A Quasi -Two Dimensional Model to Determine the Flow Over a Helicopter Fuselage in a Side Wind In and Out Of Ground Effect

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Summary

This note describes the derivation of a model using the notion of doublets to predict the flow induced by a sidewind over a helicopter fuselage allowing also the possible presence of ground effect. The fuselage and ground are replaced by image doublets and the initial results produced by the method are presented.

Introduction

The operation of a helicopter on the deck of a ship will involve the effect of sidewinds on the aerodynamic and dynamic behaviour of the rotor. Current work concerning starting and stopping the rotor in adverse wind conditions has shown that a vertical wind distribution across the rotor disc which is anti-symmetric with respect to the fuselage centre line is particularly restrictive.

It is apparent that the presence of the fuselage and ground will induce such an anti-symmetric wind distribution at the rotor disc from a simple sidewind.

To investigate this problem, a quasi two-dimensional flow model was constructed. Being based on inviscid fluid mechanics the model cannot assess flow separation on the leeward side of the fuselage. Examination of these effects will require extension of the theory. The model assumes that the flow about the fuselage does not interact with the downwash of the main rotor.

Discussion

In two dimensional incompressible flow, solid boundaries can, in many cases, be replaced by image system which the locus of the boundary as a streamline.

A familiar example of this is the image system of a circular cylinder immersed in a stream. If the cylinder is replaced by doublet of suitable strength, the circle defining the boundary is a streamline and therefore the external flows are identical. If a ground plane is introduced then this can be handled also by an image system.
Referring to Figure 1, the equivalent image systems mentioned above are illustrated in (i) and (ii), (the symbol $\rightarrow$ ) indicating a doublet. The image system of a cylinder and a ground plane is more complicated. The cylinder is replaced by doublet A. In order to account for the ground plane, doublet A’ is added. Now this doublet A’ will require an image B inside the cylinder (its appropriate inverse point). To restore the streamline around the perimeter of the cylinder to make the ground plane a streamline again doublet B’ must be added. This will now require doublet C as its image in the cylinder which in doublet C’ to correct for the ground plane. This process will clearly continue to an infinite system of image doublets. The number of image doublets can be truncated to one for the analysis of a helicopter fuselage (i.e. two doublets in total). Cross-sections of a helicopter fuselage not often truly circular and the dividing streamlines produced by the two doublets A and A’ are not markedly different from a circle the differences which make the body slightly egg shaped are not unreasonable considering type of section profiles found with helicopters. In view of this a doublet model has been developed to provide means to establish the flow pattern induced at helicopter by the fuselage and possible ground plane in a side wind. A disadvantage of truncating the image system is that the shape of the dividing streamline varies with the position of the ground plane becoming more circular as it recedes. The variation is not great and so this effect is not a limitation to the application.

The appendix shows the derivation and solution of the stream function in closed form which determines the flow pattern and it also contains the derivation of the doublet model.

Knowledge of the doublet strength and position determines completely the details of the flow. However the data available will be the dimensions of the body and its position relative to the ground plane and from this data the position and strength of the doublets have to be determined. The form
of the equations does not admit a straightforward solution; however a simple method has been derived which gives a close approximation.

The problem is to determine the strength and position of the two doublets such that dividing streamline will pass through two specified points on the axis. These points are substituted into the expression for the stream function and this results in two equations for the three unknowns, namely the stream function value, and the strength and position of the doublets. Not having a unique solution a method has to be devised to solve these equations and obtain a realistic solution.

With an image system there is an internal flow within the dividing streamline and it is possible for the value of the stream function at two points on either side of the dividing streamline to be equal (see P.245 ref 2). Because of this it is possible to obtain an unrealistic situation where the two equations are satisfied but the two points on the axis are such that one is outside the body and one inside. The solution required is the limiting case where both points lie on the boundary of the body.

For a given pair of points, the equations can be re-expressed as both the stream function and doublet strength as functions of the doublet height above the ground plane. The doublet strength has a stationary point and this being by definition the maximum doublet strength to give the function the same value at the two specified points. In the case of a single doublet this will coincide with the limiting case of both points laying on the stagnation streamline, hence the body. With two doublets however, mutual interaction will have an effect on this result and the magnitude of this effect was investigated. The separation of the doublets giving this stationary value, that maximum doublet strength and the corresponding stream function value were calculated for various pairs of points and from these values the body dimensions calculated. The exact body dimensions corresponding to the stationary value doublet pair were then compared to the pairs of points from which the stationary value approximation was calculated – see Figure 2:
The purpose of the doublet model is to calculate the flow induced by the fuselage and possible ground effect and so the error in the lower height figure is not considered restrictive. The error in the top height is very small and is also acceptable. Hence it is concluded that although the stationary value doublet approximation does not give the exact body dimensions, the difference is not considered significant considering the purpose of the model.

Results

The Lynx was chosen for the original analysis and zero coning angle was used in the calculations. The method requires knowledge of the fuselage at selected points from which the doublet strength and height can be calculated assuming that at each station the flow is two dimensional. For a general location, the doublet strength and position for the respective station is obtained by interpolation. The crossflow can then be evaluated.
The upper and lower fuselage heights together with the doublet strength and height location used by the interpolating procedure for the Lynx is shown in Table 1:

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>1.67451</td>
<td>2.401961</td>
<td>4.529412</td>
<td>3.461647</td>
</tr>
<tr>
<td>4.117647</td>
<td>1.784314</td>
<td>5.215686</td>
<td>3.471034</td>
</tr>
<tr>
<td>6.588235</td>
<td>1.715686</td>
<td>7.960784</td>
<td>4.701397</td>
</tr>
<tr>
<td>10.04706</td>
<td>1.647059</td>
<td>8.235294</td>
<td>4.77829</td>
</tr>
<tr>
<td>14.54902</td>
<td>1.509804</td>
<td>9.333333</td>
<td>5.158699</td>
</tr>
<tr>
<td>18.68039</td>
<td>1.990196</td>
<td>8.852941</td>
<td>5.28158</td>
</tr>
<tr>
<td>21.89216</td>
<td>3.088235</td>
<td>7.54902</td>
<td>5.295984</td>
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<td>4.666667</td>
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<tr>
<td>35.41176</td>
<td>4.117647</td>
<td>6.039216</td>
<td>5.077603</td>
</tr>
</tbody>
</table>

### Table 1

The basic side view of the aircraft, to which this data applies, is shown in Figure 3:

![Hover Height = 0](image.png)

**Figure 3**

The results are for the 10 cross sections shown in Figures 4-13:
Figure 6

Figure 7
Figure 10

Figure 11
Figures 14 and 15 show the effect of the Lynx fuselage on the flow with the rotor in ground effect. They are a surface and contour plot of the vertical velocity induced by the fuselage and ground at the rotor disc. Because this flow velocity distribution is anti-symmetric along the fuselage centreline, only half of the rotor disc is shown. The effect of the windscreen can be seen as a ridge around the front half of the rotor disc whilst the peak refers to the engine installation. Because of a low rotor mast, this exerts a considerable interruption to the flow.
## Glossary of Terms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Complex Potential</td>
</tr>
<tr>
<td>U</td>
<td>Undisturbed free stream velocity</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Doublet Strength</td>
</tr>
<tr>
<td>( \bar{\mu} )</td>
<td>( \mu / U )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Stream Function</td>
</tr>
<tr>
<td>( x, y )</td>
<td>Coordinates in cross section</td>
</tr>
<tr>
<td>( y_1, y_2 )</td>
<td>Heights of top and bottom of body above ground plane</td>
</tr>
<tr>
<td>( X, Y, Z )</td>
<td>Coordinates for aircraft</td>
</tr>
<tr>
<td>( r )</td>
<td>Radial Station</td>
</tr>
<tr>
<td>( \psi_s )</td>
<td>Blade Azimuth Angle (Conventional Datum)</td>
</tr>
<tr>
<td>( h_B )</td>
<td>Hub height above ground plane</td>
</tr>
<tr>
<td>( u, v )</td>
<td>Induced Velocity</td>
</tr>
</tbody>
</table>
The sign convention is shown in Figure 15:

Figure 16 – Sign Convention
Appendix

The reference furnishes the initial derivation of the complex potential of a doublet and a uniform stream.

With a uniform stream flowing in the negative x direction when undisturbed and the doublet and image placed at:

\[(0, \pm ih)\]  \hspace{1cm} (1.)

we have for the total complex potential:

\[W = -Uz - \frac{\mu}{z - ih} - \frac{\mu}{z + ih}\]  \hspace{1cm} (2.)

The velocity components can be found via:

\[\frac{dW}{dz} = u - iv\]  \hspace{1cm} (3.)

and at a stagnation point

\[\frac{dW}{dz} = 0\]  \hspace{1cm} (4.)

i.e.

\[ -U + \frac{\mu((z + ih)^2 + (z - ih)^2)}{(z^2 + h^2)^2} = 0\]  \hspace{1cm} (5.)

If

\[\mu = \frac{\mu}{U}\]  \hspace{1cm} (6.)

N.B. with a single doublet in a uniform stream the radius (r) of the equivalent circular cylinder is given by:

\[r^2 = \mu\]  \hspace{1cm} (7.)

(5) now becomes:

\[z^2 = \mu h^2 \pm \sqrt{\mu(\mu - 4h^2)}\]  \hspace{1cm} (8.)

the character of this result is governed by the sign of the term:

\[(\mu - 4h^2)\]  \hspace{1cm} (9.)
This term relates the strength of the two doublets to their physical separation.

If: 

\[
(\bar{\mu} - 4h^2) \geq 0
\]  

(10.)

then the stagnation point lies on the x axis and the dividing streamline in fact describes a single body.

When: 

\[
(\bar{\mu} - 4h^2) < 0
\]  

(11.)

the stagnation point lies off the x axis and the flow relates to a body separated from the ground plane and is, therefore, the case to be considered.

If \( \Psi \) is the stream function then:

\[
\Psi = Imag\{W\}
\]  

(12.)

\[
\frac{\psi}{U} = \psi = -y + \frac{\mu(y - h)}{x^2 + (y - h)^2} + \frac{\mu(y + h)}{x^2 + (y + h)^2}
\]

Hence:

\[
\psi + y = \frac{2\bar{\mu}y(x^2 + y^2 - h^2)}{(x^2 + (y - h)^2)(x^2 + (y + h)^2)}
\]  

(13.)

(14) becomes:

\[
(\psi + y)x^2
\]  

\[
+\{(\psi + y)(A^2 + B^2) - 2\bar{\mu}y\}x
\]

\[
+AB\{AB(\psi + y) - 2\bar{\mu}y\} = 0
\]  

(16.)
i.e. a quadratic in \(x\).

Hence the streamlines can be expressed via (16) in closed form with \(x\) as a function of \(y\).

Moving on to discussion of the doublet approximation, equation (16) will admit a solution of a point on the \(y\) axis (\(x=0\)) when the third term vanishes, i.e.

\[
AB\{AB(\psi + y) - 2\bar{\mu}y\} = 0
\]

or

\[
(y - h)(y + h)((y^2 - h^2)(\psi + y) - 2\bar{\mu}y) = 0
\]

discounting the solutions

\[
y = \pm h
\]

we have:

\[
(y^2 - h^2)(\psi + y) - 2\bar{\mu}y = 0
\]

or re-expressed as:

\[
(y^2 - h^2)\psi - 2\bar{\mu}y = -y(y^2 - h^2)
\]

This requires two values of \(y\) \((y_1\) and \(y_2\)) to satisfy equation (21) with the same value of:

\[
\psi & \bar{\mu}
\]

whence:

\[
\begin{bmatrix}
y_1^2 - h^2 \\
y_2^2 - h^2
\end{bmatrix}
\begin{bmatrix}
\psi \\
\bar{\mu}
\end{bmatrix}
=
\begin{bmatrix}
-y_1(y_1^2 - h^2) \\
y_2(y_2^2 - h^2)
\end{bmatrix}
\]

the solution of which is:

\[
\begin{align*}
\psi &= -\frac{y_1y_2(y_1 + y_2)}{(y_1y_2 + h^2)} \\
\bar{\mu} &= -\frac{(y_1^2 - h^2)(y_2^2 - h^2)}{2(y_1y_2 + h^2)}
\end{align*}
\]

There is a potential choice for values of \(\bar{\mu}\) and \(h\). It is proposed that the choice of maximum value of \(\bar{\mu}\) is made which will give the most appropriate shape to the bounding streamline defining the fuselage cross section. On that basis, the stationary value of \(\bar{\mu}\) with respect to \(h\) is given by:
this results in the expression:

\[
\frac{d\mu}{dh} = 0
\]  

(25.)

and substitution of this result in equations (16) gives for the stationary value case:

\[
h^2 = -y_1y_2 + \sqrt{y_1y_2}(y_1 + y_2)
\]  

(26.)

Referring back to equations (2 and 3) we find the velocity at a point \((x,y)\) is given by (velocities are normalised on the undisturbed uniform stream \(U\)):

\[
\frac{u}{U} = -1 + \mu\left\{ \frac{x^2 - (y - h)^2}{(x^2 + (y - h)^2)^2} + \frac{x^2 - (y + h)^2}{(x^2 + (y + h)^2)^2} \right\}
\]

\[
\frac{v}{U} = 2\mu x\left\{ \frac{(y - h)}{(x^2 + (y - h)^2)^2} + \frac{(y + h)}{(x^2 + (y + h)^2)^2} \right\}
\]

(28.)

The model for calculating the velocity induced by a sidewind on a fuselage can now be assembled. It assumes a quasi two-dimensional model of the fuselage (effectively reproducing the concept of dividing the flow into independent strips similar to wing theory. From the heights of the top and bottom surfaces of the fuselage above the ground the doublet strength and height at selected points can be calculated as described above. An interpolation routine is required to "fill in the gaps" as required and a piecewise cubic function was used. The fuselage and ground can now be represented by the doublets and referring to Figure 16, the position of a general point of a rotor blade is shown.
If the radial station is \( r \) then the coordinates of the point \( P \) are:

\[
X = r \sin \psi_s \\
Y = r \cos \psi_s \\
Z = h_B
\]  

(29.)

Where \( h_B \) is the hub height above ground (No coning angle is assumed for the initial investigation). The \( x \) component will determine the doublet strength and position via the interpolation routine. Having established \( h \) & \( \bar{\mu} \) we have:

\[
x = Y \\
Z = h_B
\]  

(30.)

the velocity components can then be calculated using equations (28).
Conclusions

An inviscid flow model has been derived to investigate the effect of a fuselage and ground plane on the flow through a disc caused by a sidewind.

A computer model has been constructed results presented. As an indication of the magnitude of the results for the Lynx fuselage the windscreen shows upwash values of around 30% of the incident horizontal sidewind and values of 90% can be seen around the engine housing, albeit at the inboard end of the rotor blades.

Considering the effect of any vertical velocity on the behaviour of a rotor blade at low rotational speed these figures are not insignificant.

Owing to the limitations of an inviscid theory, flow separation on the leeward side of the fuselage is not catered the theory will extension to allow this.

References