Synthesis of Fibre Gratings

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Outline

• Introduction (Analysis / Synthesis)

• Grating Synthesis (Design) Methods
  • Fourier-Transform Methods
  • Integral Methods
  • Differential Methods

• Layer-Peeling IS Method

• Grating Designs
  • Square Dispersionless Filters
  • Dispersion Compensators
    • 2nd & 3rd order

• Conclusions
Grating Analysis - Synthesis

Analysis

Bloch Modes
Coupled Mode Theory
Transfer Matrix Method

General Grating
[Coupling Constant $\kappa(z)$]

General Response
[Spectral & Temporal]

Synthesis

Fourier Transform
Integral Methods
Differential Methods
Grating Reflection

Multiple Distributed Scattering

Two opposite-travelling waves

Equivalent to

Coupling Constant $\kappa(z)$

Local Reflection Coefficient

$\rho(z) = \frac{B(z)}{A(z)}$

Ricatti Equation

$$\frac{dr}{dz} = -i2\beta r + \kappa - \kappa^* r^2$$
**General Grating Reflection Coefficient**

\[ r(\beta) = - \int_{z_i=0}^{L} dz_1 \kappa(z_1) e^{i\beta z_1} \]

\[ + \int_{z_2=0}^{L} dz_1 \int_{z_3=z_2}^{L} dz_2 \kappa(z_1) \kappa^*(z_2) \kappa(z_3) e^{i\beta (z_1 - z_2 + z_3)} \]

\[ - \int_{z_2=0}^{L} dz_1 \int_{z_3=z_2}^{L} dz_2 \int_{z_4=z_3}^{L} dz_3 \kappa(z_1) \kappa^*(z_2) \kappa(z_3) \kappa^*(z_4) \kappa(z_5) e^{i\beta (z_1 - z_2 + z_3 - z_4 + z_5)} \]

\[ + \ldots \quad \text{(Higher Order Reflections)} \]
Main Grating Design Methods

- Fourier Method
- Integral Methods
- Differential Methods
Grating Design - Fourier Method

\[ r(\beta) = - \int_{z_i=0}^{L} \kappa(z_1) e^{i\beta z_1} \, dz_1 \]

For weak gratings \((r \ll 1)\)

\[ \kappa(z) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} r(\beta) e^{-i2\beta z} \, d\beta \]

- Accurate only for low reflectivities
- Limited but quite useful
Grating Design - Fourier Method

Coupling Constant

Reflection Coefficient

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Integral IS Methods

[Gelfand-Levitan & Marchenko (GLM) Method]

- Invert Grating Response in Fourier Domain
- Coupling Constant in terms of Generalised FT Integral
- Exact Method - Multiple reflections accounted for
- Solution of Integral Equations
- Analytic solution exists when r(β) is rational function
- Solution usually involves (NxN) matrices
- Iterative solutions have been proposed
- High Algorithmic Complexity: $O(N^3)$
Differential IS Methods

[Layer-Peeling Methods]

- Invert Grating Response in Time Domain
- Rely on Causality
- Exact Method - Multiple reflections accounted for
- Solution of Difference Equations
- Layer-by-layer medium identification
- Replicate Scattering Physical Process
- Low Algorithmic Complexity: $O(N^2)$
Layer-Peeling Grating Design Method

- General Description
- Space-Time Diagrams
Layer-Peeling IS Algorithm

Step:
1) Impulse Response \([h_R(\tau)]\) of Desired Medium \(\rightarrow\) FT of \(r(\beta)\)

2) Impulse Response \([h_T(\tau)]\) of Identified Medium \(\rightarrow\) FT of \(r_T(\beta)\)

3) \(\kappa(\tau/2) = -2 [h_R(\tau) - h_T(\tau)]\)
Multiple Scattering Reflection
Layer-Peeling Inverse Scattering Method

\[ \Delta h(\tau) \]

Identified Medium

Actual Medium

Impulse Response

Time

[Graph showing time and space dimensions with identified medium and coupling constant denoted as \( \kappa(\tau/2) = -2 \Delta h(\tau) \).]

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Layer-Peeling Method
Grating-Design Examples

• Uniform Grating Reconstruction
• Square Dispersionless Filters
• 2nd-order Dispersion Compensators
• 3rd-order Dispersion Compensators
Uniform Grating Reconstruction

Layer-Peeling -vs- Iterative GLM

![Graph showing coupling function vs distance for different algorithms.](Southampton University)
Square Dispersionless Filter Design

\[ \text{BW}_{-1\text{dB}} = 0.45\text{nm} \]
\[ \text{BW}_{-30\text{dB}} = 0.60\text{nm} \]

75% Bandwidth Use
Square Dispersionless Filters

\[ \text{Bandwidth Utilisation} = \frac{\text{BW}_{-1}}{\text{BW}_{-30}} \]

- \( R = 0.90 \)
- \( R = 0.99 \)
Linear Dispersion Compensator Grating Design

Layer-Peeling
IS Design

Conventional
Design

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2nd order Grating Dispersion Compensator Design

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2nd order Grating Dispersion Compensator Design

Reflectivity

Time Delay

0.6 nm

0.65 nm

R = 0.50

Coupling Constant (1/mm)

0.000

0.005

0.010

0.015

0.020

Grating Length (mm)

0 50 100 150 200 250

Local Period Change (nm)

-0.3

-0.2

-0.1

0.0

0.1

0.2

0.3

Reflectivity

Time Delay

0.4 nm

0.65 nm

R = 0.50

Coupling Constant (1/mm)

0.000

0.005

0.010

0.015

0.020

Grating Length (mm)

0 50 100 150 200 250

Local Period Change (nm)

-0.3

-0.2

-0.1

0.0

0.1

0.2

0.3

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3rd Order Dispersion Compensators

- Dispersion
- Time Delay
- Reflectivity

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3rd Order Dispersion Compensator Design

- Real Coupling Coefficient
- Constant Period
3rd order DC - Physical Picture

Chirped Compensator
(with Antisymmetric Chirp)

Folded Chirped Compensator

Unchirped

Locally Varying Moire Grating

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Conclusions

• Main grating-design techniques have been reviewed

• Differential Layer-Peeling Method presented extensively
  Extremely Powerful
  Replicates physical scattering process
  Fast & Accurate method
  Gives exact solutions to exact scattering problems

• Provides Novel Exciting Grating Designs

• Enhances Grating Design Intuition