 4.28a) but this configuration would pose similar mechanical problems during the assembly as the first coreless design. Hence it was decided that the coils would be enclosed in a stainless steel can flooded with liquid nitrogen and that the pole pieces would be placed in the warm part of the rotor (Figure 4.28b). The electromagnetic design had to take into account the fact that some space is required for the superinsulation to reduce the heat leak to the cold part of the rotor from the warm part. Since the top-most coils require the flux diverter to be close, so that this can limit the perpendicular component of the field, they are placed inside the cold vessel along with the coils. In this arrangement the pole
pieces are further away from the HTS winding to allow for the superinsulation. Also this configuration serves the purpose of shaping the magnetic field, although - as it will be shown later - in a very limited manner. The pole pieces also have a significant effect on the field in the top-most coils; the effects of varying their distance from the coil stack are shown in Figure 4.29.

As would be expected, the lower position helps to reduce the perpendicular component of the magnetic field within the top-most coils but causes the parallel component to rise. The perpendicular component is the limiting factor in this
configuration and the parallel field is well within acceptable limits for the assumed current in the coils. To obtain the best field redirection the space between the HTS winding and the pole pieces has been made as small as is consistent with the design constraint imposed by the need to place the wall of the fluid vessel and sufficient superinsulation between the cold flux diverter and the warm pole piece. Figure 4.30 presents the final configuration of this design.

The rotor configuration shown in Figure 4.30 has 30 coils, which is less than the original number from the preliminary studies of that arrangement (34 originally) but the final machine has 32 coils since it was possible to add two more in the middle of the rotor without changing its size. In order to accommodate the stainless steel can and superinsulation it was necessary to remove two coils from the rotor. Two more coils were removed after the losses studies (described in Chapter 5) were conducted. The study indicated that the loss in the copper screen was too high and it was necessary to move the screen further away from the stator teeth. The whole rotor was considered to be enclosed in a 2.5 mm stainless steel tube with a copper layer sprayed on the inner surface (2.5 mm thick), later, mainly to simplify the design, the combination of two materials was replaced by one 4 mm aluminium tube which has similar resistance to the 2.5 mm copper layer. In this arrangement each of the coils has 38 turns with 120A current producing an air-gap flux density

![Figure 4.30: The final configuration of the two pole configuration of the rotor](image-url)
fundamental field component of the order of 0.41T. The further increase in the number of turns was considered to be impractical since it would inevitably lead to the non optimal use of the superconductor. It would require the current to be decreased even further. At the same time the length of one spool of superconductor allows building a 38 turn coil and bigger coils would require connecting two tape pieces which would in effect add extra losses. The other limiting factor was the available length of the superconducting tape. The original design was based on a stacked rotor configuration which would require fewer coils. Simulations predicted that it would be possible to build 60-turn coils with the current of 95A due to high parallel magnetic field component in the tapes. Such design would produce the air-gap flux density reaching 0.48T but it needs to be noted that 95A is only 53% of the tape rating (180A).

Figure 4.31: Harmonic contribution in the function of the rotor’s circumference angle

The 3D models of the final arrangement gave an air-gap flux density of 0.4T. Although this is considerably lower than the value in the coreless design studied previously, in view of mechanical and thermal constraints, this arrangement has been adopted. Despite some disadvantages, like lower air-gap flux density and higher harmonic content, this design has quite important advantages. The lower mass of the rotor, simpler construction and assembly, cheaper manufacturing of
Table 4.10: Harmonic content in the air-gap flux density for the final coreless design

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
<th>17</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.81%</td>
<td>3.42%</td>
<td>0.15%</td>
<td>0.53%</td>
<td>0.13%</td>
<td>0.10%</td>
<td>0.03%</td>
<td>0.01%</td>
<td>0.11%</td>
</tr>
</tbody>
</table>

Table 4.11: Voltage harmonic content for different stator configurations

<table>
<thead>
<tr>
<th>Order no.</th>
<th>Harmonic Magnitude</th>
<th>Winding factor</th>
<th>% harmonic contribution</th>
<th>Harmonic Magnitude</th>
<th>Winding factor</th>
<th>% harmonic contribution</th>
<th>Harmonic Magnitude</th>
<th>Winding factor</th>
<th>% harmonic contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.283561</td>
<td>0.923050</td>
<td>100.00%</td>
<td>0.88287</td>
<td>100.00%</td>
<td>0.92305</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.027835</td>
<td>-0.151021</td>
<td>-1.06%</td>
<td>0.08173</td>
<td>0.91%</td>
<td>-0.15102</td>
<td>1.61%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.040564</td>
<td>0.010065</td>
<td>0.73%</td>
<td>-0.01488</td>
<td>0.24%</td>
<td>0.010065</td>
<td>0.16%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.01001</td>
<td>-0.005225</td>
<td>0.1%</td>
<td>-0.01865</td>
<td>0.07%</td>
<td>-0.00522</td>
<td>0.02%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>9.27E-03</td>
<td>0.017677</td>
<td>0.010%</td>
<td>0.022896</td>
<td>0.00%</td>
<td>0.017677</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2.01E-03</td>
<td>-0.068324</td>
<td>0.00%</td>
<td>-0.0033</td>
<td>0.00%</td>
<td>-0.06832</td>
<td>0.01%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>6.83E-03</td>
<td>-0.006177</td>
<td>0.01%</td>
<td>0.002447</td>
<td>0.01%</td>
<td>-0.00618</td>
<td>0.02%</td>
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<td></td>
</tr>
<tr>
<td>15</td>
<td>2.19E-03</td>
<td>0.007087</td>
<td>0.01%</td>
<td>-0.00926</td>
<td>0.01%</td>
<td>0.007087</td>
<td>0.01%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>2.38E-03</td>
<td>-0.001061</td>
<td>0.00%</td>
<td>0.003787</td>
<td>0.00%</td>
<td>-0.00106</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>7.13E-04</td>
<td>0.008999</td>
<td>0.00%</td>
<td>0.001329</td>
<td>0.00%</td>
<td>0.008999</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The content of harmonics extracted from the flux linkages of the stator coils for the final design is shown in Table 4.10. It has to be noted that the effect of this high harmonic content on the output voltage might be greatly reduced if a different stator winding configuration was used. Table 4.11 shows the harmonic content of the air-gap flux density and the resulting voltage harmonics for a few possible stator winding configurations (different coil pitches). As can be seen the 3rd and 5th harmonics of the phase voltage can be reduced by choosing an appropriate configuration of the stator winding. It has to be mentioned that when star connection was used the 3rd harmonic would not be present in the output voltage, therefore the 5th harmonic is the one that should be minimised first. A graph presenting the results obtained by the use of the formula used for optimisation of the rotor shape in the previous section is shown in Figure 4.31. This shows that in order to improve the waveform it would be necessary to increase the flux below the 45° degrees. Unfortunately the configuration chosen for the rotor does not permit this because it would require adding iron or coils near the middle of the winding. Adding iron would effectively add a core to the coreless design, while adding turns to the middle coils would prevent the perpendicular component of their self field from being cancelled by that of adjacent coils and so the coils (since all of them are of the same size) are the main reasons why this arrangement has been adopted. This however does not mean that one of the other designs would not offer a competitive solution if answers to the mechanical problems were found.
might require use of flux diverters which was to be avoided in this configuration.

Two other possible changes to the design were also modelled and evaluated. It was believed that the construction of the rotor might require bolts holding the rotor that are strong enough to withstand high stresses. It was necessary to evaluate the influence of the magnetic bolts and nuts onto the air-gap flux density. If flux carried by magnetic bolts made a significant contribution to the air-gap flux density, their use would prevent the test results from being used to make a realistic assessment of the benefits of using a coreless rotor. Since the 2D simulation would not take into account the existence of the bolts an appropriate 3D model has been built and analysed. The increase in the air-gap flux density for the configuration with magnetic bolts was only 2% which was deemed unimportant; hence magnetic bolts can be used should this be considered necessary.
Chapter 5

Standstill frequency test modelling

In principle, the 3D solvers of the Vector Fields software allow solving full 3D rotating machine models and models with circuit windings. However, due to shear complexity of the stator winding, it proved to be impossible to build such models. Full 3D rotating machine models with external circuits would allow simulating a short circuit test, which would help to evaluate the machine parameters fairly easily. The expected computational cost however would probably be very high. By using external circuits in steady-state AC models, it would be possible to simulate the Standstill Frequency Response (SSFR) test in the manner described in [3]. However, the failure to build the circuit windings for the stator prevents building models of either type. A method has therefore been devised that enables the results of a SSFR test to be estimated using only AC models with current source conductors; this method is described in this chapter.

The SSFR test is based on the idea that the frequency response data describes the reaction of the machine fluxes to stator current and field voltage in both the direct and quadrature axes. These responses are taken over a range of sinusoidal excitations ranging from very low frequencies of 0.001 Hz to frequencies higher than usual 50 Hz.
5.1 Equivalent circuit parameter estimation

The design of a coreless rotor with a stack of coils and diverters has many 3D features that cannot be included in 2D models and which influence the results significantly.

When the coreless machine is compared to a conventional machine it can easily be seen that if the 2D cross section through the middle will be completely different. The coreless machine of the type analysed here has a number of magnetic rings which would be not considered in such a cross section that is used in 2D analysis. These rings provide an extra flux path for the magnetic field which is not taken into account in 2D models but can influence the results significantly. This differences will be emphasised in this chapter.

Because of low reactances of the central 2D section of the machine caused by the large air-gap, and the absence of a rotor core, increase the relative importance of the end-winding inductances, additional flux paths around the ends of the flux diverter rings complicate the analysis. These flux paths allow flux to pass from one side of the rotor to the other without flowing through the central space, increasing the quadrature reactances and making the field in the central section of the machine more three-dimensional.

It was therefore decided that 3D finite-element models should be used for everything except preliminary design investigations.

A further difficulty arises when analysing eddy currents in the copper screen, since this is a continuous sheet that does not force the current to follow well defined paths. Using an arbitrary set of mode shapes to define the current distribution in the copper screen due to stator MMF harmonics would be difficult. Consequently, the screen is not treated as a winding; instead, its conductivity is included in the finite-element models as a material property.

Although, in principle, it is now possible to build and solve a full 3D transient rotating machine model of a generator, such models are computationally very expensive. Moreover, models that include circuit windings with complex geometry usually failed to mesh. Since rotating machine models require these circuit windings to represent the unknown currents in the external circuits, the use of such models for design purposes cannot yet be considered practical with available software.

The standstill frequency response test is based on an assumption that by finding
a number of transfer functions about some operating point, describing the electrical
responses of stator and rotor quantities, parameters may be found by fitting the
obtained data to the assumed model of the proper order.

It is apparent that some errors are inherent in the use of such test results to
predict the behaviour of a machine that is rotating. These errors arise because the
field in the test only has one frequency, whereas the equivalent circuit is intended
to represent a situation in which the field on the rotor has a number of different
frequencies; the main field is at a low frequency, the field due to stator MMF
harmonics differs from this by a multiple of 6 times the supply frequency, and
stator slots modulate the field at multiples of the stator slot passing frequency.

For the machine considered, the errors introduced by using the wrong frequency
for the field driven by space harmonics of the stator MMF are likely to be particu-
larly large. There are two reasons for this: first, because the stator winding has a
very short coil pitch and so has larger MMF harmonics, and secondly, because the
damper winding is a copper sheet in the air gap rather than a set of bars buried
in slots.

In a real test, there is little that can be done about these errors other than to
make some estimates of their effect and make appropriate adjustments to the data.
However, if the test results are replaced by the results of finite-element modelling,
it is possible to model the field of the stator MMF harmonics separately at one
or more appropriate frequencies, and combine the results by superposition. The
only stator currents that need to be included in the low-frequency models are the
direct-axis and the quadrature-axis sinusoidal components.

The use of superposition implies the use of a linear model; the stator core is
therefore modelled as a linear anisotropic material. However, in the rotor iron
(flux diverters), the rotor screen ensures that the high-frequency flux components
driven by harmonics of the stator MMF are small; hence we may neglect these
high frequency fields in a non-linear model of the rotor. Other restrictions on the
use of non-linearity in the low-frequency model of the rotor will be explained later.

In addition to neglecting the effects of the high frequency fields on the per-
meability of the flux diverters, the effects of flux diverters on the high frequency
fields are also ignored. The models that are used to predict the effects of the stator
MMF harmonics on the stator flux linkage therefore have no flux diverters; the
rotor is represented by the copper screen alone (Figure 5.1). This simplification
greatly increases the symmetry of these models. It is therefore possible to employ
superposition again to predict the field of the stator MMF harmonics from that of a single pair of coils. This reduction in the number of coils in the model greatly reduces the computational effort expended to solve the finite-element models.

**Figure 5.1:** One coil model used for estimating the stator MMF harmonics influence on the stator flux

To estimate the inductive energy due to harmonics of the stator MMF, the model is first post processed to determine the flux linking each coil due to the current imposed in the one pair of coils that is included in the model. The flux linkages are calculated by integrating the normal component of B over a series of quadrilateral patches, the edges of which define 6 current paths uniformly distributed over the cross section of each 3-turn coil. The flux linkage data is scaled to form a mutual inductance vector, and expanded to form a complete mutual inductance matrix (Figure 5.2). Using this matrix, the inductive energy can be found for any distribution of stator currents by matrix multiplication. The simplest estimate of the energy due to the MMF harmonics is obtained by subtracting the energy due to the sinusoidal components (described in Table 5.1 as Sinusoidal
Table 5.1: Inductance variation with frequency

<table>
<thead>
<tr>
<th>Frequency</th>
<th>300Hz</th>
<th>600Hz</th>
<th>900Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block Distribution</td>
<td>291.8E-6 H</td>
<td>285.3E-6 H</td>
<td>283.5E-6 H</td>
</tr>
<tr>
<td>Sinusoidal Distribution</td>
<td>281.2E-6 H</td>
<td>274.9E-6 H</td>
<td>273.0E-6 H</td>
</tr>
<tr>
<td>Difference</td>
<td>10.66E-6 H</td>
<td>10.49E-6 H</td>
<td>10.46E-6 H</td>
</tr>
</tbody>
</table>

Distribution) of the stator current from that for the real (6 phase belts) current distribution (described in Table 5.1 as Block Distribution). While this method neglects the variation in inductance with frequency, the results show that for the range of frequencies involved this variation is small (Table 5.1). Since most of the additional energy is associated with rotor currents at 6 times the output frequency of the generator, the errors are negligible if the model is solved at this frequency.

\[
\begin{bmatrix}
L_{1,1} \\
M_{1,2} \\
M_{1,3} \\
\vdots \\
M_{1,n}
\end{bmatrix}
= \begin{bmatrix}
L_{1,1} & -M_{1,n} & -M_{1,n-1} & \cdots & -M_{1,2} \\
M_{1,2} & L_{1,1} & -M_{1,n} & \cdots & -M_{1,3} \\
M_{1,3} & M_{1,2} & L_{1,1} & \cdots & -M_{1,4} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
M_{1,n} & M_{1,n-1} & M_{1,n-2} & \cdots & L_{1,1}
\end{bmatrix}
\]

Figure 5.2: Formation of a full mutual inductance matrix for \( n \) pairs of coils by expansion of a mutual inductance vector

5.2 Treatment of nonlinearity and equivalent parameter finding

As noted above, the use of a separate high-frequency model to predict the field due to harmonics of the stator MMF does not preclude the use of non-linear material properties in the rotor of the low-frequency model. However, since the 3D solvers do not permit a small-signal solution to be obtained using permeability data from a previous static solution, the use of non-linear models must be restricted to the
static limiting case of the low-frequency model or to cases where there are no DC fields. In the latter case, a degree of approximation is involved, since the harmonic components of $H$ produced by the non-linearity must be neglected.

To fully model nonlinearity in the rotor, a transient solution would be required. This would increase the computational cost of the solution; however, since a rotor-frame-of-reference model does not model the movement (see Appendix I) of the stator slots past the rotor, it is possible to use a larger time step than would be required for a rotating machine model. If the field winding could be connected to an external circuit, this would allow saturated values to be obtained for the small-signal transient characteristics. However, while the software used does support this type of circuit winding, it has been found that models with complex circuit windings usually fail. Although standard (racetrack or solenoid) windings are not a problem, a circuit winding consisting of arc conductors that represents the field winding of the generator causes the model to fail. The use of non-linear models of the rotor should therefore be restricted to checking the validity of data obtained from linear models.

Both 2D and 3D methods have been used to estimate the machine characteristics for the design described above. Since the capabilities of the 2D and 3D modelling software used are significantly different, somewhat different approaches are required. Nevertheless, in both cases, the small-signal characteristics have been estimated, and no transient models have been used.

The 2D results were obtained from a set of three models: one for direct-axis MMF, one for quadrature-axis MMF, and one for field current. In each case, the model was solved for a number of frequencies using permeability data reloaded from a previous static solution with full load current in the stator. It would have been possible to remove the need for one of these models by connecting an external circuit to the field winding, but the cost of solving additional 2D models is small, and the use of separate models allows the external impedance of the field circuit to be added later.

The 3D results were obtained using the same method, but using linear models of the rotor. An appropriate value of permeability for these models was chosen as $\frac{\int\int\int B \cdot B}{\int\int\int B \cdot H}$, where the integrals were evaluated in the appropriate parts of a non-linear model with full load currents in the windings. The accuracy of this linear approximation can be checked by comparing the values of synchronous reactances obtained from the linear models with values obtained from the difference of two
nonlinear solutions using

\[ L_{diff} = \frac{\Psi_{+\delta} - \Psi_{-\delta}}{\delta i} \]  

(5.1)

This is an established technique and is described in [56]. By subtracting the 3D flux linkage from the 2D flux linkage it was possible to obtain the end winding inductance, obviously taking also into account mutual inductances. Sample relative permeability distributions for both direct and quadrature axis reactance evaluations are shown in Figure 5.3. In a 2D vector potential solution there are \( x \) and \( y \) components of flux density and \( z \) component of currents. The flux linking a rectangular loop is given by the product of the length in the \( z \) direction and the difference between the values of the vector potential \( A \) at the \( xy \) positions of the two sides of the loop. The flux linkage of a coil is defined by:

\[ \Lambda = \frac{N}{a_1} \int_{a_1} A_z da_1 - \frac{N}{a_2} \int_{a_2} A_z da_2 \]  

(5.2)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5_3a.png}
\caption{Relative permeability distribution in the model for estimating a) direct axis b) quadrature axis reactance}
\end{figure}
Table 5.2: Direct and quadrature reactances obtained from 2D and 3D simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Results from 2D</th>
<th>Results from 3D Non Linear</th>
<th>Results from 3D Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$ [P.U.]</td>
<td>0.115</td>
<td>0.116</td>
<td>0.116</td>
</tr>
<tr>
<td>$X_q$ [P.U.]</td>
<td>0.18</td>
<td>0.19</td>
<td>0.19</td>
</tr>
</tbody>
</table>

where $a_1$ and $a_2$ are the areas of outward and return conductors respectively and $N$ is the number of turns in the coil. By using this equation and including the end winding leakage inductance of 150 mH the direct reactance were found. The same method was applied to evaluate the quadrature reactance with the only difference being the position of the rotor being rotated 90° with respect to the stator.

The values of the synchronous reactances obtained from the 2D and 3D models are listed in Table 5.2 (as per unit values). It is apparent that the quadrature-axis reactance is higher than the direct-axis reactance. This result is different from what is obtained when machines of classical design are considered, where the direct axis reactance is usually higher. The table also shows that the technique used to estimate the values of relative magnetic permeability for use in linear models enables those models to be in a very good agreement with those obtained using non-linear material permeability.

![Flux distribution for a) direct b) quadrature axis](image)

**Figure 5.4:** Flux distribution for a) direct b) quadrature axis

The flux plots in Figure 5.4 illustrate the reason for this rather unusual proportion of values of $X_d$ and $X_q$. It is apparent that the quadrature-axis flux can flow for relatively long distances within a high permeability material (diverter rings), whereas the direct-axis flux must repeatedly leave the iron and flow through the
non-magnetic regions occupied by the field winding. It should also be noted that for a design without flux diverters this result may be different.

When the results from 2D and 3D modelling are compared there is a noticeable difference between the two sets, with the 2D results giving lower estimates of the synchronous reactances. The results for the 2D estimation of both reactances include the end winding inductance which has been obtained by using the one coil model (without the conductivity in the copper screen) and expanding it to the mutual inductance matrix in the same manner as described in Section V. It has been found that its value is 0.150 mH. The higher values of reactance predicted using the 3D models can be attributed to the presence of additional flux paths that cannot be included in a 2D representation of the problem, in particular: the additional iron paths available to quadrature-axis flux in the ends of the diverter rings, and the large fringing flux at the core ends due to the large air gap and the extended rotor poles. The direct and quadrature flux paths in one of the rings are shown in Figure 5.5. The AC models were solved for the range of frequencies between 0.01 and 100 Hz, and the flux linkage was extracted for each stator coil.
Figure 5.7: Equivalent 1st order circuit of the quadrature axis of the machine

and for the rotor winding. As mentioned before, linear models were used and the field due to induced current in the field winding was added to the D-axis field of the stator winding by superposition. The resistance of the field circuit is not included in the FE models; a value of 10 mΩ was assumed, which is an estimate of the resistance of the slip rings and current leads. The transfer functions as described by the following equations were then fitted

\[
\Delta \Psi_d(s) = G(s) \Delta e_{fd}(s) - L_d(s) \Delta i_d(s)
\]  

\[
\Delta \Psi_q(s) = -L_q(s) \Delta i_q(s)
\]

where \( \Psi_d \) and \( \Psi_q \) are direct and quadrature axis flux linkages, \( i_d \) and \( i_q \) are corresponding stator currents, \( e_{fd} \) is the machine field voltage at a particular operating point and \( \Delta \) means a small perturbation, \( L_d(s) \) - the direct axis operational inductance - is the Laplace transform of the ratio of direct axis armature flux linkages to the direct axis current with the field winding short-circuited, \( L_q(s) \) is analogous in quadrature axis, and \( G(s) \) the armature to field transfer function - is the Laplace transform of the ratio of the direct axis armature flux linkages to the field voltage with the armature open circuited. The results are then fitted into the function of the proper order [5] defined as
\[ x_d(s) = x_d \frac{(1 + sT'_q)(1 + sT'_d)}{(1 + sT''_d)(1 + sT''_d)} \]  \hspace{1cm} (5.5)

\[ x_q(s) = x_d \frac{(1 + sT'_q)}{(1 + sT'_q)} \]  \hspace{1cm} (5.6)

where \( T_d, T_q, T_d \) are appropriate time constants.

Transfer functions corresponding to the equivalent circuits (Figure 5.6 and Figure 5.7) were fitted to the data from the AC models. The graphs (Figure 5.8 and Figure 5.9) suggest that these equivalent circuits are sufficient to model the response of the machine. These curves were obtained using the equivalent-circuit parameters shown in Table 3. The base value of the field current was chosen arbitrarily as 15A. Although this is only 10% of the rated field current, it represents substantially more MMF than full load current in the stator. The use of significantly lower values does not permit the characteristics of the machine to be modelled by the equivalent circuit shown in Figure 5.

These values of reactances are much lower than those typical for conventional machines. Say [], for example, quotes values 2-3 times larger than the ones obtained for our HTS machine, although admittedly for much larger machines then the one
analysed here. Much smaller reactances of superconducting machines will result in a better dynamic and static stability. A small subtransient reactance means a smaller voltage drop on the machine during transients. A smaller synchronous reactance gives those machines a larger pull out torque during transients which improves static stability. A lower reactance also means that the machines works at a smaller load angle, therefore the stability range is also increased.
Chapter 6

Evaluation of Losses

The majority of losses generated inside an electric machine account for its heating. Any unnecessary load on the cryogenic cooling system cannot be accepted because at 77K removing a 1W of heat requires up to 25W of cooling power. Hence the estimation of where and how the loss-inducing harmonics penetrate the rotor becomes a critical component of evaluating the feasibility of the design. Using a superconducting winding to reduce the overall machine losses is a very appealing concept but in order to achieve the reduction a careful design process is required.

6.1 Losses in conventional machines and where superconductivity might allow improvement

In conventional machines there are few main types of losses, namely hysteresis losses and eddy current losses (both in the stator and rotor cores), $I^2R$ losses in the copper conductors, mechanical losses (friction in bearings and slip rings) and windage losses (because of shaft mounted fan or other air disturbances). It is obvious that not all of these can be reduced by using a superconducting winding in the rotor or stator or both, but it should be possible to eliminate at least some and to reduce others.

By employing superconductivity, we can reduce or remove some of the losses. However, at the same time, we may be introducing other losses or additional power consumption. In particular, power is required to drive the cryogenic cooling system. Nevertheless, as the measurements conducted by Siemens on their 4MVA machine have demonstrated [37] significant loss reduction is still achievable. The
total losses are reported to have been reduced by nearly 60%, as a result of replacing a conventional rotor with its superconducting counterpart, mainly because copper losses and iron losses of a conventional rotor were removed. Stator losses, on the other hand, remain more or less the same. The friction and windage losses were reduced by half because the HTS rotor is smaller and lighter. There are nearly no Ohmic losses in the rotor but there are some losses due to heat leak from the cryogenic part. There are hardly any additional losses, which are usually attributed to harmonics generated by the passing stator teeth, because Siemens used air-gap windings in this design. The resulting efficiency of the machine was increased by 2% to around 98.4-98.7, depending weakly on the power factor. Conventional machines, in comparison, reach values between 96.1% and 97%; the higher values are achieved mainly by the use of a shorter air-gap. Figure 6.1 depicts the comparison between a conventional generator of 4MVA output (type 1FJ4801 by Siemens) and an HTS machine of the same rating.

![Figure 6.1: Comparison of losses between conventional 4MVA machine and its HTS counterpart with the same rating](image)

6.2 Modelling losses

There is an important fact to notice about the superconductor. Due to its low resistivity, changing magnetic fields might induce high AC current losses in the field winding. Losses due to eddy currents, both in the cold metallic parts of rotor and in the superconducting winding, load the refrigeration system and so increase
the power required to drive it.

Constant flux linkage theorem states that any changes in the reluctance caused by passing slots, as well as changes in the stator MMF caused by harmonics or unbalanced load drive, would induce changes in the rotor current. This problem might be addressed by surrounding the rotor with a copper screen which effectively shields the rotor from any external sources trying to drive substantial fluctuations in its body and the field winding. With the Cu shield in place the use of constant current drive for the field winding in the models is the correct assumption, hence this could allow avoiding the definition of equivalent resistance and supply voltage for the superconducting winding.

The results presented in this chapter are mainly on the estimation of losses for different positions of this screen for the last coreless rotor design. However, some simulations were run for the first coreless design as well and the results are also included. There is a significant difference between those two designs when it comes to the losses estimation. The first one assumed that the whole rotor would be flood cooled and that would mean that the copper screen would be in the cold part. Hence excessive losses in this part would lead to a much higher strain on the cooling system of the machines. This design however was abandoned due to the mechanical problems.

Different construction of the last design sets slightly different targets for the optimisation process. Here the pole pieces and the screen are placed in the warm part of the rotor, therefore higher losses in those parts of the rotor are accepted. These however cannot be excessive because only a thin superinsulation layer is the only barrier reducing the heat leak from there to the cold part of the machine. Losses lead to the increase of the screen temperature and because the machine is air cooled there is a certain threshold that limits the highest acceptable temperature. The studies were conducted to find the most suitable position for the rotor screen and estimate the losses.

2D full transient non-linear rotating machine models were used to evaluate the losses. The rotating machine solver is a transient eddy current solver. It requires that stationary parts of the machine are separated from the moving parts by a gap region. This region is remeshed during each step of the solution. Because of the generator’s rotational symmetry around the axis only one pole pitch needed to be modelled. The boundary conditions were set to be \((A \text{ at } 180^\circ = -A \text{ at } 0^\circ)\). Figure 6.2 shows one of the first models analysed.
The copper screen prevents any large fluctuations in the field current therefore it is possible to model it as a constant current excitation which avoids the need to specify the equivalent voltage supply and resistance of the superconducting winding.

The circuit definition used to impose the load current is shown in Figure 6.3.

The model was defined with a DC field current and the fixed angular velocity of 3000 rpm. All of the materials used in these models were specified to be non-linear with proper BH characteristics and the areas that were thought to be relevant had
the conductivity parameter specified. In order to model currents at one and two
times the slot passing frequency to be modelled correctly a small time step has
been chosen of $\frac{1}{5}$ of the time required to pass one stator slot. In order to monitor
the progress of the solution, the angle, time and I1 and I2 currents in the two
loops of the circuit were outputted to the log file. The data were extracted from
the 40 last output times which spanned across 8 slots (one phase) 5 points for each
slot. The losses were extracted by integration. In addition to that the odd space
harmonic components where extracted for the mid radius of the copper screen
to evaluate the influence of the stator harmonics. The symmetry leads to the
cancellation of the even harmonics, therefore they were not extracted.

For the full load losses the symmetry dictates that the waveform period is equal
to the time needed to pass 8 stator slots and just one slot for the no load loss estimation. The data for the losses per unit length were extracted by extracting
the integrals of $\frac{dJ}{\sigma}$ over the areas with defined conductivity. In the 2D models these areas were stainless steel tube, copper screen, invar pole piece and the stainless steel can surrounding the winding.

The magnitude of each of the current harmonics is given by:

$$J_{\text{rms}} = \sqrt{a_i^2 + b_i^2} \quad (6.1)$$

where $a_i$ and $b_i$ are the magnitudes of sine and cosine components of the $i^{th}$ space
harmonics of $J$. And the loss due to harmonics is given by:

$$P = \frac{2\Pi R L t J_{\text{rms}}^2}{\sigma} \quad (6.2)$$

where $R$ is the mean radius, $t$ is thickness, $\sigma$ is conductivity of the copper screen and $L$ is the length of the stator core.

In this first model it was assumed that a copper layer will be sprayed onto the
inner wall of the stainless steel tube. This process is not perfect so based on the
supplier’s data it was assumed that such copper layer would have the conductivity
equal to 40% of the conductivity of a copper foil of the same thickness.

Since the no load simulation requires far less computational effort then the full
load modelling, the first of the analysed models was built as a no load case.

The no load loss in different parts of the rotor for this configuration is shown
in Table 6.1.
Table 6.1: No load loss in the rotor parts for the first of the analysed models

<table>
<thead>
<tr>
<th>Copper Screen</th>
<th>Stainless Steel Tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>434W</td>
<td>1283W</td>
</tr>
</tbody>
</table>

As can be seen the losses in the copper screen and the stainless steel tube are substantial even at no load condition. It has to be remembered that a too high temperature rise in this part of the rotor would make it impossible to cool it only with air. When these losses (Figure 6.4) are analysed by taking the Fourier transform of the function describing the distribution of the current density at the mean radius of the copper screen it can be seen that the close proximity of the stator slots is responsible for the loss that shows on the rotor as the 48th harmonic (2400Hz).

![Figure 6.4: Current density distribution harmonics in the copper screen in the rotor frame of reference](image)

Such high loss would mean a big rise of temperature in the copper screen. Such increase was deemed unacceptable, hence it was necessary to find a configuration that would have lower losses to allow for air cooling. Therefore a full load losses evaluation was not conducted for this arrangement.

In order to minimise the loss caused by the passing slots it was necessary to move the screen away from them. At the same time it was concluded that obtaining
a good enough layer of copper on the inside wall of the stainless steel tube would be very expensive so the screen had to be moved to the outside.

In order to move the screen further from the stator one of the rotor coils had to be removed. The rotor configuration after these modifications is shown in Figure 6.5 with an additional element added to the model (the stainless steel can enclosing the field winding). It was thought that it might be important to evaluate the losses induced in this part of the rotor since its proximity to the field winding and the fact that it is the cold vessel means that any substantial losses induced there might be a considerable strain on the refrigeration system.

![Figure 6.5: Second of the configurations analysed for the losses study](image)

Losses in different parts of the rotor are shown in Table 6.2. The table shows that the losses were substantially minimised by moving the screen further away from the stator slots and when the results of Fourier transform shown in Figure 6.6 are analysed it can be seen that the tooth ripple has a much lesser effect.

<table>
<thead>
<tr>
<th>Copper Screen</th>
<th>Stainless Steel Tube</th>
<th>Pole Piece</th>
<th>Cold Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000W</td>
<td>15W</td>
<td>100W</td>
<td>1W</td>
</tr>
</tbody>
</table>

A different configuration that was analysed is shown in Figure 6.7. Here the copper screen is wrapped around the field winding assembly. This however means that the majority of losses are induced in the stainless steel can. Thus any fields that could pass through this screen would induce losses in the pole piece and in the copper layer, which would be next to the cold vessel, hence putting strain on
the cooling system. The comparison of losses in different parts of the rotor for this configuration is shown in Table 6.3. A 100W loss in the copper screen and 4W loss in the stainless steel cold vessel was unacceptable so it was decided that the best configuration is the second analysed arrangement, with the copper screen on the rotors outer surface and the number of coils reduced to 15.

In order to allow for more direct comparison between the two types of coreless designs studied, the results for the stacked rotor design (Figure 6.8) are shown in Table 6.4 and the Fourier transform of the current density distribution in the copper screen is shown in Figure 6.9. The table shows the sum of losses in both stainless steel tubes. It can be seen that this initial arrangement is not perfect (high loss is induced in the copper screen) and it would be better to have the copper screen on the outermost wall of the rotor to shield the rotor from the harmonics. Copper, being a much better conductor, would shield the rotor more efficiently than stainless steel and the loss should not affect the rotor. It would also be much easier to absorb the heat since the screen would be in the warm part of the machine, hence it would be simpler to take away excessive heat using forced air circulation. The more detailed studies were not conducted because this design was abandoned before these loss calculation studies began.

**Figure 6.6:** Current density distribution harmonics in the copper screen in the rotor frame of reference
Chapter 6 Evaluation of Losses

Figure 6.7: Third of the analysed models with copper screen wrapped around the cold vessel

Table 6.3: Loss in the different parts of rotor for the configuration shown in Figure 6.7

<table>
<thead>
<tr>
<th></th>
<th>Copper Screen</th>
<th>Stainless Steel Tube</th>
<th>Pole Piece</th>
<th>Cold Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>183W</td>
<td>1606W</td>
<td>1147W</td>
<td>9.2W</td>
</tr>
</tbody>
</table>

Figure 6.8: Full configuration of the stacked design

Table 6.4: Losses in different parts of the rotor for the design from Figure 6.8

<table>
<thead>
<tr>
<th></th>
<th>Copper Screen</th>
<th>Stainless Steel Tube</th>
<th>Pole Piece</th>
<th>Rings</th>
<th>Cold Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1200W</td>
<td>3400W</td>
<td>20W</td>
<td>5W</td>
<td>32W</td>
</tr>
</tbody>
</table>
Figure 6.9: Current density distribution harmonics in the copper screen in the rotor frame of reference for the stacked rotor design.
Chapter 7

Scalability of the designs

The cored machine built in the previous project [9] (EPSRC Project No. GR\N21253 \01) and the coreless version presented in this work are considered to be too small for the use of the superconductor to be beneficial. However the economical benefit can be achieved for much larger machines; it is therefore important to study the possibility of scaling the designs presented in this work. The results shown in this chapter consider only the electromagnetic aspect of scaling the machine. But it has to be stressed the the main difficulty in making this type of machine larger is likely to be the mechanical design.

The possible performance of the scaled up machine is considered for a machine twice the size of the demonstrator.

![Figure 7.1: Three scaled up designs analysed](image)

Figure 7.1 shows the designs which were chosen for this investigation. The first design has a salient core and each of the coils has its own flux diverter. The final
double size configuration has 20 coils. The second model has been built with 32 coils arranged to follow the curvature of the stainless steel vessel in which the rotor is enclosed. In the last design the coils are no longer shielded from the armature fields and must therefore work at a lower current; hence a larger number of coils is required. The double size arrangement required 70 coils in the rotor.

The analysis was conducted with the assumption that the interest lies in building the machine cooled by liquid nitrogen or liquid air, therefore the temperatures at which the scalability of the designs is estimated are 70K and 66K using sub-cooled liquid nitrogen and 60K with sub-cooled liquid air.

In order to better predict the critical current, the parallel ($B_\parallel$) and perpendicular ($B_\perp$) components of the magnetic flux density within each coil are combined into one in the form of an equivalent magnetic flux density parameter. By scaling down the value of the parallel component of the field by a certain factor (depending on the superconducting tape used) it is possible to approximate the critical current density by a function of the magnitude of the resulting vector. The simple interpolation function enables the critical current to be estimated for any values of $B_\parallel$ and $B_\perp$. The equivalent flux density is assumed to be:

$$B' = \sqrt{B_\perp^2 + \left(\frac{B_\parallel}{s_f}\right)^2} \quad (7.1)$$

where $B_\parallel$ and $B_\perp$ are the relevant components of the magnetic flux density and $s_f$ is a scaling factor.

Data was extracted from the graphs showing the dependence of the critical current on the magnetic field and appropriate functions were fitted. This allowed establishing the scale factor and the equivalent flux density dependence of the critical current for each temperature. A number of 2D FE models were build in order to obtain the dependence of the equivalent flux density on the current and number of turns in the coils. By equating the critical current of the tape to the current in the coils the maximum current can be estimated.

The graphs showing the maximum current as a function of the number of turns, as well as the resulting air-gap flux density, at temperatures of 60K, 66K and 70K, are shown in Figures 7.2-7.4.

The graphs show that the best electromagnetic performance is obtained by the cored design with 80 turn coils carrying 140A and producing an air-gap flux density exceeding 1T. This may be a slightly overestimated value since the results
come from a no-load 2D model and do not account for the end effects and the effects of applying load. Nevertheless, they show a high possible gain if the machine of this design is scaled. Unfortunately it can also be seen that the curve tends to flatten. The achievable air-gap flux density rises from 0.975T at 70K to 1.01T at 60K, which confirms that saturation is the limiting factor.

**Figure 7.2:** Current and air-gap flux density as functions of the number of turns for the cored design

**Figure 7.3:** Three scaled up designs analysed
Figure 7.4: Three scaled up designs analysed

Saturation is less important in the case of the second analysed design and from the graph it can be seen that the performance of the machine can be improved further by increasing the number of turns in the coils. In this case, the difficulty lies in controlling the magnetic field in the top-most coils. The diverters and the tips of the pole piece tend to saturate because they are in the region where the perpendicular component of the magnetic field driven by the coils is highest. The flux is attracted to the stator core and that contributes to the perpendicular component of the field in the top-most coils. This design allows achieving air-gap flux densities of nearly 0.55T at 66K and gains a lot by cooling it further down to achieve the flux densities above 0.67T at 60K.

The other coreless design shows remarkable scalability possibilities, provided of course that the high cost of the superconducting material is acceptable. The penalty for adopting this design is a non optimal use of the superconducting tape. The good news is that at 66K and with 80 turn coils it is possible to achieve flux density of 0.75T in the air-gap and by cooling the superconductor further to 60K it is possible to achieve nearly 1T. But the cost of using a 200A rated HTS tape at 83A current might not be economically viable.

Electric loading of the machine depends on the stator design and so does not depend on the rotor design. The graphs presented in Figure 7.2, Figure 7.3 and
Figure 7.4 allow estimating the possible magnetic loading for any of the configurations within the range considered. When the values predicted by the simulations of a double size machine are compared with the performance of the smaller size machine models they show that for the cored design the magnetic loading can be improved while for the coreless designs saturation of the flux diverters becomes more of a problem; only the one without flux diverters shows substantial benefit from scaling.
Chapter 8

Optimisation of the next generation synchronous generator for high torque applications

The constant progress in performance of superconducting materials and especially the commercialisation of YBCO superconductor allows even higher gains from the use of HTS materials in electrical power applications. The superior performance of these new YBCO coated conductors, over the previous generation of BiSCCO materials, simplifies the construction of new generation of electrical machines. In this chapter the initial design study of the next generation superconducting synchronous multi-pole generator will be discussed.

8.1 Short overview of available global optimisation algorithms

Optimisation is the process of searching for the solution that describes the best outcome for the given problem. This solution is called the optimum. The optimisation can have single or multiple (usually conflicting) objectives. Therefore in order to minimise the effort of exploring all possible combinations of parameters special optimisation algorithms are devised. These have a task of finding a parameters’ combination that gives the best outcome in the search space (all possible combinations of the parameters). The simplest algorithms for unconstrained problems are gradient based which evaluate the slope (which parameter to change...
and how significant change in the objective function it causes) for approaching the minimum and repeat that step in each iteration until satisfactory convergence is met. The problem here however depends strongly on the starting point and if it is not close to the location of the global optimum (optimum for the whole of the search space) then the algorithm will find the best value around the starting point (local optimum) and terminate there once the termination condition is met. In order to alleviate this type of problems a different type of algorithm is required one that would not get stuck in the local optimum and instead would be able to find the global optimum.

### 8.1.1 Taxonomy of global optimisation algorithms

Global algorithms can be divided into different sets. Weise [72] suggests one possible taxonomy, which is presented in Figure 8.1. As can be seen there are two major groups of algorithms, deterministic and probabilistic. Solving problems with multiple parameters and wide range of variation using deterministic algorithms would most likely require an exhaustive search through the whole domain. These algorithms use the gradient to guide the search and that means that they will find the minimum that is closest to the start point (local minimum), despite the fact that there might be a better optimum of the function (global minimum) elsewhere. On the other hand stochastic algorithms tend to use random steps, therefore can more easily escape being trapped in local optima.
State search algorithm
The state search algorithm in its basic form has been classified by Weise as deterministic, meaning that for each run it will yield the same result. The idea behind this approach is to successively analyse possible configurations (states) with the aim of finding a goal state (state with desired properties). It can be used in global optimisation if the goal is defined in a form of $f(x) \leq \text{goal}$, where $f(x)$ is the function value and goal is a certain threshold.

Branch and bound algorithm
The branch and bound algorithm is based on successive enumeration of admissible
candidate solutions. It has three basic steps of operation, branching, bounding and pruning. The algorithm’s successful termination depends strongly on the effectiveness of those steps. The branching step creates a set of subsets so that their union covers the whole search space. The bounding step estimates lower and upper bounds of the optimised function for a given subset. If at some node A the upper bound is smaller than the lower bound of some other node B, then B can be safely discarded from further search (pruned). Usually there is a variable storing the minimum lower bound observed so far and if there is a node which has a lower bound greater then this minimum can be easily discarded.

**Genetic algorithms**
A genetic algorithm is one of the stochastic algorithms belonging to a more general class of evolutionary algorithms. Its basic principle of operation is based on the biology process of evolution of species. It starts with the creation of a population consisting of randomly sampled individuals for which the function value is evaluated and based on this solution each of the individuals is assigned fitness values. Then the fittest are chosen for a step called reproduction when their genomes are mixed together or changed randomly.

**Particle swarm and ant colony**
Particle swarm and ant colony optimisation algorithms are based on biological models. The assumption is that eventually they will find the optimal way of achieving the goal. The idea behind the ant colony method is that a number of ants start from a point and explore the space in random directions eventually finding food (the goal of search). Once the objective is achieved the ant tries to get back to the start by backtracking along its original path. While traversing the space the ant leaves a pheromone trail which makes the return easier and at the same time reinforces the trail so that other ants can follow. But other ants, although guided by this general trend, can take random diversions from the path to find different source of food or maybe a more optimal path to get to the already found source of food.

In the particle swarm algorithm the procedure is somewhat similar and is based on the idea of flock of birds which spread randomly, and move in random steps; once the possible optimum is found it is advertised to all neighbouring particles which can then approach the location from their positions. Each of the moves is described by direction and length which can be variable. Each of the members of the swarm is aware of its neighbours and stores its best value so far.

**Simulated annealing**
Simulated annealing is based on the real life annealing procedure. The idea behind it is that eventually, by periodical heating and cooling of the metal, it is possible to achieve certain material properties such as hardness. The assumption is that by periodically heating up the material its defects are removed and eventually it will settle in its equilibrium. When applied to optimisation it is usually found that this method can find the global optimum provided that appropriate initial temperature and cooling speed is chosen to stop the algorithm from getting stuck in the local minimum.

8.2 Kriging assisted surrogate modelling

From the description provided earlier in this chapter it can be seen that these algorithms for global optimisation are likely to find the global optimum of the given function but the inherent cost of this search, combined with the high cost of the objective function, might render them inapplicable to real life engineering problems. One way of alleviating this problem is to use response surface modelling (also known under the name of surrogate modelling or metamodelling). In particular, methods based on kriging such as that presented in the much quoted paper by Jones and Schonlau [31] are gaining popularity in the engineering community. Response surface methods are especially good for applications in engineering problems. These types of function can be quite easily extrapolated or interpolated over large distances of the search space; therefore typically fewer evaluations are necessary when compared with the algorithms described before. The other advantage of response surface modelling is that it provides fast and good approximation to the computer model which easily and quickly allows estimating important parameters of the model and visualisation of the relationship between the variables. Methods based on kriging also give a credible stopping criterion based on expected improvement.

8.2.1 Response surface modelling and kriging basics

In order to construct a response surface a number of initial points must first be sampled. Once we have a vector of locations \( x = (x_1 \cdots x_n) \) and corresponding responses \( y(x) = (y_1 \cdots y_n) \) we can fit a model to this data. The simplest method
Chapter 8 Optimisation of the next generation synchronous generator for high torque applications

is linear regression. The model used for fitting this data is described by:

\[ y(x_i) = \sum \beta_h f_h(x_i) + \epsilon_i \quad (i = 1, \cdots, n) \quad (8.1) \]

where \( f_h(x) \) is a linear or nonlinear function of \( x \) and \( \beta_s \) are coefficients that need to be estimated. \( \epsilon(i) \) are independent error terms that are assumed to have normal distribution with the mean zero and variance \( \sigma^2 \). The main problem with this approach is choosing appropriate shape functions for the regression terms. Using too few coefficients leads to under fitting and large error terms, while using too many leads to over fitting (when the approximation has variations that do not exist in the real function). An appropriate choice is very difficult without a priori knowledge of the function, but if it was known it might not be necessary to create such a complicated model.

Jones [31] states that the assumption, usually made in regression analysis, that errors are not correlated is false in case of the deterministic computer code. Errors are not associated with noise or measurement error but simply with the modelling error. Hence it can be stated that the error terms are really functions of left-out terms in \( x \), so \( \epsilon_i \epsilon(x_i) \). That also implies the continuity of \( \epsilon(x) \) if \( y(x) \) is continuous.

This implies that, if two points \( x_i \) are close to each other, so are their errors \( \epsilon(x_i) \), thus they are correlated. This assumption is the base for stochastic approach for response surface modelling. The correlation is assumed to be related to the distance, but instead of using Euclidean distance a special weighted distance formula is used:

\[ d(x_i, x_j) = \sum_{h=1}^{k} \Theta|x_i^h - x_j^h|^p \quad (\Theta_h \geq 0, p \in [1, 2]) \quad (8.2) \]

By using this distance the correlation between the errors of \( x_i \) and \( x_j \) can be expressed as follows:

\[ \text{Corr}[\epsilon(x_i), \epsilon(x_j)] = e^{-d(x_i, x_j)} \quad (8.3) \]

The properties of this function make the correlation close to unity when the distance between the points is small and near zero when the distance is large. The parameter \( \Theta_h \) is an indicator of how important the variable \( x^h \) is. If even small changes in \( x \) cause big changes in the function values then \( \Theta_h \) is large, hence indicating high importance of this variable. \( p_h \) describes smoothness of the function.
in direction $h$.

Jones suggests that modelling correlation in this way is so good that regression terms can be replaced with a simple constant, hence giving the following model

$$y(x_i) = \mu + \epsilon(x_i) \quad (i = 1, \ldots, n) \quad (8.4)$$

where $\mu$ is the mean of the stochastic process and $\epsilon(x_i)$ is a variable having normal distribution with mean 0 and variance $\sigma^2$. The estimates of $\mu$ and $\sigma^2$ are tightly connected with estimates of the correlation and its parameters $\Theta_h$ and $p_h$. The complete model has $2k+2$ parameters $\mu, \sigma^2, \Theta_1, \ldots, \Theta_k$ and $p_1, \ldots, p_k$. To estimate these parameters a likelihood function has to be maximised. Provided that there is a vector of observations $y$ and a vector of sampled points both of length $n$ we can build an $n \times n$ correlation matrix $R$ with entries $(i,j)$ given by $8.3$. If $1$ denotes a vector of $n$ ones then the likelihood function is given by:

$$\frac{1}{(2\pi)^{\frac{n}{2}}(\sigma^2)^{\frac{n}{2}}|R|^{\frac{1}{2}}} \exp \left[ -\frac{(y - 1\mu)'R^{-1}(y - 1\mu)}{2\sigma^2} \right] \quad (8.5)$$

and the values of $\mu$ and $\sigma^2$ that maximise the likelihood function are

$$\hat{\mu} = \frac{1'R^{-1}y}{1'R^{-1}1} \quad (8.6)$$

and

$$\hat{\sigma}^2 = \frac{(y - 1\hat{\mu})R^{-1}(y - 1\hat{\mu})}{n} \quad (8.7)$$

Substituting these equations into equation $8.5$ gives us a so called ‘concentrated likelihood’ function which depends only on $\Theta_h$ and $p_h$. Given these parameters it is possible to derive the best linear unbiased predictor (BLUP) for an untried site $x^*$ which is:

$$y(x^*) = \hat{\mu} + r'R^{-1}(y - 1\hat{\mu}) \quad (8.8)$$

where $\hat{\mu}$ is our mean, the second term is the departure from the mean base on the correlation between already sampled points, and $r'$ denotes a correlation vector between already sampled points and a new untried site. The exact derivation of the BLUP can be found in [60]. This approach gives also an estimate of the error which is denoted in the literature as:

$$s(x^*)^2 = \sigma^2 \left[ 1 - r'R^{-1}r + \frac{(1 - 1R^{-1}r)^2}{1R^{-1}r} \right] \quad (8.9)$$
(full derivation of this error can also be found in [60]) where because $x^*$ is correlated with the sampled points, the $-1R^{-1}r$ term is present (representing the reduction in prediction error, 0 in case of no correlation between the point) and because $\mu$ and $\sigma^2$ are not know exactly, but instead are estimated, the last term has to be present.

In order to use this technique in global optimisation one would want to sample as few points as it is necessary to get a reasonable starting response surface and starting from there sample more points that would improve the fit and allow to find the global minimum. Usually to improve the fit the search space should be explored at the regions of high uncertainty of the fit while to find the global minimum it should be sampled when the chance of improving current minimum is the highest. These are two conflicting objectives, hence it is necessary to find a way of guiding the process.

A figure of merit that is most widely quoted, and was first proposed by Mockus, is the expected improvement. This criterion is a reasonable way of balancing global (exploration) and local (exploitation) search of the design space. If we treat the uncertainty about $y(x)$ as some normally distributed random variable $Y$ with mean and standard deviation given by our computed model and estimated error then we can compute our expected improvement as follows:

$$E[I(x)] = (f_{\text{min}} - \hat{y})\Phi\left(\frac{f_{\text{min}} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\text{min}} - \hat{y}}{s}\right)$$

where $f_{\text{min}}$ is the current best sampled minimum $\hat{y}$ is our prediction at $x$, $s$ is the square root of the estimated square error, $\Phi$ denotes cumulative distribution function and $\phi$ probability density function (for the standard normal distribution function). It is easy to see that the first term is driven by the possibility of improving the current minimum, whereas the second term would encourage sampling at the point where there is more uncertainty about the functions value. It is also easy to show that at already tried points, where $s$ is 0, the expected improvement also takes the value of 0.

### 8.3 The optimisation algorithm

Based on the theory described in the previous section an optimisation algorithm has been been programmed. In case of the superconducting machine the physical properties of the superconducting tape have to be considered as additional
A minimum radius of 35 mm for the end of the coils was assumed. The maximum radius of the coil sides was set to 750 mm since, in the course of the current project, the cryogenics part of the research team gained confidence that they can reliably build coils with these radii. Since the analysis was restricted to a demonstrator size machine the coils length was set to 300 mm and the maximum length of the tape be of no more than 3000 m per pole, although the real region of interest due to the high cost of the superconductor is closer to 300 m per pole. The other constraints were imposed by the properties of the superconductor (current dependence on the external magnetic field for perpendicular and parallel field directions) and were included inside the finite element model.

The optimizer was allowed to vary the number of turns in coils, the number of coils and the number of coils with reduced number of turns.

The algorithm, shown in Figure 8.2, starts with the creation of vectors with all possible permutations of parameters; this ensures that all the constraints are met.

![Figure 8.2: The simplified diagram of the optimisation algorithm](image)

Using the Latin hypercube sampling method, 15 vectors are chosen from the initial set. At these points the function is evaluated and then the response
surface fitted.

At each step, when the evaluation of the real function is required, a Matlab optimisation script calls Vector Fields Opera package. In this step the script has to find the critical current for a given configuration of the number of coils and turns. This is an additional though necessary step complicating the analysis. It requires that the data for the superconducting tape is incorporated into the finite element model. Each of the curves (the critical current dependence on magnetic field density, both perpendicular and parallel components) had to be fitted to the manufacturer’s data and inserted as functions in the finite element script producing the solution.

It is necessary that the maximum critical current is found for a given set of parameters. The script runs a binary search. At each step, it evaluates both components of the magnetic field and the one which limits the current is used. The binary search starts by setting lower and upper bounds, the minimum critical current and the maximum critical current achievable by a single strip of the tape. This range is then divided in half and with that current the fields in the coils are evaluated. If the value found lies in the upper half of the interval then the lower bound is set to the value just evaluated, the domain is divided into two again and the search continues until the satisfactory convergence is achieved. If the search value is in the lower part of the interval the upper bound is set to the current value of the argument of the function. The tolerance for the search of the current was set to 10E-3 as this is considered satisfactory in the case of the current in the superconducting tape. After the search was finished the value of the function is returned to the Matlab script.

An additional difficulty for writing the optimisation algorithm was the fact that all of the parameters for the optimiser are discreet values, but none of the algorithms available in Matlab operates with discreet values. In the case of optimising the demonstrator, this problem was solved by using exhaustive search to find the best expected improvement.

Based on the prediction and its error it is possible to estimate the expected improvement and find the maximum value of it. The search for the next best point to evaluate is greatly simplified because of the existence of the vector of all possible permutations of parameters. In the case the vector length is no more than 40000 so the cost of the exhaustive search is still much lower than that of running a finite element model. The expected improvement is also a very good termination criterion for the algorithm; however, as Jones [32] suggested,
it is better to decrease the tolerance below the one which we are able to accept because the kriging interpolation method usually underestimates the error, thus leading to an underestimation of the expected improvement. The error occurs because instead of knowing exactly $\mu$ and $\sigma^2$ the parameters are estimated. In order to alleviate this problem the algorithm programmed was multiplying the error estimate by a factor at each time the expected improvement suggested that it has met the stopping criterion. The limit for the multiplication factor was set to five, so it had five possible steps. The same factor was used initially to encourage the algorithm to explore more of the search space. Expected improvement at the beginning of the process suggests the points where the uncertainty of the fit is high but after evaluating the function at this point it quickly switches to exploitation phase. By increasing the error artificially the algorithm was encouraged strongly to explore the regions of high uncertainty and therefore improve the fit.

### 8.3.1 Optimiser for the real size machine

Optimising the real size machine is a much more complicated problem. Not only the range in which the variables can vary but also the number of parameters is larger. Hence it is not practical to construct a list of all possible configuration permutations, nor is the exhaustive search for the best expected improvement possible.

At its first stage the algorithm would construct the vector of initial configurations. It is enough to have around 25 configurations for the problem with 5 variables to have an initial response surface fitted.

The other modification to the original algorithm came from the impracticality of using exhaustive search. Instead a simple random search has been applied. The search is stopped if either no significant improvement in the best value of expected improvement obtained so far is found, or if the cost of further search exceeds the cost of running an extra finite element model.

This algorithm has been adopted for simplicity because none of the algorithms available in Matlab allows using discrete values of parameters.
8.4 Investigating possible termination conditions

It is well described in the theory of DACE (Design and Analysis of Computer Experiments Jones [31]) that the error is underestimated. Hence, in order to obtain viable results it is important to use an appropriate termination condition. Jones [31] suggests that in order to get 1% accuracy it is necessary to set the termination condition to one order of magnitude less. Using a Coreless Rotor and Slotless Stator configuration at 40K as the test case a few possible combinations of termination conditions were evaluated. The finishing tolerance was varied between 100E-6 and 10E-9. Figure 8.3 shows the results for the finishing tolerance equal to 10E-3, while Figure 8.4 shows the results for the finishing tolerance set to 10E-9. As can be seen the optimiser, explored the search space a little better in the latter case by putting more points on the Pareto front, but the overall performance is not satisfactory.

![Figure 8.3: Optimisation results for Coreless Rotor and Slotless Stator at 40K with finishing tolerance set at 10E-3](image-url)
Therefore a different approach was tested. Instead of lowering the finishing tolerance the estimate of the error was multiplied by a certain factor in order to encourage the optimiser to explore the search space. The scaling factor was varied between 1 and 6 to estimate its influence on the results.
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torque applications

Figure 8.6: Multi Objective optimisation results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 10E-3 and the error scaling factor set to 5

It can be seen in Figure 8.5 that this successfully encouraged the optimiser to explore the problem space and resulted in a better mapped Pareto front. To even better map the space away from the front this factor has been set to 5 as in Figure 8.6. A scaling factor of 5 was used for all runs.

8.5 Single objective optimisation of the six pole demonstrator generator

The first problem considered for optimisation was a single objective optimisation of the demonstrator size generator. A number of possible configurations were chosen (Figure 8.7) and each of them had been analysed assuming three different operational temperatures 40K, 50K and 60K. The evaluations were required to estimate and select the default operational temperature which would give a satisfactory performance at the size considered.
It has been decided that a 6 pole machine will be the smallest possible representation of the full size multi-pole machine. For all the presented configurations except the one shown in Figure 8.7(c) and d), two stator configurations were considered: slotted and slotless. The use of a slotted stator with a completely coreless rotor would limit the performance of the machine because of tooth saturation due to high fields which might be achieved in this combination. The computer models were designed in a way that would simplify their reuse and modification. Most of the parameters relevant to any given configuration were created as variables; however, in order to simplify the analysis at this preliminary stage, not all variables were used for the optimisation process. The number of coils, the number of turns and the number of coils with reduced number of turns were used by the optimiser.

As the first group of problems, the optimisation of the demonstrator scale machine has been chosen. Figure 8.8 to Figure 8.13 show the results obtained for considered configurations at 60K.
Figure 8.9: Single Objective optimisation for Slotless stator Inner Cored rotor configuration at 60K

Figure 8.10: Single Objective optimisation for Slotless stator Partially Cored rotor configuration at 60K
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Figure 8.11: Single Objective Optimisation for Slotted stator Fully Cored rotor configuration at 60K

Figure 8.12: Single Objective Optimisation for Slotted stator Inner Cored rotor configuration at 60K
Figure 8.13: Single Objective Optimisation for Slotted stator Partially Cored rotor configuration at 60K

The results for other temperatures are shown in Appendix 1. The figures show the length of the tape as a function of the fundamental of the flux density in the air-gap of the machine. The results presented in this section are for a single objective optimisation. The objective was to maximise the fundamental component of the air-gap flux density so, as might be expected, most of the points that the optimiser wanted to evaluate were aggregated in the top most corner where this flux density is the highest; the length of HTS tape is also the highest in this region, but this cost is not considered by the optimiser.

Figure 8.12 shows the results for the fully cored rotor with a slotted stator. When compared with other results it can be seen that this configuration easily obtains the highest flux density in the air-gap. However, what is shown in the figure does not tell the whole story; because only the fundamental is shown here the effects of saturation are not easily seen. In fact lowering the operational temperature to 40K allows flux densities of over 2.5T to be obtained, but only under highly saturated conditions. The very rapid increase in the tape length required per pole can be attributed to saturation. First the teeth get saturated and then the back iron, the depth of which for this optimisation was set to 150 mm.

Figure 8.8 shows the results of optimisation obtained for the fully coreless ma-
machine. It can easily be seen that without the use of iron it is much harder to obtain high flux densities. The coreless machine can deliver an air-gap flux density of only 0.8T, whereas the cored design gives over 2.3T. Lowering the temperature to 40K allows obtaining up to 1.8T. In addition to that, because saturation is no longer a limiting factor, as can be seen from a much smaller slope of increase in the required length of tape, the potential for obtaining higher flux densities is much greater.

Inner cored rotor can offer the advantage of placing the coils even closer to the air-gap, provided that a satisfactory solution for bracing them may be found. The partially cored configuration offers the advantage of superconductor savings while still giving a reduction in mass. In these simulations the partially cored rotor configuration had a full iron body underneath the coils, mainly for simplicity, but there is nothing that should prevent using a ring of iron instead, since only a part of the core is used for conducting flux. This, however, would additionally complicate the optimisation process since the proper thickness of this ring would need to be found. Figure 8.12 and Figure 8.10 show the difference between the air-gap flux densities achievable when the slotted and slotless stator configurations are adopted. As expected, in the case of the slotted stator the air-gap flux densities are much higher. However, keeping in mind the results of loss evaluation presented in Chapter 5, it can be expected that losses in the rotor of such a machine could be quite high. Therefore, the results obtained for this simulation (slotted configuration) can only be treated as a rough estimate of what is possible. The proper simulation would require much more complicated analysis which would need to include solving a transient model in order to estimate any possible losses. This would substantially complicate the analysis and at a great cost to the objective function; therefore it was not conducted in this preliminary design study.

8.6 Multiobjective optimisation of a demonstrator size machine

The initial single objective optimisation supports the view that an increased use of HTS tape yield diminishing returns; hence the Pareto front can be expected to be convex. The optimiser is therefore made to search for points at this front by applying suitable penalty factors. A scalarization approach has been adopted and effectively multiobjective problem is transformed to a single objective one.
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Figure 8.14: Multi Objective optimisation results for Coreless Rotor with Slotless Stator at 40K with YBCO winding

Figure 8.15: Multi Objective optimisation results Coreless Rotor with Slotless Stator at 60K with YBCO winding

Figures 8.14 and 8.15 show the results of optimisation for a fully coreless machine with a slotless stator. Lowering the temperature increases the maximum flux...
density to 1.8T from 1.2T. For the length of the tape that is likely to be available for a demonstrator (about 3000 m per machine) and an appropriate coil length, we could expect a flux density of 0.75T at 40K or 0.45T at 60K.

However, in the case of the demonstrator, an interesting choice is to use partially cored rotor (Figure 8.7) where the effect of the inner iron ring is similar to having additional inner coils without the disadvantage of having the current limited by those inner coils. This reduces the cost of building a demonstrator while still achieving similar air-gap flux densities. Not only does it make it easy to insulate the core from the cryostat in which coils would be enclosed, but also simplifies the construction of a support structure for the core and coils. The choice of material for the core is not limited to the materials that are certified for cryogenic temperatures.

![Figure 8.16: Multi Objective optimisation results Partially Cored Rotor with Slotless Stator at 40K with YBCO winding](image)

Figure 8.16 shows the optimisation results for partially cored design at 40K. It shows that flux densities approaching 2T are possible at winding temperature of 40K and that in order to achieve 1.5T, 6000 m of tape would be required. In order to investigate if it is possible to gain higher flux densities with lower tape use the rotor size has been reduced by 40mm (Figure 8.17) and by 80 mm (Figure 8.18). The smaller reduction does not influence the predicted performance except at flux densities above 1.8T while the larger reduction in rotor size does reduce the
maximum achievable flux density significantly, but there is no noticeable increase in the length of the tape required to produce flux densities below 1.4T.

Figure 8.7(b) shows a configuration with rotor poles. This configuration has also been investigated; however, it would be difficult at this stage to precisely predict the design of the cryostat. Therefore two possible configurations were analyzed; the difference between them is in the clearances allowed for cryostat and thermal insulation.

**Figure 8.17:** HTS tape requirements and achievable air-gap flux densities for the partially cored rotor slotless stator design at winding temperature of 40K and the 290mm rotor radius

**Figure 8.18:** HTS tape requirements and achievable air-gap flux densities for the partially cored rotor slotless stator design at winding temperature of 40K and the 250mm rotor radius
Figure 8.19 shows the performance of the machine with 10mm clearances around the coil. This would be enough for a cold rotor and possibly a slightly warmer core, which would not be fully insulated from the cryostat and therefore would most likely operate at a temperature around 73K, while the coils could be cooled to 40K. The plot in Figure 8.20 shows the results of optimisation with a 24mm clearance around the coils which should be enough to fully insulate the core from the very low temperature of the field winding.

**Figure 8.19:** HTS tape requirements and achievable air-gap flux densities for a inner cored rotor slotless stator design at winding temperature of 40K and 10mm clearances around the winding

**Figure 8.20:** HTS tape requirements and achievable air-gap flux densities for a inner cored rotor slotless stator design at winding temperature of 40K and 24mm clearances around the winding
Some optimisation has also been conducted for the machine using a 1G superconductor and copper to show the difference in achievable flux densities. The machine with copper windings had the air-gap reduced to 5 mm (from 24 mm for superconducting devices) and all the clearances around the coils reduced to 3mm, since here only electrical insulation would be required. In addition, the model has no current in stator conductors. The current density in the winding was assumed to be 5A/mm². Figures 8.21 to 8.23 give a good comparison of what is possible with each technology. It can be seen that a copper winding cannot compete with a superconducting winding. Significant differences can also be seen between the results for the two different types of a superconductor. Here the most important factor is the critical current dependence on magnetic field. YBCO is less dependent and this can easily be seen when the plots are compared.

![Figure 8.21: Multi Objective optimisation results Coreless Rotor with Slotless Stator at 60K with YBCO winding](image-url)
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Figure 8.22: Multi Objective optimisation results Coreless Rotor with Slotless Stator at 60K with BiSCCO winding

Figure 8.23: Multi Objective optimisation results Coreless Rotor with Slotless Stator with Copper winding
8.7 Achievable performance of full size machine

Since the demonstrator is supposed to be a test case for the problems of a full size machine, optimisation of such a machine has also been conducted. True optimisation of such a machine would aim to maximize the achievable torque as a function of the costs of losses, material usage and mass (which is very important in case of wind farm application). However, these costs are hard to estimate and the results of such an optimisation would soon become out of date as the prices and performance of the materials change. These problems have been avoided by fixing the depth of the stator winding, rotor diameter and the length of the stator core, hence making stator copper losses and refrigeration losses constant. This also fixes the ratio between the torque and the air-gap flux density and, since the air-gap flux density in a conventional machine cannot be much above 1T, it enables the comparison of performance. In order to make these parameters more universal they are divided by the area of the cylindrical surface at which the air-gap flux density is evaluated.

**Figure 8.24:** Achievable air-gap flux density in a full size machine as a function of the quantities of stator iron and HTS tape used. These quantities are expressed per unit air-gap surface area

Figure 8.24 shows the results obtained by the optimiser as a function of the two principal cost components. The results were obtained for the machine with
coreless rotor and slotless stator at the winding temperature of 40K. It can be seen that with about 50000 m of tape for each square meter of the air-gap surface area, an air-gap flux density of 2T can be obtained. This is achieved with only 0.16m$^3$ of stator iron per square meter of air-gap surface. Increasing the flux density to 2.7T requires 60% more HTS tape and a similar increase in the volume of the stator core.
Chapter 9

Conclusions

This thesis has supported the research undertaken for the EPSRC project. The aim of the project was to build and test a superconducting synchronous generator with a coreless rotor using BSCCO based field winding. By the time this thesis is submitted the machine has been built and most likely tested.

The thesis presents a method for estimating machine parameters through linear modelling of a Standstill Frequency Test. In case of 3D modelling, the computation times for a fully transient 3D solution would be quite significant, while the method presented in this thesis simplifies substantially the model and reduces computation times considerably. The obtained parameters are much lower than the ones for a conventional machine, hence confirming the much better stability of superconducting machines quoted in literature.

During the research a method combining non-linear intrinsic properties of a superconducting material with kriging assisted response surface modelling was developed and used to predict the performance of future class of machines utilizing YBCO winding.

During the studies presented here it was also found that in case of using BiSCCO windings it was necessary to use flux diverters to limit the perpendicular component of flux density inside superconducting coils, otherwise the full current carrying capability of the superconductor would not be achieved. In the case of a coreless superconducting machine, however, this is not easy as it becomes difficult to support the flux diverter rings while at the same reducing the weight of a machine (which was one of the main goals of the coreless design). An alternative is of course to use a magnetic core in the rotor, which can offer the necessary support for the coils while at the same time providing magnetic path for the flux.
These problems can be easily alleviated by using YBCO windings. The results shown in Chapter 7 predict much better performance of a coreless machine utilizing YBCO field windings, thanks to their much better performance in terms of field dependence. But in order to fully utilize their potential it would be beneficial to use a slotless stator with a coreless rotor. The other benefit of using a slotless stator becomes clear when losses in various components of the machine are analysed. It is the tooth ripple that induces the highest losses in the screen shielding the rotor, hence using a slotless stator would significantly reduce these losses.

The results presented in Chapter 7 can guide the future superconducting machine developments. Inner cored and coreless design with a slotless stator have been chosen as the next possible configurations for the demonstrator size machines that might be build at the University of Southampton during the next project.

In terms of the analysis of the results and their presentation, this thesis explores the different possible designs of a superconducting machine and investigates methods that can be used to support such design. Some further detailed discussion follows.

Chapter 3 shows the evolution of the design from the initial idea of using flux diverters, to direct the flux in a way that minimises the perpendicular component, to a design which abandons this concept and relies on self shielding of superconducting coils. The first design gives the best flux density distribution in the air-gap, but at the same time is the most difficult to build. The low temperatures required by superconducting coils impose many complications on the mechanical design. Different temperature contraction coefficients of the materials in the rotor are hard to accommodate and cause large mechanical stresses. The last design has the advantage of a much simpler construction but its electromagnetic performance is noticeably worse. The chapter describes the methods used to optimise the pole shape for all of the designs considered.

In Chapter 4 a method of obtaining machine parameters is developed. This method is particularly useful when it is not possible, or it is computationally too demanding, to simulate the Standstill Frequency Response Test. It also shows that in the case of a superconducting machine the synchronous reactances are significantly smaller than for a conventional machine of similar rating. Moreover, the superconducting machine of the design analysed in Chapter 4 has a much smaller quadrature reactance.

Chapter 5 considered the losses in a superconducting machine. Losses in the
screen of the machine analysed are quite substantial and decreasing them required making the rotor smaller. Losses in superconducting machines have to be controlled carefully. The loss in the cold part of the machine contributes to an additional load on the refrigeration system. It is of crucial importance to minimise this load to increase the benefit of adopting high temperature superconducting field winding. Tooth ripple fields were identified as the reason for high losses in the copper screen shielding the rotor. If the arrangement is such that the screen is in the cryogenic part of the machine, the minimisation of these losses is of particular importance. These losses can be substantially reduced by adopting a different stator winding configuration or by using an air-gap winding for stator conductors.

Chapter 6 and 7 investigate machines with sizes larger than the demonstrator built for the current project. The double sized machines considered in Chapter 6 can achieve higher flux densities; however, when the iron in the rotor is retained it can be seen that it becomes saturated, thus limiting the benefit of using a superconducting field winding. A real improvement comes with air cored designs, although for now such machines cannot achieve the air-gap flux densities that would allow them to compete with conventional machines.

However, by using YBCO (2G) superconducting windings it is possible to achieve much higher flux densities for machines with the same configuration. Thanks to smaller field dependence it is not necessary to use flux diverters while maintaining much higher currents in coils when compared to BiSCCO superconducting tapes.

In Chapter 7 a kriging assisted method was used to investigate the potential performance of the demonstrator with an YBCO field winding. The proposed method significantly reduces the computational effort in predicting performance for a wide range of parameters’ variation. The same method was used to estimate the performance of a full size machine, although without further studies it is hard to predict if such a machine would indeed be significantly better than a conventional one of similar rating.

The task of designing a superconducting machine is not an easy one. Many difficulties are caused by extreme conditions to which the rotor assembly is exposed and thus has to be designed for. Very often - as has been hinted in the previous chapters - the difficulties arise because of the mechanical reasons. Different contraction coefficients of various materials make it extremely difficult to come up with an arrangement which would make them absorbed in a way that does not put stress on the assembly during the cooling down process and operation.
A very important aspect of designing a superconducting machine is to minimise the heat leak from the cold part. This causes many practical problems, and additionally requires the presence of clearances in the machine itself for heat insulation. The air-gap has to be extended to accommodate the cryostat and the vacuum vessel.

This thesis shows a number of methods that can be used for optimising the electromagnetic design of such machines, from minimising the harmonic content of the output waveform (Chapter 3) to optimising the rotor configuration for maximising the performance with minimum of superconducting tape required.

The thesis also shows the method for finding equivalent circuit parameters for a superconducting machine by modelling the Standstill Frequency Response Test with a set of linearised models in frequency domain. Chapter 5 presents the results of studies aimed at evaluation of the losses in the rotor. It can be seen that this part is of particular importance when trying to argue the economic benefits of a superconducting design.

The objectives of this thesis have been to provide electromagnetic modelling and simulation support to the design and development of a coreless rotor of a HTS generator project undertaken at Southampton University under an EPSRC research grant. The focus of this work was three-fold:
- to develop efficient models, using commercial package OPERA, for analysis and performance prediction of superconducting machines, with special emphasis on 3D modelling,
- to propose and develop reliable and computationally efficient, for practical purposes, algorithms for estimation of equivalent parameters and evaluation of losses,
- to explore and analyse possible future superconducting designs, through modelling and simple pareto - optimisation, to provide guidance regarding future direction of research in this field.

All these objectives have been fulfilled and the models and techniques developed provided valuable assistance for the EPSRC project. No work, however, is complete and several directions could be explored further. The following aspects are suggested for future consideration:
- The effectiveness of the screen has to be evaluated in more detail. Magnetic fields at certain frequencies can penetrate the screen and induce varying currents in the field winding, hence leading to AC loss in the superconductor.
- During transients large currents might be induced in the field winding, which
might lead to the quench of the superconductor. This situation is undesired but it should be further investigated if it may occur. Therefore the influence of transients (short circuits in particular) on the superconductor should be evaluated.

- During operation the field winding can be subjected to different types of forces, not only during steady state operation, but especially during short circuits, unbalanced loads etc. Efficient models should be developed to estimate the magnitude of such forces.

- Superconducting field windings can produce air-gap flux densities higher than normally present in convectional machines - 1T. Hence it is necessary to develop stator windings that can benefit from such high air-gap flux densities. By proper design of a stator the full potential of the superconducting rotor could be utilised.

- It is also important to develop more precise models for estimating magnetic field components in the superconducting winding in 3D models. The distribution of magnetic field in a superconducting rotor is important. The anisotropic properties of the superconductor limit its tolerance to magnetic fields at angles different from parallel to the broad face of the tape. This is of particular importance in the end regions of the rotor which are often not modelled in 2D.

- It should be helpful to develop 3D models with circuit windings. It would help to estimate the equivalent machine parameters for circuit analysis in the real system.

- It is also important to optimise the arrangement of the coils to obtain the best distribution and the highest magnetic flux in the air-gap while at the same time minimising the amount of the superconductor. After some modification the codes developed for this thesis could be used for this purpose.

- Recent research reports mention improvements of the field dependence of the superconductor, in both perpendicular and parallel components to the broad face of the tape. However, more detail examination of the results show that the critical current is lower when the field is at any other angle. Therefore it will be required to incorporate this into the finite element models.
Bibliography


[77] Wanying Zhang, Youqing Zhou, Weiming Zhao, Xuhong Zhina, Qing Zhu, Fuhai Li, and Zhongfa Li. The simulating research on a improved magnetic shielding type of htsclf. pages 1598–1602, 2008.
Appendices
1 Modelling in the rotor frame of reference

A circuit equation of a winding in any type of electrical machine can be simply represented in general form as $V=ZI$. A simultaneous solution of all winding equations will yield the currents from which the torque and the component voltages can be derived. During operational conditions all the windings are in relative motion and variation of at least some of the inductances with angular position can be expected.

It is however possible to replace time varying inductances with related, but constant inductance coefficients. It is achieved by translating the quantities into a frame of reference fixed on the rotor (so called Parks Transformation). Thanks to that transformation the analysis is greatly simplified. All transformed quantities see constant magnetic paths therefore in case of transient analysis the longer time step can be used. The transformed rotor and stator voltage equations consist of three parts, resistive voltage drop, time dependent flux variation and voltage induced due to relative rotation with respect to the rotating frame of reference.

In order to transform three phase quantities into a $dq$ rotating reference frame a transformation matrix needs to be introduced. The reverse transformation is represented by:

$$
\begin{bmatrix}
S_d \\
S_q \\
S_0
\end{bmatrix} = \begin{bmatrix}
\cos(\Theta) & \cos(\Theta - 120^\circ) & \cos(\Theta + 120^\circ) \\
-\sin(\Theta) & -\sin(\Theta - 120^\circ) & -\sin(\Theta + 120^\circ) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix}
$$

end its reverse
\[
\begin{bmatrix}
S_a \\
S_b \\
S_c \\
\end{bmatrix} =
\begin{bmatrix}
\cos(\Theta) & \sin(\Theta) & 1 \\
\cos(\Theta - 120^\circ) & \sin(\Theta - 120^\circ) & 1 \\
\cos(\Theta + 120^\circ) & \sin(\Theta + 120^\circ) & 1 \\
\end{bmatrix}
\begin{bmatrix}
S_d \\
S_q \\
S_0 \\
\end{bmatrix}
\]

The advantage of this type of transformation is that it is unitary that means that transformation matrix and its inverse are the transpose of each other. It is also power invariant that means that after using this transformation power is still a sum of appropriate components.

\[p_s = v_d i_d + v_q i_q + v_0 i_0\]

and torque becomes:

\[T = \left(\frac{\text{poles}}{2}\right) (\lambda_d i_q - \lambda_q i_d)\]

The transformation for three phase currents is derived below with the assumption that \(\Theta = \omega t\) and that alignment of rotor \textit{mmf} is aligned with stator phase A at time=0.

\[i_a = \sqrt{2}I \cos(\omega t) \quad i_b = \sqrt{2}I \cos(\omega t - 120^\circ) \quad i_c = \sqrt{2}I \cos(\omega t - 240^\circ)\]

which by using trigonometric identities \(\cos^2\alpha = \frac{1}{2}(1 + \cos 2\alpha)\) and \(\cos \alpha \sin \alpha = \frac{1}{2}\sin 2\alpha\)

\[i_d = \frac{2}{3} [i_a \cos(\omega t) + i_b \cos(\omega t - 120^\circ) + i_c \cos(\omega t - 240^\circ)]\]

\[i_q = \frac{2}{3} [-i_a \sin(\omega t) - i_b \sin(\omega t - 120^\circ) - i_c \sin(\omega t - 240^\circ)]\]

\[i_d = \frac{2}{3} \sqrt{2} I [\cos^2(\omega t) + \cos^2(\omega t - 120^\circ) + \cos^2(\omega t - 240^\circ)]\]

\[i_q = \frac{2}{3} \sqrt{2} I [-\sin(\omega t) \cos(\omega t) - \sin(\omega t - 120^\circ) \cos(\omega t - 120^\circ) - \sin(\omega t - 240^\circ) \cos(\omega t - 240^\circ)]\]
Leads to:

\[ i_d = \sqrt{2} I \]

and

\[ i_q = 0 \]

The similar procedure can be applied to stator current harmonics (here derived only for direct axis and 5\textsuperscript{th} and 7\textsuperscript{th} harmonics).

\[ i_{a5} = \sqrt{2} I \cos(5\omega t) \]
\[ i_{b5} = \sqrt{2} I \cos(\omega t - 120^\circ) \]
\[ i_{c5} = \sqrt{2} I \cos(\omega t - 240^\circ) \]

\[ i_d = \frac{2}{3} \sqrt{2} I_5 [\cos(5\omega t)\cos(\omega t) + \cos(\omega t - 120^\circ)\cos(\omega t - 120^\circ) + \cos(5\omega t - 240^\circ)\cos(\omega t - 240^\circ)] \]
\[ i_q = \frac{2}{3} \sqrt{2} I_5 [-\sin(\omega t)\cos(5\omega t) - \sin(\omega t - 120^\circ)\cos(5\omega t - 120^\circ) - \sin(\omega t - 240^\circ)\cos(5\omega t - 240^\circ)] \]
\[ i_d = \frac{2}{3} \sqrt{2} I_7 [\cos(7\omega t)\cos(\omega t) + \cos(7\omega t - 120^\circ)\cos(\omega t - 120^\circ) + \cos(7\omega t - 240^\circ)\cos(\omega t - 240^\circ)] \]
\[ i_q = \frac{2}{3} \sqrt{2} I_7 [-\sin(\omega t)\cos(7\omega t) - \sin(\omega t - 120^\circ)\cos(7\omega t - 120^\circ) - \sin(\omega t - 240^\circ)\cos(7\omega t - 240^\circ)] \]

\[
\begin{align*}
\cos u \cos v &= \frac{1}{2} [\cos(u - v) + \cos(u + v)] \\
\cos u \sin v &= \frac{1}{2} [\sin(u - v) - \sin(u + v)]
\end{align*}
\]

\[ i_d = \frac{\sqrt{2}}{3} I_5 [\cos(4\omega t) + \cos(6\omega t) + \cos(4\omega t + 240^\circ) + \cos(6\omega t) + \cos(4\omega t + 120^\circ) + \cos(6\omega t)] \]
\[ i_d = \frac{\sqrt{2}}{3} I_7 [\cos(8\omega t) + \cos(6\omega t) + \cos(8\omega t + 240^\circ) + \cos(6\omega t) + \cos(8\omega t + 120^\circ) + \cos(6\omega t)] \]
\[ i_d = \sqrt{2} (I_5 + I_7) \cos(6\omega t) \]

The proves that stator harmonics when transformed into the rotor fixed rotating frame of reference appear on the rotor as the 6\textsuperscript{th} harmonic. The same procedure can be applied to all other harmonics in effect rendering 9\textsuperscript{th} and 11\textsuperscript{th} to appear as 10\textsuperscript{th} etc.

Similar analysis can be performed for other machine variables. Voltage relations of the machine in dq0 reference frame are follows:

\[ v_d = R_a i_d + \frac{d \lambda_d}{dt} - \omega_{ms} \lambda_q \]
\[ v_q = R_a i_q + \frac{d \lambda_q}{dt} - \omega_{ms} \lambda_d \]
Appendix 1 Modelling in the rotor frame of reference

\[ v_f = R_f i_f + \frac{d\lambda_f}{dt} \]
\[ v_0 = R_a i_0 + \frac{d\lambda_0}{dt} \]

And by applying it to flux linkages and currents the following are obtained:

\[ \lambda_d = -L_d i_d + L_a f i_f \]
\[ \lambda_q = -L_q i_q \]
\[ \lambda_f = -\frac{3}{2} L_a f i_d + L_{ff} i_f \]
\[ \lambda_0 = -L_0 i_0 \]

As it can be seen the transformation converted angle varying dependence into constant coefficients therefore substantially simplifying the analysis of the machine.
2 Response Surfaces Modelling Results

Figure 1: Single Objective Optimization results for Coreless Rotor with Slotless Stator at 40K with YBCO winding
Appendix 2 Response Surfaces Modelling Results

Figure 2: Single Objective Optimization results for Inner Cored Rotor with Slotless Stator at 40K with YBCO winding

Figure 3: Single Objective Optimization results for Partially Cored Rotor with Slotless Stator at 40K with YBCO winding
Figure 4: Single Objective Optimization results for Fully Cored Rotor with Sloted Stator at 40K with YBCO winding

Figure 5: Single Objective Optimization results for Partially Cored Rotor with Sloted Stator at 40K with YBCO winding
Figure 6: Single Objective Optimization results for Coreless Rotor with Slotless Stator at 50K with YBCO winding

Figure 7: Single Objective Optimization results for Inner Cored Rotor with Slotless Stator at 50K with YBCO winding
Figure 8: Single Objective Optimization results for Partially Cored Rotor with Slotless Stator at 50K with YBCO winding.

Figure 9: Single Objective Optimization results for Inner Cored Rotor with Sloted Stator at 50K with YBCO winding.
Appendix 2 Response Surfaces Modelling Results

Figure 10: Single Objective Optimization results for Partially Cored Rotor with Sloted Stator at 50K with YBCO winding

Figure 11: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 40K with YBCO winding
Figure 12: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 40K with YBCO winding

Figure 13: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 40K with YBCO winding
Appendix 2 Response Surfaces Modelling Results

Figure 14: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 40K with YBCO winding

Figure 15: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 40K with YBCO winding
Figure 16: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 50K with YBCO winding.

Figure 17: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 50K with YBCO winding.
Figure 18: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 50K with YBCO winding

Figure 19: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 50K with YBCO winding
**Figure 20:** Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 50K with YBCO winding

**Figure 21:** Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator at 50K with YBCO winding
Figure 22: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 60K with YBCO winding

Figure 23: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 60K with YBCO winding
Figure 24: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 60K with YBCO winding

Figure 25: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 60K with YBCO winding
Figure 26: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 60K with YBCO winding.

Figure 27: Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator at 60K with YBCO winding.
Figure 28: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 40K with BiSCCO winding

Figure 29: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 40K with BiSCCO winding
**Figure 30:** Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 40K with BiSCCO winding

**Figure 31:** Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 40K with BiSCCO winding
Figure 32: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 40K with BiSCCO winding

Figure 33: Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator at 40K with BiSCCO winding
Figure 34: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 50K with BiSCCO winding

Figure 35: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 50K with BiSCCO winding
Figure 36: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 50K with BiSCCO winding

Figure 37: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 40K with BiSCCO winding
Appendix 2 Response Surfaces Modelling Results

Figure 38: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 50K with BiSCCO winding

Figure 39: Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator at 50K with BiSCCO winding
Figure 40: Multi Objective Optimization results for Coreless Rotor with Slotless Stator at 60K with BiSCCO winding

Figure 41: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator at 60K with BiSCCO winding
Appendix 2 Response Surfaces Modelling Results

Figure 42: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator at 60K with BiSCCO winding

Figure 43: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator at 60K with BiSCCO winding
Figure 44: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator at 60K with BiSCCO winding

Figure 45: Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator at 60K with BiSCCO winding
Appendix 2 Response Surfaces Modelling Results

Figure 46: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 4 and the error scaling factor set to 1E-4

Figure 47: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 3 and the error scaling factor set to 1E-4
Figure 48: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 2 and the error scaling factor set to 1E-4

Figure 49: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 1 and the error scaling factor set to 1E-4
Figure 50: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 1 and the error scaling factor set to 1E-5.

Figure 51: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 1 and the error scaling factor set to 1E-6.
Figure 52: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 1 and the error scaling factor set to 1E-7

Figure 53: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 1 and the error scaling factor set to 1E-8
Figure 54: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 5 and the error scaling factor set to 1E-4

Figure 55: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 3 and the error scaling factor set to 1E-6
Figure 56: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with YBCO winding and the finishing tolerance set to 3 and the error scaling factor set to 1E-4

Figure 57: Multi Objective Optimization results for Coreless Rotor with Slotless Stator with Copper winding
Appendix 2 Response Surfaces Modelling Results

Figure 58: Multi Objective Optimization results for Inner Cored Rotor with Slotless Stator with Copper winding

Figure 59: Multi Objective Optimization results for Partially Cored Rotor with Slotless Stator with Copper winding
Appendix 2 Response Surfaces Modelling Results

Figure 60: Multi Objective Optimization results for Fully Cored Rotor with Sloted Stator with Copper winding

Figure 61: Multi Objective Optimization results for Inner Cored Rotor with Sloted Stator with Copper winding
Figure 62: Multi Objective Optimization results for Partially Cored Rotor with Sloted Stator with Copper winding