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An Analysis of Sweep Angle Contours

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Technical Report AFM-11/05

January 2011

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Introductory Remarks

The combination of rotation and forward speed makes the local inflow distribution across a helicopter rotor disc complex. One aspect is the angle of inflow at a given point of a rotor blade at a given azimuth angle; i.e. the sweep angle. This technical note derives, in closed form, the equation governing the sweep angle contours.

Derivation of Contours

The rotor dimension is normalised to a unit radius. Also, the velocity components are normalised on the rotor tip speed. A general point of a rotor blade is at a normalised radius of (x_{RAD}) and azimuth angle (ψ).

The velocity components defining the local sweep angle is shown in Figure 1:

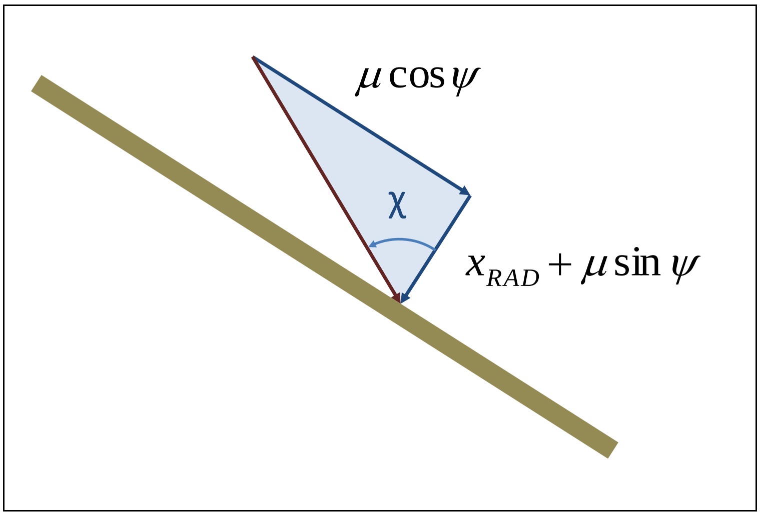

Figure 1 - Local Velocity Components

From this, the sweep angle is defined by:

$$tan\chi = \frac{\mu \cos\psi}{x_{RAD} + \mu \sin\psi} \qquad (1.)$$

In order to determine the contour equations we use the following axes system:

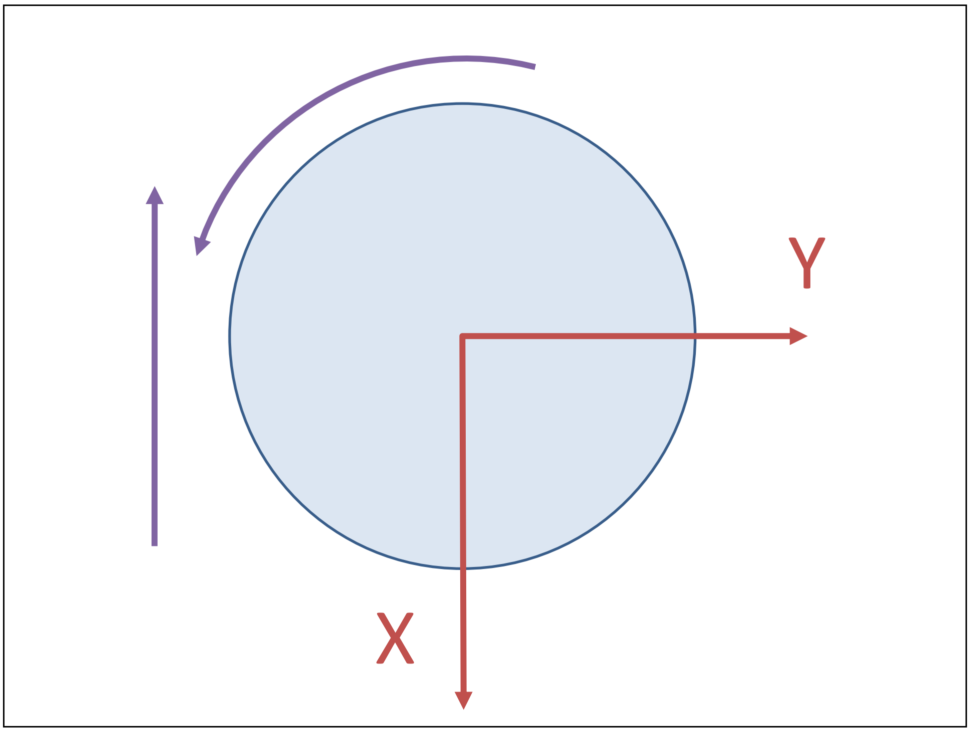

Figure 2 – Axes System Definition

The X axis (abscissa) lies in the incident airflow due to forward speed, i.e. over the tail and the y axis (ordinate) lies to starboard.

With this axes, the relationship between the normalised rotor radius (x_{RAD}), the azimuth angle (ψ) is shown in Figure 3:

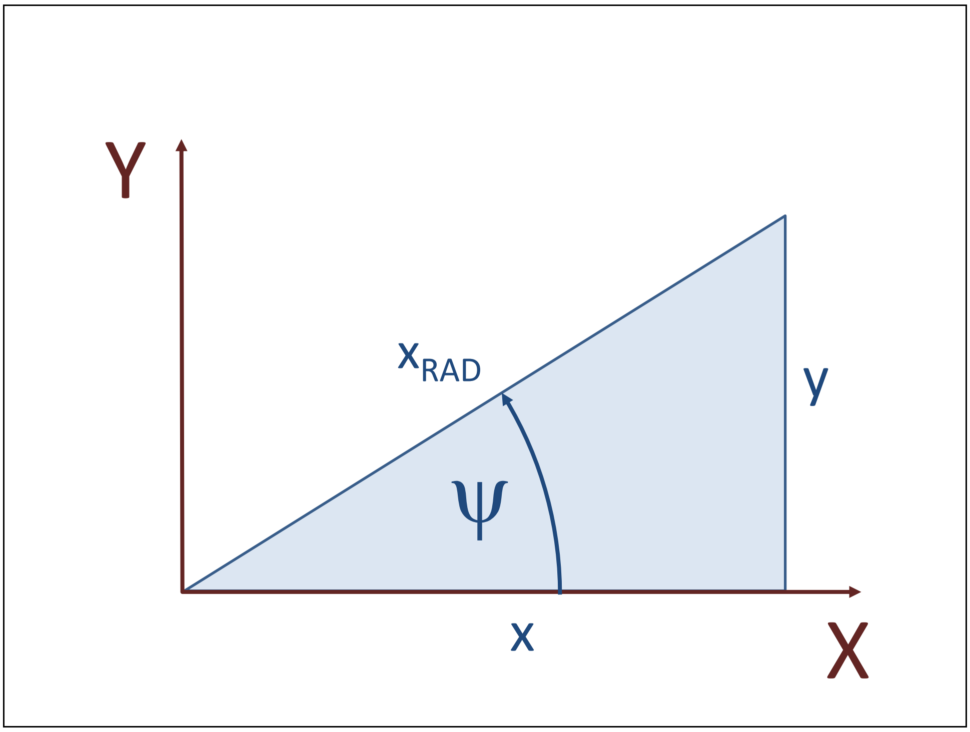

Figure 3 – Transformation between Rotor Polar and Cartesian Coordinates

The transformation equations are:

$$x_{RAD} = \sqrt{x^2 + y^2}$$

$$\cos\psi = \frac{x}{x_{RAD}} \tag{2.}$$

$$\sin\psi = \frac{y}{x_{RAD}}$$

Substituting (2) into (1) and clearing fractions gives:

$$x_{RAD} + \mu \cdot \frac{y}{x_{RAD}} = \mu \cdot \cot\chi \cdot \frac{x}{x_{RAD}} \tag{3.}$$

And hence:

$$x^2 + y^2 - \mu \cdot \cot\chi \cdot x + \mu y = 0 \tag{4.}$$

The equation of a general circle is given by:

$$x^2 + y^2 - 2gx - 2fy + c = 0 \tag{5.}$$

Where the centre of the circle is:

$$(-g, -f) \tag{6.}$$

and the radius:

$$\sqrt{g^2 + f^2 - c} \tag{7.}$$

From (5-7), the circle of equation (4) has centre:

$$\left(\frac{\mu}{2} \cdot \cot\chi\,, -\frac{\mu}{2}\right) \tag{8.}$$

and radius:

$$\sqrt{\left(\frac{\mu}{2}\cot\chi\right)^2 + \left(-\frac{\mu}{2}\right)^2} \tag{9.}$$

which reduces to:

$$\frac{\mu}{2}\csc\chi \tag{10.}$$

Geometrical Construction

This can be shown geometrically in the following Figure:

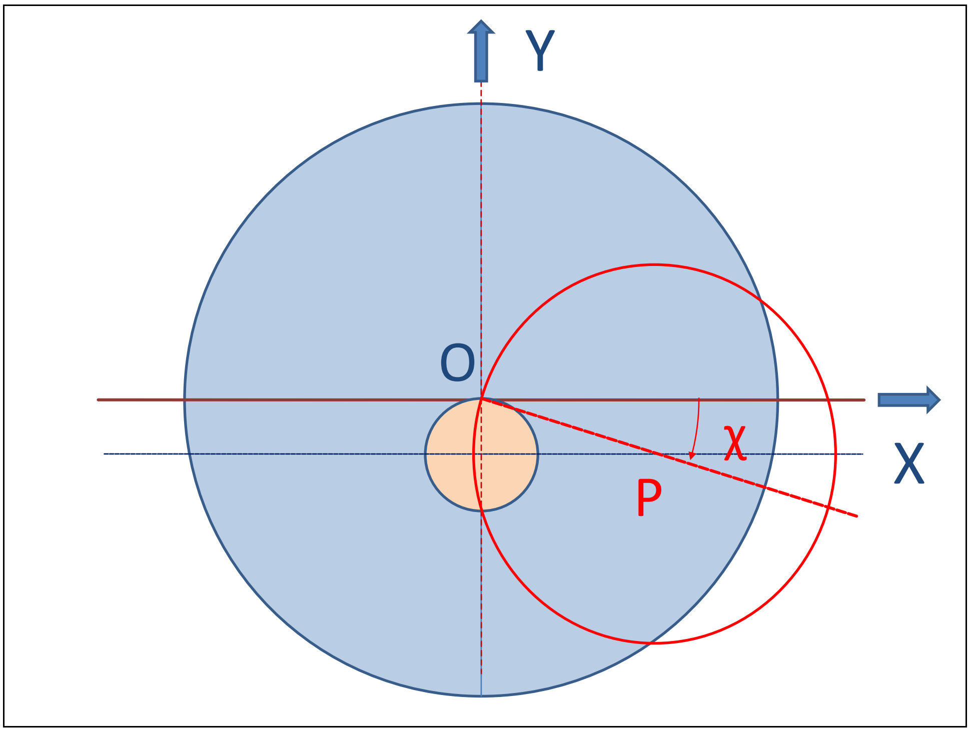

Figure 4

The construction begins with drawing a line parallel to the flight direction (x) positioned at the distance of half the advance ratio; this is in fact passing through the centre of the reverse flow region. A second line is constructed from the origin (O) at an angle to the abscissa equal to the sweep angle contour required. This intersects the first construction line at point P. Using P as the centre and OP the radius, a circle is drawn. The part of this circle within the rotor disc (the unit circle) is the required contour.

Examples

The surfaces and contours for a range of advance ratios from 0.1 – 0.5 are shown in Figures 5 – 14.

(The colour bar represents the sweep angle in degrees.)

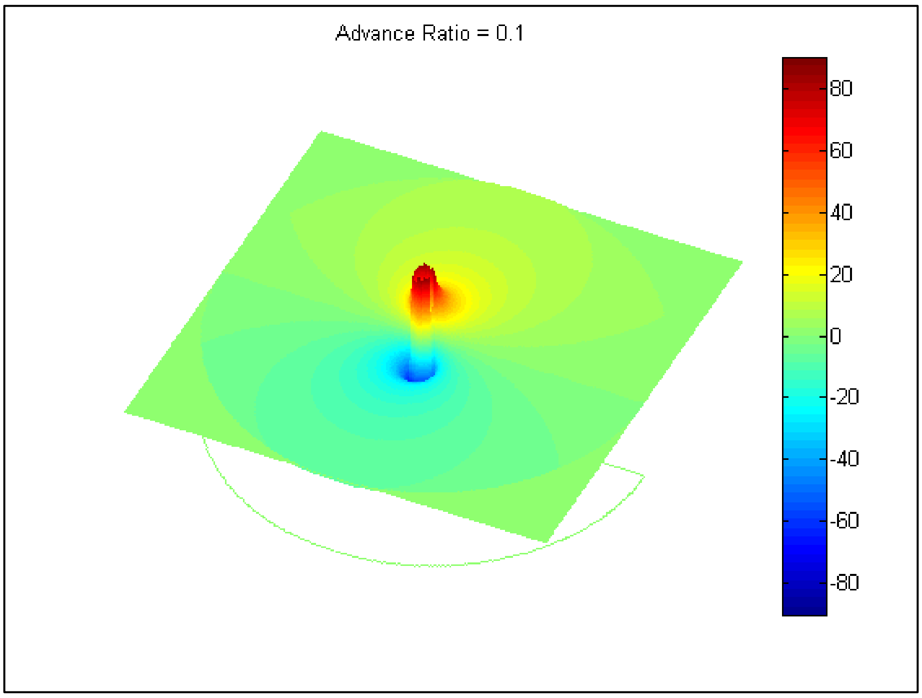

Figure 5

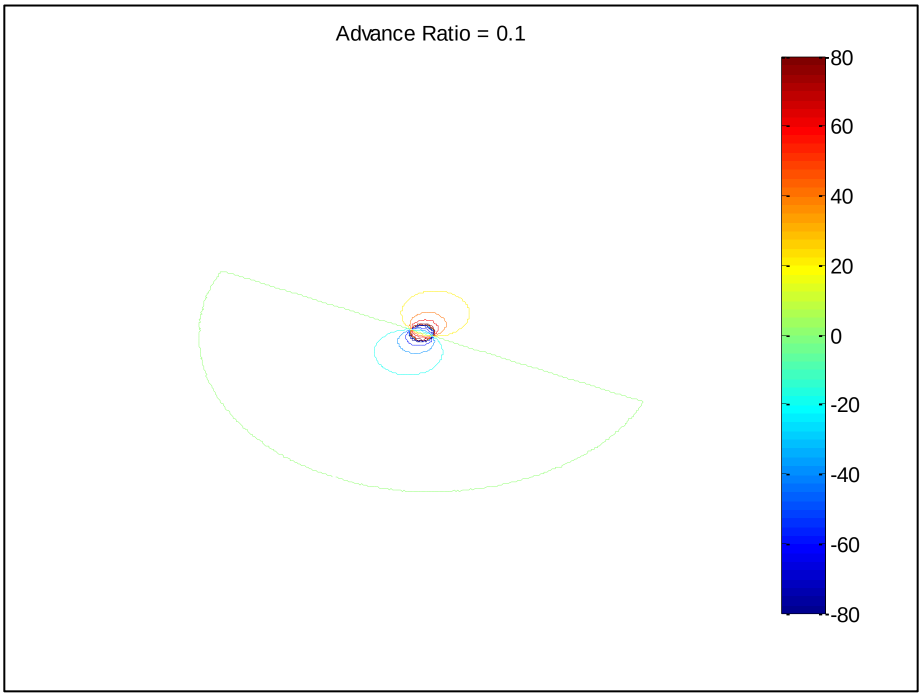

Figure 6

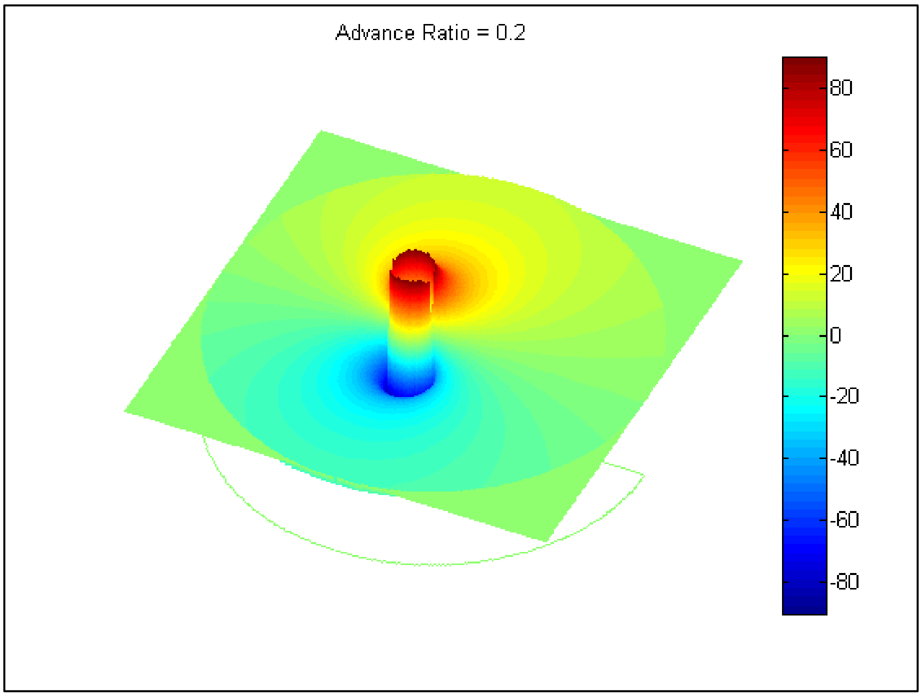

Figure 7

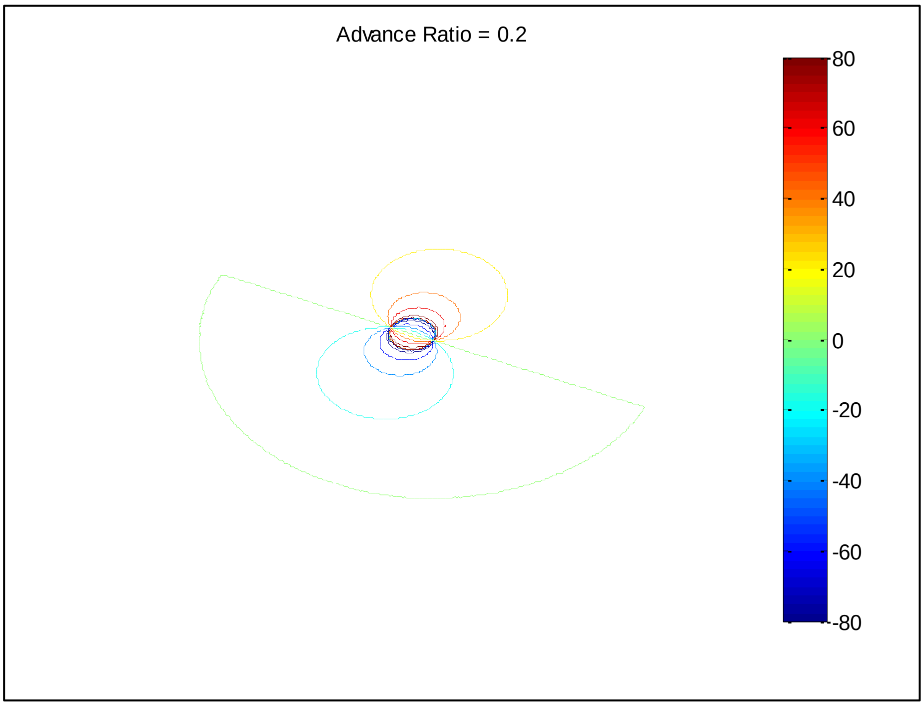

Figure 8

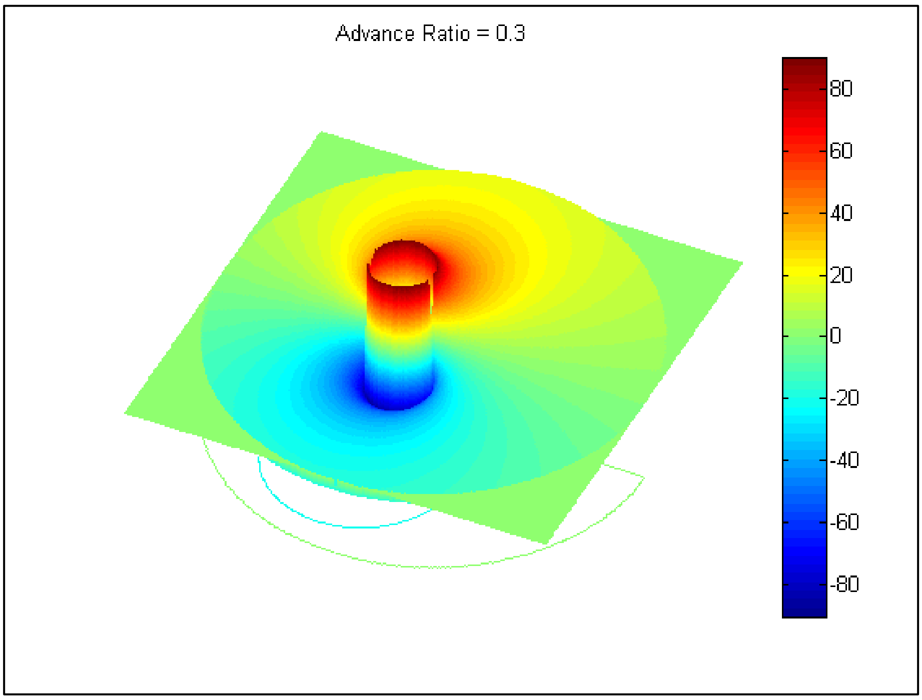

Figure 10

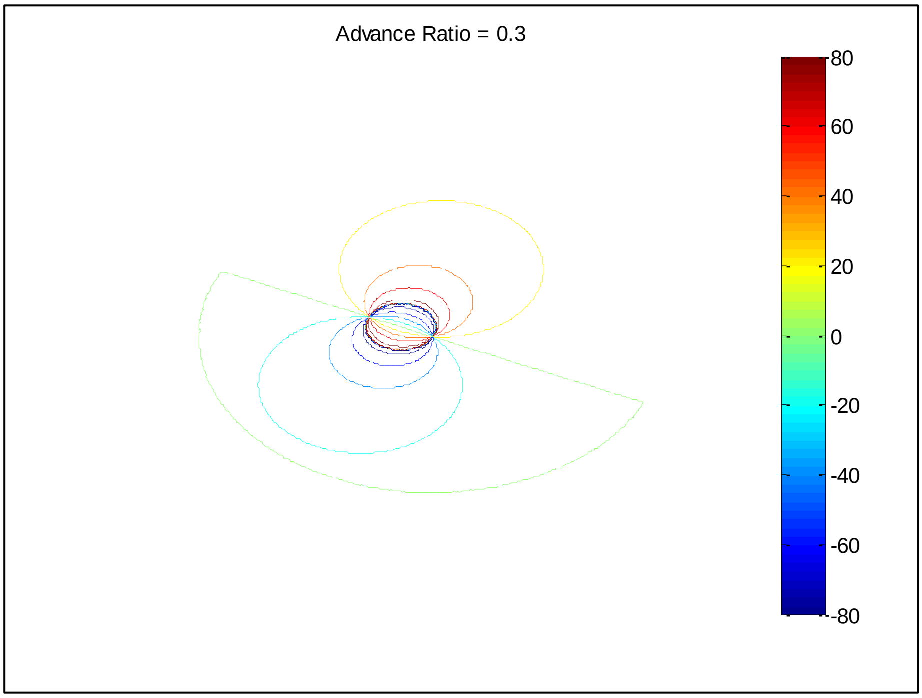

Figure 9

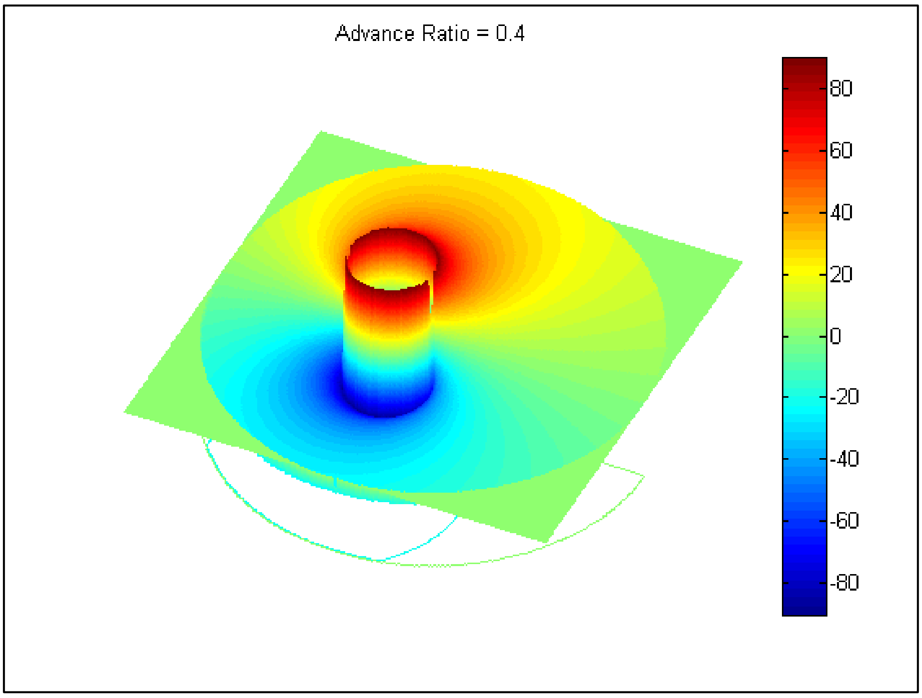

Figure 11

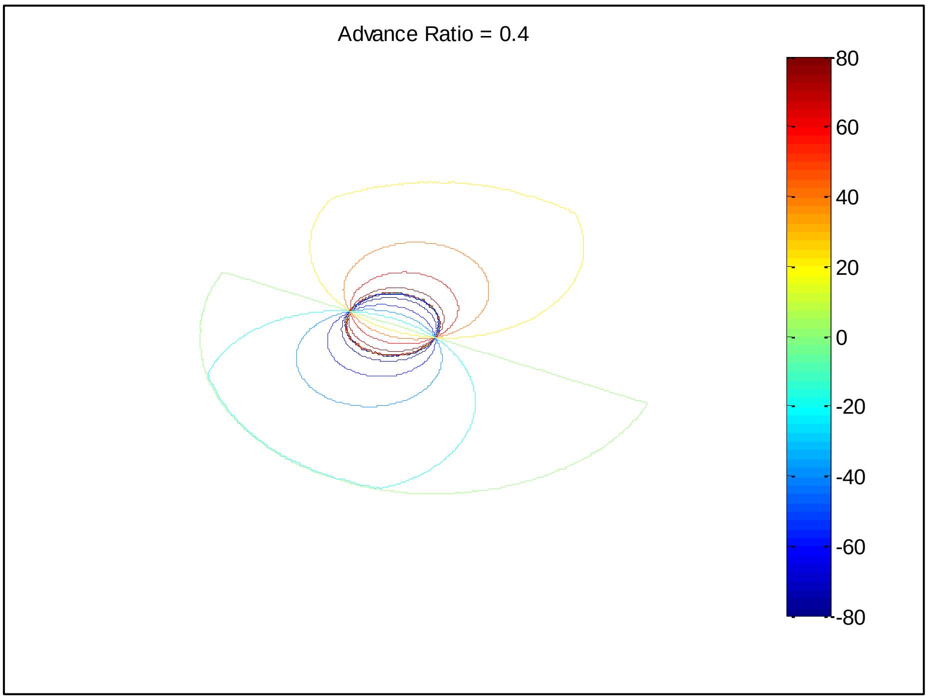

Figure 12

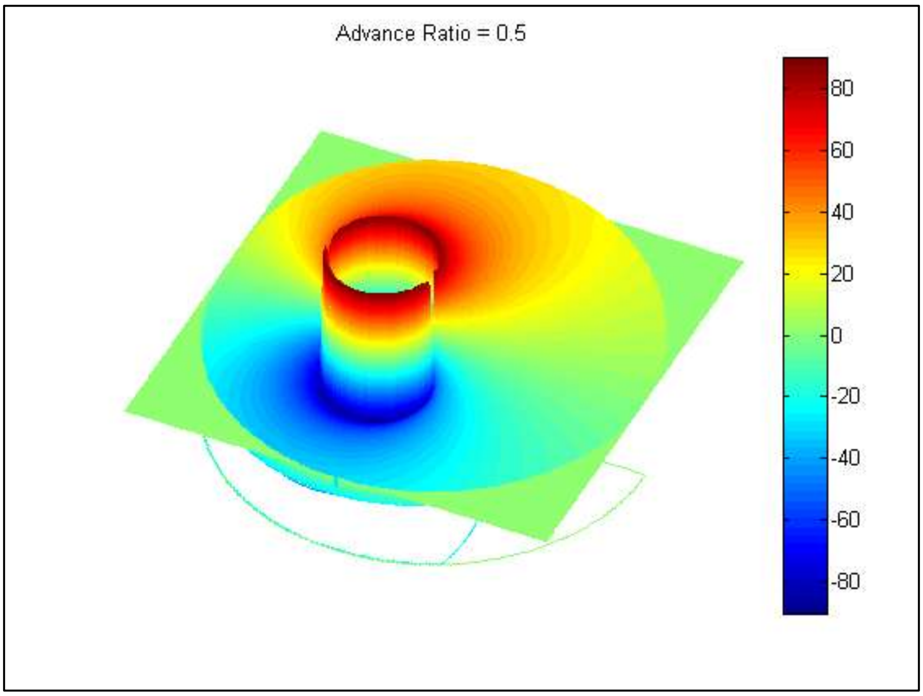

Figure 14

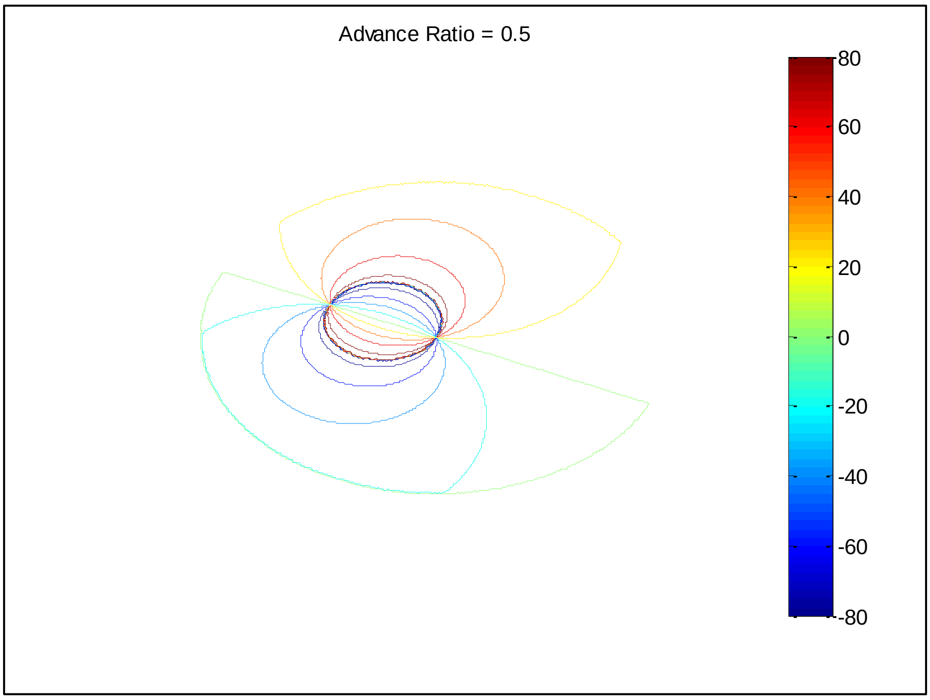

Figure 13

Conclusions

Examination of the equation governing the sweep angle contours enables the following conclusions to be made:

- The sweep angle contour is a circle.
- The zero angle contour is the advancing blade ($\psi = 0^\circ$) and the retreating blade ($\psi = 270^\circ$).
- The 90° contour is the periphery of the reverse flow region.
- All the contours pass within the reverse flow region. The sweep angle remains the same, but the flow direction is completely reversed.