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Behaviour of a Jump Start Autogyro 2D

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Preamble

The requirement for an autogyro to make a successful vertical take off is to apply an overspeed to the rotor. This enables the aircraft to lift off the ground when collective pitch is applied. As the rotor receives no power input the rotor will immediately decelerate and the vertical ascent will halt. The concept is to attain sufficient height to enable the aircraft to convert to forward, horizontal, flight before the rotor speed falls below an acceptable value.

The helicopter is on the ground when the collective pitch is applied. The equations of motion are derived below enabling the rotor speed variation and the aircraft vertical motion to be ascertained.

The assumption is made that the rotor thrust and torque consumption vary with the square of the rotor speed. This requires that the collective pitch remain fixed throughout the motion. It also means that any changes in downwash are ignored.

Nomenclature

Variable	Description
T	Rotor Thrust
W	Helicopter Weight
z	Vertical Height from the Ground
g	Acceleration due to Gravity
J	Polar Moment of Inertia of the Main Rotor
Ω	Rotor Speed
Q_A	Aerodynamic Torque
Ω_0	Rotor Speed at which the Thrust equals the Aircraft Weight (effectively in hover)
Q_0	Aerodynamic Torque at Ω_0 (effectively in hover)
k, α	Parameters used in the analysis
Ω_S	Starting Rotor Speed

Method

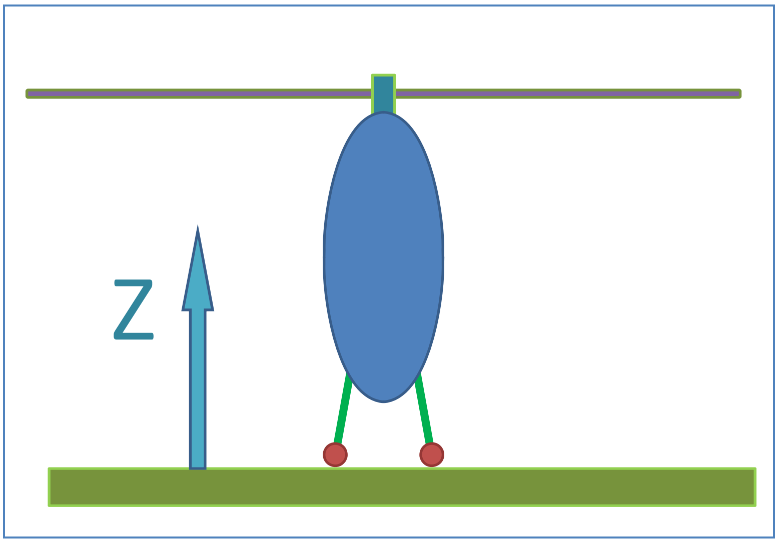

Figure 1

The assumption in the preamble can be expressed:

$$T \propto \Omega^2$$
$$Q \propto \Omega^2 \tag{1.}$$

The equation of vertical motion is-

$$\ddot{z} = g\left(\frac{T}{W} - 1\right) \tag{2.}$$

Rotor rotation equation:-

$$J\dot{\Omega} = -Q_0\left(\frac{\Omega}{\Omega_0}\right)^2 \qquad (3.)$$

$$\bar{\Omega} = \frac{\Omega}{\Omega_0} \qquad (4.)$$

$$T = W \cdot \bar{\Omega}^2$$
$$Q = Q_0 \cdot \bar{\Omega}^2 \qquad (5.)$$

If we define the rotor speed ratio as:

$$\ddot{z} = g\left(\bar{\Omega}^2 - 1\right) \qquad (6.)$$

We have the following:

Whence:

$$\frac{d\bar{\Omega}}{dt} = -\frac{Q_0}{J \cdot \Omega_0} \cdot \bar{\Omega}^2$$
$$\frac{d\bar{\Omega}}{dt} = -\alpha \cdot \bar{\Omega}^2 \qquad (7.)$$

And:

$$\alpha = \frac{Q_0}{J \cdot \Omega_0} \qquad (8.)$$

Where:

This equation has the solution:

$$\frac{1}{\bar{\Omega}} = \alpha t + \frac{1}{\bar{\Omega}_0}$$

$$\bar{\Omega} = \frac{1}{k + \alpha t} \tag{9.}$$

$$k = \frac{1}{\bar{\Omega}_0}$$

Therefore, (2), (5) & (9) give:

$$\ddot{z} = \left[\frac{1}{\left(k + \alpha t\right)^2} - 1 \right] g \tag{10.}$$

From which:

$$\begin{aligned} \dot{z} &= -g\left[\frac{1}{\alpha\left(k + \alpha t\right)} + t - \frac{1}{k\alpha} \right] \\ &= -g\left[\frac{\bar{\Omega}}{\alpha} + t - \frac{1}{k\alpha} \right] \end{aligned} \tag{11.}$$

And:

$$z = -g\left[\frac{1}{2}t^2 - \frac{t}{k\alpha} + \frac{1}{\alpha^2}\ln\left\{1 + \frac{\alpha t}{k}\right\}\right] \quad (12.)$$

Example

Variable	Value
AUM	450 kg
Ω_0	20 rads/sec
M_{BLADE}	20 kg
N	2
C	0.3 m
R	5 m
ρ	1.2256 kg/m^3
g	9.81 m/s^2
$dC_L/d\alpha$	5.8 /rad
Ω_{START}	24 rads/s

An example using these results is presented in Figure 2.

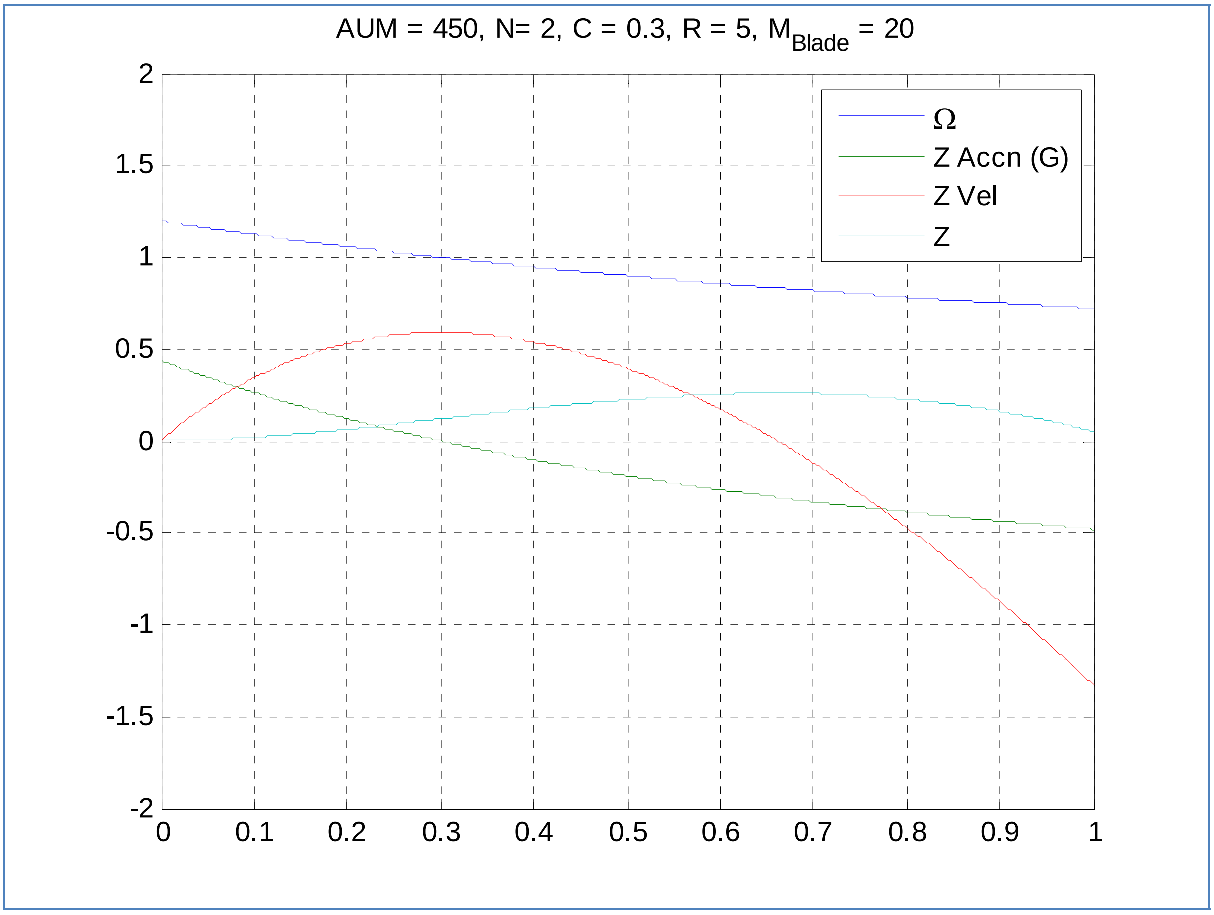
AUM = 450, N= 2, C = 0.3, R = 5, M_{Blade} = 20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Ω
Z Accn (G)
Z Vel
Z

Figure 2