

Synthesis of Fibre Gratings

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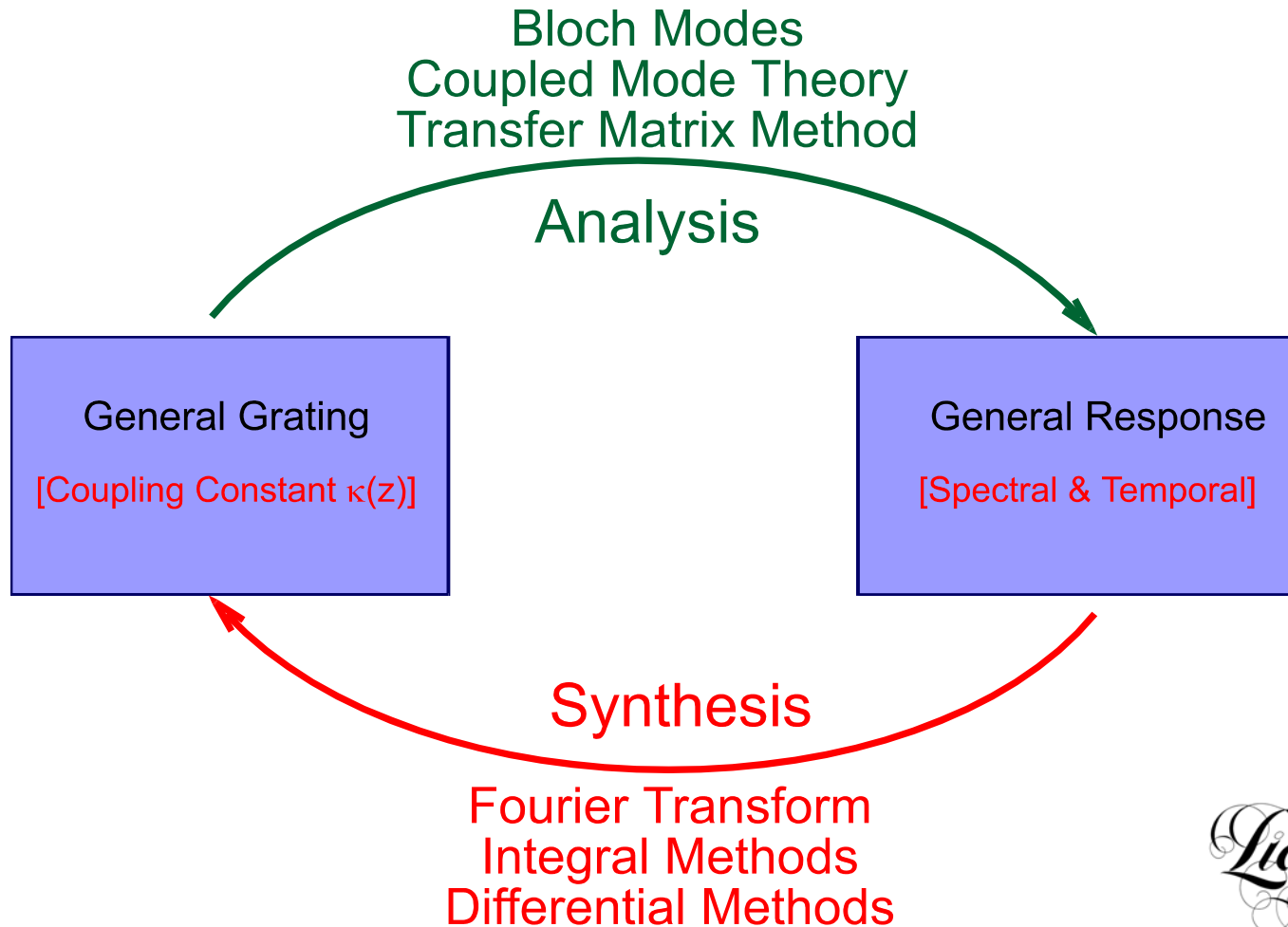
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Outline

- Introduction (Analysis / Synthesis)
- Grating Synthesis (Design) Methods
 - Fourier-Transform Methods
 - Integral Methods
 - Differential Methods
- Layer-Peeling IS Method
- Grating Designs
 - Square Dispersionless Filters
 - Dispersion Compensators
 - 2nd & 3rd order
- Conclusions

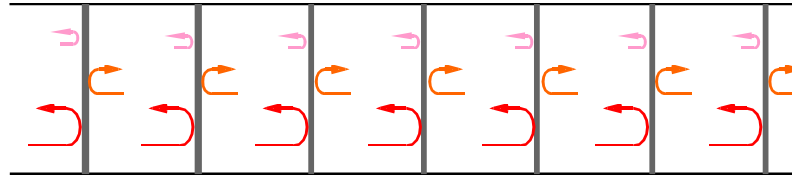


Grating Analysis - Synthesis



Grating Reflection

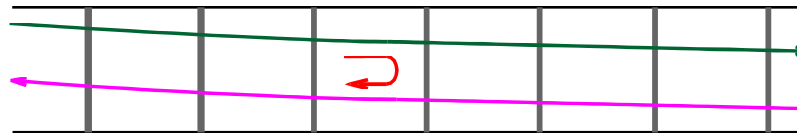
Multiple
Distributed
Scattering



Coupling
Constant
 $\kappa(z)$

equivalent to

Two opposite-
travelling waves



$A(z)$

$B(z)$

Local Reflection Coefficient

$$r(z) = \frac{B(z)}{A(z)}$$

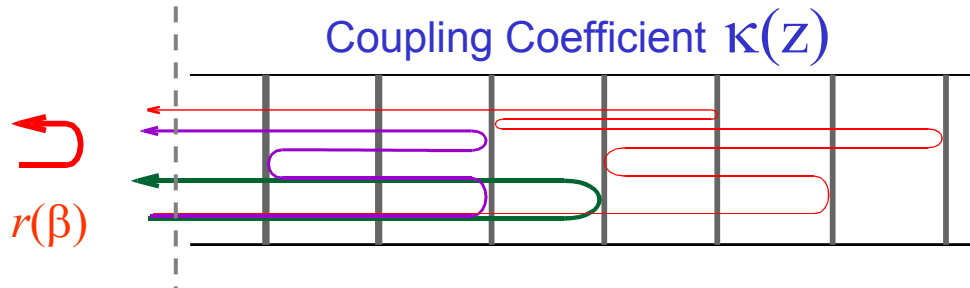
Ricatti Equation

$$\frac{dr}{dz} = -i2\beta r + \kappa - \kappa^* r^2$$



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General Grating Reflection Coefficient



$$\begin{aligned}
 r(\beta) = & - \int_{z_1=0}^L dz_1 \kappa(z_1) e^{i\beta 2z_1} \\
 & + \int_{z_1=0}^L dz_1 \int_{z_2=0}^{z_1} dz_2 \int_{z_3=z_2}^L dz_3 \kappa(z_1) \kappa^*(z_2) \kappa(z_3) e^{i\beta 2(z_1 - z_2 + z_3)} \\
 & - \int_{z_1=0}^L dz_1 \int_{z_2=0}^{z_1} dz_2 \int_{z_3=z_2}^L dz_3 \int_{z_4=0}^{z_3} dz_4 \int_{z_5=z_4}^L dz_5 \kappa(z_1) \kappa^*(z_2) \kappa(z_3) \kappa^*(z_4) \kappa(z_5) e^{i\beta 2(z_1 - z_2 + z_3 - z_4 + z_5)} \\
 & + \dots \quad (\text{Higher Order Reflections})
 \end{aligned}$$



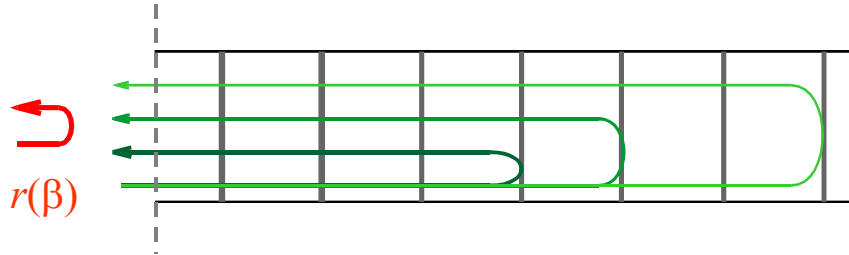
Main Grating Design Methods

- Fourier Method
- Integral Methods
- Differential Methods



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Grating Design - Fourier Method



For weak gratings
($r \ll 1$)

$$r(\beta) = - \int_{z_1=0}^L dz_1 \kappa(z_1) e^{i\beta 2z_1}$$

$$\kappa(z) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} r(\beta) e^{-i2\beta z} d\beta$$

$$r(\beta) \xleftrightarrow{\text{FT}} \kappa(z)$$

- Accurate only for **low reflectivities**
- **Limited** but quite useful



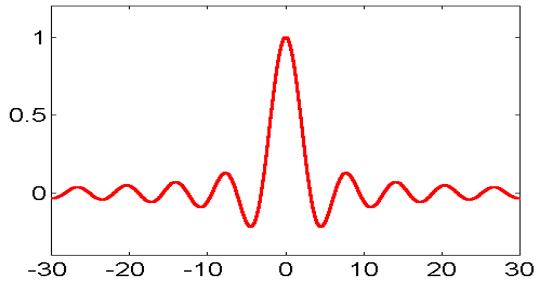
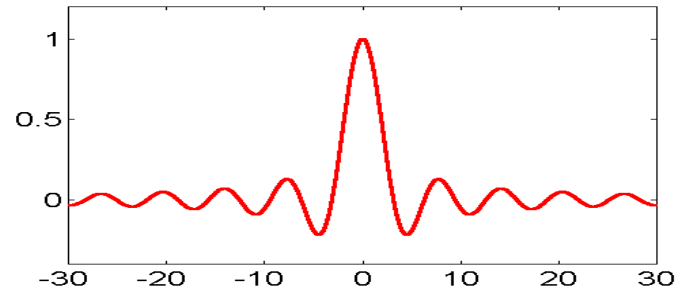
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Grating Design - Fourier Method

Coupling Constant



Reflection Coefficient



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Integral IS Methods

[Gelfand-Levitan & Marchenko (GLM) Method]

- Invert Grating Response in **Fourier Domain**
- Coupling Constant in terms of **Generalised FT Integral**
- **Exact Method** - Multiple reflections accounted for
- Solution of **Integral Equations**
- Analytic solution exists when $r(\beta)$ is rational function
- Solution usually involves $(N \times N)$ matrices
- Iterative solutions have been proposed
- **High Algorithmic Complexity: $O(N^3)$**



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Differential IS Methods

[Layer-Peeling Methods]

- Invert Grating Response in **Time Domain**
- Rely on **Causality**
- **Exact Method** - Multiple reflections accounted for
- Solution of **Difference Equations**
- **Layer-by-layer** medium identification
- Replicate Scattering Physical Process
- **Low Algorithmic Complexity: $O(N^2)$**



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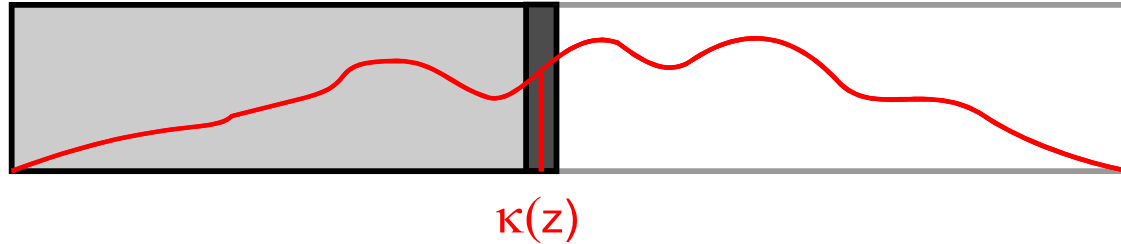
Layer-Peeling Grating Design Method

- General Description
- Space-Time Diagrams



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Layer-Peeling IS Algorithm



Step: 1) Impulse Response [$h_R(\tau)$] of Desired Medium [\rightarrow FT of $r(\beta)$]

2) Impulse Response [$h_T(\tau)$] of Identified Medium [\rightarrow FT of $r_T(\beta)$]

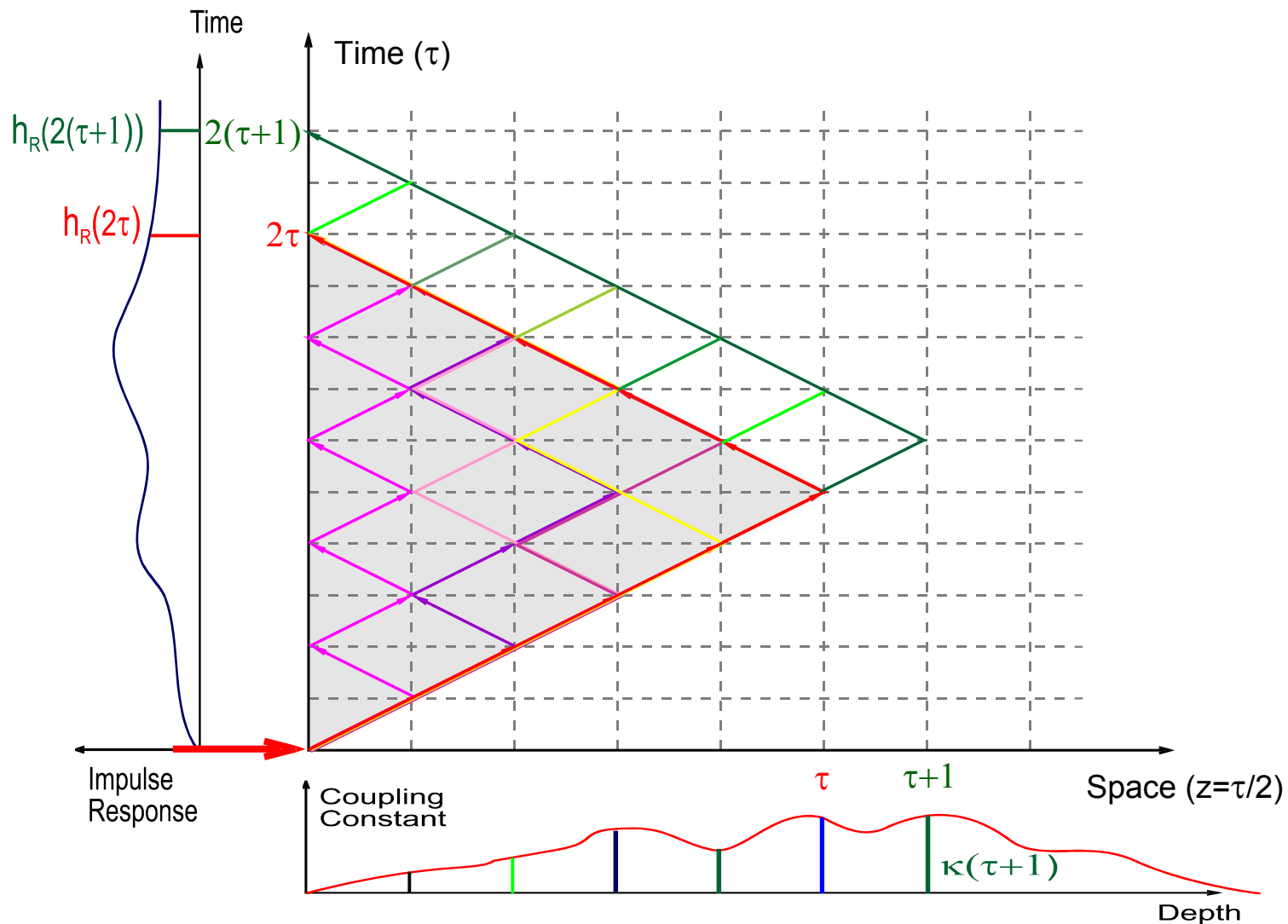
3)

$$\kappa(\tau/2) = -2 [h_R(\tau) - h_T(\tau)]$$

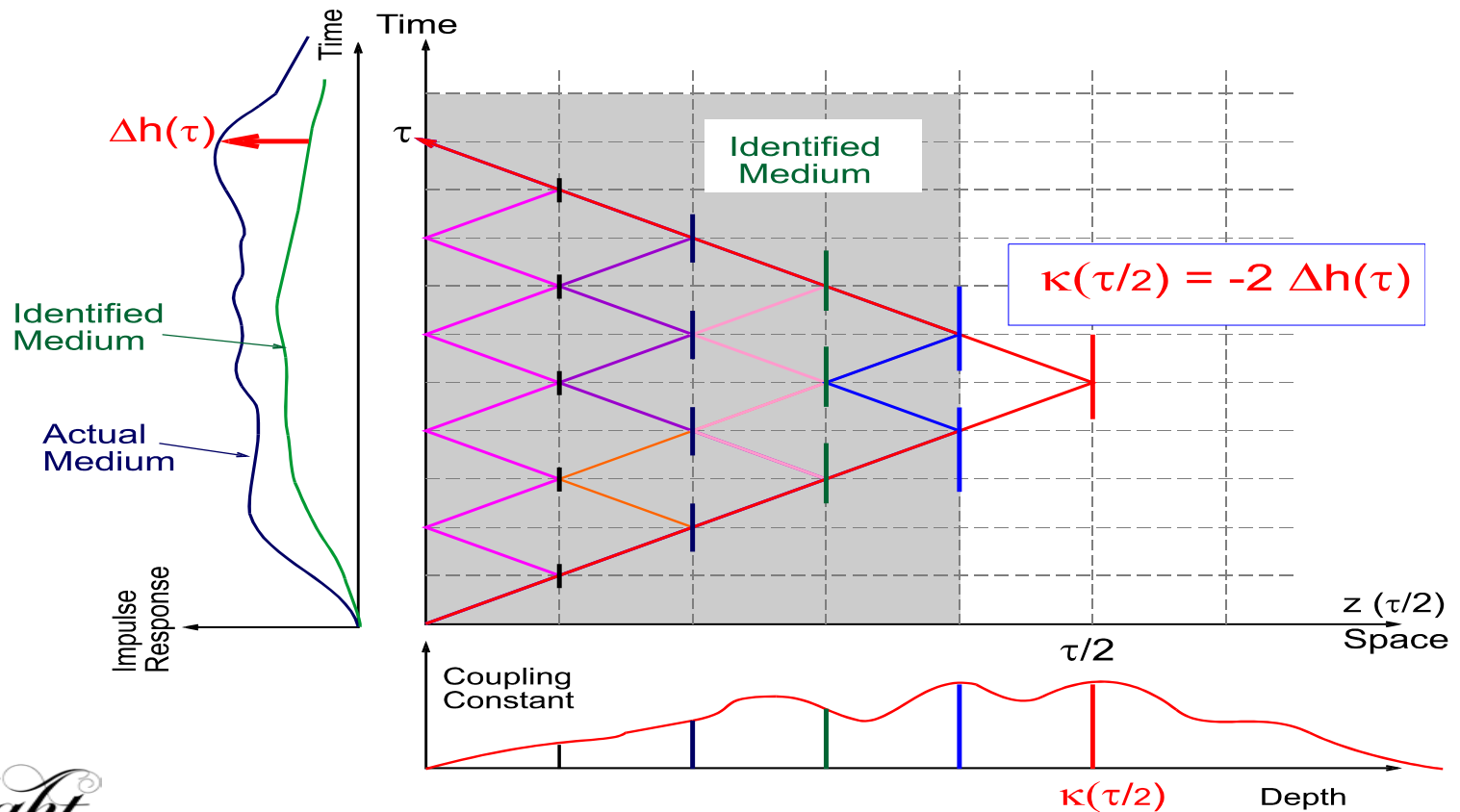


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Multiple Scattering Reflection



Layer-Peeling Inverse Scattering Method



Layer-Peeling Method

Grating-Design Examples

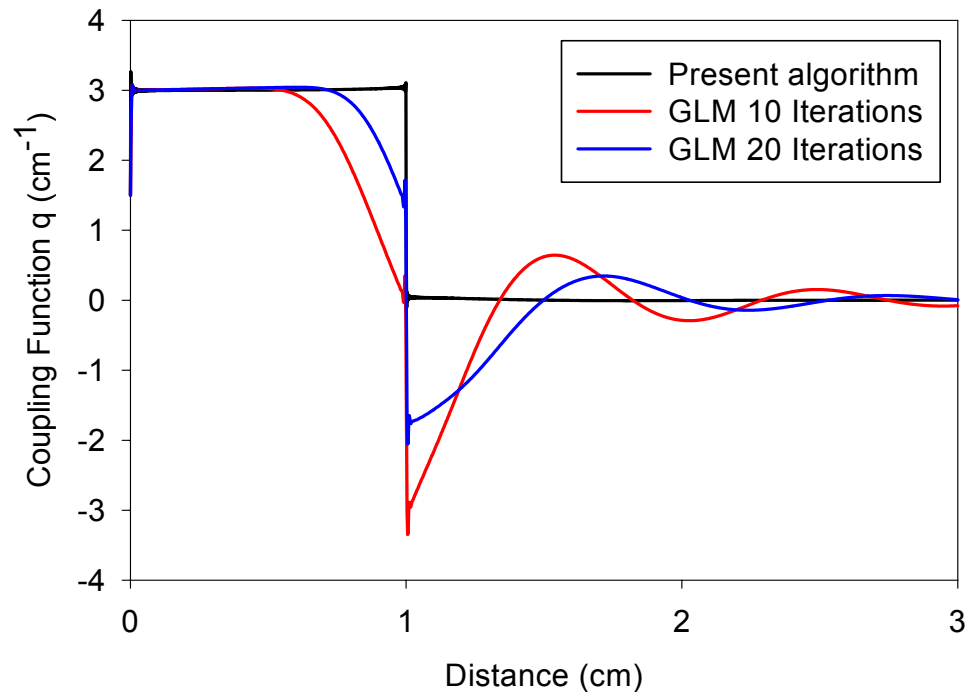
- Uniform Grating Reconstruction
- Square Dispersionless Filters
- 2nd-order Dispersion Compensators
- 3rd-order Dispersion Compensators



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Uniform Grating Reconstruction

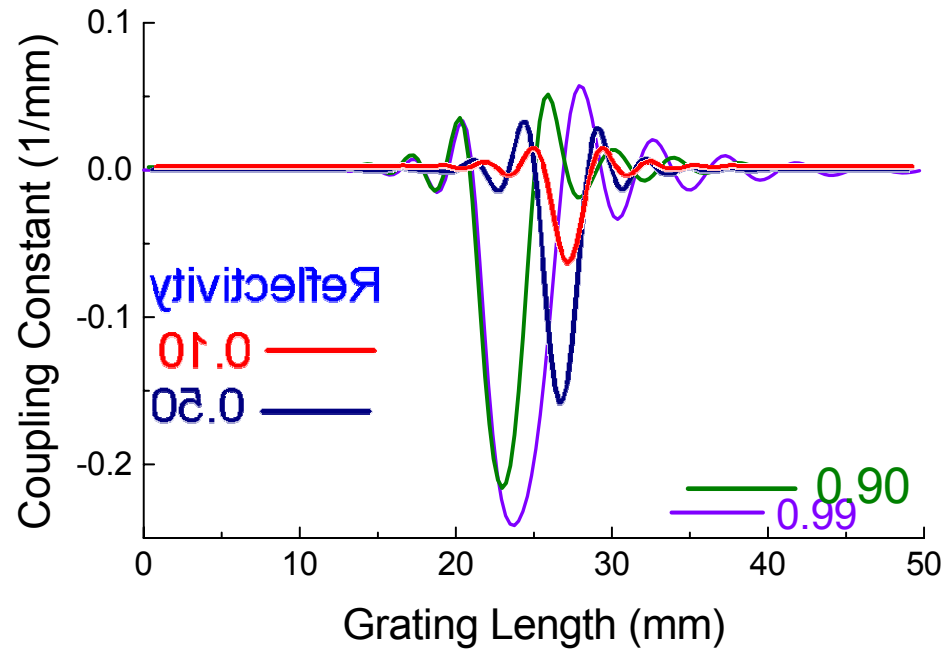
Layer-Peeling -vs- Iterative GLM



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Square Dispersionless Filter Design

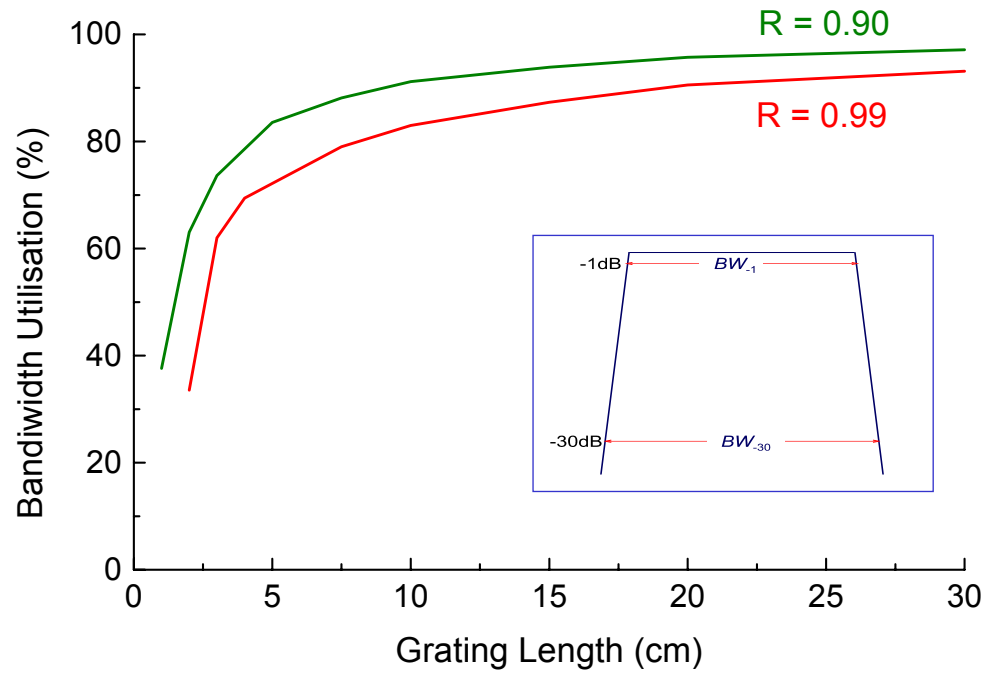
$BW_{-1dB} = 0.45\text{nm}$ \rightarrow 75% Bandwidth Use
 $BW_{-30dB} = 0.60\text{nm}$



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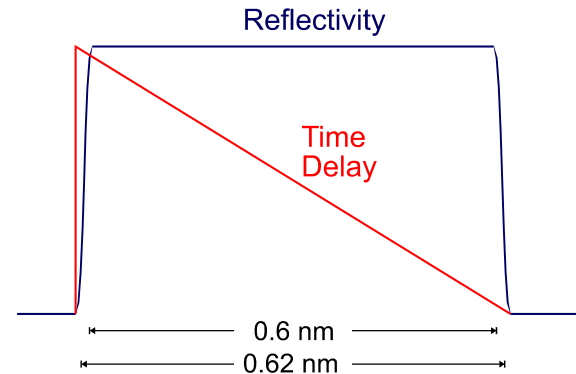
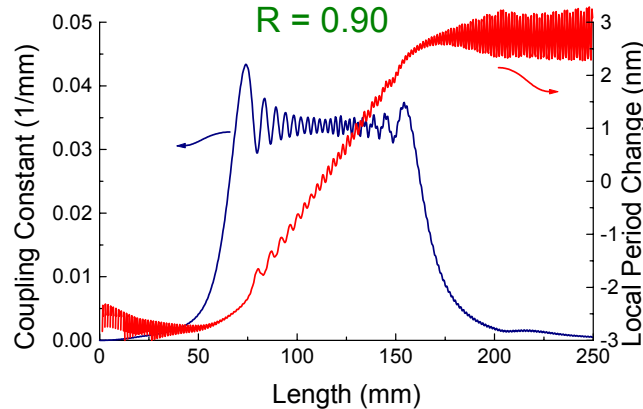
Square Dispersionless Filters

$$\text{Bandwidth Utilisation} = BW_{-1} / BW_{-30}$$

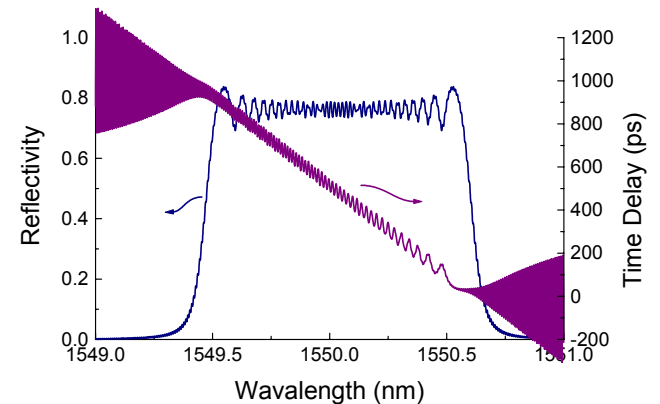
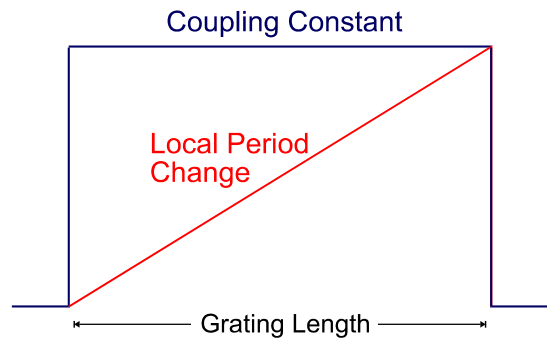


Linear Dispersion Compensator Grating Design

Layer-Peeling
IS Design

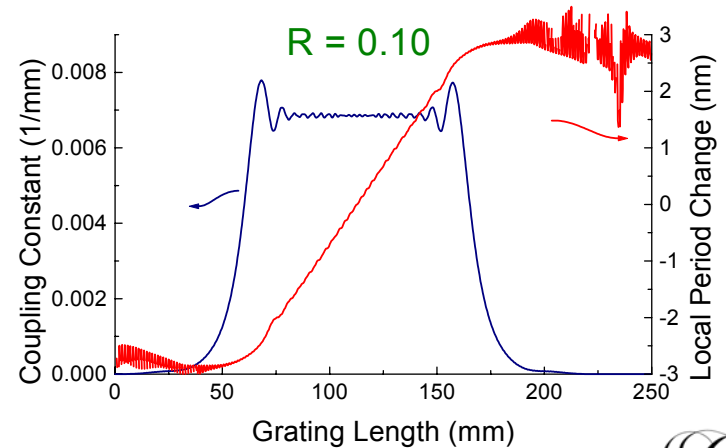
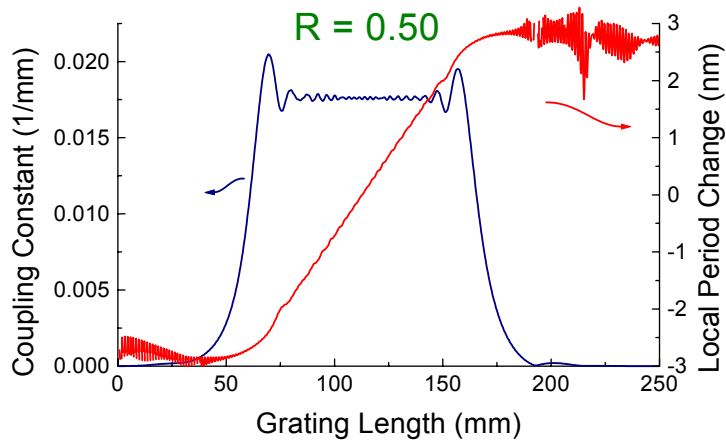
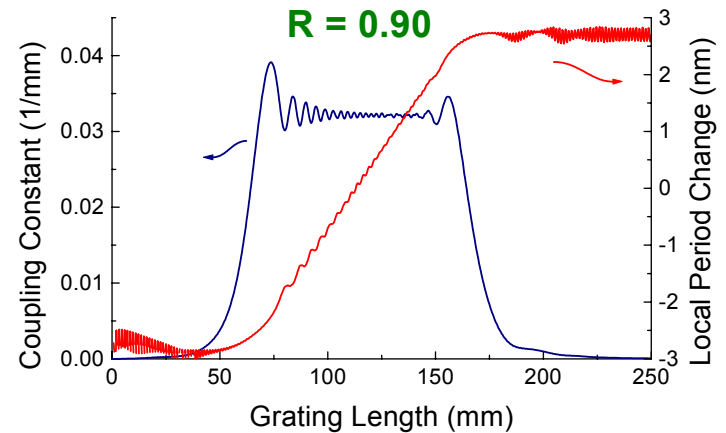
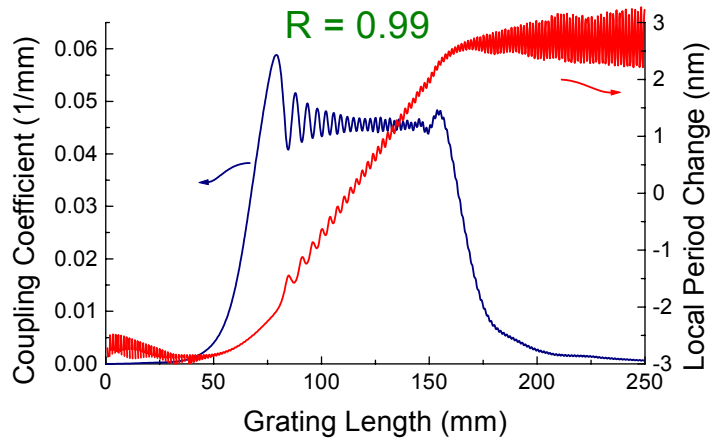


Conventional
Design

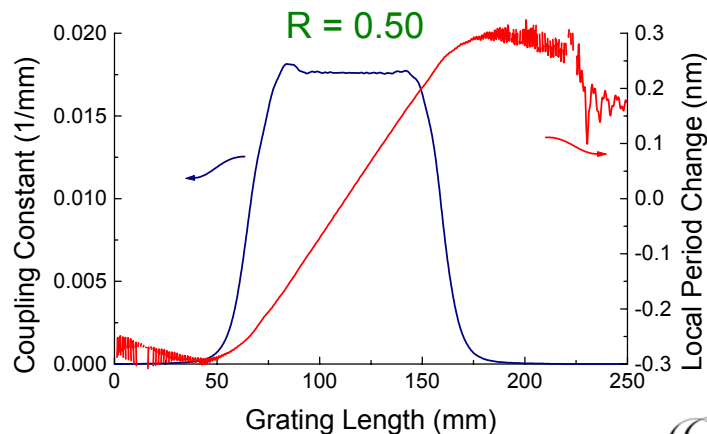
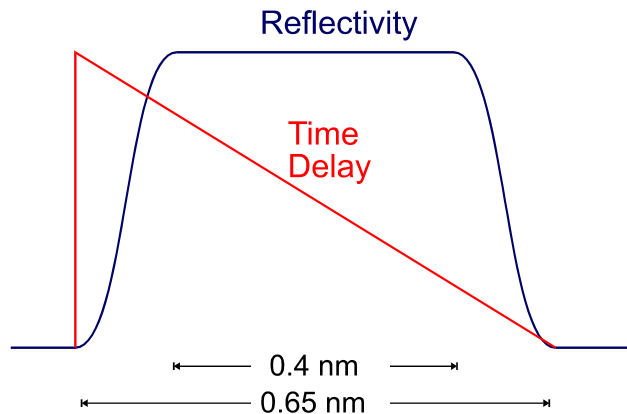
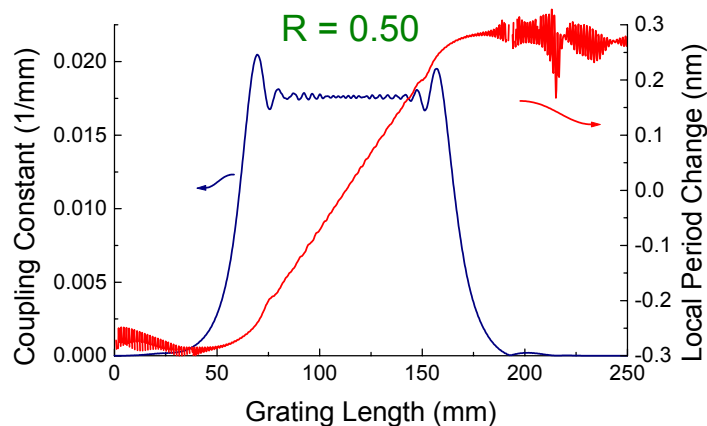
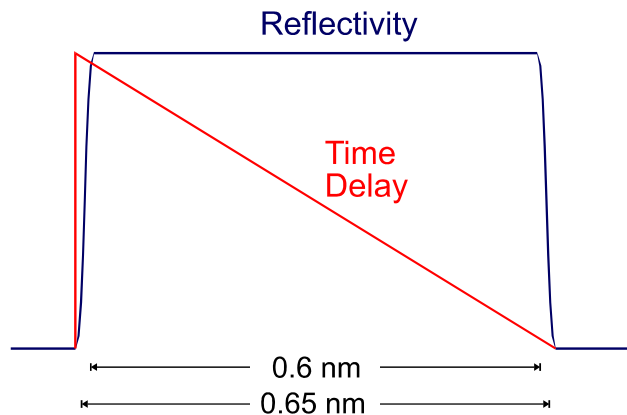


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2nd order Grating Dispersion Compensator Design

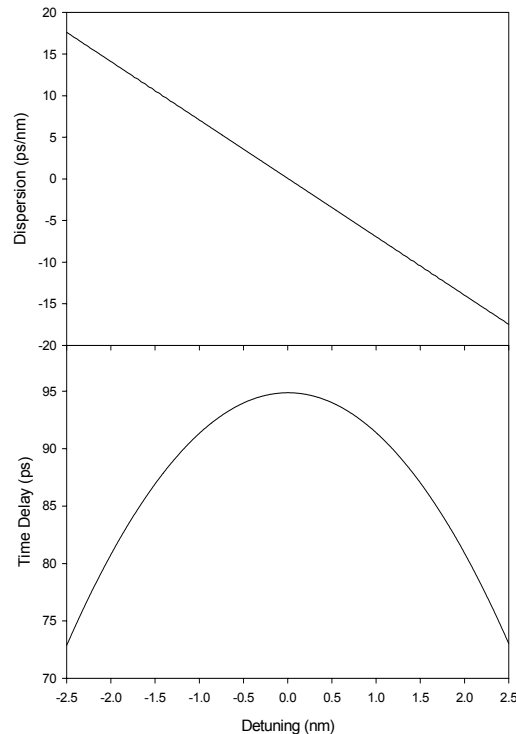


2nd order Grating Dispersion Compensator Design



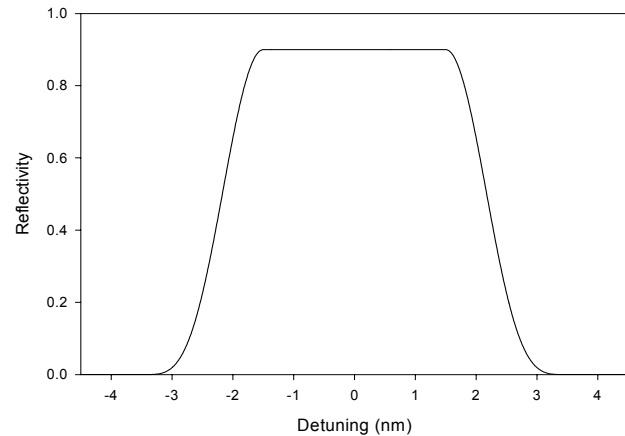
3rd Order Dispersion Compensators

Dispersion



Time Delay

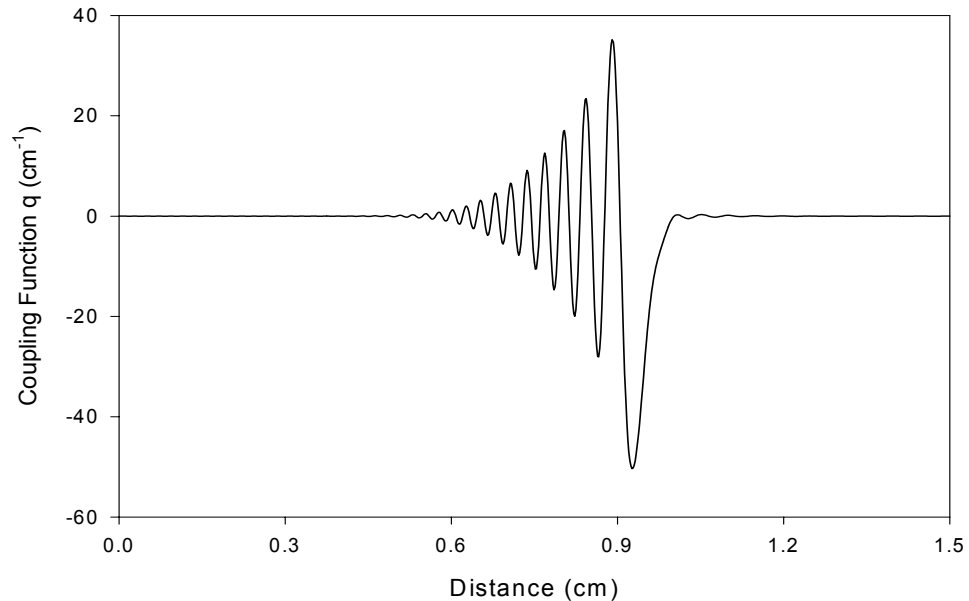
Reflectivity



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3rd Order Dispersion Compensator Design

- Real Coupling Coefficient

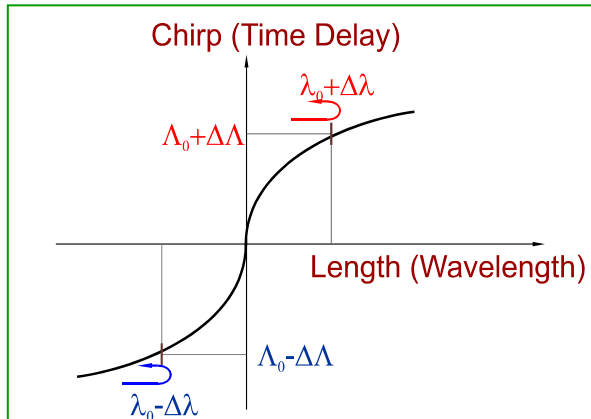


- Constant Period

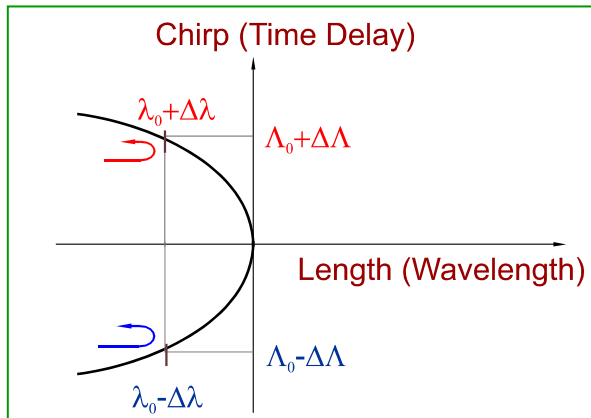


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3rd order DC - Physical Picture



Chirped Compensator
(with Antisymmetric Chirp)



Folded Chirped Compensator

Unchirped

Locally Varying Moire Grating



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Conclusions

- Main grating-design techniques have been reviewed
- Differential Layer-Peeling Method presented extensively
 - Extremely Powerful
 - Replicates physical scattering process
 - Fast & Accurate method
 - Gives exact solutions to exact scattering problems
- Provides Novel Exciting Grating Designs
- Enhances Grating Design Intuition



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