Aerodynamics & Flight Mechanics Research Group

Methods of Calculating Helicopter Power, Fuel Consumption and Mission Performance

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UNIVERSITY OF SOUTHAMPTON

SCHOOL OF ENGINEERING SCIENCES

AERODYNAMICS AND FLIGHT MECHANICS RESEARCH GROUP

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by

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Calculation of Helicopter Power & Fuel Consumption

Introduction

The previous chapters have discussed methods to calculate the power required to fly a helicopter of a given weight at a given speed. The results of these calculations, together with engine data can enable the fuel consumption to be determined and hence a mission can be "flown". This enables a project study on a proposed helicopter design to be completed. The method to be described in this chapter collates the various momentum or actuator disc theories together in a suggested calculation scheme. As explained in earlier chapters, momentum theories are the simplest available and therefore require a degree of factoring to make the results more realistic. However, because of their simplicity they should not be applied for detailed rotor studies, particularly with respect to the flight envelope. With this proviso, the simplicity does in fact allow a parametric study to be achieved quickly and cheaply. With modern personal computers, the methods described in the chapter can be readily handled with these machines and a broad picture of the helicopter design and its ability to perform a proposed mission can be quickly achieved.

Therefore the following describes a method of calculating the power required and fuel consumption of a helicopter. It is based on a momentum method used for project calculations and is only applicable for a general appraisal of the helicopter's performance and is designed for a single main and tail rotor configuration. It is unsuitable for investigating performance towards the aircraft's flight envelope.

The method is described in sections each dealing with a specific aspect of the helicopter.
They are summarised thus:-

i. The attitude of the main rotor disc and the thrust is calculated on the basis of it balancing the helicopter's drag and weight.

ii. The main rotor, induced, profile, and parasite powers are then be determined, using appropriate factors. These are summed to give the total power required to drive the main rotor.

iii. The main rotor power is then converted to a torque which determines the tail rotor thrust required for yaw trim.

iv. The tail rotor induced and profile powers can then be calculated.

v. The total helicopter power required can then be determined. by summing the main and tail rotor powers together with that required to drive auxiliary services.

vi. Transmission losses are then accounted for giving the power required of the engines.
GLOSSARY OF TERMS

The terminology used in the analysis is presented in the following glossary.

**ROTOR**

<table>
<thead>
<tr>
<th>Term</th>
<th>MAIN</th>
<th>TAIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust</td>
<td>$T_M$</td>
<td>$T_T$</td>
</tr>
<tr>
<td>Blockage Factor</td>
<td>BLOCK$_M$</td>
<td>BLOCK$_T$</td>
</tr>
<tr>
<td>Tip Speed</td>
<td>$V_{TM}$</td>
<td>$V_{TT}$</td>
</tr>
<tr>
<td>No. of Blades</td>
<td>$N$</td>
<td>$n$</td>
</tr>
<tr>
<td>Blade Chord</td>
<td>$C$</td>
<td>$c$</td>
</tr>
<tr>
<td>Rotor Radius</td>
<td>$R$</td>
<td>$r$</td>
</tr>
<tr>
<td>Disc Tilt</td>
<td>$\gamma_S$</td>
<td></td>
</tr>
<tr>
<td>Advance Ratio</td>
<td>$\mu_M$</td>
<td>$\mu_T$</td>
</tr>
<tr>
<td>Advance Ratio Component</td>
<td>$\mu_xM$</td>
<td>$\mu_xT$</td>
</tr>
<tr>
<td>Advance Ratio Component</td>
<td>$\mu_zM$</td>
<td></td>
</tr>
<tr>
<td>Downwash</td>
<td>$\lambda_{iM}$</td>
<td>$\lambda_{iT}$</td>
</tr>
</tbody>
</table>
OVERALL HELICOPTER

<table>
<thead>
<tr>
<th>Drag @ 100 Velocity Units</th>
<th>(D_{100})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auxiliary Power</td>
<td>(P_{AUX})</td>
</tr>
<tr>
<td>Transmission Loss Factor</td>
<td>TRLF</td>
</tr>
<tr>
<td>Helicopter Weight</td>
<td>(W)</td>
</tr>
<tr>
<td>Helicopter Drag</td>
<td>(D)</td>
</tr>
</tbody>
</table>

OVERALL AIRCRAFT

Calculate aircraft drag:-

Firstly the forward tilt of the main rotor needs to be established. This requires the estimation of the drag of the complete aircraft. Drag can be specified in many ways; however, this method uses as a basis the drag force at a reference speed of 100 units, \((D_{100})\) at ISA Sea Level air density. The drag of the aircraft is then calculated using a variation of drag with the square of the forward speed and
linearly with respect to the air density.

\[ D = D_{100} \left( \frac{V}{100} \right)^2 \sigma \]  \hspace{1cm} (1)

**MAIN ROTOR**

**CALCULATE MAIN ROTOR THRUST AND DISC ATTITUDE:**

The force diagram for the main rotor is shown in figure 1, force balance requires:

Resolving Vertically:

\[ T \cos(\gamma_s) = W \]  \hspace{1cm} (2)

Resolving Horizontally:

\[ T \sin(\gamma_s) = D \]  \hspace{1cm} (3)

**FROM WHICH WE OBTAIN FOR THE DISC TILT:**

\[ \gamma_s = \tan^{-1}\left( \frac{D}{W} \right) \]  \hspace{1cm} (4)
The main rotor thrust, with blockage, becomes:

\[ T = \sqrt{W^2 + D^2} \cdot BLOCK_M \]  

(5)

The blockage term is applied to the main rotor thrust as a multiplying factor and represents the download on the fuselage due to the rotor downwash.

The induced velocity of the main rotor is now required. As actuator disc theory is to be used, the main rotor advance ratio components parallel to and normal to the rotor disc plane are required together with the thrust coefficient.

The advance ratio is:

\[ \mu_M = \frac{V}{V_{TM}} \]  

(6)

Resolving parallel to the rotor disc:

\[ \mu_{xM} = \mu_M \cos(y_s) \]  

(7)

Resolving perpendicular to the rotor disc:

\[ \mu_{zM} = \mu_M \sin(y_s) \]  

(8)

**CALCULATE MAIN ROTOR DOWNWASH:**

The main rotor downwash can now be calculated using the iterative technique.
\[
\lambda_{IM,\,NEW} = \lambda_{IM,\,OLD} \left( \frac{C_T}{4 \sqrt{\mu_{\lambda M}^2 + (\mu_{\lambda M} + \lambda_{IM,\,OLD})^2}} \right) \left( 1 + \frac{(\mu_{\lambda M} + \lambda_{IM,\,OLD})C_T}{4 \sqrt{\mu_{\lambda M}^3 + (\mu_{\lambda M} + \lambda_{IM,\,OLD})^3}} \right)
\] (9)

A sensible starting value is that for hover, namely:

\[
\lambda_i\,OLD = \frac{1}{2} \sqrt{C_T}
\] (10)

This will give the downwash \( \lambda_{\lambda M} \).

**ASSEMBLE MAIN ROTOR POWERS:**

The main rotor power components are now calculated.

**Induced (note the inclusion of the induced power factor \( k_{iM} \))**:

\[
P_{iM} = k_{iM} \cdot T_M \cdot V_{TM} \cdot \lambda_{iM}
\] (11)

**Profile**:

\[
P_{pM} = \frac{1}{8} \rho (V_{TM})^3 \cdot NCR \cdot C_{D0M} \cdot (1 + 3.0 \mu_{\lambda M}^2)
\] (12)

**Parasite**:

\[
P_{paraM} = D \cdot V
\] (13)

**Total**:
\[ P_{TOTM} = P_{iM} + P_{pM} + P_{paraM} \]  \hfill (14)

It should be noted that in equations (12,19) the \( \mu \times 2 \) term is multiplied by 3.0. This is the value which was obtained in v.40. The extra effects raising this to 4.7 can be simply dealt with by amending the calculation method.

**TAIL ROTOR**

The main rotor power is now converted to a torque from which the required tail rotor thrust for torque balance is calculated.

**CALCULATE TAIL ROTOR THRUST:**

\[ T_T = \left( \frac{P_{TOTM}}{\Omega_M \cdot l_{BOOM}} \right) BLOCK_T \]  \hfill (15)

The induced velocity of the tail rotor is also required, whence the following are calculated:-

\[ \mu_{xT} = \frac{V}{V_{TT}} \]  \hfill (16)

\[ \mu_{zT} = 0 \]  \hfill (17)

Since the tail rotor is only required to develop a thrust with no disc tilt, the disc plane is assumed parallel to the flight path.
**CALCULATE TAIL ROTOR DOWNWASH**:

Using the same iterative method, the tail rotor downwash, $\lambda iT$, can be calculated.

**ASSEMBLE TAIL ROTOR POWERS**:

The tail rotor power components can now be calculated.

Induced:

$$P_{iT} = k_{iT} \cdot T_T \cdot V_{TT} \cdot \lambda_{iT}$$  \hspace{1cm} (18)
Profile:-

\[ P_{pT} = \frac{1}{8} \rho (V_{TT})^3 \cdot ncr \cdot C_{D0T} \cdot (1 + 3.0 \mu_{XT}^2) \] \hspace{1cm} (19)

TOTAL:-

\[ P_{TOTT} = P_{iT} + P_{pT} \] \hspace{1cm} (20)

Note that the main rotor is responsible for overcoming the parasite drag of the aircraft.

**COMPLETE AIRCRAFT**

The total power required can now be calculated by summing the total powers for each rotor together with any auxiliary services \((P_{Aux})\) such as oil pumps and electrical generators. Since some losses will occur in the transmission, a factor is applied to account for this giving the power required from the engines.

**ASSEMBLE OVERALL AIRCRAFT POWERS AND ALLOW FOR TRANSMISSION LOSSES:-**

\[ P_{TOTAL} = [P_{TOTM} + P_{TOTT} + P_{Aux}] \cdot TRLF \] \hspace{1cm} (21)
EXAMPLE OF PARAMETER VALUES

The following table gives example values of the various factors applied.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>MAIN ROTOR</th>
<th>TAIL ROTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Blockage</td>
<td>BLOCK</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Induced Power Factor</td>
<td>k_i</td>
<td>1.10</td>
<td>1.20</td>
</tr>
<tr>
<td>Profile Drag Coefficient</td>
<td>C_{D0}</td>
<td>0.011</td>
<td>0.012</td>
</tr>
</tbody>
</table>

The blockage effect on each rotor will vary with forward speed. The main rotor blockage is a result of the main rotor downwash impinging on the fuselage causing a download. Therefore, in order to achieve the required component of force from the rotor, the thrust must exceed this by an amount equal to the download on the fuselage. The blockage factor multiplies the desired net thrust to account for the loss, however, as the forward speed of the helicopter increases, the rotor downwash is carried increasingly downstream of the fuselage eventually causing no real fuselage download from the main rotor wake. In this case the blockage factor becomes unity. The variation specified for this example is based on advance ratio and the blockage value refers to the hover condition and linearly decreases to unity at an advance ratio of 0.05. Thereafter it remains at unity as illustrated in figure 2.
CALCULATION OF ENGINE FUEL CONSUMPTION

Having determined the power required from the engine(s) to fly a given helicopter flight condition, the fuel consumption can now be calculated.

Engine fuel consumption data is often given in terms of specific fuel consumption (sfc) (kg/hr/kw) for a corresponding power setting (P). A small conversion of the data allows a very simple method of calculating the fuel consumption of a gas turbine engine developing a specified amount of power.

The fuel flow of an engine (\(W_f\)) (kg/hr) can be obtained as the product of the sfc and the respective power. Plotting fuel flow against power produces a variation very close to linear and can be fitted by means of linear regression (least squares). Because of this simple linear variation, the fuel consumption calculation becomes elementary.

To enable full use to be made of the method, the operating altitude and temperature will need to be built in to the calculation. Each atmospheric condition will produce an individual straight line fit, but if the fuel flow and engine power are normalised by:

\[ \frac{\delta}{\sqrt{T}} \]

Where:-

\(\delta\) is the pressure ratio.

\(T\) is the absolute temperature ratio.

(both relative to ISA Sea Level Atmosphere conditions.)

the various lines defining the engine performance at a given atmospheric condition close together and the straight line law for the fuel flow becomes practically one line.

i.e. The engine fuel consumption law for any atmospheric condition becomes:-

\[
\frac{W_f}{\delta \sqrt{T}} = A_E + B_E \left[ \frac{P}{\delta \sqrt{T}} \right]
\]

(22)
This gives complete flexibility as regards changing atmospheric conditions. It should be noted that this type of engine calculation assumes that the power unit is not operating close to a limit.

The straight line fit possesses a positive intercept on the fuel flow axis \( (A_E) \) which results in an important fact concerning the optimising of fuel consumption for a multi-engined helicopter.

Consider a helicopter which is fitted with \( N \) engines combining to give a total power production of \( P \). Each engine must then produce a power of \( P/N \) and the corresponding fuel consumption for all \( N \) engines is therefore:

\[
\frac{W_f}{\delta \sqrt{\theta}} = N \left( A_E + B_E \left[ \frac{P}{N} \times \frac{1}{\delta \sqrt{\theta}} \right] \right)
\]

i.e.:-

\[
W_f = N \cdot A_E \cdot \delta \sqrt{\theta} + B_E \cdot P
\]

Because of the first term of the right hand side of (24), it can be seen that for a given power production, a smaller number of engines will give a lower fuel consumption. Consequently a helicopter designer, when optimising for fuel consumption, will use a minimum number of engines capable of providing sufficient power. Other requirements such as helicopter performance with an engine failure tend to work in the opposite direction and, on this type of basis, the optimum design will have a maximum number of engines. Hence, choice of engine for a multi engine helicopter design is not as clear cut as might at first be thought.

**ENGINE LIMITS**

The previous discussions have considered the engine performance purely from a fuel consumption point of view. In reality, a gas turbine engine will have limitations which are usually based on the permissible operating temperature of the turbine section. However, it is possible to operate the engine at higher power settings for a limited period of time without causing permanent damage. In emergency situations, some excessive power settings can be demanded for very limited time periods at the expense of accelerated engine wear or damage.
Typical examples of power settings are described below:

**Maximum Continuous Power Rating**

This is the maximum power that an engine can develop without a time constraint, and consequently operate continuously.

**Take Off or 1 Hour Power Rating**

This is used for the higher power conditions such as take off and hover at high altitude and/or ambient temperature. It can be used for periods of approximately 1 hour (sometimes ½ hour) before the power demand must be reduced.

**Maximum Contingency or 2½ Minute Power Rating**

This is used with engine loss and other contingency situations when the power can be used for short periods of 2 to 3 minutes.

An engine inspection after such use is usually considered.

**Emergency or ½ Minute Power Rating**

This is the highest and consequently the most damaging condition. It is used for situations where loss of the aircraft is a real possibility.
An example is when a twin-engined naval helicopter suffers an engine loss in a high power condition, (high all up weight and in the hover), and is forced to ditch in the sea. After jettisoning as much weight as possible, a take off on a single engine from the water will be necessary to save the aircraft. Emergency power will be required for such a dire emergency and the engine will probably require extensive maintenance and refurbishment, if not scrapping, due to the use of such an excessive but necessary power demand.

### CALCULATION OF THE PERFORMANCE OF A HELICOPTER

To illustrate the method, the following calculations, based on a small utility helicopter are presented.

The input data required is shown below:-

<table>
<thead>
<tr>
<th>Rotor Data</th>
<th>Main Rotor</th>
<th>Tail Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Blades</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Chord (m)</td>
<td>0.394</td>
<td>0.180</td>
</tr>
<tr>
<td>Radius (m)</td>
<td>6.4</td>
<td>1.105</td>
</tr>
<tr>
<td>Tip Speed (m/s)</td>
<td>218.69</td>
<td>218.69</td>
</tr>
<tr>
<td>Blockage</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Induced Power Factor (k_i)</td>
<td>1.10</td>
<td>1.20</td>
</tr>
<tr>
<td>Profile Drag Coefficient (C_{D0})</td>
<td>0.011</td>
<td>0.012</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuselage Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail Boom Length (m)</td>
<td>7.66</td>
</tr>
<tr>
<td>Description</td>
<td>Value</td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$D_{100}$ (N)</td>
<td>6226.9</td>
</tr>
<tr>
<td>Auxiliary Power PAUX (kw)</td>
<td>26.1</td>
</tr>
<tr>
<td>Transmission Loss Factor (TRLF)</td>
<td>1.04</td>
</tr>
</tbody>
</table>
No. of Engines = 2

The fuel flow law, presented in (25), was obtained from public domain information using linear regression:\n\[
\frac{W_f}{\delta \theta} = 46.5 + 0.24 \frac{P}{\delta \theta}
\] \hspace{1cm} (25)\n
With the above data, the variation of the main rotor power components with forward speed is shown in figure 3, namely induced, profile, parasite, and total.

It is instructive to see these results when shown cumulatively as in figure 4, The following are then added to the main rotor power, tail rotor power, auxiliary power, and the influence of transmission losses to give the total overall power required of the powerplants.

The total power variation of the complete helicopter with forward speed is shown in figure 5. From this power distribution the fuel flow can now be calculated and converted to endurance and range estimates. Endurance is the time required to consume a specified amount of fuel, whilst range is the corresponding distance covered. Endurance is concerned principally with minimising the rate of fuel usage with time, whilst range is a compromise between time and forward speed. (A fuel usage of 100 kg is assumed for these calculations.) The endurance is shown in figure 6, and the range (km) is shown in figure 7. Two plots are presented in each figure corresponding to the full fuel flow law defined in equation (1), and a modified law where the intercept ($A_E$, or fuel flow at zero power) is set to zero. (This corresponds to a constant sfc.) The lower curve corresponds to the full law ($A_E \neq 0$) and the upper curve to the modified law ($A_E = 0$).

The positions of maximum endurance and range are indicated in the figures. As can be seen, changing the fuel flow law does not alter the optimum endurance speed of 38 m/s (A), which corresponds to minimum power. However, the change does have an influence on the optimum range speed increasing it from 65 m/s (B) to 80 m/s (C).

Figure 8 shows the three extrema indicated by A, B, and C on the total power v forward speed curve.
In fact, the three points A, B, and C, corresponding to the various extrema, can be determined geometrically as the following analysis shows, using the following definitions:

\[ W_{\text{FUEL}} = \text{Fuel Weight (fixed)} \]
\[ T = \text{Time} \]
\[ S = \text{sfc} \]
\[ P = \text{Power} \]
\[ V = \text{Forward Speed} \]
\[ E = \text{Endurance} \]
\[ R = \text{Range} \]

we have the following:

Point A (maximum endurance)

The definition for sfc is:

\[ S = \frac{W_{\text{FUEL}}}{P \cdot T} \quad (26) \]

i.e.:

\[ T = \frac{W_{\text{FUEL}} \cdot \frac{1}{P}}{S} \quad (27) \]

As \( W_{\text{FUEL}} \) and \( S \) are fixed, \( T \) is maximum when \( P \) is minimum, i.e. point A is the minimum point on the Power v Velocity curve.
Point B (maximum range - constant sfc, $A_t=0$)

The range is given by:

$$R = T \cdot V = \frac{V}{P} \cdot \frac{W_{FUEL}}{S}$$

(28)

i.e. R is a maximum when $P/V$ is a minimum, so point B is positioned where the tangent, drawn from the origin, touches the power curve.

Point C (maximum range - full fuel flow law)

For this case we have:

$$W_{FUEL} = T \left( A_E + B_E P \right)$$

(29)

Therefore:

$$R = V \cdot T = \frac{V}{A_E + B_E P} \cdot W_{FUEL} = \frac{W_{FUEL} \cdot V}{B E \cdot \frac{A_E}{B E} + P}$$

(30)

This is very similar to point B except that the additional term in the denominator $(A_E/B_E)$ requires that the tangent be drawn from the point $(0,-A_E/B_E)$. The construction for the three points is shown in figure 9.

**Wind**

If the helicopter is flying into a headwind of $V_w$, the above formula for range becomes:
\[ R = (V - V_w) \cdot T \]
\[ = \frac{V - V_w}{A_E + B_E} \cdot W_{FUEL} \]
\[ = \frac{W_{FUEL}}{B_E} \cdot \frac{V - V_w}{A_E + P} \]  

This means that the tangent should be drawn from the point \((V_w, -A_E/B_E)\).
MISSION ANALYSIS

The calculation of engine power and fuel consumption can be extended to "fly" a mission in a computer which is of direct use to a project assessment. In flying a mission, the helicopter weight will be changing due to fuel usage consequently this must be accounted for in the calculations and the method described here uses a technique which is simple to implement. The mission is divided into legs, each being calculated separately, in sequence, with the value of the aircraft weight at the end of a particular leg becoming the start value for the succeeding leg. (Each leg will describe a fixed flight condition, particularly altitude and/or forward speed.) This also allows discrete weight changes of payload to be incorporated, such as changes in passenger/cargo payload or the deployment of ordnance.

For each leg the power and fuel consumption at the start weight is calculated. Knowing the duration of the leg a first estimate of the weight change over the leg is obtained. Taking the mean value of aircraft weight the process is repeated and the fuel usage of the leg at the two weights compared. If they lie within a specified tolerance, the final estimate is taken. If not, a revised mean aircraft weight is adopted (using the latest fuel usage figure) and the process repeated iteratively until convergence to within the required tolerance is achieved.

CALCULATION METHOD

A summary of the calculation method is presented and can be used as a basis for a flow diagram. Each part of the calculation is presented as a complete entity with the input and output data being specified. This imparts a segmented structure to the method which is recommended for implementation in a computer program.

ATMOSPHERIC PARAMETERS

This allows the air characteristics required by the calculations to be determined from the altitude and ambient temperature.
The helicopter is assumed to remain within the troposphere.

**Input**

- Altitude,
- Sea Level Air Temperature,
- Sea Level Air Density.

**Output**

- Density Ratio ($\sigma$),
- Absolute Temperature Ratio ($\theta$),
- Pressure Ratio ($\delta$).

**Calculation**

The absolute temperature ratio is given by:

$$\theta = \frac{T_{\text{sea level}} - \text{Altitude} \cdot \text{Lapse Rate}}{T_{\text{sea level}}}$$

The pressure ratio by:

$$\delta = \theta^{5.256}$$
The relative density by:

\[ \sigma = \theta^{4.256} \]  \hspace{1cm} (34)

The lapse rate of 6.5°C per kilometre is used in the calculations.
DOWNWASH CALCULATION

This is the application of momentum theory to the downwash calculation.

Input

- Thrust Coefficient ($C_T$),
- Advance Ratio components,
  - parallel to the rotor disc ($\mu_x$),
  - perpendicular to the rotor disc ($\mu_z$).

Output

- Downwash ($\lambda_i$)

Calculation

Start value (hover)

\[
\lambda_{i\,OLD} = \frac{1}{2}\sqrt{C_T}
\]  

(35)
\[
\Delta \lambda_i = \left( \lambda_i^{\text{OLD}} - \frac{C_T}{4 \sqrt{\mu_x^2 + \left( \mu_z + \lambda_i^{\text{OLD}} \right)^2}} \right) \left( 1 + \frac{(\mu_z + \lambda_i^{\text{OLD}}) C_T}{4 \left[ \mu_x^2 + (\mu_z + \lambda_i^{\text{OLD}})^2 \right]^{1.5}} \right)
\] (36)

\[
\lambda_i^{\text{NEW}} = \lambda_i^{\text{OLD}} - \Delta \lambda_i
\] (37)

Has the iteration reached convergence?

\[
|\lambda_i^{\text{NEW}} - \lambda_i^{\text{OLD}}| < \text{Tolerance}?
\] (38)

If NO then reset the downwash value and calculate the next estimate:

\[
\lambda_i^{\text{OLD}} = \lambda_i^{\text{NEW}}
\] (39)

GO TO †

If YES then convergence has been achieved:

\[
\text{DOWNWASH} = \lambda_i^{\text{NEW}}
\] (40)

EXIT
HELICOPTER POWER

This is the helicopter power calculation.

Input

- Aircraft All up Weight
- Forward Speed
- Atmospheric Data
- Aircraft Data

Suffix M refers to the Main Rotor, whilst T refers to the Tail Rotor.

Output

- Helicopter Power

Calculation

Fuselage Drag, and Aircraft Weight

Main Rotor Thrust, Disc Tilt & Forward Speed
\[ \text{CTM} \mu_{\lambda M}, \mu_{\lambda M} \]
\[ \downarrow \]
\[ \lambda_{\lambda M} \]

\[
\begin{bmatrix}
P_{IM} \\
+ P_{pM} \\
+ P_{paraM} \\
\hline
P_{TOTM}
\end{bmatrix}
\]  \hspace{1cm} (41)

\[
\text{Tail Rotor Thrust} = \frac{\text{Main Rotor Torque}}{\text{Tail Boom Length}}
\]  \hspace{1cm} (42)
Tail Rotor Thrust (assume no disc tilt) & forward speed

\[
\downarrow
\]

\[ C_{TT}, \mu_{xT} (\beta_{2T}=0) \]

\[
\downarrow
\]

\[ \lambda_{iT} \]

\[
\begin{bmatrix}
P_{iT} \\
+ P_{IT} \\
\hline
P_{TOTT}
\end{bmatrix}
\]

\[
\begin{bmatrix}
P_{TOTT} \\
+ P_{TOTT} \\
+ P_{AUX} \\
\hline
P_{TOTAL}
\end{bmatrix}
\]

\[ P_{TOTM} = \text{Transmission Loss Factor (TRLF)} \cdot P_{TOTAL} \quad (45) \]

**EXIT**
FUEL FLOW

This calculates the fuel consumption rate from the power requirements.

Input

- Engine Power (P)
- No. of Engines (N)
- Atmospheric Pressure Ratio (δ)
- Atmospheric Temperature Ratio (θ)
- Engine Performance Coefficients (A_E, B_E)

Output

- Fuel Flow for the stated power required (W_f)

Calculation

\[
Fuel \ Flow \ (W_f) = \delta \sqrt{\theta} \cdot N \cdot A_E + P \cdot B_E \tag{46}
\]

EXIT
MISSION LEG

This calculates the fuel usage over a mission component, or leg.

Input

- Start and Finish Altitudes
- (Power, and fuel flow are averaged between these two conditions)
- Start All Up Weight (AUWSTART)
- Forward Speed (V)
- Time or Distance of Leg
- (eg. 5 min hover or 20 km cruise at 100 m/s)

Output

- Fuel used during mission leg

Calculation

*Set variables to the conditions at the start of the leg,*

\[ \text{AUW}_{\text{START}}, V \]
Calculate Power

\[ \Downarrow \]

Calculate Fuel Flow

\[ \Downarrow \]

Calculate fuel usage - Fuel Flow \( \star \) Time \( \Rightarrow \) Fuel Used\(_{OLD}\)

\[ \Downarrow \]

Calculate average weight - AUW=AUW\(_{START}\) - \( \frac{1}{2} \) Fuel Used\(_{OLD}\)

\[ \Downarrow \]

\[ \dagger \]

*Calculate the fuel usage with the revised helicopter weight.*

AUW, V

\[ \Downarrow \]

Power

\[ \Downarrow \]

Fuel Flow

\[ \Downarrow \]

Fuel Flow \( \star \) Time \( \Rightarrow \) Fuel Used\(_{NEW}\)
Has the fuel usage estimate converged?

\[ |\text{Fuel Used}_\text{OLD} - \text{Fuel Used}_\text{NEW}| < \text{Tolerance} \]  

(47)

If NO, then reset the helicopter weight and proceed to a new estimate:

\[ \text{Fuel Used}_\text{OLD} = \text{Fuel Used}_\text{NEW} \]
\[ \downarrow \]

\[ \text{AUW} = \text{AUW}_\text{START} - \frac{1}{2} \text{Fuel Used}_\text{NEW} \]
\[ \downarrow \]

GO TO †

If YES, then fuel usage calculation has converged and therefore the weight change over the leg can be determined:

\[ \text{AUW}_\text{FINISH} = \text{AUW}_\text{START} - \text{Fuel Used}_\text{NEW} \]

\[ \downarrow \]

EXIT
EXAMPLES OF MISSION CALCULATIONS

To see the application of the calculation to a mission, two examples are presented here. They are for the WG13 aircraft flying:

i. A mission performed mainly at high speed, such as anti-tank.
ii. A mission containing much hover time, such as anti-submarine (ASW).

They do not represent existing missions but are used merely to examine the performance of a given helicopter engaged on missions which contain either significant elements of high speed operation or hover time.

The mission calculation was performed using the Westland WG13/Lynx as a datum aircraft for five helicopter designs, each illustrating the type of parametric change which a designer could choose. The results show the effect of such changes on the fuel usage of the Lynx aircraft.

Case 1  This is the basic aircraft.

Case 2  This has the aircraft parasitic drag doubled via the $D_{100}$ term.

Case 3  This has the main and tail rotor radii increased by 0.5 m, with the consequent increase in tail boom length of 1 m.

Case 4  The number of engines are reduced to 1. No allowance has been made for the ability of the single engine to generate sufficient power. Only the fuel consumption has been studied.

Case 5  The number of engines are increased to 3. The data used for the power calculation of the Westland Lynx is used as a basis for these calculations.
The five sets of parameter changes are tabulated below:

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{100}$</td>
<td>6227</td>
<td>12454</td>
<td>6227</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main Rotor Radius</td>
<td>6.401</td>
<td>6.901</td>
<td>6.401</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tail Rotor Radius</td>
<td>1.105</td>
<td>1.605</td>
<td>1.105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tail Boom Length</td>
<td>7.66</td>
<td>8.66</td>
<td>7.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Engines</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WESTLAND LYNX - ANTI TANK MISSION

DESCRIPTION OF THE MISSION

The mission is divided into nine distinct legs which are tabulated below:

<table>
<thead>
<tr>
<th>Leg</th>
<th>Phase</th>
<th>Altitude</th>
<th>Speed</th>
<th>Time</th>
<th>Distance</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Take Off</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Cruise</td>
<td>Sea Level</td>
<td>70 m/s</td>
<td></td>
<td>100 km</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Climb</td>
<td>Sea Level - 2500m</td>
<td>50 m/s</td>
<td>2 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity</td>
<td>Distance</td>
<td>Speed</td>
<td>Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>----------</td>
<td>----------</td>
<td>-------</td>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Loiter</td>
<td>2500m</td>
<td>35 m/s</td>
<td>15 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Descent</td>
<td>2500m - Sea Level</td>
<td>55 m/s</td>
<td>10 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Ambush</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Attack</td>
<td>Sea Level</td>
<td>80 m/s</td>
<td>5 mins</td>
<td>Drop 130 kg of ordnance</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Return</td>
<td>Sea Level</td>
<td>70 m/s</td>
<td>100 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Land</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**PROGRESSION OF CALCULATION**

The method commences with the helicopter at the start of leg 1, at its takeoff weight of 4500 kg. A tolerance of 5 kg of fuel usage for each leg is used.

LEG 1 - Hover 5 mins (0.083 hrs)

<table>
<thead>
<tr>
<th>Pass #</th>
<th>Aircraft Weight (kg)</th>
<th>Power (kw)</th>
<th>Fuel Flow (kg/hr)</th>
<th>Fuel Used (kg)</th>
<th>Average Aircraft Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4500</td>
<td>949</td>
<td>322</td>
<td>27</td>
<td>4487</td>
</tr>
<tr>
<td>2</td>
<td>4487</td>
<td>946</td>
<td>321</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

The "fuel used" calculation has therefore converged after the second pass at 27 kg.

Aircraft weight at the end of the first leg and hence the beginning of the second leg is:

\[
4500 - 27 = 4473 \text{ kg}
\]

The calculation of the second leg now gives:

LEG 2 - Cruise 100 km at 70 m/s, time = 0.397 hrs
<table>
<thead>
<tr>
<th>Pass #</th>
<th>Aircraft Weight (kg)</th>
<th>Power (kw)</th>
<th>Fuel Flow (kg/hr)</th>
<th>Fuel Used (kg)</th>
<th>Average Aircraft Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4473</td>
<td>620</td>
<td>242</td>
<td>96</td>
<td>4425</td>
</tr>
<tr>
<td>2</td>
<td>4425</td>
<td>617</td>
<td>242</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

The "fuel used" calculation has converged after the second pass at 96 kg.

Aircraft weight at the end of the second leg and hence the beginning of the third leg is:

\[ 4473 - 96 = 4377 \text{ kg} \]

This calculation now proceeds through the remaining seven legs of the mission.
FUEL CONSUMPTION

The summary of the fuel consumption for all nine mission legs and for each of the five helicopter configurations is tabulated below. The total fuel consumption is included, expressed as a mass and as a percentage relative to the basic helicopter (Case 1).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
<td>27</td>
<td>26</td>
<td>23</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>96</td>
<td>118</td>
<td>97</td>
<td>77</td>
<td>114</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>46</td>
<td>43</td>
<td>37</td>
<td>53</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>29</td>
<td>25</td>
<td>18</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
<td>26</td>
<td>25</td>
<td>22</td>
<td>29</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>29</td>
<td>23</td>
<td>19</td>
<td>26</td>
</tr>
<tr>
<td>8</td>
<td>94</td>
<td>116</td>
<td>96</td>
<td>76</td>
<td>113</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>TOTAL</td>
<td>373</td>
<td>430</td>
<td>373</td>
<td>306</td>
<td>441</td>
</tr>
<tr>
<td>% of Case 1</td>
<td>100</td>
<td>115</td>
<td>100</td>
<td>82</td>
<td>118</td>
</tr>
</tbody>
</table>

As can be seen in Case 2, changes in $D_{100}$ increase the fuel consumption by 15% consistent with the amount of time spent at high speed where parasite power dominates. Changes to rotor size, in Case 3, produce virtually no change in fuel usage. This is explained by the fact that an increase in disc area reduces the hover power (induced component), but the attendant increase in blade area increases the profile power, particularly at high speed, and for this mission the two effects cancel out. The changes in the number of engines, Cases 4 and 5, are apparently the most influential, showing the advantage, from a fuel consumption viewpoint, of operating on a single engine, and on the same basis, the disadvantage of carrying a third engine.

These results are plotted in the following figures 10-15. Figure 10 shows the fuel usage for the nine
legs and under each leg, the five helicopter configurations. Figures 11-15 show the helicopter weight variation with time for each configuration over the complete mission.
WESTLAND LYNX - ASW MISSION

As a comparison, a second mission is presented. The anti tank mission previously discussed uses high speed to a great degree. As was seen in the results of the anti tank mission, the influence of rotor radius is not particularly influential in this mission which consists mainly of conditions of high speed where parasite power is dominant. The ASW mission, described below, however contains much hover time and so will illustrate the importance of rotor radius for this type of role.

DESCRIPTION OF THE MISSION

<table>
<thead>
<tr>
<th>Leg</th>
<th>Phase</th>
<th>Altitude</th>
<th>Speed</th>
<th>Time</th>
<th>Distance</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Take Off</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Cruise</td>
<td>Sea Level</td>
<td>40 m/s</td>
<td>20 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3A</td>
<td>Dunk</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td>The dunk, dash sequence is performed 9 times. 3A - 11B</td>
</tr>
<tr>
<td>3B</td>
<td>Dash</td>
<td>Sea Level</td>
<td>60 m/s</td>
<td>2 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Dunk</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Dash</td>
<td>Sea Level</td>
<td>80 m/s</td>
<td>5 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Attack</td>
<td>Sea Level</td>
<td>50 m/s</td>
<td>5 mins</td>
<td></td>
<td>Drop 300 kg of torpedoes</td>
</tr>
<tr>
<td>15</td>
<td>Return</td>
<td>Sea Level</td>
<td>70 m/s</td>
<td>20 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Land</td>
<td>Sea Level</td>
<td>Hover</td>
<td>5 mins</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the exception of increasing the start AUW from 4500 kg to 4770 kg, the aircraft data and the parameter changes are identical to the Anti Tank mission.
**FUEL CONSUMPTION**

The fuel consumption for all sixteen mission legs and the five configurations is tabulated below and in figures 16-21:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>28</td>
<td>27</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>31</td>
<td>29</td>
<td>23</td>
<td>36</td>
</tr>
<tr>
<td>3A - 11B</td>
<td>264</td>
<td>266</td>
<td>253</td>
<td>226</td>
<td>300</td>
</tr>
<tr>
<td>12</td>
<td>26</td>
<td>26</td>
<td>25</td>
<td>23</td>
<td>30</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>19</td>
<td>17</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>15</td>
<td>19</td>
<td>23</td>
<td>19</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>16</td>
<td>25</td>
<td>25</td>
<td>24</td>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>TOTAL</td>
<td>413</td>
<td>424</td>
<td>399</td>
<td>350</td>
<td>475</td>
</tr>
<tr>
<td>% of Case 1</td>
<td>100</td>
<td>103</td>
<td>97</td>
<td>85</td>
<td>115</td>
</tr>
</tbody>
</table>

The effect of the change in D_{100} (Case 2), is reduced from the anti-tank mission because the mission is primarily concerned with low speed hovering flight. This reason also accounts for the 3% advantage gained from increasing the size of the rotors and tail boom (Case 3).

The arguments about the number of engines still apply.

In summary, the parasite drag has a great influence on the performance if the mission contains substantial periods of high speed. This is particularly true if external stores are being carried. Increasing rotor size has a beneficial effect if hover forms an important part of a mission. However, should the blade chord remain unaltered, then the increase in blade area will cause an increase in profile power and hence work in opposition to the benefits gained from the lower disc loading. The net effect of rotor size will therefore depend on the balance between low and high speed operation throughout the particular mission.
The effect of the number of engines on fuel consumption is of major importance.

FIGURES

Figure 1 - Force Balance of the Main Rotor
Figure 2 - Rotor Blockage Variation with Advance Ratio

Figure 3 - Variation of Main Rotor Powers Components in Forward Flight
Figure 4 - Total Power Variation in Forward Flight

Figure 5 – Endurance (hr) Variation in Forward Flight
Figure 6 – Range (km) Variation in Forward Flight

Figure 7 - Location of Endurance (hr) and Range (km) Maxima
Figure 8 - Graphical Construction of Performance Extrema

Anti Tank Mission Fuel Usage (kg)
Figure 9 - Case 1 - Basic Aircraft

Figure 10 - Case 2 - $D_{100}$ doubled
Figure 11 - Case 3 - Rotor Radii +.5m, Tail Boom +1m

Figure 12 - Case 4 - No. of Engines = 1
Figure 13 - Case 5 - No. of Engines = 3

ASW Mission Fuel Usage (kg)

Figure 14 - Case 1 - Basic
Figure 15 - Case 2 - $D_{100}$ doubled

Figure 16 - Case 3 - Rotor radii +.5m, Tail Boom +1m
Figure 17 - Case 4 - No. of Engines = 1

Figure 18 - Case 5 - No. of Engines = 3