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The Vortex Ring Filament in Ground Effect

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Induced Velocity of a Vortex Ring

The analysis of this is detailed in AFM Technical Report 11/03. The vortex ring has radius R, is placed in the XOY plane with the centre at the origin, O. The control point, P, without loss of generality, can be considered as lying in the XOZ plane with coordinates (r,0,h). In order to evaluate the overall induced velocity, we need to consider a vortex element at azimuth, ψ.

This is shown in Figure 1.

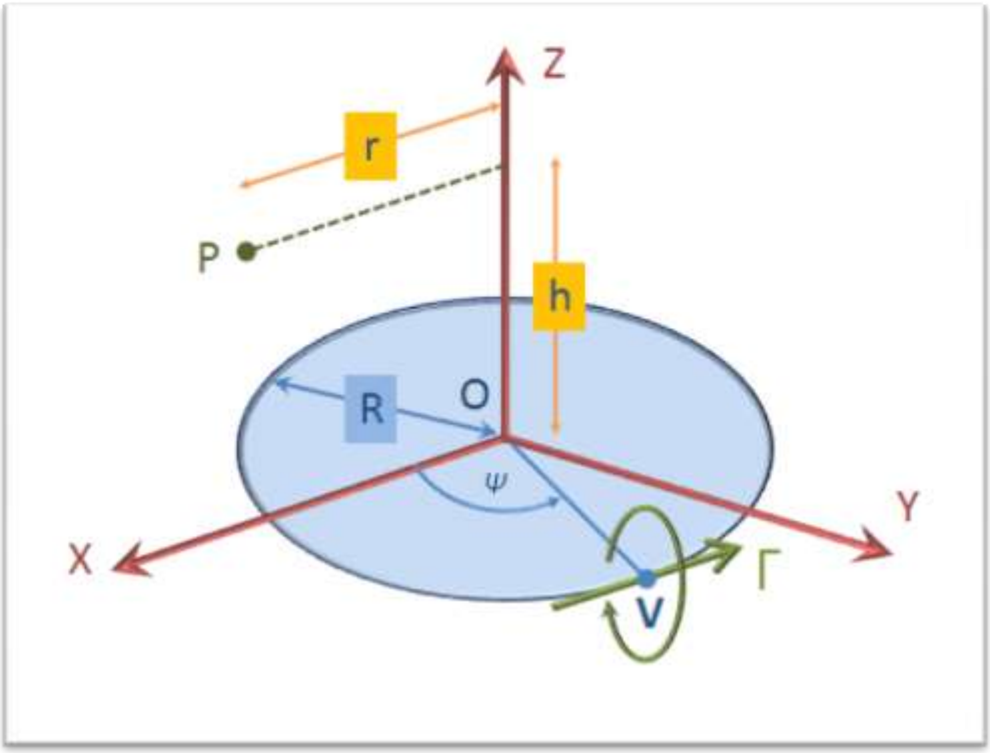


Figure 1

The induced velocity of the vortex ring in terms of the centre velocity can be expressed in terms of elliptic integrals, namely:

$$\begin{bmatrix} q_x \\ q_z \end{bmatrix} = q_{z0} \frac{2}{\pi \cdot k'^2 \left\{(1+x)^2 + \bar{h}^2\right\}^{3/2}} \begin{bmatrix} \bar{h}(2K - E - 2D) \\ E - x(2K - E - 2D) \end{bmatrix} \qquad (1.)$$

Where:

$$k^2 = \frac{4x}{(1+x)^2 + \overline{h}^2} \tag{2.}$$

$$k'^2 = 1 - k^2 \tag{3.}$$

$$\begin{aligned} K &= \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}} \\ E &= \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \phi}\, d\phi \\ D &= \int_0^{\frac{\pi}{2}} \frac{\sin^2 \phi\, d\phi}{\sqrt{1 - k^2 \sin^2 \phi}} = \frac{K - E}{k^2} \end{aligned} \tag{4.}$$

For the special case of:

$$k = 0 \tag{5.}$$

the elliptic integrals become:

$$\begin{aligned} K &= \frac{\pi}{2} \\ E &= \frac{\pi}{2} \\ D &= \frac{\pi}{4} \end{aligned} \tag{6.}$$

General Location

Consider a vortex ring at a height h above the ground plane. The horizontal component of the induced velocity at a general point in the ground plane is as shown in figure 2. This figure also shows and incident horizontal wind.

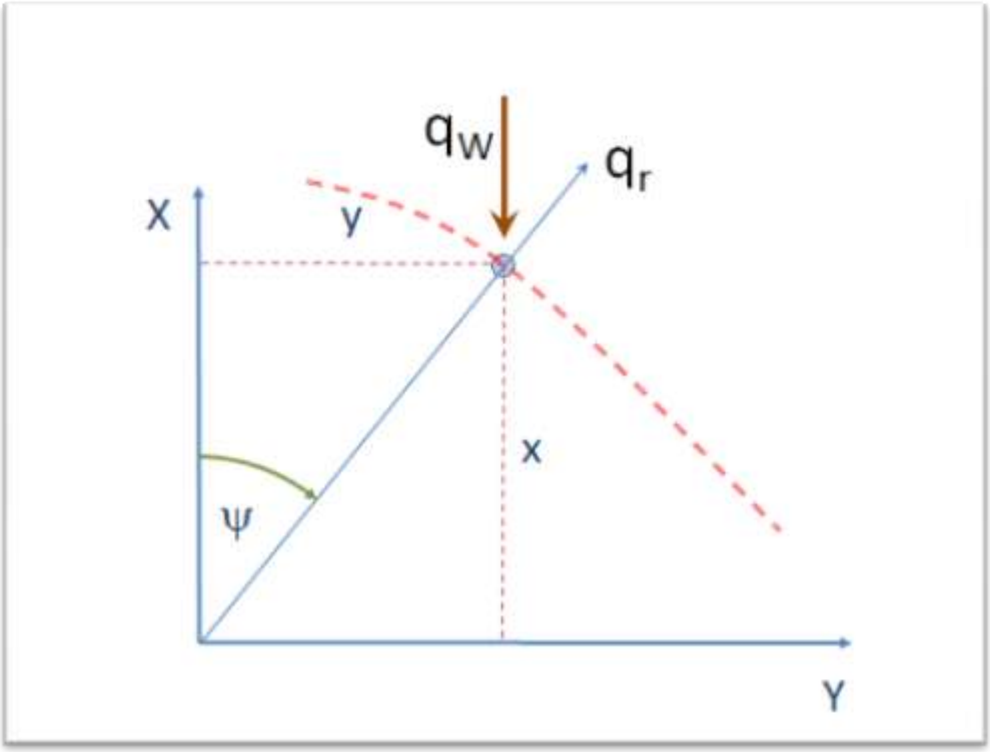


Figure 2

The radial velocity is given by:

$$q_r = q_{z0} \frac{4\bar{h}(2K - E - 2D)}{\pi \cdot k'^2 \left\{(1+x)^2 + \bar{h}^2\right\}^{3/2}} \quad (7.)$$

(Note the doubling of the formula to account for the image ring.)

Due to the velocity relationship, the method is best assembled by using the radial variable, x, as the independent variable. In figure 2, the boundary of where the flow separates from the ground surface. The condition for this is the velocity along the radial line is zero, i.e.:

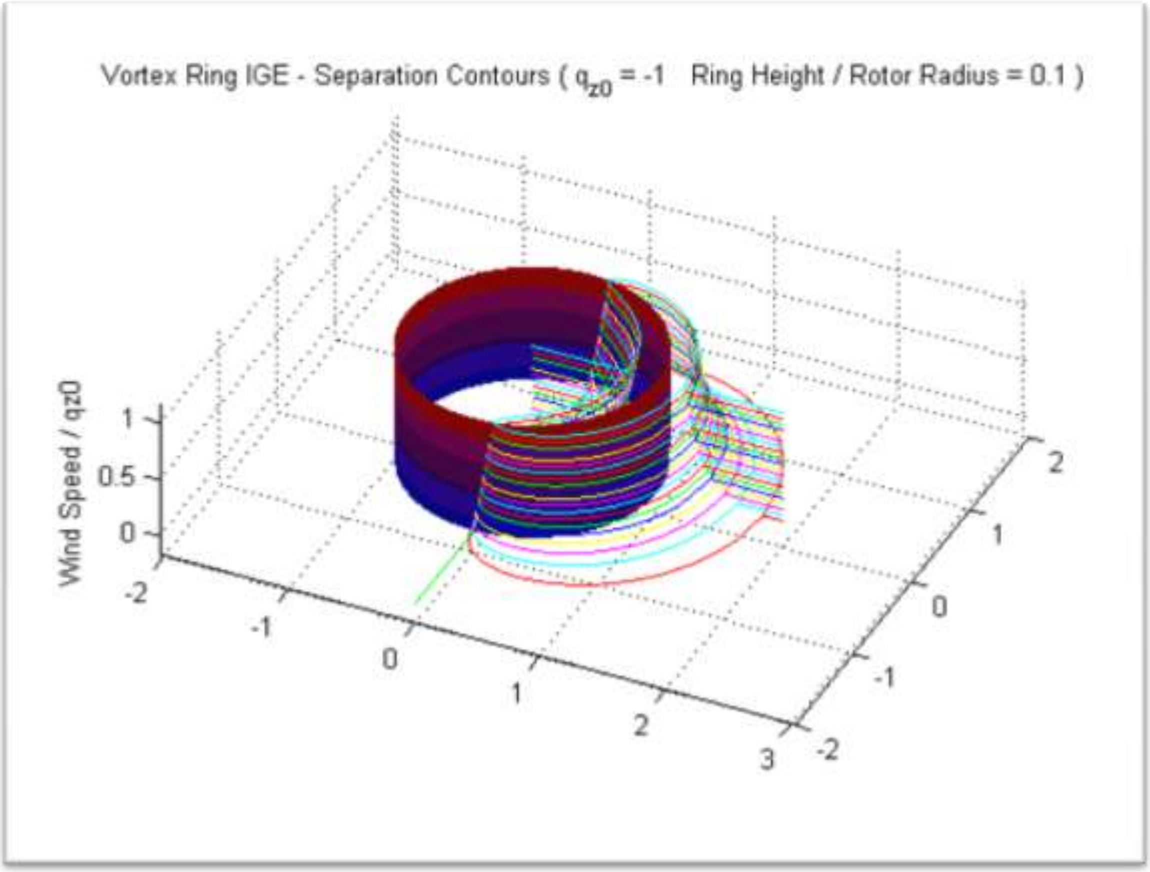
$$q_r = q_W \cdot \cos\psi \tag{8.}$$

This then allows a radial plot (r,ψ) to be made.

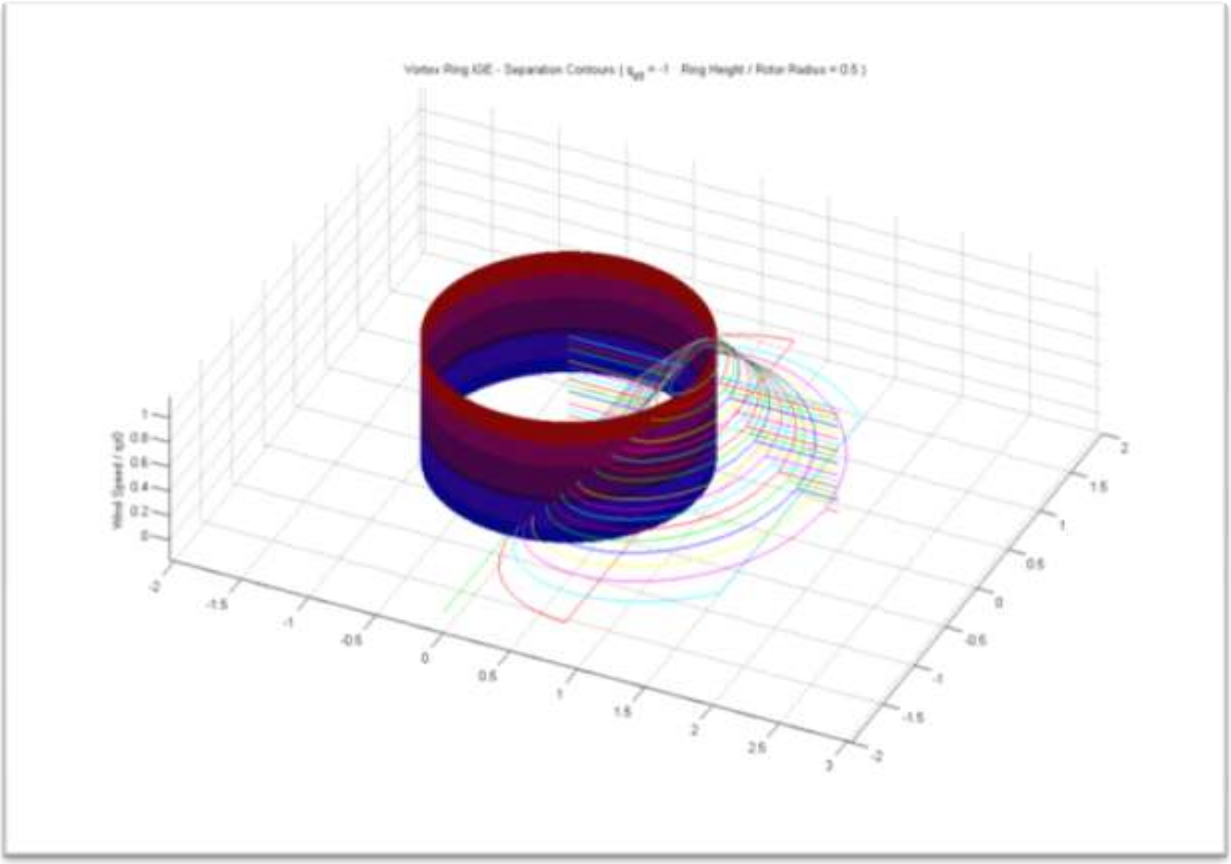
Examples

The incident wind varies from 0 to the value of the centre induced velocity.

Ring Height = 10% Rotor Radius



Ring Height = 50% Rotor Radius



Ring Height = 100% Rotor Radius